



Frequency Estimation for Low Earth Orbit Satellites

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CERTIFICATE

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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To My Parents

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List of Symbols

a	Azimuth
α_x	X-rotation
α_y	Y-rotation
β	Off-boresight error
δ	The offset between true frequency line and the closest bin
$\hat{\delta}$	Estimate of δ
D_r	Normalised Doppler rate
D_s	Normalised Doppler shift
$\Phi(x)$	Standard Normal density function, $= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
φ_{ES}	Earth station latitude
φ_s	Satellite latitude
f	Signal frequency
\hat{f}	Frequency estimate
f_s	Sampling frequency
γ	Central angle between earth station and satellite
G	Gravitational constant $\approx 6.672 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
GM_e	Earth figure $= 3.986004418 \times 10^{14}$
i	Satellite orbit inclination
\mathbf{I}	The identity matrix
$\Im(z)$	Imaginary part of z

$\binom{K}{l}$	Binomial coefficients
λ_{ES}	Earth station longitude
λ_s	Satellite longitude
$\ln(x)$	Natural logarithm of x
$\log(x)$	Logarithm base 10 of x
m	Index of the bin with the highest magnitude
M_e	Mass of the Earth $\approx 5.9736 \times 10^{24}$ kg
N	Number of samples
ω	Angular frequency of a sinusoidal signal $= 2\pi f$
ω_e	Angular velocity of the Earth
ω_F	Satellite angular velocity in the ECEF frame
ω_I	Satellite angular velocity in the ECI frame
ψ	Angular displacement of the satellite from the point of closest approach
Q	Number of iterations
$Q(x)$	The standard Q -function $= \int_x^\infty \Phi(t) dt$
q	Probability of an outlier
ρ	Signal to Noise Ratio
$\rho(\text{dB})$	Signal to Noise Ratio in decibels
\mathbb{R}	The set of real numbers
$\Re(z)$	Real part of z
σ	Standard deviation of a random variable
$s(k)$	k^{th} sample of sinusoidal signal
$S(n)$	Fourier transform of $s(k)$

$s(t)$	Earth station to satellite slant range at time t
θ	Elevation
θ_v	Minimum elevation for visibility
\mathbf{v}^T	Transpose of \mathbf{v}
T_s	Sampling time = $\frac{1}{f_s}$
T_v	Satellite visibility duration
$w(k)$	k^{th} sample of additive white Gaussian noise
$W(n)$	Fourier transform of $w(k)$
x^*	Complex Conjugate of x
$x(k)$	Signal plus noise = $s(k) + w(k)$
$X(n)$	Fourier transform of $x(k)$
$Y(n)$	Magnitude of $X(n)$
$N(0,1)$	The standard normal distribution, $= \int_{-\infty}^x \Phi(x) dx$

List of Abbreviations

ACRB	Asymptotic Cramer Rao Bound
a.s.	Almost Surely
ARMA	Auto-Regressive Moving Average
AWGN	Additive White Gaussian Noise
Az	Azimuth
BPSK	Binary Phase Shift Keying
CLT	Central Limit Theorem
CRB	Cramer Rao Bound
CRCSS	Cooperative Research Centre for Satellite Systems
DFT	Discrete Fourier Transform
DSP	Digital Signal Processing
ECEF	Earth Centred Earth Fixed
ECI	Earth Centred Inertial
EKF	Extended Kalman Filter
El	Elevation
FAST	Frequency Assisted Spatial Tracking
FFCI	Fractional Fourier Coefficients Interpolation
FFT	Fast Fourier Transform
GPS	Geographical Positioning System
IEKF	Iterative Extended Kalman Filter
IFFCI	Iterative Fractional Fourier Coefficients Interpolation
i.i.d.	Independent and Identically Distributed
IMOJ	Iterative Magnitudes Only Interpolation
IMSI	Iterative Magnitudes Squared Interpolation

LEO	Low Earth Orbit
LO	Local Oscillator
MBS	Maximum Bin Search
ML	Maximum Likelihood
MOI	Magnitudes Only Interpolation
MSI	Magnitudes Squared Interpolation
MMSI	Modified Magnitudes Squared Interpolation
MSE	Mean Squared Error
NORAD	North American Aerospace Command
pdf	Probability Density Function
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RMSE	Root Mean Squared Error
SNR	Signal to Noise Ratio
STK™	Satellite Tool Kit
TLEs	Two Line Elements
TT&C	Tracking, Telemetry and Command

Abstract

Low Earth Orbit (LEO) satellites have received increased attention in recent years. They have been proposed as a viable solution for remote sensing, telemedicine, weather monitoring, search and rescue and communications to name a few applications. LEO satellites move with respect to an earth station. Thus, the station must be capable of tracking the satellite both spatially and in frequency. In addition, as the spectrum becomes more congested, links are being designed at higher frequencies such as Ka band. These frequencies experience larger attenuations and therefore the system must be capable of operating at low signal to noise ratios.

In this dissertation we report on the research conducted on the following problems. Firstly, we study the estimation of the frequency of a sinusoid for the purpose of acquiring and tracking the frequency of the received signal. Secondly, we propose the use of the frequency measurements to assist the spatial tracking of the satellite.

The highly dynamic environment of a LEO system, combined with the high Ka band frequencies result in large Doppler rates. This limits the available processing time and, consequently, the fundamental resolution of a frequency estimator. The frequency estimation strategy that is adopted in the thesis consists of a coarse estimator followed by a fine estimation stage. The coarse estimator is implemented using the maximum of the periodogram. The threshold effect is studied and the derivation of an approximate expression of the signal to noise ratio at which the threshold occurs is examined.

The maximum of the periodogram produces a frequency estimate with an accuracy that is $O(N^{-1})$, where N is the number of data samples used in the FFT. The lower bound for the estimation of the frequency of a sinusoid, given by the Cramer-Rao bound (CRB), is $O\left(N^{-\frac{3}{2}}\right)$. This motivates the use of a second stage in order to improve the estimation resolution. A family of new frequency estimation algorithms that interpolate on the fractional Fourier coefficients is proposed. The new estimators can be implemented iteratively to give a performance that is uniform in frequency. The iterative algorithms are analysed and their asymptotic properties derived. The

asymptotic variance of the iterative estimators is only 1.0147 times the asymptotic CRB.

Another method of refining the frequency estimate is the Dichotomous search of the periodogram peak. This is essentially a binary search algorithm. However, the estimator must be padded with zeroes in order to achieve a performance that is comparable to the CRB. An insight into this is offered and a modified form that does not require the zero-padding is proposed. The new algorithm is referred to as the modified dichotomous search. A new hybrid technique that combines the dichotomous search with an interpolation technique in order to improve its performance is also suggested.

The second research aim was to study the possibility of applying the frequency measurements to obtain spatial tracking information. This is called the frequency assisted spatial tracking (FAST) concept. A simple orbital model is presented and the resulting equations are used to show that the Doppler shift and rate uniquely specify the satellite's position for the purpose of antenna pointing. Assuming the maximum elevation of the pass is known, the FAST concept is implemented using a scalar Extended Kalman Filter (EKF). The EKF performance was simulated at a signal to noise ratio of 0dB. The off-boresight error was found better than 0.1° for elevations higher than 30° .