

THE UNIVERSITY OF  
TECHNOLOGY, SYDNEY



Faculty of Engineering

**An Investigation of the Dynamic Characteristics of  
Hydraulic Power Steering Systems**

A thesis submitted for the degree of

**Doctor of Philosophy  
in Engineering**

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## CERTIFICATE OF AUTHORSHIP / ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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## **Abstract**

The dynamic characteristics of the rack and pinion hydraulic power steering systems have been investigated in the time domain and the frequency domain by using modelling, simulation and testing methods. A model of the hydraulic power steering system and the numerical solution scheme were developed in order to obtain the time domain responses of the system. For the frequency domain analysis, a state space representation of the system incorporating the dynamics of the mechanical and hydraulic subsystems was proposed and the system coefficient matrices were derived. A root searching method was developed to determine the natural frequencies and corresponding modes of the steering system. Applications to a typical hydraulic power steering system were conducted to validate the developed models and computational schemes. The results obtained from the frequency domain analysis agreed well with those obtained from the transient analysis. The examples illustrated the dynamic coupling between the mechanical and hydraulic subsystems. In addition, the dynamic characteristics of the variable ratio rack and pinion steering system and the speed sensitive steering system were investigated.

A hydraulic power steering system test rig, which provided an approximately realistic working environment for the hydraulic steering system, was designed and built to validate the mathematical models. A series of experiments including the impact testing on the front wheel, the steering shudder testing and the pressure ripple testing were carried out. The experimental results confirmed the presented modelling and simulation analysis.

The models and test rig may assist automotive engineers in performing theoretical noise and vibration analysis of the steering system for optimising its performance. The modelling methods and numerical solution scheme which reveal the mechanical and hydraulic coupling action can be applied to the dynamics study of other complex mechanical/hydraulic systems.

