HTS High Q Resonant Controller

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Abstract. High T_c superconductor (HTS) technology has been used to develop an advanced high Q resonant circuit and its devices. With a HTS, a very high Q circuit can be achieved; consequently special aspects such as high voltage generation and high current control can be theoretically and practically realized. Theoretical study has been carried out, as well as a practical approach has been made for the concept verification. This paper describes the theory of this high Q resonant circuit and the operational principle of its high voltage generation and current control.

Introduction

A resistor R - capacitor C - inductor L series resonant circuit has been explored with regard to its voltage aspects of using a high T_c superconductor (HTS) [1,2]. The relation between the circuit quality factor Q and its voltage aspects has been studied, and formed a method of high voltage generation. Then a method of generating high voltage from a low voltage source has been explored. This high voltage generator using the resonant circuit mainly consists of an inductor, a capacitor, a DC battery source, and an electronic switch. As a fundamental principle, the resonant circuit generates the voltage which is proportional to the circuit Q value. As a basic principle of operational approach, a low voltage DC power source can be used and its polarity is reversed at a certain frequency, and this control is achieved with an electronic switch. Resistance in the circuit will limit the Q value and therefore voltages that can be achieved in practice; however HTS technology can dramatically reduce the resistance and present a very high Q value. Both theoretical analysis and practical device operation principle will be presented in this paper with the advantages of using a high Q circuit made by HTS Bi-2223/Ag multifilament wires.

Operation Theory

Q Value and Voltage Feature of a Resonant Circuit. The build-up voltage in a resistive R-C-L series resonant circuit, as shown in Fig. 1, is related to the circuit quality factor Q. The quality factor of a resonant circuit is $Q = \omega_o L/R$, sometimes called the magnification factor, and $\omega_o = [(1/LC)-(R^2/4L^2)]^{1/2}$ is the resonant frequency in rad s^{-1}. The potential difference across the capacitor at resonance is Q times as large as the applied emf V_p (rms). For a sinusoidal power supply, the voltage across the capacitor V_{Cmax} at resonance is given by

$$V_{Cmax} = Q \ V_p \tag{1}$$

For using a DC power supply with an electronic switch to reverse the polarity, the maximum voltage V_{Cmax} can be expressed as

$$V_{Cmax} = Q\ V_B' \tag{2}$$

where V_B' is the effective voltage of the low voltage power source. If the used switching controller has rectangular wave form with low voltage source polarity switching frequency f, the build-up voltage wave form F(t) can be expressed by Fourier series as

$$F(t) = (4V_B/\pi)[\sin\omega t + (1/3)\sin 3\omega t + (1/5)\sin 5\omega t + ...] \tag{3}$$

The generator only resonates on the first harmonic with amplitude of $4V_B/\pi$. Therefore

$$V_B' = 4\ V_B/\pi \tag{4}$$

and the maximum build-up voltage is related to the R-C-L resonant circuit Q value by

$$V_{Cmax} = Q\ (4V_B/\pi) \tag{5}$$

Reducing the R-C-L resonant circuit resistance is achieved by introducing the superconducting inductor; the circuit Q value will be then dramatically increased, which leads to a very high voltage across the capacitor.

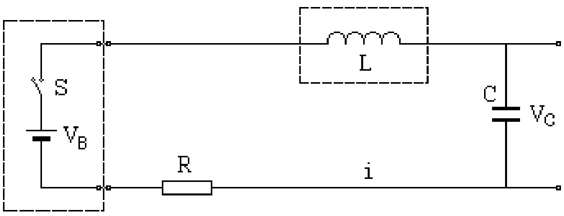

Fig.1. A R-C-L series resonant circuit with a DC source.

Resistance-Less Circuit. In a R-C-L resonant circuit as shown in Fig. 1, when switch S is closed in this circuit, the instantaneous current i(t) and capacitor voltage $V_C(t)$ solutions respectively are

$$i(t) = e^{\frac{-Rt}{2L}}\left[\frac{(V_{CO} + V_B)\sin\omega t}{\omega L}\right] \tag{6}$$

$$V_C(t) = (V_{CO} + V_B)\left[1 - e^{\frac{-Rt}{2L}}\left(\cos\omega t + \frac{R}{2L\omega}\sin\omega t\right)\right] - V_{CO} \tag{7}$$

where $R < 2(L/C)^{1/2}$, V_{CO} is the initial capacitor voltage. Both equations describe decaying sinusoids with $V_C(t)$ approaching a steady state value of V_B, and i(t) approaching a steady state value of zero. For the circuit using a superconducting inductor and no separate resistor, then R will become very small. If R = 0 Ω, then Eq. 6 and Eq. 7 can be simplified to

$$i(t) = \frac{(V_{CO} + V_B)\sin\omega t}{\omega L} \tag{8}$$

$$V_C(t) = -(V_{CO} + V_B)\cos\omega t + V_B \tag{9}$$

where $\omega = (LC)^{-1/2}$ rad s^{-1}. These two equations describe constant magnitude sinusoids with the average values of i(t) and $V_C(t)$ being zero and V_B respectively.
When switch S in Fig. 1 is closed, $V_C(t) = V_C(0) = -V_{CO}$. One half a resonant cycle later, this voltage will have increased to

$$V_C(t) = V_C(\pi/\omega) = -(V_{CO} + V_B)(-1) + V_B = V_{CO} + 2V_B \tag{10}$$

If the battery is disconnected at this point of time, and then reconnected in the opposite polarity for the next half cycle, then the initial capacitor voltage V_{CO} is changed to $V_{CO\text{-}new}$, and it is given by

$$V_{CO\text{-}new} = -V_C(t) = -(V_{CO} + 2V_B) \tag{11}$$

Half a cycle later, $V_C(t) = V_C(2\pi/\omega)$ becomes

$$V_C(t) = -[-(V_{CO} + 2V_B) + (-V_B)](-1) + (-V_B) = -(V_{CO} + 4V_B) \tag{12}$$

If the battery is reversed every half cycle thereafter, then the $V_C(t)$ is

$$V_C(t) = (V_{CO} + 6V_B);\ -(V_{CO} + 8V_B);\ (V_{CO} + 10V_B);\ \ldots\text{etc} \tag{13}$$

This is the build-up voltage for an ideal non-resistive circuit. Consequently the positive and negative peak voltages can be described by the following equation, i.e. in a resistance-less circuit, the voltage across the capacitor C after n cycles will be

$$V_C\,(n) = (-1)^{n+1}\ (V_{CO} + 2nV_B) \tag{14}$$

where n is the iteration number, and V_{CO} is the initial capacitor voltage.

Practical Resistive Circuit. From Eq. 6, when $t = \pi/\omega$, $i = 0$, if the electronic bridge in the Fig. 1 changes DC source polarity, the capacitor voltage is given by

$$V_{C1} = (V_{CO}+V_B)(1+e^{-R\pi/2L\omega})-V_{CO} \tag{15}$$

After the polarity is changed n times, the capacitor voltage becomes

$$V_{Cn} = (V_{Cn-1}+V_B)(1+e^{-R\pi/2L\omega})-V_{Cn-1} \tag{16}$$

If $V_{CO} = 0$, then

$$\begin{aligned}
V_{C1} &= V_B(1+e^{-R\pi/2L\omega})\\
V_{C2} &= (V_{C1}+V_B)(1+e^{-R\pi/2L\omega})-V_{C1}\\
&= V_B(1+2e^{-R\pi/2L\omega}+e^{-2R\pi/2L\omega})\\
&\ldots\\
V_{Cn} &= (V_{Cn-1}+V_B)(1+e^{-R\pi/2L\omega})-V_{Cn-1}\\
&= V_B(1+2e^{-R\pi/2L\omega}+2e^{-2R\pi/2L\omega}\ldots+2e^{-(n-1)R\pi/2L\omega}+e^{-nR\pi/2L\omega})\\
&= V_B(1+e^{-nR\pi/2L\omega})+2V_B\sum_{i=1}^{n-1} e^{-iR\pi/2L\omega}
\end{aligned} \tag{17}$$

Assuming that the electronic bridge changes the power supply polarity at $t = n\pi/\omega$, after the polarity is changed n times, the build-up voltage V_{Cn} is given by

$$V_{Cn} = V_B(1+e^{-nR\pi/2L\omega}) + 2V_B \frac{e^{-R\pi/2L\omega} - e^{-(n-1)R\pi/2L\omega}}{1-e^{-R\pi/2L\omega}} \quad (18)$$

When $n \to \propto, V_{Cn} \to V_{Cmax}$, therefore

$$V_{Cmax} = \lim_{n\to\infty} V_{Cn} = V_B(1+\frac{2}{e^{R\pi/2L\omega}-1}) \quad (19)$$

From Eq. 19, when $R \to 0$, $V_{Cmax} \to \propto$. When $R \to \propto$, $V_{Cmax} \to V_B$. Since $Q = \omega L/R$, therefore Eq. 19 can be expressed as

$$V_{Cmax} = V_B + \frac{2V_B}{e^{\pi/2Q}-1} \quad (20)$$

Practical Operation Theory

A low voltage DC source is readily to be used as the resonant circuit power supply. A practical device central circuitry comprises an electronic bridge which applies and periodically reverses the DC source polarity to the circuit, e.g. a bridge of four silicon controlled rectifiers (SCRs) connected as shown in Fig. 2. This is accomplished by a control circuit triggering the alternate pairs of SCRs at a selected rate.

If the initial state of the bridge circuit in Fig. 2 is: SCRs off, C discharged, no current flowing; then if S1 and S4 are both triggered, battery voltage V_B is applied to the series resonant circuit comprising L and C. If the trigger pulse is maintained until the SCR latching current is reached and ignoring any losses, a current will sinusoidally rise to a maximum and down to zero whereupon the SCRs will cease conducting due to load commutation. The length of this charge pulse is one half of the natural resonant period of the C-L circuit. The voltage left on C will be twice the battery voltage V_B. If S2 and S3 (the other pair of SCRs) are triggered at a later time, the battery voltage will be placed in series with the voltage left on C and this cycle will again add twice the battery voltage to C with opposite sign. Thus $2V_B$, $-4V_B$, $6V_B$, $-8V_B$, $10V_B$, etc., will be the sequence of voltages produced across the capacitor. Consequently repeated cycles will raise the absolute voltage of C until losses in the resonant circuit cause a voltage plateau to be reached in a practical resistive circuit. If the period between SCR switching is enlarged, i.e. lower cycle repetition rate, the slow voltage loss in C between charge sinusoids will cause the voltage left on C to be stabilized at lower levels. This gives some means of voltage control, if the Q is changed while the system is running, the voltages and currents will change as a result. Fig. 3 shows a graph illustrating the voltage build up and the principle of operation; and Fig. 4 shows a practically designed device circuit frame.

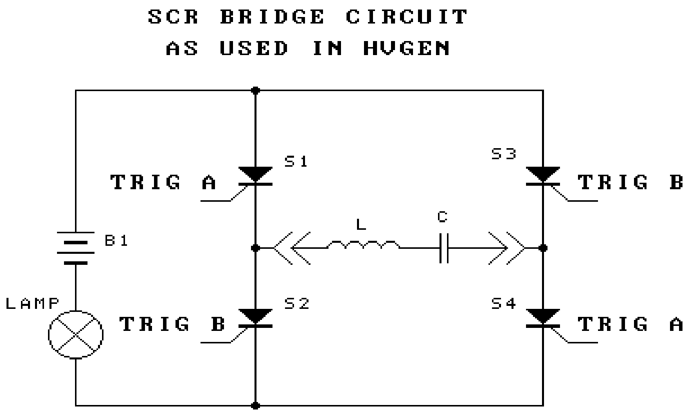

Fig. 2. The resonant circuit with an electronic switch.

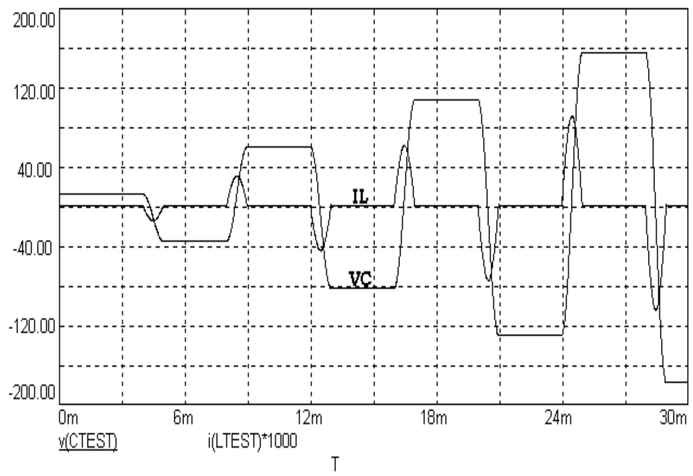

Fig. 3. Build-up of the voltage V_c(V) and the circuit current i_L(A).

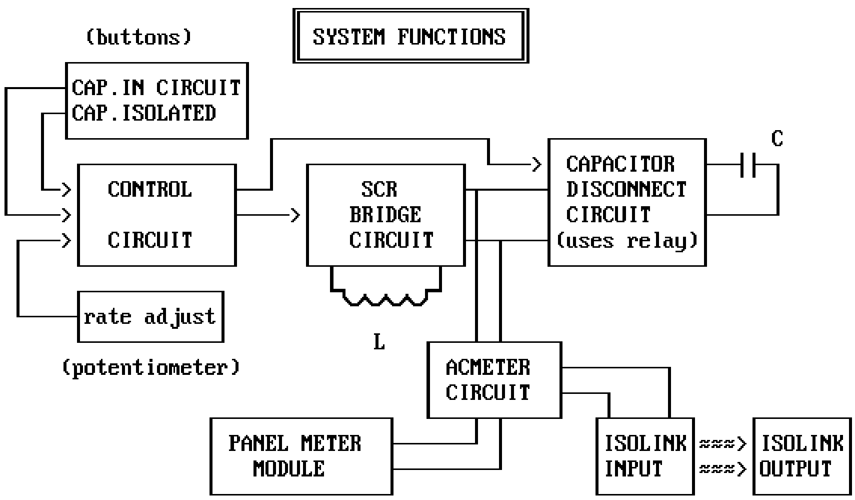

Fig. 4. An overall designed system diagram.

Discussion

As the analysis above, with a HTS the resonant circuit can be developed to be a method of generating high voltage from a low voltage source. This method can achieve high voltages by using a high Q circuit with the appropriate choice of circuit components, as an example shown in Fig. 5 for build-up voltages. From Eq. 19, when $n \rightarrow \infty$ and $R \rightarrow 0$, then $V_C \rightarrow \infty$. By limiting the number of iterations, or by employing voltage sensing controls, it is possible to generate a pre-determined set voltage.

HTS wire can be used to achieve the very high Q inductor to make this method viable; on the other hand the conventional inductor technique can not make this method applicable. The high Q inductor is able to be realized by using Ag-clad $(Bi,Pb)_2Sr_2Ca_2Cu_3O_{10+x}$ HTS wires, which have potential capability with high I_c to make the required inductor winding as an example shown in Fig. 6 [3]. This HTS wire has high engineering critical current density $J_e > 10^4$ A/cm^2 in 77 K, high magnetic field tolerance when the operational temperature is lower, mechanical flexibility, and long length; and can be employed for design of the HTS inductor [4-6]. The HTS inductor virtually has no resistance in a DC current operation; however loss will be generated even at a relative low value for low frequency AC application, which does not affect this application significantly [7]. The circuit current i, which can be calculated from V_C, is required for the design of a HTS inductor, and the circuit di/dt is also required for the HTS inductor design and the design of the electronic switch.

Practically, any circuit resistance causes energy dissipation, and for each reversal of the DC source polarity the corresponding increase in voltage is less than $2V_B$, and the magnitude of the voltage increase gets smaller with each iteration. Any resistance in the circuit will limit the final achievable voltage. The circuit resistance is mainly caused by the inductor as well as the DC source, electronic switches, wires and connections. In practice, the final voltage across the capacitor would reach a finite maximum value. This would occur as a result of the small voltage increase per cycle to be balanced by the same magnitude voltage loss per cycle due to leakage effects.

In a series resonant circuit with a power supply $V_p = V_m \sin\omega t$, the resonant frequency is given by $\omega_o = [(LC)^{-1} - (R/2L)^2]^{1/2}$. To reduce the circuit resistance to near zero leads to an infinite circuit quality factor Q (= $\omega_o L/R = X/R$) at resonance. The rms current I_{max} in the circuit at resonance is

$$I_{max} = V_m/(R\sqrt{2}) \tag{21}$$

and the voltage across the capacitor V_{Cmax} (rms) is

$$V_{Cmax} = X_C\, I_{max} = (\omega_o C)^{-1}\, [V_m/(R\sqrt{2})] \tag{22}$$

By assuming that the circuit resistance R is zero, the circuit then has infinite Q at resonant frequency $\omega_o = 1/\sqrt{(LC)}$ rad s^{-1}. This leads to an infinite value of V_C generated and a very large potential current. This device is therefore able to provide controls of both high voltages and high currents. Fig. 7. shows an experimental HTS resonant controller prototype developed recently having rated output of 1 kV / 20 A.

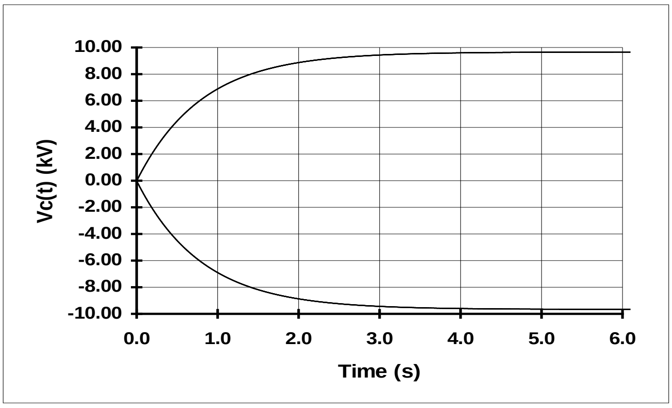

Fig. 5. Envelope of $V_C(t)$ for a sample circuit with R = 0.05 Ω, C = 20 μF and L = 20 mH.

Fig. 6. An experimental HTS inductor made by a Bi-2223/Ag wire.

Fig. 7. A prototype of HTS resonant controller.

Conclusion

A resonant circuit with a power electronic controller has been verified by theoretical and practical operation analysis, which enables voltages to be increased rapidly to a value many times greater than the input low voltage source, and is also able to control the circuit current in a wide range. The resonant controller using the HTS technology can be developed to be a useful method for high voltage and current generations and controls.

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