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Uncertainty Analysis and Optimization by Using the Orthogonal Polynomials

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Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Publications and Conference Contributions

International scientific journal publications

[1] **Jinglai Wu**, Yunqing Zhang, Liping Chen, Zhen Luo, Interval method with Chebyshev series for dynamic response of nonlinear systems, Applied Mathematical Modelling, 37(4578-4591), 2013.(Chapter 3)

[2] **Jinglai Wu**, Zhen Luo, Yunqing Zhang, Nong Zhang, Interval uncertain analysis for multi-body mechanical systems with Chebyshev inclusion functions, International Journal for Numerical Methods in Engineering, 95(608-630), 2013.(Chapter 3)

[3] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new uncertain analysis method and its application in vehicle dynamics, Mechanical Systems and Signal Processing, 50-51 (659-675), 2015.(Chapter 4)

[4] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new interval uncertain optimization method for structures using Chebyshev surrogate models, Computers and Structures, 146(185-196), 2015.(Chapter 5)

[5] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, An interval uncertain optimization method for vehicle suspensions using Chebyshev metamodels, Applied Mathematical Modelling, 38(3706-3723), 2014. (Chapter 5)

[6] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new sampling scheme for developing metamodels with the zeros of Chebyshev polynomials, Engineering

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[7] **Jinglai Wu**, Zhen Luo, Nong Zhang, Incremental modelling for high-order polynomial surrogate models, Applied Mathematical Modelling, 2015, DOI: 10.1016/j.apm.2015.12.002.

[8] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, Robust topology optimization for structures under interval uncertainty, Advances in Engineering Software, under review, 2015. (Chapter 5).

[9] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A new uncertain optimization method for structures using orthogonal series expansions, Computers & Structures, under review, 2015. (Chapter 6)

International scientific conference publications

[10] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, Hybrid uncertainties optimization using the orthogonal polynomials expansion, the 8th Australian Congress on Applied Mechanics (ACAM 8), 23-26 November 2014, Melbourne, Australia.

[11] **Jinglai Wu**, Zhen Luo, Nong Zhang, Structural optimization under a description of hybrid uncertainties, the 11th World Congress on Computational Mechanics (WCCM2014), July 20-25, 2014, Barcelona, Spain.

[12] **Jinglai Wu**, Zhen Luo, Nong Zhang, Interval method for solving multi-body dynamics problems with uncertain parameters, Proceedings: the 5th Asia Pacific Congress on Computational Mechanics (APCOM2013) and 4th International

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[15] **Jinglai Wu**, Zhen Luo, Yunqing Zhang, Liping Chen, Interval analysis method for dynamics systems with uncertain parameters, Proceedings: 4th International Conference on Computational Methods, Paper ID: 057, 15-28 Nov 2012, Gold Coast, Australia.

[16] **Jinglai Wu**, Zhen Luo, Nong Zhang, Yunqing Zhang, A Chebyshev meta-model for uncertain optimization of vehicle suspensions, Proceedings: 4th International Conference on Computational Methods, Paper ID: 061, 15-28 Nov 2012, Gold Coast, Australia.

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Abbreviations

PC	Polynomial Chaos
SRSM	Stochastic Response Surface Method
ODEs	Ordinary Differential Equations
DAEs	Differential Algebraic Equations
K-L	Karhunen-Loeve
RBDO	Reliability-based Design Optimization
RDO	Robust Design Optimization
LSM	Least Square Method
PCCI	Polynomial-Chaos-Chebyshev-Interval
IM	Interval Mean
IV	Interval Variance
MLB	Mean of Lower Bound
MUB	Mean of Upper Bound
VLB	Variance of Lower Bound
VUB	Variance of Upper Bound
DOF	Degree of Freedom
ASO	Active Set Optimization
MIGA	Multi-Island Genetic Algorithm
SQP	Sequential Quadratic Programming
RTO	Robust Topology Optimization
MMA	Method of Moving Asymptotes
PDF	Probability Density Function
CDF	Cumulative Distribution Function
MCST	Monte Carlo Scanning Test

Abstract

Engineering problems are generally described by mathematic models, and the parameters in mathematic models are usually assumed to be deterministic when solving these models. However, many parameters are hard to obtain accurately in practical application, which leads to the uncertainty of parameters. The uncertain parameters may induce the response of theoretical analysis that is quite different from the actual instance. In order to characterize the response of system more accurately, the uncertainty analysis methods need to be introduced. For the design optimization, considering the uncertainty may help to improve the reliability and robustness of design solution. This thesis investigates both the aleatory (random) uncertainty and epistemic uncertainty (expressed by interval variables in the thesis), by using the Polynomial Chaos (PC) expansion theory and Chebyshev polynomials approximation theory, respectively. Since there are many cases that both types of uncertainty are existed simultaneously, the hybrid uncertainty is also investigated in this thesis. A new hybrid uncertainty analysis method based on the orthogonal series expansion is proposed in this study, which solves the two types of uncertainty in one integral framework. The design optimization under uncertainty is also investigated based on the proposed uncertainty analysis method. The detailed content of this thesis is shown as follows.

The interval uncertainty analysis theory is firstly studied in this thesis. By using the Chebyshev polynomials that have high accuracy in the approximation theory of polynomials, a new Chebyshev inclusion function based on the Chebyshev series expansion is proposed. The Chebyshev inclusion function can compress the wrapping effect of interval arithmetic more efficiently than the traditional Taylor inclusion function, especially for the interval computation of non-monotonic functions. On the other hand,

the Chebyshev inclusion function does not require the derivatives information which has to be given in the computation of Taylor inclusion function. Therefore, the proposed Chebyshev inclusion function is quite easier to implement than the Taylor inclusion function. The Chebyshev inclusion function is applied to solve the ordinary differential equations (ODEs) and differential algebraic equations (DAEs) with interval parameters, which are used to solve the mechanical dynamic systems with interval parameters.

Secondly, the random uncertainty analysis based on the PC expansion is investigated, where the polynomials series are used to approximate the response of a system with respect to the random variables. The hybrid uncertainty analysis method using the orthogonal series expansion is proposed, termed as Polynomial-Chaos-Chebyshev-Interval (PCCI) method, which is the combination of PC expansion method and Chebyshev interval method. Since both the polynomials used in PC expansion and Chebyshev polynomials belong to the orthogonal polynomials, the PCCI method investigates the random uncertainty and interval uncertainty under one integral framework. Two types of evaluation index of hybrid uncertainty are also proposed in the PCCI method, which is then used in the analysis of vehicle dynamics containing hybrid uncertainty.

Thirdly, considering the interval uncertain parameters or variables existed in the optimization problems, the interval optimization design is investigated. To improve the computational efficiency of traditional nested optimization procedure in uncertainty optimization, the interval arithmetic is employed to delete its inner loop optimization. A new Chebyshev polynomials-based surrogate model is proposed to improve the computational efficiency in further. The numerical examples for the vehicle suspension design and truss structure design indicate that the interval optimization method has a good balance between the accuracy and efficiency. The interval optimization method is also

employed to solve the continuous structural topology optimization problem with uncertain load conditions, which gives a more robust solution than the traditional deterministic topology optimization method.

Lastly, the hybrid uncertainty optimization model is proposed by combining the PCCI method and the classical optimization algorithms. To use the traditional mathematical programming method, the sensitivity of objectives and constraints with uncertain parameters are derived. For the application, the proposed hybrid uncertainty optimization method is used in the optimization of a planar truss and a space truss structures. Compared with the deterministic optimization and pure random uncertainty optimization, the hybrid uncertainty optimization provides a more feasible solution.

Key words: interval uncertainty; hybrid uncertainty; Chebyshev polynomials; Polynomial Chaos expansion; orthogonal polynomials.

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