



Faculty of Engineering and Information Technology

# **Uncertainty Analysis and Optimization by Using the Orthogonal Polynomials**

A thesis submitted for the degree of

**Doctor of Philosophy**

**JINGLAI WU**

(2015)

**Title of the thesis:**

Uncertainty Analysis and Optimization by Using the Orthogonal Polynomials

**Ph.D. student:**

Jinglai Wu

E-mail: [jinglai.wu@student.uts.edu.au](mailto:jinglai.wu@student.uts.edu.au)

**Supervisor:**

Dr Zhen Luo

E-mail: [zhen.luo@uts.edu.au](mailto:zhen.luo@uts.edu.au)

**Co-Supervisor:**

Prof Nong Zhang

E-mail: [nong.zhang@uts.edu.au](mailto:nong.zhang@uts.edu.au)

**Address:**

School of Electrical, Mechanical and Mechatronic Systems

The University of Technology, Sydney, NSW 2007, Australia

# **Certificate of Original Authorship**

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition,

I certify that all information sources and literature used are indicated in the thesis.

Signature of Student:

**JINGLAI WU**

Date:

# Acknowledgments

I would like to take this opportunity to thank many people for their assistance, encouragement and support throughout my candidature.

First and foremost, I extend my deep gratitude to my principal supervisor Dr Zhen Luo. He supported me with outstanding guidance in research direction and academic writing and gave me many encouragements and other help in my study and life. His knowledge, hard-working and research attitude fostered my development in various aspects. I also sincerely thank my co-supervisor, Prof Nong Zhang, for his assistance, guidance and support over past few years. His extensive experience has been much appreciated through my research.

I would like to thank Dr Paul Walker and my colleagues Xingxing Zhou, Yu Wang, Lifu Wang, Sangzhi Zhu, Tianxiao Zhang, Guangzhong Xu, JiagengRuan, Yuhong Fang, Li Sun, Bo Zhu, who supported me in study or life. I also wish to acknowledge gratefully the consistent financial support of the following agents: China Scholarship Council (CSC) and University of Technology, Sydney.

Most especially to my family, words alone cannot express what I own them for their encouragement and whose patient love enabled me to complete this thesis.

Jinglai Wu

Sydney, August, 2015

# Publications and Conference

## Contributions

### International scientific journal publications

- [1] **Jinglai Wu**, Yunqing Zhang, Liping Chen, Zhen Luo, Interval method with Chebyshev series for dynamic response of nonlinear systems, Applied Mathematical Modelling, 37(4578-4591), 2013.(Chapter 3)
- [2] **Jinglai Wu**, Zhen Luo, Yunqing Zhang, Nong Zhang, Interval uncertain analysis for multi-body mechanical systems with Chebyshev inclusion functions, International Journal for Numerical Methods in Engineering, 95(608-630), 2013. (Chapter 3)
- [3] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new uncertain analysis method and its application in vehicle dynamics, Mechanical Systems and Signal Processing, 50-51 (659-675), 2015.(Chapter 4)
- [4] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new interval uncertain optimization method for structures using Chebyshev surrogate models, Computers and Structures, 146(185-196), 2015.(Chapter 5)
- [5] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, An interval uncertain optimization method for vehicle suspensions using Chebyshev metamodels, Applied Mathematical Modelling, 38(3706-3723), 2014. (Chapter 5)
- [6] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new sampling scheme for developing metamodels with the zeros of Chebyshev polynomials, Engineering

Optimization, 1-25, 2014. (Chapter 5)

- [7] **Jinglai Wu**, Zhen Luo, Nong Zhang, Incremental modelling for high-order polynomial surrogate models, Applied Mathematical Modelling, 2015, DOI: 10.1016/j.apm.2015.12.002.
- [8] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, Robust topology optimization for structures under interval uncertainty, Advances in Engineering Software, under review, 2015. (Chapter 5).
- [9] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new uncertain optimization method for structures using orthogonal series expansions, Computers & Structures, under review, 2015. (Chapter 6)

#### **International scientific conference publications**

- [10] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, Hybrid uncertainties optimization using the orthogonal polynomials expansion, the 8th Australian Congress on Applied Mechanics (ACAM 8), 23-26 November 2014, Melbourne, Australia.
- [11] **Jinglai Wu**, Zhen Luo, Nong Zhang, Structural optimization under a description of hybrid uncertainties, the 11th World Congress on Computational Mechanics (WCCM2014), July 20-25, 2014, Barcelona, Spain.
- [12] **Jinglai Wu**, Zhen Luo, Nong Zhang, Interval method for solving multi-body dynamics problems with uncertain parameters, Proceedings: the 5th Asia Pacific Congress on Computational Mechanics (APCOM2013) and 4th International

Symposium on Computational Mechanics (ISCM2013), 11-14 Dec 2013, Singapore.

- [13] **Jinglai Wu**, Zhen Luo, Nong Zhang, A new method for building high order polynomial regression model, Proceedings: the 1st Australasian Conference on Computational Mechanics (ACCM2013), eds. G.P. Steven, and Q. Li, 3-4 October, 2013, Sydney, Australia.
- [14] **Jinglai Wu**, Zhen Luo, Nong Zhang, Interval uncertain optimization of structures using Chebyshev meta-models, Proceedings: the 10th World Congress on Structural and Multidisciplinary Optimization (WCSMO10), May 19-24, 2013, Orlando, USA.
- [15] **Jinglai Wu**, Zhen Luo, Yunqing Zhang, Liping Chen, Interval analysis method for dynamics systems with uncertain parameters, Proceedings: 4th International Conference on Computational Methods, Paper ID: 057, 15-28 Nov 2012, Gold Coast, Australia.
- [16] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A Chebyshev meta-model for uncertain optimization of vehicle suspensions, Proceedings: 4th International Conference on Computational Methods, Paper ID: 061, 15-28 Nov 2012, Gold Coast, Australia.

# List of figures

Figure 2- 1(a) probabilistic variable (b) fuzzy variable (c) interval variable ..... 8

Figure 3-1 The computation of interval Cosine function..... 31

Figure 3-2 The interval inclusion function from  $R^2$  to  $R^2$ ..... 32

Figure 3-3 Overestimation of interval arithmetic ..... 33

Figure 3-4 Three natural inclusion function for the same function ..... 35

Figure 3-5 Errors of  $\arctan x$  for Chebyshev series and Taylor series ..... 45

Figure 3-6 The plot of Chebyshev polynomials ..... 48

Figure 3-7 (a) The plot of  $y_1$ ; (b) The plot of  $y_2$ ..... 53

Figure 3-8 The flow chart of Chebyshev inclusion function solving interval ODEs ..... 57

Figure 3-9 Schematic of a double pendulum ..... 58

Figure 3-10 The angle of pendulum: (a) top pendulum; (b) bottom pendulum..... 59

Figure 3-11 The calculation flow for uncertain multi-body system ..... 67

Figure 3-12 Schematic of slider crank ..... 68

Figure 3-13 (a) Rotation angle of crank with uncertain length of the crank; (b) Rotation angle of connecting rod with uncertain length of the crank; (c) Displacement of piston with uncertain length of the crank ..... 70

Figure 3-14 (a) Rotation angle of crank with uncertain torque; (b) Rotation angle of connecting rod with uncertain torque; (c) Displacement of the piston with uncertain torque ..... 72

Figure 4-1 The 2 DOF Bicycle model for vehicle ..... 78

Figure 4-2 Steering angle step input ..... 80

Figure 4-3 Mean value for yaw velocity ..... 81

Figure 4-4 Mean value for lateral acceleration ..... 81

Figure 4-5 Standard deviation for yaw velocity ..... 82

Figure 4-6 Standard deviation for lateral acceleration..... 82

Figure 4-7 The flowchart of the PCCI method ..... 92

Figure 4-8 The roll plan model of a vehicle..... 95

Figure 4-9 The road input ..... 97

Figure 4-10 IM of  $z_1$  ..... 98

Figure 4-11 IM of  $z_2$  ..... 98

Figure 4-12 IV of  $z_1$  ..... 99

Figure 4-13 IV of  $z_2$  ..... 99

Figure 4-14 MLB and MUB of  $z_1$  ..... 100

Figure 4-15 MLB and MUB of  $z_2$  ..... 100

Figure 4-16 VLB of  $z_1$  ..... 101

Figure 4-17 VUB of  $z_1$  ..... 101

Figure 4-18 VLB of  $z_2$ ..... 101



Figure 4-19 VUB of z2 .....	102
Figure 5-1 Interval objective function .....	106
Figure 5-2 Interval constraints .....	106
Figure 5-3 Difference of the deterministic optimization and uncertain optimization .....	108
Figure 5-4 Flowchart of MIGA .....	109
Figure 5-5 Flowchart of optimization under interval uncertainty.....	112
Figure 5-6 Flowchart of Chebyshev surrogate model .....	117
Figure 5-7 The flowchart of interval optimization .....	118
Figure 5-8 18-bar planar truss structure.....	119
Figure 5-9 The model of double wishbone suspension .....	122
Figure 5-10 Camber angle, caster angle, kingpin inclination angle, and toe angle .....	122
Figure 5-11 Camber angle.....	123
Figure 5-12 Caster angle.....	123
Figure 5-13 Kingpin inclination angle.....	124
Figure 5-14 Toe angle .....	124
Figure 5-15 The average objective. ....	126
Figure 5-16 The best objective .....	127
Figure 5-17 Comparison of camber angle .....	128
Figure 5-18 Comparison of caster angle.....	129
Figure 5-19 Comparison of kingpin inclination angle.....	129
Figure 5-20 Comparison of toe angle .....	129
Figure 5-21 The plot of $\text{sign}(x)$ and $\tanh(\beta x)$ .....	137
Figure 5-22 Mitchell-type structure design .....	140
Figure 5-23 Deterministic topology optimization .....	140
Figure 5-24 The RTO of Mitchell-type structure.....	141
Figure 5-25 The iteration history of the RTO for Mitchell beam .....	142
Figure 6-1 (a) The PDF of two bounds; (b) The CDF of two bounds .....	152
Figure 6-2 The CDF of two bounds for pure interval function.....	153
Figure 6-3 The PDF and CDF of $F(\xi, \eta)$ for different types of random variable .....	154
Figure 6-4 The iteration history of the objective function .....	159
Figure 6-5 PDF of f.....	160
Figure 6-6 CDF of f .....	160
Figure 6-7 PDF of constraint g1 .....	161
Figure 6-8 CDF of constraint g1 .....	161
Figure 6-9 CDF of constraint g2.....	162
Figure 6-10 25-bar space truss structure.....	163
Figure 6-11 The iteration history of objective function .....	166
Figure 6-12 PDF of objective $f$ .....	166
Figure 6-13 CDF of objective $f$ .....	167
Figure 6-14 CDF of constraint $g_6$ .....	167

Figure 6-15 PDF of constraint  $g_8$  ..... 168  
Figure 6-16 CDF of objective  $g_8$ ..... 168

## List of tables

Table 3-1 Comparing inclusion functions..... 39  
Table 3-2 Parameters of slider crank mechanisms..... 68

Table 4- 1 Parameters for a car ..... 80  
Table 4-2 Parameters of the roll plan model..... 96

Table 5-1 The optimization results of 18-bar planar truss ..... 121  
Table 5-2 Optimization results of vehicle suspension ..... 128  
Table 5-3 The compliance of Mitchell-type under the worst condition..... 141

Table 6-1 Optimization results of planar truss structure..... 159  
Table 6-2 Loading conditions ..... 163  
Table 6-3 Member stress limitations..... 164  
Table 6-4 Optimization results..... 165

# Abbreviations

PC	Polynomial Chaos
SRSM	Stochastic Response Surface Method
ODEs	Ordinary Differential Equations
DAEs	Differential Algebraic Equations
K-L	Karhunen-Loeve
RBDO	Reliability-based Design Optimization
RDO	Robust Design Optimization
LSM	Least Square Method
PCCI	Polynomial-Chaos-Chebyshev-Interval
IM	Interval Mean
IV	Interval Variance
MLB	Mean of Lower Bound
MUB	Mean of Upper Bound
VLB	Variance of Lower Bound
VUB	Variance of Upper Bound
DOF	Degree of Freedom
ASO	Active Set Optimization
MIGA	Multi-Island Genetic Algorithm
SQP	Sequential Quadratic Programming
RTO	Robust Topology Optimization
MMA	Method of Moving Asymptotes
PDF	Probability Density Function
CDF	Cumulative Distribution Function
MCST	Monte Carlo Scanning Test

# Abstract

Engineering problems are generally described by mathematic models, and the parameters in mathematic models are usually assumed to be deterministic when solving these models. However, many parameters are hard to obtain accurately in practical application, which leads to the uncertainty of parameters. The uncertain parameters may induce the response of theoretical analysis that is quite different from the actual instance. In order to characterize the response of system more accurately, the uncertainty analysis methods need to be introduced. For the design optimization, considering the uncertainty may help to improve the reliability and robustness of design solution. This thesis investigates both the aleatory (random) uncertainty and epistemic uncertainty (expressed by interval variables in the thesis), by using the Polynomial Chaos (PC) expansion theory and Chebyshev polynomials approximation theory, respectively. Since there are many cases that both types of uncertainty are existed simultaneously, the hybrid uncertainty is also investigated in this thesis. A new hybrid uncertainty analysis method based on the orthogonal series expansion is proposed in this study, which solves the two types of uncertainty in one integral framework. The design optimization under uncertainty is also investigated based on the proposed uncertainty analysis method. The detailed content of this thesis is shown as follows.

The interval uncertainty analysis theory is firstly studied in this thesis. By using the Chebyshev polynomials that have high accuracy in the approximation theory of polynomials, a new Chebyshev inclusion function based on the Chebyshev series expansion is proposed. The Chebyshev inclusion function can compress the wrapping effect of interval arithmetic more efficiently than the traditional Taylor inclusion function, especially for the interval computation of non-monotonic functions. On the other hand,

the Chebyshev inclusion function does not require the derivatives information which has to be given in the computation of Taylor inclusion function. Therefore, the proposed Chebyshev inclusion function is quite easier to implement than the Taylor inclusion function. The Chebyshev inclusion function is applied to solve the ordinary differential equations (ODEs) and differential algebraic equations (DAEs) with interval parameters, which are used to solve the mechanical dynamic systems with interval parameters.

Secondly, the random uncertainty analysis based on the PC expansion is investigated, where the polynomials series are used to approximate the response of a system with respect to the random variables. The hybrid uncertainty analysis method using the orthogonal series expansion is proposed, termed as Polynomial-Chaos-Chebyshev-Interval (PCCI) method, which is the combination of PC expansion method and Chebyshev interval method. Since both the polynomials used in PC expansion and Chebyshev polynomials belong to the orthogonal polynomials, the PCCI method investigates the random uncertainty and interval uncertainty under one integral framework. Two types of evaluation index of hybrid uncertainty are also proposed in the PCCI method, which is then used in the analysis of vehicle dynamics containing hybrid uncertainty.

Thirdly, considering the interval uncertain parameters or variables existed in the optimization problems, the interval optimization design is investigated. To improve the computational efficiency of traditional nested optimization procedure in uncertainty optimization, the interval arithmetic is employed to delete its inner loop optimization. A new Chebyshev polynomials-based surrogate model is proposed to improve the computational efficiency in further. The numerical examples for the vehicle suspension design and truss structure design indicate that the interval optimization method has a good balance between the accuracy and efficiency. The interval optimization method is also

employed to solve the continuous structural topology optimization problem with uncertain load conditions, which gives a more robust solution than the traditional deterministic topology optimization method.

Lastly, the hybrid uncertainty optimization model is proposed by combining the PCCI method and the classical optimization algorithms. To use the traditional mathematical programming method, the sensitivity of objectives and constraints with uncertain parameters are derived. For the application, the proposed hybrid uncertainty optimization method is used in the optimization of a planar truss and a space truss structures. Compared with the deterministic optimization and pure random uncertainty optimization, the hybrid uncertainty optimization provides a more feasible solution.

**Key words:** interval uncertainty; hybrid uncertainty; Chebyshev polynomials; Polynomial Chaos expansion; orthogonal polynomials.

# Contents

<b>CERTIFICATE OF ORIGINAL AUTHORSHIP .....</b>	<b>I</b>
<b>ACKNOWLEDGMENTS .....</b>	<b>II</b>
<b>PUBLICATIONS AND CONFERENCE CONTRIBUTIONS .....</b>	<b>III</b>
<b>LIST OF FIGURES .....</b>	<b>VI</b>
<b>LIST OF TABLES.....</b>	<b>VIII</b>
<b>ABBREVIATIONS .....</b>	<b>IX</b>
<b>ABSTRACT.....</b>	<b>X</b>
<b>CHAPTER 1 INTRODUCTION .....</b>	<b>1</b>
1.1 OVERVIEW OF THE PROJECT .....	1
1.2 RESEARCH OBJECTIVE AND CONTRIBUTION TO KNOWLEDGE .....	4
1.3 OUTLINE OF THE THESIS .....	5
<b>CHAPTER 2 BACKGROUND AND LITERATURE REVIEW .....</b>	<b>7</b>
2.1 PROBABILISTIC UNCERTAINTY ANALYSIS .....	9
2.2 INTERVAL UNCERTAINTY ANALYSIS .....	14
2.2.1 Interval arithmetic solving the static problems.....	16
2.2.2 Interval arithmetic solving the dynamic problems .....	19
2.3 UNCERTAINTY OPTIMIZATION .....	23
<b>CHAPTER 3 INTERVAL UNCERTAINTY ANALYSIS BASED ON CHEBYSHEV SERIES .....</b>	<b>29</b>
3.1 INTERVAL ARITHMETIC.....	29
3.1.1 Basic theory of interval arithmetic .....	29
3.1.2 Interval inclusion function.....	31
3.2 THE CHEBYSHEV INCLUSION FUNCTION .....	40
3.2.1 Chebyshev polynomial approximation theory.....	40
3.2.2 Chebyshev inclusion function .....	46
3.3 THE ALGORITHM FOR SOLVING INTERVAL ODES.....	49
3.3.1 The interval ODEs solved by Taylor inclusion function .....	50
3.3.2 The interval ODEs solved by Chebyshev inclusion function.....	54
3.3.3 The numerical example .....	57
3.4 CHEBYSHEV INCLUSION FUNCTION SOLVING MULTIBODY SYSTEM.....	60
3.4.1 The Modeling for multibody dynamic system .....	60
3.4.2 The interval DAEs solved by Chebyshev inclusion function.....	63

3.4.3 The numerical example .....	68
3.5 SUMMARY .....	72
<b>CHAPTER 4 HYBRID UNCERTAINTY ANALYSIS USING ORTHOGONAL SERIES EXPANSION .....</b>	<b>74</b>
4.1 THE POLYNOMIAL CHAOS EXPANSION THEORY .....	74
4.1.1 The generalized Polynomial Chaos expansion theory .....	74
4.1.2 The stochastic response surface method .....	77
4.1.3 Numerical example .....	78
4.2 THE HYBRID UNCERTAIN ANALYSIS METHOD .....	82
4.2.1 The Chebyshev interval analysis using LSM .....	82
4.2.2 The statistical evaluation based on interval variables .....	86
4.2.3 The bounds evaluation based on random variables .....	89
4.3 APPLICATION OF HYBRID UNCERTAIN ANALYSIS IN VEHICLE DYNAMICS .....	94
4.3.1 Model description .....	94
4.3.2 Hybrid uncertain analysis of vehicle model .....	96
4.4 SUMMARY .....	102
<b>CHAPTER 5 OPTIMIZATION UNDER INTERVAL UNCERTAINTY .....</b>	<b>104</b>
5.1 THE INTERVAL UNCERTAIN OPTIMIZATION MODEL .....	104
5.2 NESTED OPTIMIZATION METHOD .....	108
5.3 THE INTERVAL OPTIMIZATION METHOD .....	112
5.3.1 Chebyshev surrogate model .....	112
5.3.2 Optimization algorithm using interval arithmetic .....	117
5.4 APPLICATION OF INTERVAL OPTIMIZATION .....	119
5.4.1 The 18-bar planar truss optimization .....	119
5.4.2 Vehicle suspension optimization .....	121
5.5 ROBUST TOPOLOGY OPTIMIZATION UNDER INTERVAL UNCERTAINTY .....	130
5.5.1 Material Density based approach for topology optimization .....	133
5.5.2 Robust topology optimization under interval uncertainty .....	134
5.5.3 Numerical implementation of RTO using interval arithmetic .....	136
5.5.4 Numerical example .....	140
5.6 SUMMARY .....	143
<b>CHAPTER 6 OPTIMIZATION UNDER HYBRID UNCERTAINTY .....</b>	<b>144</b>
6.1 THE HYBRID UNCERTAIN OPTIMIZATION MODEL .....	145
6.2 THE REALIZATION OF HYBRID UNCERTAIN OPTIMIZATION .....	147



6.2.1 Hybrid uncertainty analysis model .....	147
6.2.2 Quantification of hybrid uncertainty .....	150
6.3 THE SENSITIVITY OF HYBRID UNCERTAIN OPTIMIZATION MODEL .....	154
6.4 NUMERICAL EXAMPLES .....	157
6.4.1 Planar truss structure .....	157
6.4.2 Space truss structure .....	162
6.5 SUMMARY .....	169
<b>CHAPTER 7 SUMMARY AND PROSPECT .....</b>	<b>170</b>
7.1 SUMMARY .....	170
7.2 PERSPECTIVE FOR FUTURE WORK .....	172
<b>REFERENCES.....</b>	<b>174</b>

