An Analysis of Assortment Choice in Grocery Retailing

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Abstract

Consumers in grocery retailing commonly buy bundles of products to accommodate current and future consumption. When all products in a particular bundle share common attributes (and are selected from the same product category), the consumer is said to assemble an assortment. This research examines the impact of assortment variety on the assortment choice process. In particular, we test the prediction that consumers demand less variety for higher quality items. To investigate this relationship, we employ a flexible choice model, suitable for the analysis of assortment choice. The model, based upon the assumption that the utility of purchase of one item in an assortment depends upon the set of items already selected, allows for a general utility structure across the assortment items. We apply the model to household assortment choice histories from the yogurt product category. Substantively, we show that yogurt choice is affected by brand quality perceptions (quality-tier competition). Moreover, we show that reaction to reductions in variety (number of yogurt flavors) is mediated by brand quality perceptions. Taken together, these empirical facts paint a picture of a consumer who is willing to trade-off variety against product quality in assortment choice.

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Introduction

For frequently purchased products such as grocery items, consumers can more easily anticipate short-run preferences than long-run preferences (Kahneman and Snell 1990; Luce 1992). In order to balance the trade-off between short-term and long-term preferences, consumers assemble bundles of products during each shopping trip. Of particular interest is the product assortment, a bundle consisting of items selected from a single product category (such as breakfast cereals or yogurt). From a choice behavior perspective, assortments allow consumers to buy goods that will be consumed over time. This provides the consumer flexibility in planning for future consumption events and in accommodating the preferences of multiple users in the household (Chernev 2011). From a retail management perspective, understanding the consumer decision process behind assortments provides guidelines for the number and variety of items that should be stocked in a given product category (Corstjens and Corstjens 2002).

Variety of an Assortment

An assortment can be viewed as a choice strategy that allows consumers to pursue multiple decision objectives. Due to the transaction costs of grocery shopping, it is more efficient for the consumer to buy an assortment of products that will be consumed in the future. Assortments permit consumers to minimize decision conflict on a given trip (Simonson 1990), account for uncertainty in future preference (Kahn and Lehmann 1991; Shin and Ariely 2004; Walsh 1995), and address attribute satiation in consumption (Kim, Allenby, and Rossi 2002). All these studies emphasize that assortments are constructed to maximize variety with respect to item attributes.

Research in the store choice and consumer behavior literature argues that assortment variety is a psychological perception that plays a key role in the purchase decision (e.g., Broniarczyk 2008; Hoch, Bradlow, and Wansink 1999). Variety perception has been found to depend upon the size of the assortment (e.g., Broniarczyk, Hoyer, and McAlister 1998; Kahn 1995),...
the distinctiveness of alternatives (Gourville and Soman 2005; Markman and Gentner 1993; Van Herpen and Pieters 2007), and the proximity and display of alternatives (e.g., Hoch, Bradlow, and Wansink 1999). While larger assortments are typically believed to offer greater variety, decreasing the number of alternatives may actually increase the purchase likelihood (Broniarczyk, Hoyer, and McAlister 1998; Kahn 1995; Kuksov and Villas-Boas 2010). This may be attributed to the time and effort trade-off consumers resort to while making grocery purchases (Hoyer 1984). Assortments with greater similarity among items are perceived to have lower variety, thus lowering the probability of purchase. However, items that are displayed (or organized) together are often perceived to offer greater variety (Chernev 2011). Factors such as the marginal utility of items (Kahn, Moore, and Glazer 1987; Kahnenman and Snell 1992) and overall assortment attractiveness (e.g., inclusion of favorite item/brand) (Botti and McGill 2006) supplement variety perceptions in motivating assortment preference and purchase.

Variety and Product Quality

Recent work by Chernev and Hamilton (2009) proposes a theory of choice behavior that links assortment size to perceived product quality. This theory is based upon the trade-off between the cognitive effort in evaluating choice alternatives (Shugan 1980) and the desire for more variety. Two assumptions underlie this theory. First, cognitive effort is proportional to the number of items in the assortment, but has no relationship to perceived quality of the items. Second, utility for item variety is assumed to be a concave function. Due to decreasing marginal returns, each new item in the assortment adds less incremental utility. The consumer will add new items to the assortment until the increasing mental costs overwhelm the added utility of more variety.

Suppose that the consumer constructs assortments from either high quality items or low quality items (not both). Chernev and Hamilton (2009) propose that for assortments of equal size, the marginal benefit of adding an item to a high quality assortment is smaller than marginal benefit of adding an item to a low quality assortment. The argument, in essence, is that each item of a high quality assortment is sufficiently valued that decreasing marginal returns become binding more quickly. Thus, the prediction is quite simple: consumers will prefer a small high-quality assortment over a large low-quality assortment. The goal of our research is to evaluate the evidence for this relationship between variety and product quality.

Modeling Assortment Choice

To test this prediction, it is necessary to construct a general model of assortment choice. The academic literature on product bundles provides insights on how a model might be constructed. Multi-attribute models of bundle choice assume that the researcher has a list of product attributes for each item in the bundle and understands how these attributes interact with one another in determining consumer preferences (Rao 2004). For example, Farquhar and Rao (1976) develop a bundle utility model based upon the assumption that the researcher can code each attribute in terms of “more is better” (maximize the bundle mean) or “heterogeneity is better” (maximize the bundle variance). Preference for the bundle, then, reflects consumer preference for global features that are derived from the attributes of all items in the bundle. Harlam and Lodish (1995) draw upon this global feature logic, but assume that the bundle is assembled sequentially (across grocery shopping trips), with the probability of choosing the next product contingent on the products already selected. Because order of purchase is generally not reported in grocery scanner datasets, the authors assume that higher preference products are purchased earlier. More generally, bundle utility models can be constructed using conjoint measurement procedures (Green, Jain, and Wind 1972).

Although these models have many attractive features, they are not suitable for empirical studies in a grocery shopping environment. The key reason is that the researcher does not typically have an exhaustive list of product features. Moreover, it is clear that the modeling framework must be able to simultaneously accommodate various demand relationships, including complementarity and substitution. For these reasons, we model assortment choice using a version of the multivariate logistic (MVL) category incidence model developed by Russell and Petersen (2000). The model implies that the probability of buying an assortment on a shopping trip depends upon the household’s valuation of each item in the assortment, adjusted for the demand relationships among the items. As we show subsequently, the model provides a strong foundation for studying the linkage between product quality and assortment variety.

Overview of Research

The remainder of this paper is organized as follows. We first provide a brief review of the multivariate logistic (MVL) choice model, explaining how the model can be extended to build a general model of assortment choice. Drawing upon this theory, we construct an assortment choice model for the yogurt product category that links product characteristics (brand and flavor) to the pattern of household purchase decisions. This choice model, which assumes perfect substitution across brand names and pick-any choice among flavors within a brand, provides a structure for studying the role of brand quality perceptions in assortment choice. Substantively, we show that the pattern of competition among yogurt items is affected by brand quality perceptions (Blatberg and Wisniewski 1989; Sivakumar and Raj 1997). Moreover, we show that reaction to reductions in variety (number of yogurt flavors) is mediated by brand quality perceptions. The overall pattern of our findings is consistent with the Chernev and Hamilton (2009) theory. We conclude with suggestions for future research.

Multivariate Logistic Model of Assortment Choice

In this section, we describe an assortment model that allows for a very flexible pattern of product competition. The model is built upon the conditional utility of choice of a single item, dependent on its own marketing mix and the other item choices in the chosen assortment. The development here, which builds
upon the discussion in Russell and Petersen (2000), yields a choice model specification that is a special case of the multivariate logistic distribution proposed by Cox (1972). However, the model is considerably richer than the derivation implies. The model is consistent with theoretical frameworks in both economics and psychology, and has sufficient flexibility to accommodate complex demand patterns among items.

Model Description

We begin with a discussion of bundle choice, keeping in mind that an *assortment* is a bundle of products, all of which are members of the same product category. Suppose that household $h$ selects an assortment $b$ at time $t$. An assortment $b$ contains multiple items (zero, one or more items) from multiple clusters. We use the word cluster to mean a set of items with some attribute in common. For example, in a study of assortments in the laundry detergent category, clusters might be market designations (liquid detergents), family brand names (e.g., Tide), or a collection of product features (scented liquid detergents).

Denote $J_c$ as an item $j (= 1, 2, \ldots, J_c)$ in cluster $c (= 1, 2, \ldots, C)$. The total number of alternatives is $M = \sum_{c=1}^{C} J_c$. Define an assortment as a $M \times 1$ vector of binary variables indicating the presence of the item in an assortment

$$z^b_{ht} = \left[ z^1_{ht}, z^2_{ht}, \ldots, z^1_{ht1}, z^2_{ht1}, \ldots, z^1_{htJ_c}, z^2_{htJ_c}, \ldots, z^1_{htJ_c}, z^2_{htJ_c}, \ldots, z^1_{htC}, z^2_{htC} \right]'$$

(1)

where $z^{c}_{ht} = 1$ if household $h$ selects item $j$ in cluster $c$ at time $t$, and equals zero otherwise. Under the assumption of pick-any choice, there are $2^M$ possible assortments that could be selected (including the null assortment containing no purchases). Without further restriction, the model subsequently assigns a choice probability to each of these $2^M$ assortments.

We now define the conditional choice probability for an item, given choices of all other items in the assortment. The conditional utility consists of two parts: (a) a baseline utility dependent on item characteristics and (b) demand interactions with other items in the assortment. The utility of item $j$ in cluster $c$, conditional on the observed purchase outcomes of other items under consideration, is assumed to have the form

$$U(j_c|\text{all else}) = \pi^c_{hjt} + \sum_{c_s} \sum_{k} \theta^c hjk \cdot z^{c_s}_{hkt} + \varepsilon_{hjt}$$

(2)

where $\pi^c_{hjt}$ is a baseline utility for item $j$ in cluster $c$ of household $h$, at time $t$ and $\varepsilon_{hjt}$ is a Gumbel distributed random error. The baseline utility depends both upon household preferences and marketing mix elements, and can be regarded as the inherent utility for the item, net of demand interactions with other items.

These interactions are represented by the double summation term on the right hand side of Eq. (2). Individual items are denoted by the notation $[c,j]$ for item $j$ nested within cluster $c$. For purposes of model identification, we assume that $\theta^c hjk = 0$, for $[c,j] = [c^*, k]$ (We discuss the necessity of this restriction later in this section). The parameter $\theta^c hjk$ captures the influence of item $[c^*, k]$ on $[c,j]$. Accordingly, $\theta^c hjk > 0$ if item $k$ has positive impact (complementarity effect) on item $j$’s utility, $\theta^c hjk < 0$ if item $k$ negatively influences item $j$’s utility (substitution effect), and $\theta^c hjk = 0$ if there is no influence between items (independence). For reasons that will become clear subsequently, we require that the $\theta^c hjk$ parameters be symmetric in items $[c,j]$ and $[c^*, k]$.

To simplify the presentation of the model, it is useful to define the symmetric matrix, $\Theta_h$, containing all the elements of $\theta^c hjk$, as

$$\Theta_h = [\Theta^1_{h11}, \Theta^1_{h21}, \ldots, \Theta^2_{h1J_c}, \ldots, \Theta^C_{hJ_c J_c}]_{N_J \times 1}$$

$$= \begin{bmatrix}
\theta^1_{h11} & \cdots & \theta^1_{h1J_c} \\
\theta^1_{h21} & 0 & \cdots & \theta^1_{h2J_c} \\
\vdots & \ddots & \ddots & \vdots \\
\theta^C_{h1J_c} & \theta^C_{hJ_c 1} & \cdots & 0
\end{bmatrix}_{N_J \times N_J}$$

(3)

where $\Theta^c_{hj}$ is the column vector for the corresponding item $j$ in cluster $c$ of the matrix $\Theta_h$. Using matrix (3) and choice vector (1), we can now rewrite Eq. (2) as

$$U(j_c|\text{all else}) = \pi^c_{hjt} + \Theta^c_{hj} z^b_{ht} + \varepsilon_{hjt}$$

(4)

Assuming that the error in (4) follows a Gumbel distribution, the conditional probability of purchasing item $j$ in cluster $c$, given the known purchase outcomes for all other items, has the form of the binary logit model

$$Pr(z^c_{ht}|\text{all other zs}) = \frac{[\exp(\pi^c_{hjt} + \Theta^c_{hj} z^b_{ht})]^{z^c_{ht}}}{1 + \exp(\pi^c_{hjt} + \Theta^c_{hj} z^b_{ht})}$$

(5)

where, as noted earlier, $z^c_{ht} \in \{0, 1\}$ is an indicator variable reporting whether or not item $j$ is chosen.

Eq. (5) is known in the spatial statistics literature as an autologistic regression model (Anselin 2002; Besag 1974; Cressie 1993). From the standpoint of statistical theory, Eq. (5) can be seen to be the full conditional distribution of one binary purchase variable (corresponding to item $[c,j]$), given the known outcomes of the binary purchase variables for all other items in the assortment. Note that each of the $M$ possible items has a full conditional distribution in the general form of Eq. (5), and all of these $M$ models collectively describe the choice process of the household. The technical problem is to write down a global probability model that is consistent with this system of interlocking equations.

This problem has been analyzed and solved by researchers working in spatial statistics (Besag 1974). We provide an overview of the key results in Appendix A. As Besag (1974) notes, there exists a global probability model consistent with (5) only when the cross-effect matrix is symmetric ($\theta^c hjk = \theta^c hkj$).
With this symmetry assumption, the global choice model takes the form

\[ Pr(z_{ht} = z_{ht}^b) = \frac{\exp(V_{ht})}{\sum_b \exp(V_{ht}^b)} = \frac{\exp \left( \sum_h \pi_{ht}^b z_{ht}^b + \frac{1}{2} \{ z_{ht}^b, \Theta_h z_{ht}^b \} \right)}{\sum_b \exp \left( \sum_h \pi_{ht}^b z_{ht}^b + \frac{1}{2} \{ z_{ht}^b, \Theta_h z_{ht}^b \} \right)} \]

(6)

where \( \pi_{ht}^b = [\pi_{ht1}^b, \pi_{ht2}^b, \ldots, \pi_{htj1}^b, \pi_{htj2}^b, \ldots, \pi_{htjc1}^b, \ldots, \pi_{htjc2}^b, \ldots, \pi_{htjc}^b] \) is a \( M \times 1 \) column vector containing the individual (full) conditional baseline utilities for items in the chosen assortment \( b \). It is important to understand that the system of equations given by (5) implies the assortment choice model given by (6). That is, once the researcher assumes the validity of the \( M \) equations in (5), Eq. (6) follows as a logical consequence. Conversely, (6) implies the set of full conditional distributions in (5). Using statistical terminology, Eq. (6) is the joint (multivariate) distribution of the \( M \) binary purchase indicator variables \( z_{ht} \). Because (6) is known to statisticians as the multivariate logistic distribution (Cox 1972), we refer to (6) as the multivariate logistic (MVL) choice model.\(^1\)

As discussed earlier, the cross-effect matrix \( \Theta_h \) obeys two constraints. First, the diagonal of the matrix is set to zero. Second, the matrix is symmetric. Both are technical restrictions to ensure that the model is properly specified. Given the form of Eq. (6), it is easily shown that zeros on the diagonal of the cross-effect matrix are necessary to allow the parameters in the \( \pi_{ht}^b \) main effect terms to be identified. The symmetry restriction on the cross-effect matrix is necessary to conclude that the set of equations in (5) implies the global model in (6). Put another way, if cross-effects were asymmetric, then no global model consistent with Eq. (5) exists (Besag 1974).

From a more intuitive point of view, the cross effect matrix is a signed (plus or minus) measure of similarity among items in the assortment. In the spatial statistics literature, the elements of an autologistic cross-effect matrix are modeled as functions of the relative geographic distance between response locations, with closer locations having a larger (positive) cross-effect term (Anselin 2002; Cressie 1993). Conceptually, it is useful to regard the items in the assortment as being located on a mental map in which closer items have stronger interactions with respect to the consumer’s buying behavior (Moon and Russell 2008; Russell and Petersen 2000). The elements of \( \Theta_h \) are not correlations. However, using the properties of the multivariate logistic distribution (Cox 1972), it can be shown that the correlations in choice outcomes among the items in the assortment are determined by the cross-effect matrix. Thus, symmetry in cross-effects is necessary because of the link between \( \Theta_h \) and the correlation structure of the global choice model.

The symmetry of the cross-effect matrix also has another interesting interpretation. As explained in Appendix A, the MVL model derivation assumes that the consumer’s valuation of an assortment depends only on the final composition of the assortment – not upon the order in which the items in the assortment were actually selected. Put another way, a consumer who values an assortment solely on the basis of its contents must have a symmetric \( \Theta_h \) matrix. This order independence property is quite attractive in grocery retailing applications in which purchase sequence is not observed.

**Model Interpretation**

The MVL choice model can be viewed from a variety of perspectives. In the spatial statistics literature, the model is an example of a conditional autoregressive (CAR) stochastic process in which the researcher uses a set of full conditional distributions to derive the form of a joint distribution (Cressie 1993). This interpretation emphasizes the fact that the local choice behavior in Eq. (5) determines the global choice behavior in Eq. (6). The CAR approach provides a way for the researcher to specify a complex system by breaking the system into \( M \) interlocking parts, each of which can be developed separately.

From the standpoint of the marketing science literature, the MVL choice model can be given two different interpretations. The first interpretation treats Eq. (5) as a linkage between the purchase decision for one item and the actual choices of all other items. Anselin (2002), in a review of the spatial econometrics literature, calls the system of equations represented by (5) reaction functions and the implied global model in (6) the system equilibrium. In other words, if the consumer simultaneously makes individual item choices using a random utility process given by (5), then the observed global choice behavior (for the assortment) will be observed to follow (6). In fact, recent work in the marketing science literature by Yang et al. (2010) uses economic theory to argue that MVL choice model in (6) represents a decision process equilibrium.

The second marketing science interpretation specifies the choice model in (6) directly by making assumptions about the nature of the utility function of an assortment. Song and Chintagunta (2006), using a utility theory argument due to Gentzkow (2007), adopt this approach. These authors make two key assumptions. First, they assume that the expected utility for an assortment of goods \( V_{ht} \) has the general form \( \pi_{ht}^b z_{ht}^b + (1/2) \{ z_{ht}^b, \Theta_h z_{ht}^b \} \). Here, \( V_{ht} \) can be considered to be a discrete variable second-order (quadratic) approximation to the true (indirect) utility of assortment \( b \). Second, they assume that the actual utilities fluctuate around \( V_{ht} \) due to independent, identically distributed, extreme value errors. These assumptions, in the context of a random utility theory framework, also yield the MVL choice model in (6).

Intuitively, the MVL choice model states that the utility of an assortment depends upon household valuations for each item in the assortment, adjusted for interactions among the selected items. This theory exhibits clear relationships with work in the consumer behavior literature dealing with product assortments. If all elements of \( \Theta_h \) are set to zero, then \( V_{ht} \) is just the sum of

\(^1\) Marketing science applications related to the MVL model include Boztug and Hildebrandt (2008), Moon and Russell (2008), Niraj, Padmanabhan, and Seetharaman (2008), and Yang et al. (2010).
the utilities of the items in the assortment. This type of model was proposed by Janiszewski and Cunha (2004) in a study of consumer reactions to price anchors. More broadly, the notion that the valuation of an assortment depends upon main effects and interactions of item attributes has strong support in the marketing literature (Chung and Rao 2003; Farquhar and Rao 1976).

Restricting the Model

The MVL choice model has an important property that can be exploited to tailor the general model to particular applications (such as assortment choice with respect to a particular product category). Given that Eq. (6) has the form of a multivariate logistic distribution (Cox 1972), it may be viewed as the probability distribution of an $M$-dimensional multivariate random variable. This implies, in particular, that these binary variables (representing the choice outcomes of different items) will be correlated. Interestingly, it is also clear from the form of (6) that the model can be viewed as a simple logit model defined with respect to assortments. Accordingly, the MVL model assumes that choice is characterized by an IIA (independence of irrelevant alternatives) property relative to assortments. It is important to understand that this IIA property corresponds to assortments—not to the individual items within an assortment.

In general, given the interlocking full conditional expressions in (5), the relationships between specific items will be characterized by arbitrary deviations from IIA.

The IIA assortment property makes it very easy to infer the form of a choice model subject to various restrictions. First, write the general form of the MVL choice model for all items in the choice set. Second, discard all assortments that will never be observed (because they are not in the set of possible assortments). Third, renormalize the model so that the probabilities of all remaining assortments sum to unity. Whatever remains after this pruning process must be the correct model for the choice process. For example, suppose that the choice task is such that at least one item must be chosen from a set of $M$ items. This means that all assortments are possible, except for the null assortment in which no items are chosen. Thus, the appropriate choice model is

$$P_r(x_{hi} = z_{hi}^b) = \frac{\exp(\pi_{hi}^b z_{hi}^b + (1/2)\{z_{hi}^b/\Theta, z_{hi}^b\})}{\sum_{b^* \in \text{all nonempty bundles}} \exp(\pi_{hi} z_{hi}^b + (1/2)\{z_{hi}^b/\Theta, z_{hi}^b\})}$$

subject to the restriction that $z_{hi}^b$ cannot be the zero vector. Implicitly, the probability of any outcome not in the set of feasible assortments—i.e., the null assortment—is set to zero.

Constraining the $\Theta$ Matrix

A useful way of understanding model restrictions is to place constraints directly on the $\Theta$ matrix. Suppose, for example, that we are presented with assortment choice task with three clusters and three items each within each cluster. Suppose, as well, that clusters are considered substitutes, while items within a cluster have an unspecified level of demand dependence on one another. Although this structure may not appear intuitive, it can nevertheless be useful in some settings. For example, in the yogurt category, there are multiple flavors within each brand name. Households typically buy multiple flavors on a shopping trip, but restrict their purchases to items having the same brand name (see, e.g., Kim, Allenby, and Rossi 2002). If we allow brand names to define clusters and the flavors of a brand to define items, then the structure shown in Fig. 1 is an appropriate description of the market structure within the yogurt category.

To accommodate this structure, we impose restrictions on the symmetric matrix $\Theta$ in the following manner. Assuming three brands ($A$, $B$, $C$) with three flavors within each brand ($1$, $2$, $3$), the $\Theta$ matrix takes the form

$$\Theta = \begin{bmatrix}
0 & \theta_{21}^{A} & \theta_{31}^{A} & \theta_{22}^{A} & \theta_{32}^{A} & \theta_{33}^{A} & 0 & 0 & 0 \\
\theta_{21}^{A} & \theta_{22}^{A} & \theta_{23}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta_{31}^{A} & \theta_{32}^{A} & \theta_{33}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta_{22}^{A} & \theta_{23}^{A} & \theta_{22}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta_{32}^{A} & \theta_{33}^{A} & \theta_{32}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta_{33}^{A} & \theta_{32}^{A} & \theta_{33}^{A} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

(8)

where $\theta_{21}^{A}$ denotes the cross-effect parameter between flavors 1 and 2, both from brand $A$. The symbol $-\infty$ (negative infinity) implies perfect substitution. In words, Eq. (8) indicates that consumers construct assortments by selecting one brand (pick-one choice task) and then selecting an assortment of flavors within each brand (pick-any choice task). Because the parameters of the $\Theta$ matrix within each brand are unrestricted, varying degrees of substitution and complementarity among flavors within brands are possible.

From the standpoint of the consumer, assortments containing certain combinations of items (e.g., \{Brand $A$, flavor 1\} and \{Brand $B$, flavor 2\}) are excluded from consideration because the choice probability of the assortment equals zero (i.e., the numerator of Eq. (6) becomes $\exp(-\infty) = 0$). With the $\Theta$ structure in Eq. (8), there are only 21 (obtained as $3 \times (3^2 - 1)$) assortments for the consumer to consider. In model calibration, the parameters set to $-\infty$ are not actually estimated. Rather, the model specification in Eq. (6) is rewritten to exclude assortments with zero purchase probability.
Summary

At this point, the important properties of the MVL choice model should be clear. From the standpoint of assortment choice, the model has two desirable features. First, it implies that the marginal utility of each item in an assortment depends upon the characteristics of all other items in the assortment (Eq. (2)). The MVL choice model (Eq. (6)) can be viewed as the equilibrium outcome at which all the marginal utility equations are satisfied simultaneously. Second, the model is extremely flexible. Because the parameters of the \( \Theta \) matrix can be positive or negative, various patterns of substitution and complementarity can be imposed. If necessary, assortments that will never be observed can be excluded, allowing a parsimonious model specification. In our empirical work, we take advantage of this flexibility in studying the role of product quality in assortment choice.

Assortment Choice in the Yogurt Category

We employ the MVL choice model to analyze household purchase histories from the yogurt product category using the structure of the \( \Theta \) matrix corresponding to Fig. 1 (as depicted in Eq. (8) above). Our main goal is substantive. We use the model to explore the predictions of the Chernev and Hamilton (2009) theory relating demand for variety to product quality. In this section, we discuss technical aspects of model calibration. Readers who are primarily interested in the implications of model forecasts for the Chernev–Hamilton theory may wish to skip to the next section.

Data Description

The data are taken from an ERIM scanner panel of yogurt purchase histories from the Sioux Falls area over a two and one-half year period. We focus on the low fat, fruit on the bottom assortment. We select eight common flavors from both brands and collapse three flavors into one composite flavor (Others). Taken together, we have total of twelve SKU’s (six SKUs for each brand) in the analysis.\(^2\) The six common flavors used in the analysis are Cherry, Mixed Berry, Peach, Raspberry, Strawberry, and Others. By constructing the SKU set in this manner, we are able to study the role of brand and flavor in yogurt choice decisions.

We split the data into three consecutive periods. The first 8 months (240 days) of the data were used to create household-specific SKU loyalty variables. The remainder of the data was split into two sets: a model calibration period (1443 shopping trips over 580 days) and a holdout period (185 shopping trips over 90 days). Table 1 shows descriptive statistics for the two brands across these periods. Due to retailer marketing decisions, there is no variation in marketing mix activity across flavor SKU’s having the same brand name at the same time. There was no display activity at all for Dannon during the observed periods. Of the 113 households used for model calibration,\(^3\) 32 households did not make any purchases of these two brands during the holdout period. For this reason, only 81 households enter the holdout data.

Table 2 presents a contingency table showing items that are purchased together. Diagonal elements are purchase frequencies for individual SKU’s in the calibration data period. The table shows no cross purchases across brands, but many instances of joint purchasing of flavors within a brand. We assume the choice process for Dannon and Nordic follows the pattern shown in Fig. 2: pick-one choice across brands, and pick-any choice for flavors within brands. This type of demand pattern corresponds to the structure of the \( \Theta \) matrix discussed earlier in relation to Eq. (8).

In calibrating the model (discussed below), we also make two other adjustments. Since the choice histories are restricted to only those instances in which either Dannon or Nordic is purchased, we never observe a null (empty) assortment. Consequently, we eliminate the null assortment from the denominator of Eq. (6), thereby fixing the probability of observing the null assortment to zero. (As explained earlier, this procedure is justified by the IIA property of the choice model with respect to product assortments.) In addition, some items are not available

\(^2\) Multiple purchases of a single SKU are considered to be one SKU choice.

\(^3\) Only eleven of 124 households made purchases across brands on a single shopping occasion. This accounts for about 1% of total purchase events. These households were excluded from the dataset prior to model calibration. Kim, Allenby, and Rossi (2002) also report that almost all yogurt purchase bundles consist of items having the same brand name.
in the market at certain time points due to stock-outs, and so forth. We also adjusted the model specification to accommodate such occasions.

**Model Specification**

We first define individual SKU’s baseline utility $\pi_{hjt}^p$ as

$$\pi_{hjt}^p = \omega_{hjt}^p + \beta_{1,hjt}^p \text{PRICE}_{ht} + \beta_{2,hjt}^p \text{DISP}_{ht} + \beta_{3,hjt}^p \text{FEAT}_{ht} + \beta_{4,hjt}^p \text{LOY}_{ht}^p$$

where $p$ indicates a brand name, here taking on two values, 1 for Nordica and 2 for Dannon. This specification allows for the influence of price (PRICE), in-store display (DISP) and feature advertising (FEAT). The notation $\text{LOY}_{ht}^p$ denotes a loyalty variable that adjusts for the household’s long-run propensity to buy the item (flavor) $j$ within a cluster (brand). We define $\text{LOY}_{ht}^p = \log((n_{hjt}^p + 0.5)/(n_{hjt}^p + 0.5N_J))$ where $n_{hjt}^p$ is the number of purchases of the item (flavor) $p$ across the household’s $N_J$ shopping trips in the initial eight months of the dataset, and $N_J$ is the number of items in the household’s $n_J$ flavor SKU’s within each brand name ($N_J = 6$). Because marketing mix variables (such as price) are always the same for all flavors of a particular brand, we assume that marketing mix parameters vary only by brand and household.

Next, we specify the structure of the cross-effect matrix $\Theta_h$, following the structural restrictions shown in Fig. 2. The two brands are treated as perfect substitutes, implying that any assortment containing both Nordica and Dannon products will never be observed. That is, we restrict the cross-effects $\theta_{hjk}$ between any two items having different brand names to be $-\infty$, thus setting the choice probabilities of these assortments to zero. However, the cross-effects for flavors within a brand $(\theta_{hjk}^N, \theta_{hjk}^D)$ are unrestricted (allowing for varying degrees of substitution, complementarity or independence).

These restrictions reduce the total number of possible assortments from 4095 ($= 2^{12} - 1$) to 126 ($= 2 \times (2^6 - 1)$). The corresponding cross-effect matrix for all brands and flavors has the general form

$$\Theta_h = \begin{bmatrix} \Theta_h^N & -\infty \\ -\infty & \Theta_h^D \end{bmatrix}_{12x12}$$

where $\Theta_h^N$ and $\Theta_h^D$ are $6 \times 6$ matrices for the flavors within each brand (Nordica and Dannon, respectively). These sub-matrices must be symmetric, but need not necessarily be equal.

We complete the model specification by developing assumptions about the pattern of household response parameter heterogeneity. Define the vector containing all unique cross-effect parameters as $\theta_h$. Using this notation, we assume that household response heterogeneity is governed by the relationships

$$\alpha_h \sim N(\bar{\alpha}, \Sigma_\alpha)$$

$$\beta_h \sim N(\bar{\beta}, \Sigma_\beta)$$

$$\theta_h \sim N(\bar{\theta}, \Sigma_\theta)$$

where $\alpha_h$ and $\beta_h$ are vectors of item specific intercepts and marketing mix parameters.

In general, we can estimate any type of covariance structure for the parameters by properly specifying the variance-covariance matrices in (11). In this application, we assume, for simplicity, that all variance-covariance matrices are diagonal. Although this assumption requires that parameters
independently vary across households, it does not imply that marketing actions of one item have no impact on other items. As we show below, such cross-item dependencies exist due to the fact that the cross-effect matrix in Eq. (10) is not equal to zero.

Model Calibration

We introduce the indicator variable \( y_h^b \), which equals one if the assortment \( b (z_{ht}^b) \) is chosen and zero otherwise. Ignoring household parameter heterogeneity, the likelihood takes on the standard form

\[
L = \prod_{h} \prod_{t} \prod_{b} \left[ \Pr(z_{ht} = z_{ht}^b) \right]^{y_{ht}^b}
\]

(12)

where, as noted earlier, \( h \) denotes household and \( t \) denotes time. However, given our heterogeneity assumptions, it is necessary to construct the likelihood of each household by integrating out the random components of the parameters as

\[
L_h = \int_{\xi} \prod_{t} \prod_{b} \left[ \Pr(z_{ht} = z_{ht}^b | \xi) \right]^{y_{ht}^b} f(\xi) d\xi
\]

(13)

where \( \xi \) represents all random elements in the model. This integral can be approximated using \( R \) draws of simulated values of household-varying parameters. In this way, we obtain

\[
L_h \approx \frac{1}{R} \sum_{r} L_h^{(r)} = \frac{1}{R} \sum_{r} \prod_{t} \prod_{b} \left[ \Pr(z_{ht} = z_{ht}^b | \xi^{(r)}) \right]^{y_{ht}^b}
\]

(14)

which, in turn, leads to \( \text{LL}(\alpha, \beta, \theta) \approx \sum_h \log L_h(\alpha_h, \beta_h, \theta_h) \), an approximation to the true log likelihood of all households and time points.

Maximization of the approximate log likelihood with respect to all model parameters generates Maximum Simulated Likelihood (MSL) estimates. Parameter estimates obtained in this fashion are known to be consistent (Lee 1995). In this research, we implemented the MSL procedure with \( R = 250 \) simulates obtained using Randomized Halton Sequence methods (Train 2003). Simulates drawn from a Halton Sequence are negatively correlated and evenly cover the support of the parameter distribution. Both features substantially improve the accuracy of the MSL procedure. Train (2003) reports that MSL parameter estimates obtained using 100 draws from a Halton Sequence can be more accurate than MSL parameter estimates obtained with using 1,000 randomly drawn simulates. Moreover, in the context of random coefficient logit models, Huber and Train (2001) report excellent correspondence between estimates of household level parameters obtained using MSL and Hierarchical Bayes methods. For these reasons, MSL was deemed an appropriate estimation methodology for our model.

Model Comparison

Using the MSL methodology, we tested several variations of our proposed model on the yogurt purchase histories by altering the market structure implied by the \( \Theta_h \) matrix. In all model comparisons, we retained the assumption that brands are perfect substitutes. The first benchmark model, called No Cross-Effects, assumes independence among flavors within each brand, restricting all elements in the cross-effect matrix \( (\Theta_h) \) to be zero. In this instance, we only estimate marketing mix parameters including item specific intercepts. The second benchmark model, called Same Cross-Effects, allows cross-effects parameters to be estimated, imposing the restriction that cross-effects are based on flavor (not on brand). This makes the cross-effect sub-matrices identical across brands, that is, \( \Theta_h^N = \Theta_h^p = \Theta_h^b \). The third benchmark model, called Differential Cross-Effects, assumes that cross-effect parameters vary across flavors and brands, that is, \( \Theta_h^N \neq \Theta_h^p \).

Table 3 displays the fit statistics for different models. We report values of the log-likelihood (for both calibration and holdout data), \( \rho^2 \), AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion). With the exception of the BIC criterion, all fit statistics point toward the Differential Cross-Effects model (shown in the last row of Table 3) in calibration as well as holdout data. Given these results, we restrict attention to the Differential Cross-Effects model.

Parameter Estimates

Table 4 presents parameter estimates for baseline utilities for the Differential Cross-Effects model. Recall that coefficients for price, feature and display vary by brand, but not by flavor. Moreover, the display variable for Dannon is omitted from the model because Dannon (in our dataset) never uses display in its promotions. The parameter values suggest that Dannon is, in general, the preferred brand (larger brand intercepts).
Moreover, households are relatively more sensitive to price changes in Dannon than Nordica. All marketing mix coefficients have the correct signs (negative for price, positive for feature and display). As explained earlier, all model coefficients are allowed to vary across households. For this reason, we present the standard deviation of the parameters in Table 4.

Table 5 displays the cross-effect $\Theta_h$ matrix for the Differential Cross-Effects model. Although these coefficients also vary across households, for expositional purposes, we only present estimates of the mean of the population. Some parameters, denoted by the value $-\infty$ (negative infinity), are not estimated by the MSL procedure. Rather, these parameters, implying perfect substitution across brand names, reflect restrictions imposed by the model specification. In addition, the zeros on the diagonal of the matrix are structural restrictions imposed for model identification. Again, these values are not estimated. Overall, the market structure is characterized by perfect substitution across brands and varying levels of dependence (both substitution and complementarity) across flavors within brands.

### Variety and Quality in the Yogurt Category

Having described the basic results of the yogurt assortment model, we now turn to the relationship between perceived product quality and assortment variety. As noted earlier, the prediction from the theory proposed by Chernev and Hamilton (2009) is that consumers will prefer a small high-quality assortment over a large low-quality assortment. To test the theory, we first show that quality tiers exist in the yogurt category. We then explore the mediating role of quality in consumer demand for assortment variety.

### Quality Tiers in the Yogurt Category

There exists strong evidence that Nordica and Dannon fall into two different quality tiers. We note that Dannon is priced higher than Nordica, and relies less on promotional techniques such as display and feature (Table 1). Both factors argue that Dannon is perceived by consumers to be of higher quality than Nordica. In fact, in the United States, Dannon and Yoplait (not included in this study) largely define the premium national brand segment of the yogurt market (Orgish 2002).

A more rigorous way of assessing quality tiers is to examine the predicted pattern of cross-price competition between Nordica and Dannon. Quality tier theory argues that brands that are perceived to be of higher quality (and typically, are higher priced) have more impact on lower quality (lower priced) brands, than vice versa (Blattberg and Wisniewski 1989; Sivakumar 2000; Sivakumar and Raj 1997). For this purpose, we derive individual household level price elasticities and analytically aggregate them to develop aggregate own- and cross-price market share elasticities (cf. Russell and Kamakura 1994; Russell and Petersen 2000). This is done by inferring household-level parameters from the maximum simulated likelihood output (see Train 2003, Chapter 11), and then applying the elasticity expressions shown in Appendix B.
Aggregate cross-price elasticities for the Differential Cross-Effects model are presented in Table 6. Two general patterns are evident. First, cross elasticity patterns are not symmetric even though the cross-effect matrix (Table 5) is symmetric. Across brands, the perfect substitution assumption leads to positive cross-price elasticities. Within each brand (across flavors), cross-elasticity patterns are mixed in terms of signs. Complementarity effects (negative signs) predominate for Nordica, while substitution effects (positive signs) predominate for Dannon. Second, in general, between-brand cross-elasticities are relatively large (due to strong substitution effects).

According to quality-tier theory, Dannon, the high-quality brand, should dominate Nordica, the low-quality brand, in terms of brand price competition. In fact, this pattern is clearly evident in Table 6: Dannon price changes have a larger impact on Nordica choice share than vice versa. For example, a one percent price decrease of Dannon raspberry will lead to a 0.2–0.5% decrease in market share of Nordica products. In contrast, a one percent price decrease in Nordica raspberry leads to a 0.05–0.08% decrease in the share of Dannon products. Fig. 3 provides a summary of this pattern. Each bar of the histogram represents the average cross-price elasticity reporting the impact of price changes of a given brand and flavor combination on all other flavors of the other brand. The figure clearly shows that Dannon dominates Nordica with respect to inter-brand price competition. Thus, following quality-tier theory, we can infer that Dannon has higher perceived quality than Nordica.

### Table 5
Cross-effect matrix.

<table>
<thead>
<tr>
<th></th>
<th>Nordica</th>
<th>Dannon</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH</td>
<td>CH</td>
<td>CH</td>
</tr>
<tr>
<td>MX BY</td>
<td>0.0</td>
<td>−∞</td>
</tr>
<tr>
<td>PCH</td>
<td>0.912***</td>
<td>−∞</td>
</tr>
<tr>
<td>RBY</td>
<td>0.343</td>
<td>−∞</td>
</tr>
<tr>
<td>ST</td>
<td>0.105</td>
<td>−∞</td>
</tr>
<tr>
<td>OTHER</td>
<td>0.459*</td>
<td>−∞</td>
</tr>
</tbody>
</table>

### Table 6
Price elasticity matrix.

<table>
<thead>
<tr>
<th></th>
<th>Nordica</th>
<th>Dannon</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH</td>
<td>(-1.691)</td>
<td>0.696</td>
</tr>
<tr>
<td>MX BY</td>
<td>(-0.337)</td>
<td>0.716</td>
</tr>
<tr>
<td>PCH</td>
<td>(-0.177)</td>
<td>0.931</td>
</tr>
<tr>
<td>RBY</td>
<td>(-0.174)</td>
<td>0.787</td>
</tr>
<tr>
<td>ST</td>
<td>(-0.141)</td>
<td>0.646</td>
</tr>
<tr>
<td>OTHER</td>
<td>(-0.095)</td>
<td>0.545</td>
</tr>
</tbody>
</table>

### Note
- Significance levels are denoted as \(p < 0.05\) (*) and \(p < 0.01\) (**). For clarity, the standard deviations of parameters are not shown. The zeros on the diagonal of the table and the \(−∞\) values are structural restrictions of the model. These coefficients are not estimated from the data. Codes for flavors are as follows: CH = cherry, MX BY = mixed berry, PCH = peach, RBY = raspberry, ST = strawberry, and OT = others.
- Table displays the percentage change in the aggregate choice share of the row SKU with respect to a one percent increase in the price of the column SKU.
- Own-price elasticities are shown in boldface. Codes for flavors are as follows: CH = cherry, MX BY = mixed berry, PCH = peach, RBY = raspberry, ST = strawberry, and OT = others.
Assortment CH Nordica

Table 7
Assortment variety distributions.

<table>
<thead>
<tr>
<th>Number of flavors per assortment</th>
<th>Nordica</th>
<th>Dannon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46.3%</td>
<td>58.7%</td>
</tr>
<tr>
<td>2</td>
<td>29.5%</td>
<td>28.7%</td>
</tr>
<tr>
<td>3</td>
<td>18.3%</td>
<td>10.6%</td>
</tr>
<tr>
<td>4</td>
<td>4.7%</td>
<td>1.9%</td>
</tr>
<tr>
<td>5</td>
<td>1.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>6</td>
<td>0.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Number of assortments</td>
<td>579</td>
<td>864</td>
</tr>
<tr>
<td>Mean number of flavors</td>
<td>1.85</td>
<td>1.56</td>
</tr>
<tr>
<td>Standard deviation of flavors</td>
<td>0.97</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Note: Table shows the number of yogurt flavors per assortment, averaged across all households in the calibration sample. A chi-squared test indicates that the flavor distributions vary significantly by brand name (p < .0001).

Product Quality and Assortment Variety

Recall that the Chernev–Hamilton theory predicts that consumers should prefer small assortments when perceived quality is high and large assortments when perceived quality is low. Globally, this prediction is validated by our data. In Table 7, we display the distribution of assortment variety (number of yogurt flavors) for our dataset. As expected, Dannon (high quality) tends to have smaller-sized assortments than Nordica (low quality). A chi-squared test shows that this difference is statistically significant (p < .0001).

An alternative way of testing the Chernev–Hamilton prediction is to consider what would happen if the retailer were to alter the number of items in the product category. It is well known that variety impacts consumer perceptions of category attractiveness (Boatwright and Nunes 2001; Broniarczyk, Hoyer, and McAlister 1998) and consequently is a key determinant of store choice (Briesch, Chintagunta, and Fox 2009; Corstjens and Corstjens 2002). Here, we conduct an SKU deletion experiment to study the relative strengths of the Nordica and Dannon brands with respect to variety. Given the Chernev–Hamilton theory, we would expect that Dannon should be more resistant to the effects of SKU deletion due the fact that consumers demand less variety for high quality product assortments.

As noted earlier, the MVL choice model assumes IIA with respect to assortments, but not with respect to the individual products within the assortment. This property allows us to infer the new choice process that arises when a single SKU is deleted from the consideration set. We follow the same general procedure discussed earlier in connection with Eq. (7). We start with the choice model structure developed for the yogurt analysis. To analyze the impact of the deletion of a given SKU, we delete from the MVL choice model all assortments containing the SKU and then renormalize the probabilities of the remaining assortments. This has the effect of setting the probability of choice for the given SKU to zero and resetting the choice probabilities of the remaining alternatives. Using this adjusted choice model and the household-level choice model parameters estimated earlier, we then average the choice probabilities across all consumers to obtain new market shares. This process is repeated 12 times, once for each of the 12 SKUs in our study.

Table 8
Percentage change in market share due to SKU deletion.

<table>
<thead>
<tr>
<th></th>
<th>Nordica</th>
<th>Dannon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CH</td>
<td>MX BY</td>
</tr>
<tr>
<td>Nordica</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CH</td>
<td>−6.8</td>
<td>−2.4</td>
</tr>
<tr>
<td>MX BY</td>
<td>−7.2</td>
<td>1.4</td>
</tr>
<tr>
<td>PCH</td>
<td>−2.5</td>
<td>1.4</td>
</tr>
<tr>
<td>RBY</td>
<td>−1.4</td>
<td>0.2</td>
</tr>
<tr>
<td>ST</td>
<td>−3.2</td>
<td>−0.8</td>
</tr>
<tr>
<td>OTHER</td>
<td>−1.8</td>
<td>−3.2</td>
</tr>
<tr>
<td>Dannon</td>
<td></td>
<td>CH</td>
</tr>
<tr>
<td>CH</td>
<td>5.5</td>
<td>4.4</td>
</tr>
<tr>
<td>MX BY</td>
<td>4.5</td>
<td>5.2</td>
</tr>
<tr>
<td>PCH</td>
<td>4.1</td>
<td>3.6</td>
</tr>
<tr>
<td>RBY</td>
<td>4.3</td>
<td>3.6</td>
</tr>
<tr>
<td>ST</td>
<td>4.0</td>
<td>3.8</td>
</tr>
<tr>
<td>OTHER</td>
<td>3.7</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Note: Each entry is the percentage change in the aggregate choice share of the row SKU with respect to removal of the column SKU from the choice set. An entry should only be compared to other entries in the same column. Codes for flavors are as follows: CH = cherry, MX BY = mixed berry, PCH = peach, RBY = raspberry, ST = strawberry, and OT = other. Entries corresponding to the same flavor as the deleted product are shown in boldface.
A summary of the results of this SKU deletion experiment can be found in Table 8. We report the percentage change in market share of the row SKU caused by the deletion of the column SKU. Only numbers within a given column should be compared. For example, deleting the Cherry flavor of Nordica (first column of Table 8) reduces the choice shares of all other Nordica flavors, but increases the sales of all flavors of Dannon. In contrast, deleting the Cherry flavor of Dannon increases the sales of all other yogurt SKU’s.

The most interesting observation is that deleting a Nordica SKU generally decreases the shares of other flavors of Nordica, while deleting a Dannon SKU generally increases the shares of other flavors of Dannon. That is, reducing the variety of Nordica causes many consumers to switch to Dannon, thus reducing Nordica share. In contrast, reducing the variety of Dannon causes some consumers to switch to Nordica, but others to stay with the brand by switching to other flavors of Dannon. In this fashion, Dannon sales are more resistant to changes in the variety of the Dannon product line.

Summary

The key finding of this empirical work is that assortment choice behavior and product quality are linked. The high quality product (Dannon) dominates inter-brand price competition, shows less variety (number of flavors) in the typical chosen assortment, and is more resistant to reductions in variety (SKU) deletion in the choice set. Consistent with the Chernov–Hamilton theory, these empirical facts paint a picture of a consumer who is willing to trade-off variety against product quality in assortment choice.

Conclusions

This work examines the impact of product quality on the variety of an assortment. Using the multivariate logistic (MVL) choice model, we analyzed purchase histories in which consumers buy an assortment of yogurt flavors on the same shopping trip. The Chernov–Hamilton (2009) theory predicts that consumers will prefer smaller assortments (less variety) when perceived product quality is higher. Our empirical analysis provides clear evidence in favor of the theory.

Substantive Contributions

We used the MVL model structure to fit a non-standard model of assortment choice to the yogurt product category. The model implies that households treat brand names as strong substitutes, and flavors (within a brand) as weak substitutes and complements. We showed that the general pattern of price competition is consistent with the Blatberg and Wisniewski (1989) theory of quality-tier competition. The estimated cross-price elasticities indicate that price changes of the premium brand (Dannon) have more impact on the lower-tier brand (Nordica) than vice versa. We also demonstrated that item deletion (reduction in variety) has a stronger impact on Nordica share than on Dannon share. The overall pattern of these results is consistent with work in consumer behavior linking quality perceptions to the desired variety of the assortment (Chernov and Hamilton 2009).

This relationship between demand for variety and perceived product quality has clear implications for retail assortment planning. Assortment planning is important because assortment (along with pricing policy and store location) drives both store positioning and store choice (Corstjens and Corstjens 2002). The link between product quality and variety implies that high product quality allows the retailer to carry less of the manufacturer’s product line. Advice of this sort, however, must be interpreted cautiously. Although SKU reduction can improve retailer profitability (Boatwright and Nunes 2001), retailers should always consider retail competition before making major changes in product offerings. Failure to carry items that are typically on the consumer’s shopping list can lead to store switching (Bell and Lattin 1998). Nevertheless, the fact that product quality moderates demand for variety is a useful insight for retailers.

Modeling Assortment Choice

The MVL model adopted here has a number of advantages for modeling assortment choice. First, the model assumes that households evaluate assortments of products by summing evaluations of individual items and then adjusting the joint value for demand interactions (perceived substitutability or complementarity). This theory is attractive because it is consistent both with economic theory (Ma, Seetharaman, and Narasimhan 2005; Song and Chintagunta 2006) and with consumer behavior research on bundling (Janiszewski and Cunha 2004). Second, due to the closed form of the model, we can easily study the patterns of price competition and predict changes in share due to item deletion. Such flexibility enables retailers to interpret complex demand patterns and to develop marketing policies consistent with market structure. Finally, by accommodating the deletion of assortments that do not occur, this methodology lends itself to parsimonious specifications.

Limitations and Extensions

The current study has several limitations, each of which provides opportunities for further research. First, our empirical application is limited to two key brands in one submarket (fruit-on-bottom) of the yogurt category. Clearly, a larger scale study (multiple submarkets as well as different product categories) would be desirable to assess the generality of the role of quality in assortment choice. Second, the concept of variety itself is not well-defined. In our research, flavor is the obvious choice because flavor is the only attribute that varies within a particular product form of a yogurt producer. Developing a general approach to defining variety would provide researchers and retailers with a useful tool for analyzing assortment choice (cf. Boatwright and Nunes 2001; Hoch, Bradlow, and Wansink 1999, 2002; Kahn and Wansink 2004).

Third, the multivariate logistic assortment choice model faces issues of tractability as the number of assortment items increase. In part, this issue arises because the denominator of the assortment choice function (Eq. (6)) contains as many
terms as possible assortments. In this research, we avoided the dimensionality problem by using structural assumptions to limit the number of possible assortments. Currently, simulation estimation technologies that approximate the denominator of Eq. (6) exist, but are confined to models without parameter heterogeneity (see, e.g., Wu and Huffer 1997). However, recent work by Kamakura and Kwak (2012) involving sampling of alternatives (Ben-Akiva and Lerman 1985) and latent class analysis (Kamakura and Russell 1989) shows promise in developing a practical method of model calibration.

In addition, the number of cross-effect parameters increases geometrically with the number of items in the study. The obvious solution here is to constrain the pattern of the cross-effect matrix to reduce the number of parameters in the model. For example, cross-effect parameters can be projected into a reduced dimensional space (Kamakura and Schimmel 2013). Alternatively, the cross-effect pattern can be made to correspond to the distances on a psychometric map representing similarity among items (Moon and Russell 2008). Successfully addressing these problems would allow retailers to use the model to study choice for large numbers of assortments.

Appendix A. Derivation of Assortment Model

Let \( z = (z_1, z_2, \ldots, z_J) \) be any assortment of item choices. Brook’s Lemma, cited in Besag (1974), states that the joint distribution of the random variable \( z_j \), namely \( Pr(z_1, z_2, \ldots, z_J) \), is proportional to a series of ratios through a certain constant \( Pr(z_{10}, z_{20}, \ldots, z_{J0}) \) where \( z_{j0} \) is an arbitrary reference value of the random variable \( z_j \). Specifically, Brook’s Lemmas states that

\[
\frac{Pr(z_1, z_2, \ldots, z_J)}{Pr(z_{10}, z_{20}, \ldots, z_{J0})} = \frac{Pr(z_1|z_{20}, \ldots, z_{J0})}{Pr(z_{10}|z_{20}, \ldots, z_{J0})} \cdot \frac{Pr(z_2|z_{10}, z_3, \ldots, z_J)}{Pr(z_{20}|z_{10}, z_3, \ldots, z_J)} \cdots \frac{Pr(z_J|z_{10}, z_{20}, \ldots, z_{J-1,0})}{Pr(z_{J0}|z_{10}, z_{20}, \ldots, z_{J-1,0})}
\]

In this research, we define the joint distribution \( Pr(z_1, z_2, \ldots, z_J) \) to be the probability of choosing an assortment of products.

We derive an assortment choice model in Eq. (6) in the following way. First, we assume (without loss of generality) that the reference values \( z_{j0} = 0 \) so that the proportionality constant \( Pr(z_{10}, z_{20}, \ldots, z_{J0}) \) is equal to \( Pr(0, 0, \ldots, 0) \). Second, we assume that all the conditional probabilities on the right hand side of (A1) have the form of the binary logit expressions found in Eq. (5). Third, we assume that all elements of the \( \Theta_k \) matrix in Eq. (3) are symmetric. This last assumption essentially means that the probability of observing an assortment depends only on the contents of the assortment – not upon the order in which the various products in the assortment are purchased.

The assortment choice probability given in Eq. (6) is obtained by inserting these assumptions into (A1) and using the fact that the sum of the probabilities over all assortments must add to one. The symmetry of \( \Theta_k \) ensures that the assortment model generated by (A1) always has the same form, regardless of the order in which the SKU’s are placed in the \( z = (z_1, z_2, \ldots, z_J) \) vector. The model implicitly assumes that any assortment – including the null assortment with probability equal to \( Pr(0, 0, \ldots, 0) \) – may be observed. However, the analyst can restrict the model to a subset of assortments by restricting the possible values of \( z \) and then renormalizing the resulting model. In the analysis of the yogurt data, we followed this procedure in order to exclude assortments that cannot occur (e.g., assortments containing items with different brand names).

Appendix B. Derivation of Price Elasticities

We first derive household-level price elasticities. Let \( j \) and \( k \) denote two different SKU’s. At specific time point \( t \) for household \( h \), elasticities are computed as

\[
E(j, j)_{ht} = \frac{\partial [\ln Pr(j)_{ht}]}{\partial [\ln \text{PRICE}(j)_{ht}]} = \frac{\partial Pr(j)_{ht}}{\partial \text{PRICE}(j)_{ht}} \cdot \frac{\text{PRICE}(j)_{ht}}{Pr(j)_{ht}} 
\]

(B1)

\[
E(j, k_{c*})_{ht} = \frac{\partial [\ln Pr(j)_{ht}]}{\partial [\ln \text{PRICE}(k_{c*})_{ht}]} = \frac{\partial Pr(j)_{ht}}{\partial \text{PRICE}(k_{c*})_{ht}} \cdot \frac{\text{PRICE}(k_{c*})_{ht}}{Pr(j)_{ht}} 
\]

(B2)

Given the form of the model, we can write probability of purchasing item \( j \) by enumerating all possibilities of assortments containing item \( j \). We can also define the probability of choosing item \( j \) and \( k \) in one assortment in the same way. These can be written as

\[
Pr(j)_{ht} = \sum_{b \text{ containing } j} \sum_{b*} \exp \left( \pi_{ht} z_{ht}^{*b} + \frac{1}{2} \{ z_{ht}^{*b}/Theta_{ht} z_{ht}^{*b} \} \right) 
\]

(B3)

\[
Pr(j, k)_{ht} = \sum_{b \text{ containing } j} \sum_{b*} \exp \left( \pi_{ht} z_{ht}^{*b} + \frac{1}{2} \{ z_{ht}^{*b}/Theta_{ht} z_{ht}^{*b} \} \right) 
\]

(B4)

where \( \pi_{ht} = \alpha_{ht} + \beta_{ht}^{p} \cdot \text{PRICE}_{hpt} + \beta_{2,ht}^{p} \cdot \text{DISP}_{hpt} + \beta_{3,ht}^{p} \cdot \text{FEAT}_{hpt} + \beta_{4,ht}^{p} \cdot \text{LOY}_{ij} \) and marketing mix parameters are brand specific, not item specific.
Using simple algebra, we can derive the following derivatives:

\[
\frac{\partial P\left(j_p\right)_{ht}}{\partial \text{PRICE}_{ij}} = \beta^p_{j_i} \text{PRICE}_{ij} \left(1 - P\left(j_p\right)_{ht}\right) \quad \text{if } j \text{ has brand } p
\]

\[
\frac{\partial P\left(j_p\right)_{ht}}{\partial \text{PRICE}_{kj}} = -\beta^q_{j_k} P\left(j_p\right)_{ht} \text{PRICE}_{kj} \quad \text{if brand } p \neq q
\]

\[
\frac{\partial P\left(j_p\right)_{ht}}{\partial \text{PRICE}_{kj}} = \beta^p_{j_k} \left[P\left(j_p\right)_{ht} - P\left(j_p\right)_{ht} P\left(k_p\right)_{ht}\right] \quad \text{if } j \neq k \text{ for different brands}
\]

Applying elasticity definitions in (B1) and (B2), the corresponding household-level elasticities are computed as:

\[
E\left(j_p, j_p\right)_{ht} = \beta^p_{\text{price}} \left(1 - P\left(j_p\right)_{ht}\right) \text{PRICE}_{ij}^p
\]

\[
E\left(j_p, k_q\right)_{ht} = -\beta^q_{\text{price}} P\left(k_q\right)_{ht} \text{PRICE}_{kj}^q \quad \text{if } p \neq q
\]

\[
E\left(j_p, k_q\right)_{ht} = \beta^p_{\text{price}} P\left(k_q\right)_{ht} \left[S\left(j_p, k_p\right)_{ht} - 1\right] \text{PRICE}_{kj}^p \quad \text{if } j \neq k \text{ have the same brand}
\]

where \(S\left(j, k\right)_{ht} = P\left(j, k\right)_{ht}/P\left(j\right)_{ht} P\left(k\right)_{ht}\). When there are more than two items in a category, cross-elasticity between two items in the same category is not solely determined by cross-effect terms but by \(S\left(j, k\right)\). Notice that \(S\left(j, k\right) > 0\) for complements, \(S\left(j, k\right) < 0\) for substitutes, and \(S\left(j, k\right) = 0\) for independence.

To compute aggregate elasticities, we first define the overall choice shares as \(MS_j = \sum_h \sum_t P\left(j\right)_{ht}/NT\) where \(NT\) (the number of households \(N\) times average number of shopping trips \(T\) per household) is the number of total observations in a particular data set. Following procedures outlined in Russell and Kamakura (1994), we differentiate this expression and use the household level derivatives in (B5) to infer market share elasticities. This process yields the expressions

\[
\eta_{j} = \frac{\% \Delta MS\left(j_p\right)}{\% \text{PRICE}_j^p} = \left[\frac{\sum_h \beta^p_{\text{price}} P\left(j_p\right)_{ht} \left[1 - P\left(j_p\right)_{ht}\right]}{MS\left(j_p\right)} \right] \text{PRICE}_j^p
\]

\[
\eta_{jk}^p = \frac{\% \Delta MS\left(j_p\right)}{\% \text{PRICE}_k^p} = -\left[\frac{\sum_h \beta^q_{\text{price}} P\left(k_q\right)_{ht} P\left(k_q\right)_{ht} / N}{MS\left(j_p\right)} \right] \text{PRICE}_k^p
\]

\[
\eta_{jk}^p = \frac{\% \Delta MS\left(j_p\right)}{\% \text{PRICE}_k^p} = -\left[\frac{\sum_h \beta^p_{\text{price}} P\left(j_p, k_p\right)_{ht} - P\left(j_p\right)_{ht} P\left(k_p\right)_{ht}}{MS\left(j_p\right)} \right] \text{PRICE}_k^p
\]

for brands \(p \neq q\)

\(j \neq k\) for the same brand

References


