# Frequency Estimation for Low Earth Orbit Satellites

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# CERTIFICATE

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate

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**To My Parents**

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# List of Symbols

| | |
|---|---|
| $a$ | Azimuth |
| $\alpha_x$ | X-rotation |
| $\alpha_y$ | Y-rotation |
| $\beta$ | Off-boresight error |
| $\delta$ | The offset between true frequency line and the closest bin |
| $\hat{\delta}$ | Estimate of $\delta$ |
| $D_r$ | Normalised Doppler rate |
| $D_s$ | Normalised Doppler shift |
| $\Phi(x)$ | Standard Normal density function, $= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ |
| $\varphi_{ES}$ | Earth station latitude |
| $\varphi_s$ | Satellite latitude |
| $f$ | Signal frequency |
| $\hat{f}$ | Frequency estimate |
| $f_s$ | Sampling frequency |
| $\gamma$ | Central angle between earth station and satellite |
| $G$ | Gravitational constant $\approx 6.672 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$ |
| $GM_e$ | Earth figure $= 3.986004418 \times 10^{14}$ |
| $i$ | Satellite orbit inclination |
| $\mathbf{I}$ | The identity matrix |
| $\Im(z)$ | Imaginary part of $z$ |

| | |
|---|---|
| $\binom{K}{l}$ | Binomial coefficients |
| $\lambda_{ES}$ | Earth station longitude |
| $\lambda_s$ | Satellite longitude |
| $\ln(x)$ | Natural logarithm of $x$ |
| $\log(x)$ | Logarithm base 10 of $x$ |
| $m$ | Index of the bin with the highest magnitude |
| $M_e$ | Mass of the Earth $\approx 5.9736 \times 10^{24}$ kg |
| $N$ | Number of samples |
| $\omega$ | Angular frequency of a sinusoidal signal $= 2\pi f$ |
| $\omega_e$ | Angular velocity of the Earth |
| $\omega_F$ | Satellite angular velocity in the ECEF frame |
| $\omega_I$ | Satellite angular velocity in the ECI frame |
| $\psi$ | Angular displacement of the satellite from the point of closest approach |
| $Q$ | Number of iterations |
| $Q(x)$ | The standard $Q$-function $= \int_x^\infty \Phi(t)dt$ |
| $q$ | Probability of an outlier |
| $\rho$ | Signal to Noise Ratio |
| $\rho$(dB) | Signal to Noise Ratio in decibels |
| $\mathbb{R}$ | The set of real numbers |
| $\Re(z)$ | Real part of $z$ |
| $\sigma$ | Standard deviation of a random variable |
| $s(k)$ | $k^{th}$ sample of sinusoidal signal |
| $S(n)$ | Fourier transform of $s(k)$ |

| | |
|---|---|
| $s(t)$ | Earth station to satellite slant range at time t |
| $\theta$ | Elevation |
| $\theta_v$ | Minimum elevation for visibility |
| $\mathbf{v}^T$ | Transpose of $\mathbf{v}$ |
| $T_s$ | Sampling time $= 1/f_s$ |
| $T_v$ | Satellite visibility duration |
| $w(k)$ | $k^{th}$ sample of additive white Gaussian noise |
| $W(n)$ | Fourier transform of $w(k)$ |
| $x^*$ | Complex Conjugate of $x$ |
| $x(k)$ | Signal plus noise $= s(k) + w(k)$ |
| $X(n)$ | Fourier transform of $x(k)$ |
| $Y(n)$ | Magnitude of $X(n)$ |
| N(0,1) | The standard normal distribution, $= \int_{-\infty}^{x} \Phi(x)dx$ |

# List of Abbreviations

| | |
|---|---|
| ACRB | Asymptotic Cramer Rao Bound |
| a.s. | Almost Surely |
| ARMA | Auto-Regressive Moving Average |
| AWGN | Additive White Gaussian Noise |
| Az | Azimuth |
| BPSK | Binary Phase Shift Keying |
| CLT | Central Limit Theorem |
| CRB | Cramer Rao Bound |
| CRCSS | Cooperative Research Centre for Satellite Systems |
| DFT | Discrete Fourier Transform |
| DSP | Digital Signal Processing |
| ECEF | Earth Centred Earth Fixed |
| ECI | Earth Centred Inertial |
| EKF | Extended Kalman Filter |
| El | Elevation |
| FAST | Frequency Assisted Spatial Tracking |
| FFCI | Fractional Fourier Coefficients Interpolation |
| FFT | Fast Fourier Transform |
| GPS | Geographical Positioning System |
| IEKF | Iterative Extended Kalman Filter |
| IFFCI | Iterative Fractional Fourier Coefficients Interpolation |
| i.i.d. | Independent and Identically Distributed |
| IMOI | Iterative Magnitudes Only Interpolation |
| IMSI | Iterative Magnitudes Squared Interpolation |

| | |
|---|---|
| LEO | Low Earth Orbit |
| LO | Local Oscillator |
| MBS | Maximum Bin Search |
| ML | Maximum Likelihood |
| MOI | Magnitudes Only Interpolation |
| MSI | Magnitudes Squared Interpolation |
| MMSI | Modified Magnitudes Squared Interpolation |
| MSE | Mean Squared Error |
| NORAD | North American Aerospace Command |
| pdf | Probability Density Function |
| PSK | Phase Shift Keying |
| QAM | Quadrature Amplitude Modulation |
| QPSK | Quadrature Phase Shift Keying |
| RMSE | Root Mean Squared Error |
| SNR | Signal to Noise Ratio |
| STK™ | Satellite Tool Kit |
| TLEs | Two Line Elements |
| TT&C | Tracking, Telemetry and Command |

# Abstract


Low Earth Orbit (LEO) satellites have received increased attention in recent years. They have been proposed as a viable solution for remote sensing, telemedicine, weather monitoring, search and rescue and communications to name a few applications. LEO satellites move with respect to an earth station. Thus, the station must be capable of tracking the satellite both spatially and in frequency. In addition, as the spectrum becomes more congested, links are being designed at higher frequencies such as Ka band. These frequencies experience larger attenuations and therefore the system must be capable of operating at low signal to noise ratios.

In this dissertation we report on the research conducted on the following problems. Firstly, we study the estimation of the frequency of a sinusoid for the purpose of acquiring and tracking the frequency of the received signal. Secondly, we propose the use of the frequency measurements to assist the spatial tracking of the satellite.

The highly dynamic environment of a LEO system, combined with the high Ka band frequencies result in large Doppler rates. This limits the available processing time and, consequently, the fundamental resolution of a frequency estimator. The frequency estimation strategy that is adopted in the thesis consists of a coarse estimator followed by a fine estimation stage. The coarse estimator is implemented using the maximum of the periodogram. The threshold effect is studied and the derivation of an approximate expression of the signal to noise ratio at which the threshold occurs is examined.

The maximum of the periodogram produces a frequency estimate with an accuracy that is $O\left(N^{-1}\right)$, where $N$ is the number of data samples used in the FFT. The lower bound for the estimation of the frequency of a sinusoid, given by the Cramer-Rao bound (CRB), is $O\left(N^{-\frac{3}{2}}\right)$. This motivates the use of a second stage in order to improve the estimation resolution. A family of new frequency estimation algorithms that interpolate on the fractional Fourier coefficients is proposed. The new estimators can be implemented iteratively to give a performance that is uniform in frequency. The iterative algorithms are analysed and their asymptotic properties derived. The

asymptotic variance of the iterative estimators is only 1.0147 times the asymptotic CRB.

Another method of refining the frequency estimate is the Dichotomous search of the periodogram peak. This is essentially a binary search algorithm. However, the estimator must be padded with zeroes in order to achieve a performance that is comparable to the CRB. An insight into this is offered and a modified form that does not require the zero-padding is proposed. The new algorithm is referred to as the modified dichotomous search. A new hybrid technique that combines the dichotomous search with an interpolation technique in order to improve its performance is also suggested.

The second research aim was to study the possibility of applying the frequency measurements to obtain spatial tracking information. This is called the frequency assisted spatial tracking (FAST) concept. A simple orbital model is presented and the resulting equations are used to show that the Doppler shift and rate uniquely specify the satellite's position for the purpose of antenna pointing. Assuming the maximum elevation of the pass is known, the FAST concept is implemented using a scalar Extended Kalman Filter (EKF). The EKF performance was simulated at a signal to noise ratio of 0dB. The off-boresight error was found better than 0.1º for elevations higher than 30º.