Animal Spirits and Financial Instability - a disequilibrium macroeconomic perspective

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A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy in the Finance Discipline Group, School of Business

April 2016
Declaration of Authorship

I, Tianhao Zhi, declare that this thesis titled, 'Animal Spirits and Financial Instability - a disequilibrium macroeconomic perspective' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.

- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.

- Where I have consulted the published work of others, this is always clearly attributed.

- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

- I have acknowledged all main sources of help.

- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

Date:
“Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.”

- John Maynard Keynes

“There are two basic motivating forces: fear and love. When we are afraid, we pull back from life. When we are in love, we open to all that life has to offer with passion, excitement, and acceptance.”

- John Lennon

“Stability leads to instability. The more stable things become and the longer things are stable, the more unstable they will be when the crisis hits.”

- Hyman Minsky
Abstract

Finance Discipline Group
School of Business

Doctor of Philosophy

Animal Spirits and Financial Instability - a disequilibrium macroeconomic perspective

by Tianhao Zhi
This thesis aims to develop a set of dynamic models that (i) systematically investigate the fiscal/credit-driven monetary dynamics under an alternative quantitative framework and (ii) analyze how bounded rationality of credit-creating financial institutions induces credit cycles and propagates macroeconomic instability.

Chapter 2 surveys the literature of Keynesian disequilibrium macroeconomics by examining the relationship between the origin of disequilibrium macroeconomic thinking and the development of disequilibrium macroeconomic models. Keynes alludes to the idea of disequilibrium macroeconomics in his *General Theory*. It has inspired the ongoing development of disequilibrium macroeconomic literature that strives to formalize Keynes’s disequilibrium thinking with the use of advanced tools in nonlinear dynamic systems. We discuss two particular strands of modelling approaches: the Keynes-Metzler-Goodwin approach and the Weidlich-Haag-Lux approach.

Chapter 3 takes a critical look at the “Modern Money Theory” (a.k.a Neo-Chartalism) from a dynamic perspective. In this chapter, we propose a set of dynamic models that aim to investigate the monetary dynamics as well as its real and inflationary consequences. We contend that some of the MMT’s claims are questionable due to a general lack of formal quantitative analysis that justifies and measures its policy effects and consequences, especially the long run inflationary consequence of the fiscal/credit-driven monetary expansion. This chapter also provides a modelling framework that is further adopted in the subsequent two chapters of the thesis.

Chapter 4-5 systematically construct the CDGZ model that aims to examine (i) how bounded rationality and heterogeneity in the banking sector induces credit cycles due to “animal spirits”-driven bounded rationality of financial institutions and how it propagates macroeconomic instability and (ii) the dynamics of interbank market over the course of credit cycles. The idea of “animal spirits” has been widely treated in the literature with particular reference to investment in the productive sector. Chapter 4 takes a different view and analyses from a theoretical perspective the role of banks’ collective behaviour in the creation of credit that, ultimately, drives the credit cycles. We further investigate the mechanisms of interbank market in chapter 5. The policy implications regarding UMP as well as Tobin-type tax are thoroughly discussed. The analytical as well as numerical simulations of the final 8D CDGZ model may potentially be useful in providing relevant policy recommendations.
Acknowledgements

This thesis sums up my academic and life’s journey in the last four years. While studying and writing about the manias, panics and crashes of the world economy, I have simultaneously experienced a boom-bust cycle in my own personal life. Here I would like to express my sincere gratitude to a number of people without whom this tumultuous journey would never reach its final destination.

My boom period had started from the initial two and a half years since the commencement of my PhD journey in 2012, when the U.S. and Europe were busy dealing with their mess in the financial system and Hollywood had just dramatized the end of the world earlier on. I developed a keen interest in Agent-Based Model and Keynesian economics while teaching at University of Queensland. I was lucky enough to meet my former principle supervisor Prof. Carl Chiarella who offered me the opportunity to do my PhD here at UTS. He gives me not only an excellent academic guidance, but also supports me to travel to various international conferences and present my work. Working under his supervision I enjoyed the freedom to pursue my own research interest. He is a great mentor to me and gives me pivotal guidance throughout my graduate studies. I am also grateful for the academic support from my co-supervisor Tony Xue-Zhong He. I am thankful to UTS Graduate Research School (GRS) and Business Faculty for the generous financial support in the first three and a half years, which covers not only my tuition fee and living stipend during this period, but also allows me to travel to various international conferences through numerous travel grants. I am grateful for Ms Stephanie Ough for her wonderful job in helping me with the administrative work. I am thankful to the presence and companion of my fellow PhD students in our mutual journey. I am grateful for my former mentor Dr. Frederique Bracoud at University of Queensland who had ignited my initial interest into the field of banking theory and financial economics, which leads to the ongoing development of this thesis.

My bust period had started from the late 2014. While the world economy was still in depression and Greece was on the brink of a colossal economic collapse, I started to experience an onset of severe depression due to unforeseen life’s circumstances. For a period of time I was unable to fulfill my research work and every simple task of daily life seemed to be a great struggle. My future was clouded with dreadful instability and radical uncertainty. Yet, during this very dark period of my life when I nearly decided to give up my PhD (and almost everything), I received so much help from UTS counselling service, RPA hospital, as well as members of Central Baptist Church in my battle against depression. I have eventually triumphed over the demon and regained my unconditional love for life. I would like to mention in particular Susan Colbert, Matthew Abdelahad, and Nathaniel Kong from CBC for their love and kindness. I am grateful for many
alternative therapists whom I have met in Sydney. I am grateful for my friend Sara
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encountered at my local pub Friend-In-Hand\textsuperscript{1} in Glebe who cared about me during my
emotional struggle. I am grateful for the ongoing financial and emotional support from
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work by Heath Henn. I am greatly inspired by my friend Wanting Xiong whom I met in
2014 INET plenary conference in Toronto, Canada. Her passion in economics and her
establishment of New Economic Reading Group has not only helped many like-minded
students to study economics from an alternative perspective, but also revived my interest
in pursuing my study through participation of reading groups. Most importantly, it is
the immense patience and guidance of my current principal supervisor Dr. Corrado Di
Guilmi who gives me tremendous courage to finish the last part of my thesis. He is
truly a wonderful mentor and it is a great honour to complete this journey through his
guidance. To him I dedicate this thesis.

I have learnt the insight of Keynes' idea through my academic studies as well as through
my own life's struggle, which is reminiscent of what Keynes elaborates in his \textit{General
Theory}: "most, probably, of our decisions to do something positive, the full consequences
of which will be drawn out over many days to come, can only be taken as the result of
animal spirits". I have come to the realization through reading Hyman Minsky as well
as through my own relationship struggles that stability is intrinsically unstable. It is
vital to take unconventional action or policy intervention during unconventional times.
However, it will not work out if the economy is structurally broken, and the "animal
spirits" of individuals is not revived to a state of optimism from pessimism. The looming
instability and radical uncertainty still prevails in our daily life. Yet a state of optimism
and love is the only light that guides us through the dark tunnel of financial instability
and radical uncertainty.

\textsuperscript{1}The acronym of this pub has always reminded me of one major theme in my thesis, i.e. Minsky's
Financial Instability Hypothesis.
Contents

Declaration of Authorship i

Abstract iii

Acknowledgements v

Contents vii

List of Figures x

List of Tables xii

Abbreviations xiii

1 Introduction 1
  1.1 Background .................................................. 1
  1.1.1 Fluctuations or cycles? - a perennial debate ............... 1
  1.1.2 The “Great Moderation” .................................... 2
  1.1.3 The 2007-2008 Global Financial Crisis ................... 3
  1.2 Aim and structure of the thesis .......................... 4

2 The theory and models of Keynesian disequilibrium macroeconomics 7
  2.1 Introduction .................................................. 7
  2.2 The origin of disequilibrium macroeconomic thinking .......... 10
    2.2.1 The “disequilibrium” thinking of John Maynard Keynes ... 10
    2.2.2 Minsky’s interpretation of Keynes ........................ 12
  2.3 Models of Keynesian disequilibrium macroeconomics ........... 14
    2.3.1 The Keynes-Metzler-Goodwin model ....................... 14
    2.3.2 Modelling the dynamics of “animal spirits” ................ 17
      2.3.2.1 Modelling speculative behaviour: the “fundamentalist- 
               chartist” approach ........................................ 17
      2.3.2.2 Modelling herd behaviour and “animal spirits”: the Weidlich- 
               Haag-Lux approach ....................................... 18
  2.4 Conclusion .................................................. 21
### 3 The dynamics of endogenous money, banking, and public finance: a critical look at Modern Money Theory

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Introduction</td>
<td>22</td>
</tr>
<tr>
<td>3.2 The process of endogenous money creation and interbank settlement</td>
<td>25</td>
</tr>
<tr>
<td>3.3 The disequilibrium dynamics of Open Market Operation (OMO)</td>
<td>28</td>
</tr>
<tr>
<td>3.4 The monetary effect of fiscal policy</td>
<td>30</td>
</tr>
<tr>
<td>3.5 The long-run dynamics of monetary growth in a fiat money system</td>
<td>32</td>
</tr>
<tr>
<td>3.5.1 Genesis: from a special case to a general case</td>
<td>32</td>
</tr>
<tr>
<td>3.5.1.1 A digression on the hedging assumption: a new perspective on the government’s budget constraint</td>
<td>34</td>
</tr>
<tr>
<td>3.5.2 The time-dependent cyclical dynamics of $\Lambda(\cdot)$: a special case</td>
<td>39</td>
</tr>
<tr>
<td>3.5.3 The endogenous dynamics of monetary growth: a generalized case</td>
<td>42</td>
</tr>
<tr>
<td>3.6 The long-run effect of monetary expansion: speculative vs. fiscal inflation</td>
<td>43</td>
</tr>
<tr>
<td>3.7 Conclusion</td>
<td>51</td>
</tr>
</tbody>
</table>

### 4 Modelling the “animal spirits” of bank’s lending behaviour

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Introduction</td>
<td>54</td>
</tr>
<tr>
<td>4.2 The 2D CDGZ Model</td>
<td>58</td>
</tr>
<tr>
<td>4.2.1 The basic set-up</td>
<td>59</td>
</tr>
<tr>
<td>4.2.2 Opinion dynamics</td>
<td>60</td>
</tr>
<tr>
<td>4.2.3 Analysis of the two-dimensional system</td>
<td>62</td>
</tr>
<tr>
<td>4.3 The 3D CDGZ model: herding amongst optimistic banks</td>
<td>63</td>
</tr>
<tr>
<td>4.3.1 Sensitivity and bifurcation analysis of the behavioural parameters</td>
<td>68</td>
</tr>
<tr>
<td>4.4 The 4D CDGZ model: the convergence and divergence of heterogeneous lending strategies</td>
<td>71</td>
</tr>
<tr>
<td>4.5 The Kaldorian Investment-Saving disequilibrium dynamics with credit-driven investment sector</td>
<td>75</td>
</tr>
<tr>
<td>4.5.1 The credit-driven Kaldorian investment function</td>
<td>76</td>
</tr>
<tr>
<td>4.5.2 The 4D credit-driven investment sector with Kaldorian I-S disequilibrium</td>
<td>79</td>
</tr>
<tr>
<td>4.6 The 7D CDGZ model: the interaction between banks and a speculative financial sector</td>
<td>80</td>
</tr>
<tr>
<td>4.7 Conclusion</td>
<td>84</td>
</tr>
</tbody>
</table>

### 5 Credit bubble, “monetary famine”, and stock market crash: a disequilibrium dynamic model of interbank market

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Introduction</td>
<td>86</td>
</tr>
<tr>
<td>5.2 The baseline model</td>
<td>90</td>
</tr>
<tr>
<td>5.2.1 The baseline 4D CDGZ model</td>
<td>90</td>
</tr>
<tr>
<td>5.2.2 The loan-to-reserve ratio after interbank borrowing/lending</td>
<td>91</td>
</tr>
<tr>
<td>5.2.3 The excess demand of reserve in the banking system</td>
<td>92</td>
</tr>
<tr>
<td>5.2.4 The dynamics of interbank borrowing rate</td>
<td>93</td>
</tr>
<tr>
<td>5.2.5 The extended 5D CDGZ model with an interbank lending market</td>
<td>93</td>
</tr>
<tr>
<td>5.3 Analysis of sub-dynamical system</td>
<td>95</td>
</tr>
<tr>
<td>5.3.1 The 2D sub-dynamics with $x$ and $r$</td>
<td>95</td>
</tr>
<tr>
<td>5.3.2 The 3D sub-dynamics of $x, r,$ and $y$</td>
<td>97</td>
</tr>
<tr>
<td>5.4 Numerical simulations</td>
<td>98</td>
</tr>
<tr>
<td>5.4.1 Simulations of the 3D sub-dynamics</td>
<td>98</td>
</tr>
</tbody>
</table>
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>The time-dependent cyclical Minskyan dynamics</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>$\omega_e$ vs. $\omega_G$</td>
<td>41</td>
</tr>
<tr>
<td>3.3</td>
<td>$\omega_e$ with varying $\Lambda$</td>
<td>41</td>
</tr>
<tr>
<td>3.4</td>
<td>Quantity Theory of Money vs. Endogenous Money</td>
<td>45</td>
</tr>
<tr>
<td>3.5</td>
<td>The formation of speculative inflation</td>
<td>45</td>
</tr>
<tr>
<td>3.6</td>
<td>The inflationary effect of excess government spending</td>
<td>46</td>
</tr>
<tr>
<td>3.7</td>
<td>The bounded fluctuations of $(\pi, s_p, g)$</td>
<td>49</td>
</tr>
<tr>
<td>3.8</td>
<td>The unbounded growth of speculative inflation</td>
<td>50</td>
</tr>
<tr>
<td>4.1</td>
<td>The effect of Quantitative Easing on Money Base and M2. Source: the Federal Reserve statistics release.</td>
<td>56</td>
</tr>
<tr>
<td>4.2</td>
<td>The isocline of 2D model with $\dot{y} = 0$ (straight line) and $\dot{x} = 0$</td>
<td>63</td>
</tr>
<tr>
<td>4.3</td>
<td>Simulations of the baseline model</td>
<td>63</td>
</tr>
<tr>
<td>4.4</td>
<td>The feedback loop of the 3D system</td>
<td>64</td>
</tr>
<tr>
<td>4.5</td>
<td>The 3D CDGZ model: representative simulation of a stable (left) and an unstable (right) scenario.</td>
<td>67</td>
</tr>
<tr>
<td>4.6</td>
<td>The dynamics of Debt/GDP ratio in the 3D system: $a_1 = 0.5$ (dashed limit cycle), $a_1 = 1.2$ (solid rectangular), $a_1 = 1.7$ (dashed triangular).</td>
<td>68</td>
</tr>
<tr>
<td>4.7</td>
<td>The 3D CDGZ model: effect of varying $a_1$ on output: $a_1 = 0.3$ (solid), $a_1 = 0.7$ (dashed), $a_1 = 1.5$ (dash-dot)</td>
<td>69</td>
</tr>
<tr>
<td>4.8</td>
<td>The 3D CDGZ model: bifurcation diagram for $a_1$.</td>
<td>70</td>
</tr>
<tr>
<td>4.9</td>
<td>The 3D CDGZ model: bifurcation for non-herding parameters.</td>
<td>70</td>
</tr>
<tr>
<td>4.10</td>
<td>The 4D CDGZ model: representative simulation (top left panel: simulation over 600 periods; other panels: magnification over 100 periods.)</td>
<td>75</td>
</tr>
<tr>
<td>4.11</td>
<td>The 4D CDGZ model: bifurcation diagram</td>
<td>76</td>
</tr>
<tr>
<td>4.12</td>
<td>The Kaldorian I-S disequilibrium</td>
<td>78</td>
</tr>
<tr>
<td>4.13</td>
<td>The 4D CDGZ model with Kaldorian I-S Disequilibrium</td>
<td>78</td>
</tr>
<tr>
<td>4.14</td>
<td>The extended 7D CDGZ model with a speculative financial sector</td>
<td>83</td>
</tr>
<tr>
<td>4.15</td>
<td>Bifurcation diagram: the effect of Tobin-type tax</td>
<td>83</td>
</tr>
<tr>
<td>5.1</td>
<td>The ratio between $M_2$ and $M_0$ in China. Source: Statistical Release of the People’s Bank of China</td>
<td>88</td>
</tr>
<tr>
<td>5.2</td>
<td>An illustration</td>
<td>94</td>
</tr>
<tr>
<td>5.3</td>
<td>The isocline of the 2D sub-dynamics with $x$ and $r - r^*$</td>
<td>96</td>
</tr>
<tr>
<td>5.4</td>
<td>Representative simulation of the 3D dynamics</td>
<td>99</td>
</tr>
<tr>
<td>5.5</td>
<td>Bifurcation diagram of the 3D dynamics</td>
<td>100</td>
</tr>
<tr>
<td>5.6</td>
<td>Representative simulation of the 5D dynamics</td>
<td>101</td>
</tr>
<tr>
<td>5.7</td>
<td>The average opinion $x$ vs. the interbank rate $r$</td>
<td>102</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.8</td>
<td>$x$, $y$, and $r$</td>
<td>102</td>
</tr>
<tr>
<td>5.9</td>
<td>Phase plot of interbank rate with varying $\theta_0$</td>
<td>103</td>
</tr>
<tr>
<td>5.10</td>
<td>The uncertainty of deposit withdrawal I</td>
<td>105</td>
</tr>
<tr>
<td>5.11</td>
<td>The uncertainty of deposit withdrawal II</td>
<td>106</td>
</tr>
<tr>
<td>5.12</td>
<td>$\dot{q} = \frac{\alpha_1}{1 + \exp(\alpha_2 x)}$</td>
<td>106</td>
</tr>
<tr>
<td>5.13</td>
<td>The effect of UMP in 4D system</td>
<td>107</td>
</tr>
<tr>
<td>5.14</td>
<td>The effect of UMP in 6D system I</td>
<td>108</td>
</tr>
<tr>
<td>5.15</td>
<td>The effect of UMP in 6D system II</td>
<td>108</td>
</tr>
<tr>
<td>5.16</td>
<td>Representative simulation of the extended 8D model with a speculative financial market</td>
<td>111</td>
</tr>
<tr>
<td>5.17</td>
<td>The “leverage tornado”</td>
<td>112</td>
</tr>
<tr>
<td>5.18</td>
<td>The effect of Tobin-type Tax</td>
<td>113</td>
</tr>
</tbody>
</table>
List of Tables

1.1 The structure of the CDGZ model ...................... 6
1.2 The key features of the CDGZ Model ..................... 6
3.1 Loans create deposits ................................ 25
3.2 The interbank lending and borrowing ..................... 26
3.3 The effect of Open Market Operation .................... 28
3.4 The monetary effect of government spending (left) and taxation (right) . 31
3.5 A special scenario ................................ 33
4.1 A simplified balance sheet of commercial bank ............... 59
5.1 The change of balance sheet after interbank transaction ............ 90
A.1 Name of variables/functions in the CDGZ model ............... 119
A.2 Name of parameters in the CDGZ model .................... 120
### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG</td>
<td>Beja Goldman</td>
</tr>
<tr>
<td>DSGD</td>
<td>Dynamic Stochastic General Disequilibrium</td>
</tr>
<tr>
<td>DSGE</td>
<td>Dynamic Stochastic General Equilibrium</td>
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<tr>
<td>FIH</td>
<td>Financial Instability Hypothesis</td>
</tr>
<tr>
<td>GFC</td>
<td>Global Financial Crisis</td>
</tr>
<tr>
<td>HAM</td>
<td>Heterogeneous Agent Model</td>
</tr>
<tr>
<td>HPM</td>
<td>High Powered Money</td>
</tr>
<tr>
<td>KMG</td>
<td>Keynes Metzler Goodwin</td>
</tr>
<tr>
<td>MMT</td>
<td>Modern Money Theory</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>OMO</td>
<td>Open Market Operation</td>
</tr>
<tr>
<td>QE</td>
<td>Quantitative Easing</td>
</tr>
<tr>
<td>UMP</td>
<td>Unconventional Monetary Policy</td>
</tr>
<tr>
<td>WHL</td>
<td>Weidlich Haag Lux</td>
</tr>
</tbody>
</table>
Dedicated to Dr. Corrado Di Guilmi for being a wonderful mentor and friend. Without him this thesis may never be finished.
Chapter 1

Introduction

“The study of the business cycle, fluctuations in aggregate measures of economic activity and prices over periods from one to ten years or so, constitutes or motivates a large part of what we call macroeconomics”

- Sims (1980): Macroeconomics and Reality

1.1 Background

1.1.1 Fluctuations or cycles? - a perennial debate

The nature and causes of the continually recurring fluctuations in economic aggregates, since the advent of industrialization and the development of modern financial system, have been subject to a constant debate between two main schools of thought. The “classical school”, which follows the insight of Adam Smith, believes that the economy ought to be in, or converging towards a stable general equilibrium as a result of the rational behaviour of economic agents. According to this view, the economy is most appropriately modelled as a linear, stable dynamical system yet constantly perturbed by exogenous shocks. This approach is originally advocated by Frisch (1933) and Slutsky (1937), more often known as the Frisch-Slutsky paradigm.

Yet another view, which is more or less influenced by the writings of John Maynard Keynes, holds that instability is an inherent nature of business cycle due to market failures. Thus it should be modelled in a deterministic fashion, since random variables play non-essential role. We may label this school the “endogenous school”. In mathematical terms, the endogenous school believes that the business cycle is a self-sustaining oscillation generated by the delayed reaction of economic variables and the nonlinear relations
between them (Baur, 2012, Chiarella, 1992). Commonly this strand of literature employs systems of ordinary differential equations to capture the nonlinearity and disequilibrium dynamics of business cycles. Classical examples can be found by the works of Kaldor (1940) and Goodwin (1951).

1.1.2 The “Great Moderation”

Nearly four decades later since Keynes published the General Theory, the 1980s and 1990s across the major OECD countries have been marked by a significant decline of volatility in the real sector, despite the continuing turbulence that had occurred in the financial sector. In the US, the standard deviation of annual real GDP growth rate dropped from 2.7% (1960-1983) to 1.6% (1984-2001) (Stock and Watson, 2002). Numerous studies in the early 2000s also documented similar evidence found in other OECD countries in terms of the volatility decline in output and inflation (Blanchard and Simon, 2001). The two decades of economic tranquillity reinforced the optimism amongst the majority of macroeconomists who adhere to the classical school. Robert Lucas, in his 2003 presidential address to the American Economic Association, claimed that:

“...macroeconomics in this original sense has succeeded: Its central problem of depression-prevention has been solved, for all practical purposes, and has in fact been solved for many decades. There remain important gains in welfare from better fiscal policies, but I argue that these are gains from providing people with better incentives to work and to save, not from better fine tuning of spending flows. Taking U.S. performance over the past 50 years as a benchmark, the potential for welfare gains from better long-run, supply side policies exceeds by far the potential from further improvements in short-run demand management.”

- Lucas (2003), Macroeconomic priorities

One year later, Ben Bernanke, in his 2004 Federal Reserve Board speech titled “The Great Moderation”, explained why the economy had entered a period of great moderation:

“Three types of explanations have been suggested for this dramatic change; for brevity, I will refer to these classes of explanations as structural change, improved macroeconomic policies, and good luck. Explanations focusing on
structural change suggest that changes in economic institutions, technology, business practices, or other structural features of the economy have improved the ability of the economy to absorb shocks... The second class of explanations focuses on the arguably improved performance of macroeconomic policies, particularly monetary policy. The historical pattern of changes in the volatilities of output growth and inflation gives some credence to the idea that better monetary policy may have been a major contributor to increased economic stability... The third class of explanations suggests that the Great Moderation did not result primarily from changes in the structure of the economy or improvements in policymaking but occurred because the shocks hitting the economy became smaller and more infrequent...”

- Bernanke (2004), The Great Moderation

In the debate over the causes of business cycles, the adherents of the Frisch-Slutsky paradigm had triumphed over the endogenous school, in absolute terms. Intellectual attention had been diverted from determinism to stochasticism in the philosophy of macroeconomic modelling. More sophisticated structural VAR-type econometric tools have been widely developed since Sims (1980). In the theoretical study, models that assume rational expectations, representative agents, general equilibrium, and inter-temporal maximizing behaviour have evolved and refined to the highest modern form, in the name of neo-classical Real Business Cycle (RBC) theory and subsequently the neo-“Keynesian” Dynamic Stochastic General Equilibrium (DSGE) Model (Kydland and Prescott, 1982, Smets and Wouters, 2003). The old Keynesian-style macroeconomic models, which were once popular in the 1940s and 1950s, were heavily criticized by their classical opponents in two fundamental aspects: i), in this nonlinear, disequilibrium approach, the agents’ expectations are, in most circumstances, inconsistent with agents’ assumed rational behaviour (Muth, 1961); ii), this approach is simply refuted by empirical observation (Granger and Newbold, 1977). As a result, this type of models diminished rapidly in this period.

1.1.3 The 2007-2008 Global Financial Crisis

The “Great Moderation” came to an abrupt halt in the dawn of the 2007-2008 Global Financial Crisis. For decades of optimism, both economists and the public were blindfolded by the danger that had gradually unfolded in the financial system. The collapse of Bretton-Woods system in 1971 had planted a seed for the ever-worsening US current account deficit, which potentially posed a downward pressure on interest rates. In the
beginning of year 2000, the Federal Reserve started to lower interest rate, in the wake of dot.com bubble. The Federal Fund Rate reached a historical low of 1% in 2004. The wide availability of credit and the encouragement of home ownership by the US government had triggered a boom in the housing market. With the repeal of Glass-Steagall act by the Clinton administration and waves of financial innovation and deregulation in the last three decades, banks had become increasingly active in the capital market. Securitization, which involves selling bank loans “off balance sheet” to an investment bank which syndicates the Special Purpose Vehicle (SPV) that, in turn, issues asset backed security (ABS), allows bank to free its illiquid assets and creates more loans beyond its regulatory limit. This is commonly known as the “originate and distribute” model. Securitization had allowed both US investors and investors overseas to search for a higher yield outside treasury bonds by investing in the US housing market. It had also prompted banks to lend excessively into the housing sector, which had ultimately fuelled the housing bubble. In 2006, large waves of default started to occur, which eventually caused the collapse of housing price and triggered the subsequent world-wide crisis.

The series of events during the GFC also triggered a swing from optimism to pessimism amongst numerous mainstream macroeconomists who had casted doubts over the current macroeconomic paradigm. Yet it also serves as a timely reminder of what is missing in conventional macroeconomic thinking. Traditional Keynesian economics becomes highly relevant in explaining and dealing with the current economic crisis in at least three aspects: (i) Keynes advocates radical policy interventions in order to restore aggregate demand; (ii) Keynes examines how psychological factors affect agent’s behaviour in the face of radical uncertainty and propagate the macroeconomic fluctuations; (iii) Keynes’ follower Minsky narrates upon the intrinsic link between real and financial sectors and explains how instability from the financial sector transmits to the real sector that induces macroeconomic instability. On the other hand, there is a resurgence of studies in disequilibrium macroeconomic model of Keynesian tradition. It seems potentially a gap to be filled in the area of disequilibrium dynamic modelling that formalizes Keynes’ analysis of boom-bust cycles. It can potentially be useful in examining the efficacy of government policies, at least in the form of qualitative analysis.

1.2 Aim and structure of the thesis

Inspired by the recent financial crisis, as well as the ongoing development of disequilibrium macro-dynamic literature, this thesis aims to develop a set of dynamic models that (i) systematically investigate the operational details of fiscal/credit-driven monetary dynamics under an alternative quantitative framework and (ii) studies how bounded
rationality of credit-creating financial institutions induces credit cycles and propagates macroeconomic instability.\footnote{The ongoing four chapters are based on four potential publications, the earlier versions of which had been presented in various conferences. Chapter 2 is based on Zhi (2015a). Chapter 3 is adopted from Zhi (2015b). The earlier versions were presented at the 2014 INET plenary conference in Toronto, Canada; the 14th Annual Society of Heterodox Economists (SHE) Conference in Sydney, Australia. Chapter 4 is based on Chiarella et al. (2015a) co-authored with C. Chiarella and C. Di Guilmi. The earlier versions were presented at the following conferences: WEHIA2013 in Reykjavik, Iceland; NED2013 in Siena, Italy; and SNDE2014 in New York. Chapter 5 is based on Chiarella et al. (2015b) co-authored with C. Chiarella and C. Di Guilmi. The earlier versions were presented at WEHIA2014 conference in Tianjin, China and the 2015 inaugural UTS Business School PhD Student conference in Sydney, Australia.}

In chapter 2, we discuss the literature in disequilibrium macroeconomics by examining the relationship between the origin of disequilibrium macroeconomic thinking and the development of Keynesian disequilibrium macroeconomic models. Keynes alludes the idea of disequilibrium macroeconomics in his \textit{General Theory}. It has inspired the ongoing development of disequilibrium macroeconomic literature that strives to formalize Keynes’s disequilibrium thinking with the use of advanced tools in nonlinear dynamic systems. We discuss two particular strand of modelling approaches: the Keynes-Metzler-Goodwin (KMG) approach and the Weidlich-Haag-Lux (WHL) approach. In the KMG framework, “Keynes” refers to the savings-investment disequilibrium, the sticky adjustment of prices and wages, and the link between financial and real markets; “Metzler” to inventory dynamics and “Goodwin” to the dynamics of distributive shares. The WHL approach, on the other hand, formalizes the modelling of self-fulfilling “animal spirits” and herd behaviour under a highly stylized setting.

Chapter 3 takes a critical look at the much controversial “Modern Money Theory” (\textit{a.k.a Neo-Chartalism}) from a dynamic perspective. In this chapter, we propose a set of dynamic models that aims to investigate the monetary dynamics as well as its real and inflationary consequences due to the complex interactions between public and private banking and non-banking sectors. We contend that some of the MMT’s claims are questionable due to a general lack of formal quantitative analysis that justifies and measures its policy effects and consequences, especially the long run inflationary consequence of the fiscal/credit-driven monetary expansion. This chapter also provides a modelling framework that is further adopted in the subsequent two chapters of the thesis.

Chapter 4-5 aim to examine (i) how heterogeneity in the banking sector induces credit cycles due to “animal spirits”-driven bounded rationality of financial institutions and how it propagates macroeconomic instability and (ii) the dynamics of interbank market over the course of credit cycles. The idea of “animal spirits” has been widely treated in the literature with particular reference to investment in the productive sector. Chapter 4 takes a different view and analyses from a theoretical perspective the role of banks’ collective behaviour in the creation of credit that, ultimately, drives the credit cycles.
We further investigate the mechanisms of interbank market in chapter 5. The analytical as well as numerical simulations of the final 8D CDGZ model may potentially be useful in providing relevant policy recommendations. We summarize the structure and key features of the CDGZ model in Table 1.1 and 1.2 below.

Table 1.1: The structure of the CDGZ model

<table>
<thead>
<tr>
<th>Name</th>
<th>Key Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D CDGZ ((x, y))</td>
<td>switching strategy</td>
</tr>
<tr>
<td>3D CDGZ ((x, y, \lambda_+))</td>
<td>three-stage cycles</td>
</tr>
<tr>
<td>4D CDGZ ((x, y, \lambda_+, \lambda_-))</td>
<td>convergence &amp; divergence of lending behaviour</td>
</tr>
<tr>
<td>5D CDGZ ((x, y, \lambda_+, \lambda_-, r))</td>
<td>the Kaldorian investment-saving disequilibrium</td>
</tr>
<tr>
<td>6D CDGZ (UMP I) ((x, y, r, q))</td>
<td>credit creation magnifies stock market volatility</td>
</tr>
<tr>
<td>7D CDGZ ((x, y, \lambda_+ \lambda_-, \lambda_+, \lambda_-))</td>
<td>the emergence of interbank rate hikes during the downturn</td>
</tr>
<tr>
<td>8D CDGZ (UMP II) ((x, y, \lambda_+, \lambda_-, r, q))</td>
<td>the necessity and timely consideration of UMP</td>
</tr>
<tr>
<td>9D CDGZ ((x, y, \lambda_+, \lambda_-, r, p_k, \pi_k, x_p))</td>
<td>UMP and the zero bound</td>
</tr>
</tbody>
</table>

Table 1.2: The key features of the CDGZ Model

Chiarella, Di Guilmi, and Zhi.
Chapter 2

The theory and models of Keynesian disequilibrium macroeconomics

2.1 Introduction

Progress in any scientific discipline is very much a function of the mathematical concepts and tools which are at hand to forge and articulate the paradigm models which are an essential part of scientific understanding...Therefore to understand the development of a scientific discipline it is important to not only understand the development of the paradigms of the discipline but also the development of the mathematical ideas which are interacting with these paradigms.

- Chiarella (1990), pp.5

This chapter examines the interplay between the origin of disequilibrium macroeconomic thinking from John Maynard Keynes in his General Theory, and the subsequent development of Keynesian disequilibrium macroeconomic models. Given that these two strands of literature are both plentiful, I will focus on the essence of Keynes’s disequilibrium thinking, and discuss how it influences the development of relevant disequilibrium macroeconomic models, mainly in the context of deterministic nonlinear dynamics of Keynes-Metzler-Goodwin and Weidlich-Haag-Lux approaches.

Disequilibrium macroeconomics was originally envisioned by Keynes (1936). It had once enjoyed a brief popularity in the 1970s since Leijonhufvud (1968) published his famous
book “On Keynesian Economics and the Economics of Keynes”\textsuperscript{1}. Yet it had almost disappeared in the 1980s due to the popularization of theory of rational expectations and the emergence of Neo-classical synthesis.

Yet it becomes apparent in the aftermath of the 2007-2008 GFC and the subsequent world-wide recession that disequilibrium macroeconomics offers a better explanation about macroeconomic instability. Indeed, Keynes wrote the General Theory nearly 80 years ago in his effort to explain the Great Depression. Although his analytical framework was static in nature, he had in mind a dynamic theory, since disequilibrium is essential in understanding Keynes’s General Theory: saving and investment are independently determined; wages and prices are rigid in the short run; and expectations are formed by the self-fulfilling sentiments and herd mentality. Later on, Minsky deepens Keynes’s analysis by emphasizing the crucial role of finance in propagating macroeconomic instability.

On the other hand, the development of theories in non-linear dynamics provides a mathematical backbone for the study of disequilibrium macroeconomic models. Three areas are of particular importance: \textit{(i)} the seminal work of Poincare (1899), originally applied in the study of celestial mechanics, which had laid the foundation for the modern qualitative-geometric approach to the analysis of non-linear dynamical systems; \textit{(ii)} the development of Bifurcation Theory, especially the Hopf Bifurcation theorem (Kuznetsov, 2004); \textit{(iii)} the discovery of chaos in deterministic dynamic system (Lorenz, 1963, 1993). The use of Hopf bifurcation theorem to demonstrate the existence of a limit cycle, as a parameter of interest passes through a critical bifurcation point, has been widely applied in numerous fields in natural science. To a lesser degree in economics, it is applied in identifying and modelling endogenous business cycles of various kind\textsuperscript{2}; the discovery of deterministic chaos, characterized by the irregular fluctuations in the Lorenz system, had inspired the study of complexity economics, which looks into the emergence of complex phenomena as a result of local interactions of heterogeneous agents with simple, deterministic laws - mimicking many complex phenomena observed in reality such as herd behaviour.

An important use of disequilibrium macroeconomic models is to understand the dynamic and heterogeneous interactions amongst the core sectors of the aggregate economy, in order to see which parameters are stabilizing or destabilizing, and which parameters,\footnote{Leijonhufvud is perhaps the first author that interprets Keynes in terms of disequilibrium phenomena. He argues that the traditional IS/LM formulation of Keynes’s theory fails to explain phenomena such as “involuntary unemployment”, which is central for Keynes’s explanation of unemployment and depression. Leijonhufvud advocates a “cybernetic” approach to macroeconomics where the dynamic adjustments of prices and quantities are explicitly considered without imposing the standard Walrasian equilibrium concept.}{Leijonhufvud is perhaps the first author that interprets Keynes in terms of disequilibrium phenomena. He argues that the traditional IS/LM formulation of Keynes’s theory fails to explain phenomena such as “involuntary unemployment”, which is central for Keynes’s explanation of unemployment and depression. Leijonhufvud advocates a “cybernetic” approach to macroeconomics where the dynamic adjustments of prices and quantities are explicitly considered without imposing the standard Walrasian equilibrium concept.}\footnote{See Chiarella (1990) for detailed discussion.}{See Chiarella (1990) for detailed discussion.}
particularly parameters that are associated with policy interventions such as Tobin-type taxes, have the most influence in switching the economy between the regions of stability and instability, thus offering us an insight over the effectiveness of policy interventions in qualitative terms. In the context of deterministic models, the term “stability” (or “instability”) refers to the local properties of the steady state, and the existence of transition from stability to instability (in the form of persistent, self-sustaining fluctuation) can be mathematically formulated and proven with the use of Hopf bifurcation theorem. Given its aggregative nature, this type of models is sometimes coined “macro-founded”. It offers a particular advantage over the traditional “micro-founded” models of stochastic intertemporal optimization type such as RBC or New-Keynesian DSGE models, which are difficult to be analyzed in terms of the dynamic linkages and feedbacks between various sectors of the macro-economy (Chiarella et al., 2005).

This chapter expounds two fundamental approaches of modelling Keynesian disequilibrium macro-dynamics: the Keynes-Metzler-Goodwin (KMG) approach and the Weidlich-Haag-Lux (WHL) approach. the former is only briefly discussed and the latter is emphasized in terms of mathematical details, since it is the primary focus of this thesis. These two approaches represent one of the most stylized and most recent development of Keynesian disequilibrium macro-dynamics. They are also probably the most relevant in addressing Leijonhufvud’s critique of standard IS-LM interpretation of Keynes that lacked disequilibrium phenomena. The KMG model is formulated by Chiarella and Flaschel (2000) and later on, Chiarella et al. (2005). “Keynes” refers to the disequilibrium between savings and investment, the sticky adjustment of prices and wages, and the causal nexus from financial to real markets; “Metzler” to inventory dynamics and “Goodwin” to the dynamics of distributive shares. It synthesizes and extends the disequilibrium macro-dynamic models in the 1960s and 1970s in a systematic and hierarchical manner. The interaction of three agents (households, firms, and government) across five markets (labour market, goods market, money market, bond market, and equities market) are considered. On the other hand, the WHL approach attempts to model the dynamics of “animal spirits”. It is originally inspired by earlier work of Weidlich and Haag (1983) that models the interacting population dynamics. Lux (1995), in his seminal work, studies how herd behaviour and sentiment contagion contributes to asset price bubbles/crashes. The framework is further applied in Franke (2012) and Charpe et al. (2012), which study the dynamics of “animal spirits” and real-financial interaction in a macroeconomic setting.

The rest of the chapter is organized as follows: section 2.2 traces back to the origin of Keynes’s disequilibrium macroeconomic thinking, as well as Minsky’s interpretation of
Chapter 2. *The theory and models of Keynesian disequilibrium macroeconomics* 10

Keynes. Section 2.3 introduces the modelling of Keynesian disequilibrium macroeconomic models in terms of KMG and WHL approaches, particularly the latter one. Section 2.4 concludes with a discussion regarding the interplay between these two strands of literature. It also paves a few path for future research.

2.2 The origin of disequilibrium macroeconomic thinking

The second half of the 19th century and the beginning of the 20th century had witnessed an increasing instability of capitalism, in the form of more severe economic crises and an increasing number of bankruptcies. The worsening macroeconomic instability culminates in the “Black Thursday” on the 24th of October, 1929 and the subsequent Great Depression of the 1930s, which is marked as the most prolonged and severe depression of the 20th century.

Classical economists at that time, however, believed that crises would not occur, and full employment is guaranteed due to the self-adjusting market. They based their analysis on the Say’s law: goods produced will be be sold since “supply creates its own demand”. Saving from household always equals business investment. Furthermore, wages and prices are assumed to be highly flexible. The increase of product prices will quickly be matched by a rise of costs, which eliminates the incentive to expand output. The belief of Say’s law, coupled with the assumption of flexible price, insures that business will be able to sell their goods to either consumers or investment; full-employment is automatically maintained, as long as there is no involuntary unemployment - workers are always willing to work at any wages.

The Great Depression is clearly a significant event that had casted doubt over the myth of the self-adjusting markets. The Classical doctrines had outlived its ideological usefulness in explaining the severe depression. When it came to policy recommendations, the Classical economists could recommend nothing but a general cut in all wages. It was obvious that unconventional policies were needed to restore aggregate demand and business confidence on a scale that could only be achieved by a drastic government intervention (Hunt and Lautzenheiser, 2011).

2.2.1 The “disequilibrium” thinking of John Maynard Keynes

Keynes’s *General Theory* was born out of the need to understand and explain the Great Depression. In a nutshell, Keynes made his departure from the Classical doctrine mainly in three aspects:
Chapter 2. The theory and models of Keynesian disequilibrium macroeconomics

- The rejection of Say’s law;
- The rejection of flexible wages and price;
- “Animal spirits”.

The first departure of Keynes’s analysis lies in his rejection from the Say’s law. While Say’s law says “supply creates its own demand”, Keynes runs the causation other way around. Keynes believes that an active management of aggregate demand by means of monetary and fiscal policy is crucial in maintaining full employment. Otherwise involuntary employment would occur and the economy would reach a sub-optimal equilibrium, prevailing with prolonged unemployment. With regard to interest rate determination, Keynes refutes the concept of self-adjusting interest rates mechanism driven by saving-investment equilibrium. He argues that saving does not necessarily equal to investment, since household income is the major determinant of saving; firms’ investment decisions, on the other hand, are mainly determined by profit expectations. Interest rate is primarily determined by liquidity preference in the money market.

The second departure of Keynes’s analysis, which is more received and becomes the main ingredient of the New-Keynesian macroeconomics, is his rejection of flexible price and wage assumption. Price would be rigid, or in a disequilibrium dynamical adjustment process due to the existence of monopoly and the resistance of workers for cutting wages.

Perhaps the most fundamental departure of Keynes’s thinking from the Classical school lies in his view over how non-rational, psychological factors play a role in influencing human behaviours and propagating aggregate business cycles, due to the presence of radical uncertainty. Keynes coined the term “animal spirits” to address this aspect in Chapter 12 - The State of Long-term Expectation, in which he writes:

“Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits - a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.”

Akerlof and Shiller (2009), in their book “Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism” goes further beyond and
identifies five types of animal spirits: confidence, fairness, corruption, money illusion, and stories. This thesis will, however, only focus on the confidence factor given its primary importance. Akerlof and Shiller used the term “confidence multiplier”, which is borrowed from the concept of Keynesian consumption multiplier, in describing the self-fulfilling nature of confidence: on an individual level, an optimistic attitude leads to positive actions that reinforces the confidence; on an aggregate level, confidence is contagious in the form of herd-type behaviours. This is well depicted in Keynes’s famous beauty contest metaphor:

“It is not a case of choosing those [faces] that, to the best of one’s judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.”

“Animal spirit” alludes to a state in disequilibrium - a constant deviation of human expectations and behaviours from the ones being rationally determined. It is crucial in Keynes’s explanation of economic expansion and recession: a state of over-optimism leads to inflated asset prices, over-investment and over-consumption, as well as over-expansion of credit in the financial sector, which eventually leads to its own defeat.

Although Keynes’s theory is conservative in nature: his analytical framework does not deviate from the Walrasian general equilibrium theory, the three aspects of Keynes’s disequilibrium thinking had clearly paved a path for the emergence of disequilibrium macroeconomics during the 1960s, and later on, inspired the work of Hyman Minsky.

2.2.2 Minsky’s interpretation of Keynes

“Keynesian economics as the economics of disequilibrium is the economics of permanent disequilibrium.”

- Minsky (1975), pp.66

It is the work of Hyman Minsky that brings a new light over a conventional, Hicksian IS/LM interpretation of Keynes’s General Theory. Minsky views Keynes's theory as an

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3See Backhouse and Boianovsky (2005) for a detailed survey over the development of disequilibrium macroeconomics during the 1960s.
endogenous theory of business cycle that explicitly considers the crucial role of the financial instability in propagating macroeconomic fluctuations, as he writes in his famous book “John Maynard Keynes”:

“The missing step in the standard Keynesian theory is the explicit consideration of capitalist finance within a cyclical and speculative context...finance sets the pace for the economy. As recovery approaches full employment...soothsayers will proclaim that the business cycle has been banished and debts can be taken on...But in truth neither the boom, nor the debt deflation...and certainly not a recovery can go on forever. Each state nurtures forces that lead to its own destruction.”

The instability and interconnection between the real and financial sector lies mainly in two aspects in Minsky’s analysis: (i) the role of speculative and Ponzi borrowers; (ii) the role of financial institutions. The first aspect is elaborated in Minsky’s well-known “Financial Instability Hypothesis”. Minsky argues that stability is inherently unstable: the economic boom in the preceding period nurtures a self-fulfilling sense of optimism that leads to an increasing portion of speculative and Ponzi borrowers in the financial market. However, the speculative boom cannot last forever. The “Minsky moment” occurs when investors are over-indebted. As they start to sell off their asset in order to meet debt repayments, asset price turns south. Consequently, the financial sector transits from stability to instability. Over the course of business cycle, it is the expansionary phase that ultimately leads to its own destruction. The financial and real cycles are inherently interconnected.

The second aspect, however, is more or less implicit in Minsky’s discussion. Financial institutions, particularly banks, play a crucial role in propagating credit cycle, since banks create money by creating loans that simultaneously create deposits, as is discussed by Minsky’s predecessor Joseph Schumpeter and Knut Wicksell. Specifically, Minsky stresses on the role of banker’s confidence over the state of credit, as he wrote:

“The state of credit reflects banker’s view toward borrowers...A revision by banker of their views about the appropriate leverage to use in financing positions in capital assets will not necessarily cause an immediate revision in

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4Minsky defines three categories of borrowers, i.e. the hedging borrower, whose current cash flow can serve both interest and principle; the speculative borrower, who can only serve the interest payable; and the Ponzi borrower, who thrives on an inflating asset price and has to rely on borrowing in order to pay back both interest and principle.

This aspect is further developed by Minsky’s followers, in the Post-Keynesian literature of endogenous money theory. It can be generally categorized into three groups: the horizontalist approach, the structuralist approach, and the state money approach (Wray, 2007). Horizontalist approach is initially advocated by Basil Moore\(^6\) in the 1970s and 1980s that stresses the endogenous nature of money and reserves, arguing that “loans make deposits, deposits make reserves”. Structuralists, which had emerged in 1990s and advocated by Charles Goodhart\(^7\), on the other hand, take a more active role of banks’ lending behaviour, given the profit-seeking nature of banks. The state money approach, advocated by Wray (2012) and Mosler (2010) amongst several other so-called “neo-chartalists”, agrees upon most aspects of the horizontalist and structuralist proposition. Yet it stresses the mechanism of fiat money system of the public sector and studies the macroeconomic impacts of taxation and government deficit from an accounting perspective. In a historical context, Charles Kindleberger (1989) vividly depicts the boom/bust of credit cycles and the subsequent economic expansions/recessions over a course of historical events, which further vindicates Minsky’s insights over the role of financial institutions in propagating financial and macroeconomic instability.

To sum up, the disequilibrium thinking of Keynes involves four aspects:

1. Saving ≠ investment;
2. Price and wages rigidity;
3. “Animal spirits”;
4. Financial instability and real-financial nexus.

2.3 Models of Keynesian disequilibrium macroeconomics

2.3.1 The Keynes-Metzler-Goodwin model

The three decades since 1960s had witnessed an emergence of literature in the mathematical modelling of disequilibrium monetary macro-dynamics, both in Neo-classical and Keynesian schools of thought. This strand of literature is further synthesized and extended by Chiarella and Flaschel (2000) and Chiarella et al. (2005) in the form of

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\(^6\)See Moore (1988).
\(^7\)See Goodhart (1989).
Chapter 2. The theory and models of Keynesian disequilibrium macroeconomics

Neo-classical Tobinian model, the Keynes-Wicksell model, and the textbook Keynesian AS-AD model of Turnovsky (1977) and Sargent (1987). It leads to the formulation of Keynes-Metzler-Goodwin (KMG) Model that overcomes several drawbacks of these previous model types.

The (neo)-classical discussion of monetary growth can be traced back to Tobin (1955, 1965) and its subsequent extensions. In Tobinian model, money is typically considered as an asset in addition to real capital, and the money market disequilibrium is the core driving force behind inflation and inflationary expectations. Goods market, on the other hand, is assumed to follow the Say’s Law and always stays in equilibrium (Saving = Investment). These features are captured in the following equations of a prototype Tobinian model of monetary growth:

\[ W = \frac{M}{p} + K, \]  
\[ I = S = Y - \delta K - C - G = \dot{K}, \]  
\[ \dot{w} = \beta_w(V - \dot{V}) + \kappa_w\dot{p} + (1 - \kappa_w)\pi, \]  
\[ \dot{p} = \beta_p(\frac{M - M^d}{pK}) + \eta\pi + (1 - \eta)(\mu_0 - n), \]  
\[ \dot{\pi} = \beta_{\pi_1}(\dot{p} - \pi) + \beta_{\pi_2}(\mu_0 - n - \pi), \]

where \( W \) is the real wealth, \( M \) is the money supply (index \( d \): demand, growth rate \( \mu_0 \)), \( p \) is price, \( K \) is capital stock, \( I(S) \) is investment(saving), \( w \) is nominal wage, \( \pi \) is expected rate of inflation, \( V \) is the rate of employment, \( n \) is the natural growth rate, and \( \beta_x, \kappa_x, \) and \( \eta \) are the adjustment parameters.

The assumption of Say’s Law, coupled with the role of monetary market disequilibrium in determining inflation are surely questionable features of the Tobin-type model from a Keynesian disequilibrium perspective. The Keynes-Wicksell model, which synthesizes Goodwin (1967) and Rose (1967), dispenses the Say’s Law by introducing independently

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8See Orphanides and Solow (1990) for a detailed survey.
9The equations presented in the following sections only describe partial structure. See Chiarella and Flaschel (2000) for full descriptions of the model.
10Here we only list the key variables and parameters of the model. See Chiarella and Flaschel (2000) for a detailed discussion.
determined saving and investment functions, and uses both the Labour and Goods market disequilibrium as the cause of inflation, as is represented by the following equations:

\[
\begin{align*}
W & = \frac{(M + B + p_eE)}{P}, \\
S & = Y - \delta K - C - G, \\
I & = i(\rho - r + \pi)K + nK, \\
\dot{w} & = \beta_w(V - \bar{V}) + \kappa_w\tilde{p} + (1 - \kappa_w)\pi, \\
\dot{\pi} & = \beta_p\left(\frac{I - S}{K}\right) + \kappa_p\dot{w} + (1 - \kappa_p)\pi,
\end{align*}
\]

where \(p_e\) is the price of equities, \(\rho\) is the rate of profit, and \(r\) is the nominal rate of interest.

The wealth function now has two additional assets (private equity and government bond) that replace the capital \((K)\) in the prototype Tobin model. Most importantly, the investment function is independently determined by the rate of profit, rather than the saving function.

The Keynes-Wicksell model, however, still suffers from an important drawback: it is a supply-side model, since full-capacity growth is assumed. It is a problematic feature since \((i)\) the price adjusts sluggishly while the quantity adjust instantly; \((ii)\) for a true “Keynesian” model it requires a detailed treatment on the demand-driven mechanism. It leads to the formulation of the Keynes-Metzler-Goodwin (KMG) model, which is derived from a more traditional IS-LM approach, yet with IS disequilibrium. The Metzlerian inventory dynamics is introduced to address the Saving-Investment disequilibrium. The demand-side dynamic multiplier, coupled with Metzlerian inventory dynamics, are captured by the following equations:

\[
\begin{align*}
\dot{Y} & = \gamma Y_e + \beta_{Ye}(Y^d - Y^e), \\
\dot{N} & = Y - Y^d = S - I,
\end{align*}
\]

where \(Y^e\) is the expected aggregate demand and \(N\) is the stock of inventories.

In a stylized manner, the KMG model captures the Keynesian demand-side multiplier with saving-investment disequilibrium, the Metzlerian inventory dynamics, as well as the Goodwinian profit-squeeze mechanism. The formulation of KMG model is an important development toward a rigorous mathematical formalization of Keynes’s disequilibrium thinking: it provides a “macro-foundation” as a viable alternative to the more received micro-founded New-Keynesian model types.
2.3.2 Modelling the dynamics of “animal spirits”

An important drawback of the KMG model discussed in the previous section, is that it overlooks the “animal spirits” aspect of Keynesian disequilibrium thinking, which is essential in Keynes’s explanation of business cycles. In part, this is due to the “macro-founded” nature of the KMG model that overlooks certain behavioural aspects at the micro level. Arguably, however, a true Keynesian model should take into account the dynamics of “animal spirits”, i.e. how waves of optimism and pessimism of economic agents at micro level cause speculative and herd behaviours, which ultimately lead to fluctuations at macro level.

2.3.2.1 Modelling speculative behaviour: the “fundamentalist-chartist” approach

An emerging number of empirical studies in finance literature has found abnormality in asset prices in terms of the existence of serial correlations and excess volatilities, which casts doubts over the traditional asset pricing models of efficient market hypothesis and rational expectations school. It has inspired the development of Heterogeneous Agent Models (HAMs) that was initially aimed to study speculative behaviours in the financial market and later on, it has found applications to a broader scope of issues in Keynesian disequilibrium macroeconomics\(^\text{11}\). In this strand of literature, agents are typically categorized into two groups: the fundamentalists and the chartists. The fundamentalists represent the rational agents that provide a stabilizing force toward the fundamental price, whereas the chartists represent trend chasers that are self-fulfilling and destabilizing.

Beja and Goldman (1980) is amongst the first work that takes this “fundamentalist-chartist” approach. The stabilizing fundamentalists’ excess demand is driven by the deviation of actual price from the fundamental price, while the destabilizing chartists’ excess demand is driven by the perceived historical price trends:

\[
\dot{p} = D^f_t + D^c_t, \tag{2.13}
\]

\[
D^f_t = a(w - p), \tag{2.14}
\]

\[
D^c_t = b(\psi - g), \tag{2.15}
\]

where \(D^f_t \) (\(D^c_t \)) is the excess demand of fundamentalists (chartists), \(w \) is the underlying equilibrium price, \(p \) is the current price, \(\psi \) is the speculators’ assessment of the price.

\(^{11}\)See Hommes (2005) for a detailed survey of HAMs.
Chapter 2. The theory and models of Keynesian disequilibrium macroeconomics

2.3.2.2 Modelling herd behaviour and “animal spirits”: the Weidlich-Haag-Lux approach

In addition to the speculative behaviour captured by Beja and Goldman (1980) type model at an individual level, herd behaviour is another important aspect of self-fulfilling animal spirits, at an aggregate level. The seminal work of Lux (1995) is perhaps the first work that formalizes the dynamics of herd behaviour and mutual mimetic contagion in speculative financial market. This approach has its origin from social science in the earlier work of Weidlich and Haag (1983), which attempts to model the interacting populations in a more general context. The basic idea of Weidlich-Haag-Lux approach is to model a population of agents that choose and switch between two attitudes in probabilistic terms. The agents interact with each other based on average opinion, as well as other non-herding factors. A Master equation that captures the average opinion as a mean-field variable is applied to simplify the analysis of the stochastic system. The Lux (1995) model also inspires numerous works in macroeconomics that incorporates “animal spirits” as the core driving factor of business fluctuations.

The basic set-up in Lux (1995) model is as follows: there are $2N$ speculative traders who hold either optimistic or pessimistic sentiment. There are $n_+$ number of optimists and $n_-$ number of pessimists such that $n_+ + n_- = 2N$. Let $n \equiv 0.5(n_+ - n_-)$ and $x \equiv n/N$, we have an index $x \in [-1, 1]$ that describes the average opinion of traders. Hence $x > 0(<0)$ corresponds to a situation of predominant optimism (pessimism).

The contagion process in Lux model is modelled in terms of transitory probability. Let $p_{+-}$ be the probability that a pessimist would switch to an optimist, and likewise for $p_{-+}$. It is plausible that these two transitory probabilities will depend on the average opinion: a predominant optimism will lead to a higher likelihood that pessimists switch to optimists, and vice versa. Hence at any point in time we expect $n_- p_{-+}$ to switch from $n_-$ to $n_+$, while $n_+ p_{+-}$ to switch from $n_+$ to $n_-$. It follows that $dn_+/dt = n_- p_{-+} - n_+ p_{+-}$ and $dn_-/dt = n_+ p_{+-} - n_- p_{-+}$. Since $n = 0.5(n_+ - n_-)$ and $x = n/N$, we obtain\(^\text{12}\):

$$\dot{x} = (1 - x)p_{+-}(x) - (1 + x)p_{-+}(x). \tag{2.16}$$

\(^{12}\)This equation can also be derived more formally as an approximative mean value equation for the original stochastic system in terms of Master equation approach (Lux, 1995).
Lux makes three assumptions over the transitory probabilities $p_+^-$ and $p_-^+$: (i) they must be positive; (ii) the transition from pessimism to optimism is larger than the opposite direction if the predominant sentiment is already optimistic and vice versa; and (iii) the relative change in the probability to switch from pessimism to optimism increases linearly with changes in $x$, in a symmetric manner in both directions ($dp_+^-/p_+^- = adx$, and $dp_-^+/p_-^+ = -adx$). Hence the most appropriate functional form would be:

$$p_+^-(x) = v exp(ax),$$

$$p_-^+(x) = v exp(-ax).$$

(2.17)

(2.18)

By substituting equation (2.17-2.18) into (2.16) we have

$$\dot{x} = v[(1-x)exp(ax) - (1+x)exp(-ax)].$$

(2.19)

Equation (2.19) forms the core of the contagion mechanism. The parameter $a$ plays a crucial role in determining the local stability condition. When $a \leq 1$, equation (2.19) has a unique equilibrium at $x = 0$; when $a \geq 1$, this equilibrium becomes unstable and two additional, stable equilibria emerges.

Lux further extends the model by adding a speculative element, which is in line with Beja and Goldman (1980): it is assumed that price is driven by the fundamental and speculative demand ($\dot{p} = \beta(D_N + D_F)$ where $D_N$ is the speculative demand and $D_F$ is the fundamental demand. Lux further adds the price change to the opinion formation process, since a rising price will make pessimists more likely to switch to optimists, and vice versa. Hence,

$$p_+^- = v exp(a_1\dot{p}/v + a_2x),$$

$$p_-^+ = v exp(-a_1\dot{p}/v - a_2x).$$

(2.20)

(2.21)

Franke (2012) adopts the Weidlich-Haag-Lux approach to model firm’s sentiments in a disequilibrium macro-dynamical model. The model is characterized by a population of heterogeneous firms that constantly switch between optimistic and pessimistic attitudes. The sentiment, measured by the average opinion of firms, influences the investment decisions of firms and leads to variations of output gap. The model also incorporates a Phillips curve to capture the inflation climate and a Taylor rule equation to capture the
interest rates. The two dimensional ODE system is given by Equation (2.22-2.23):

\[
\dot{x} = v[(1-x)\exp(as) - (1+x)\exp(-as)], \quad (2.22)
\]
\[
\dot{\pi}^c = \alpha[\gamma\pi^* + (1-\gamma)(\pi^c + \kappa\eta x) - \pi^c], \quad (2.23)
\]

where \(\pi^c\) is the inflation climate, \(\gamma\) captures the credibility of the central bank, \(\kappa\) is the slope of the Phillips curve, \(\eta\) is the proportionality factor linking the output gap to \(x\).

The Franke paper is a highly-stylized framework that formalizes the modelling of “animal spirits” in this small-scale macroeconomic model. It can be viewed as providing an alternative micro-foundation for macro-dynamic model from a Keynesian perspective, which is fundamentally different from the dominating paradigm of representative, utility-maximizing agent approach. This approach is sufficiently simple and flexible to be applied to a broader scope of decision problem in economics.

Another stimulating work is Charpe et al. (2012), which further extends Lux (1995) and Franke (2012) and proposes a so-called “Dynamical Stochastic General Disequilibrium (DSGD)” model. It features the continuous dynamical adjustment process on interacting real and financial markets. The financial market is destabilizing in the presence of speculative behaviour of heterogeneous agents driven by “Animal Spirits”. The financial market is populated with a changing portion of fundamentalists and chartists. The real market, on the other hand, follows a much simplified version of KMG model, in the sense that the real side is always stable in the absence of speculative activities on the financial side. The baseline 4D system is written as:

\[
\dot{Y} = \beta_y[(a_y - 1)(Y - Y_o) + a_k(p_k - \hat{p}_k)K + A], \quad (2.24)
\]
\[
\dot{\hat{p}}_k = \beta_k\alpha_k[f(Y, p_k, \pi^e_k) - 1], \quad (2.25)
\]
\[
\dot{\pi}^e_k = \beta_{\pi^e_k}[\frac{1+x}{2}\hat{p}_k(Y, p_k, \pi^e_k) - \pi^e_k], \quad (2.26)
\]
\[
\dot{x} = \beta[(1-x)\exp(as) - (1+x)\exp(-as)], \quad (2.27)
\]

where \(a_y\) is the propensity to spend, \(a_k\) measures the reaction of investment demand to deviations between the actual and the steady state, \(A\) is the autonomous consumption, \(\alpha_k\) captures the partial demand for capital that actually enters the financial market, and \(\pi^e_k\) is the expected rate of return.

The DSGD approach is another viable step toward a more Keynesian macro-dynamic model that incorporates “animal spirits” in inducing financial instability and macroeconomic fluctuations. It captures the highly inter-connected real and financial markets: the former is treated as stable and the latter unstable due to a population of speculative, contagious agents with fundamentalist-charist type interaction. This approach
paves a path for numerous areas of ongoing research in Keynesian disequilibrium macro-dynamics. For example, it is subsequently adopted to an open-economy scenario in Flaschel et al. (2014), which investigates the exchange rate dynamics in a two-country framework\textsuperscript{13}.

2.4 Conclusion

In his attempt to reorient economic theory and explain the Great Depression, Keynes envisioned the idea of disequilibrium macroeconomics in his General Theory, which consists of four crucial aspects: (i) the independently determined saving and investment; (ii) the price and wage rigidity; (iii) the self-fulfilling “animal spirits” and (iv), the intrinsic connection and interaction between real sector and financial sector. It had inspired not only economic thinkers such as Axel Leijonhufvud (who is amongst the first economist that interprets Keynes in disequilibrium terms) and Hyman Minsky (who brings modern capitalist finance in the interpretation of Keynes), but also mathematicians and dynamic modellers who strive to formalize Keynes’s disequilibrium thinking with the use of advanced tools in nonlinear dynamic systems.

This chapter discusses two stylized approaches of modelling Keynesian disequilibrium macro-dynamics: the Keynes-Metzler-Goodwin (KMG) approach and the Weidlich-Haag-Lux (WHL) approach that are complementary to each other. The former emphasizes on the investment-saving disequilibrium, wage-price spiral, and sluggish inventory adjustments; whereas the latter emphasizes on the role of “animal spirits” in inducing macroeconomic fluctuations.

One particular area that could potentially benefit from this line of research is the modelling of instability from the irrational behaviours of financial institutions. It is evident that the irrational and predatory behaviours of commercial banks and investment banks, coupled with a loosening regulatory environments are the key contributors to the recent 2007-2008 GFC. It would be worthwhile to incorporate the modelling of “animal spirits” of financial institutions, which will be further explored in the rest of the thesis.

\textsuperscript{13}This model is in line with Dornbusch (1976).
Chapter 3

The dynamics of endogenous money, banking, and public finance: a critical look at Modern Money Theory

3.1 Introduction

[Arthur] Burns loved to provoke disagreements among his graduate students. One day, in a class about inflation’s corrosive effect on national wealth, he went around the room asking, “what causes inflation?” None of us could give him an answer. Professor Burns puffed on his pipe, then took it out of his mouth and delared, “Excess government spending causes inflation!”

- Alan Greenspan (2007), The Age of Turbulence, page 35

It is almost an unchallenged view in the contemporary macroeconomic thinking that money is originated from commodity. It evolves to become the medium of exchange from a barter economy in order to facilitate the exchange of a growing large number of commodities. Money is nothing but a veil over the real economic activities. This common wisdom is formalized by the Quantity Theory of Money and its variations, which leads to the following two well-received propositions that are taught from Econ101 to more advanced New-Keynesian DSGE model: (i) the source of inflation is ultimately from the growth rate of money supply, hence the expansionary monetary policy of the central bank will contribute to nothing but a higher inflation in the long run. In the short run,
however, it temporarily raises output since prices are sticky; (ii) the supply of money is best treated as an exogenous variable in the long run, since it plays a non-essential role in the real activity. The real variables: real output or real interest rates for example, are determined by real factors such as time preference of money and technology.

Indeed, this theory is fundamentally right in an ancient, hypothetical society where commodity money (specie) is circulated and banks are yet to be developed. However, it is questionable in the contemporary system of fiat money, where the definition of money in itself is subject to endless debate (Werner, 2012). In such a system, the government’s spending essentially increases the monetary base (in the form of bank reserves) in the banking system, which is asset for commercial banks and liability for the central bank; the reserves in turn provide a basis for the commercial banks’ credit expansion. The creation of loans leads to further creation of private deposits, which is liability for the banks and asset for the depositors. The sum of reserves, currency (in circulation), and deposits (of various kind) thus becomes a working measure of the quantity of money by most central banks around the world. In other words, money is intrinsically debt.

Over a century ago, George Friedrich Knapp (1905) wrote his seminal work “The State Theory of Money” that systematically investigates the mechanism and consequence of using fiat money by the sovereign government and established the Chartalism school of thought. Recently there is an emergence of “Neo-chartalist” movement that strives to follow Knapp’s idea and re-investigate the mechanism of fiat money system under the modern day setting, which leads to the establishment of the “Modern Money Theory (MMT)”. The operational narration of MMT can best be summarized by the following five principles:

1. **Loans create deposits**: money supply is endogenous, since commercial banks have the power to create money through issuing loans. The banks lend first and borrow later (from the interbank market or from the central bank), in order to maintain the reserve requirement.

2. **government spending creates reserves, taxation eliminates reserves**: government spending leads to a simultaneous increase of reserves and private deposits in the commercial bank. The reverse happens when tax is paid from the private sector.

3. **Interest rate is exogenous, the quantity of reserve is endogenous**: since the central bank has a commitment to maintain an interest rate target through controlling the quantity of reserves via Open Market Operations or discount lending, the short-term interest rate is exogenous (in a control sense) and the quantity of reserves is endogenous.
4. **Government spending lowers the short-term interest rate:** government spending increases the aggregate supply of excess reserves in the banking sector, thus pushing down the short-term interest rate.

5. **Tax drives money:** since the government can finance anything with sovereign debt, why still tax? MMTers generally argue that *first*, taxation in the national government’s own currency will create demand for that currency and *second*, tax serves as a counter-cyclical fiscal tool that reduces aggregate demand and stabilizes the economy.

The first three principles have more or less reached a consensus amongst academics of Post-Keynesian strand and, to a certain degree, amongst policy makers. The fourth and fifth principles are somehow controversial, since the fourth principle contradicts the conventional IS-LM model that claims government spending “crowds out” private investment, thus pushing up the interest rates; the fifth principle contradicts common wisdom that the government should maintain a balanced budget.

Perhaps the most controversial claim made by neo-chartalists is that “governments with the power to issue their own currency are always solvent, and can afford to buy anything for sale in their domestic unit of account even though they may face inflationary and political constraints” (Tymoigne and Wray, 2013). This claim has spurred wide criticisms toward neo-chartalists *web-wide* from various schools of thought. Paul Krugman (2011) points out that neo-chartalists goes too far in its support for government budget deficits regardless of its inflationary consequence. A more severe criticism is made by Murphy (2011), who outright denies the validity of MMT, stating that “the MMT world view doesn’t live up to its promises” and that it seems to be “dead wrong”.

The descriptive part of MMT is not essentially wrong: it is no more than a factual demonstration of the operational details in the transactions between government, central bank, commercial banks, and non-banking private sector. MMTers are generally resistant (in some cases, hostile) toward critiques from non-MMTers (Lavoie, 2011). Indeed, some of the criticisms merely stem from a misunderstanding of MMT, as Randall Wray stresses: “just because government can afford to spend does not mean government ought to spend”, since “too much spending can cause inflation” (Wray, 2012) (page 187-188). Yet the normative claims and policy prescriptions made by neo-chartalists are highly debatable due to a general lack of formal quantitative analysis to justify and measure the policy claims and consequences, especially the long run inflationary consequences of fiscal/credit-driven monetary expansion. Hence it is crucial to go beyond

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the static MMT narration by incorporating a detailed quantitative and macro-dynamic discussion in order to reach more viable and accurate policy recommendations.

In a preliminary manner, this chapter proposes a set of dynamic models to fulfill this goal. It involves minimal technicality in mathematical terms, yet we are able to use these simple tools to demonstrate the dynamic effects of those operational details discussed by neo-chartalists and to contradict some of its own controversial claims. The rest of the chapter is organized as follows: section 3.2 proposes a mathematical framework that describes the process of endogenous money creation and interbank settlement; section 3.3 discusses the mechanism of open market operations and formulates a set of differential equations to describe the dynamics of short-term interbank rate, which is then applied in the subsequent discussion over the monetary effect of fiscal policy in section 3.4; section 3.5 discusses the long-term dynamics of monetary growth; finally, section 3.6 examines the real and inflationary effects of fiscal/credit-driven monetary growth and the relevant policy implications.

3.2 The process of endogenous money creation and interbank settlement

<table>
<thead>
<tr>
<th>Commercial Bank</th>
<th>Asset</th>
<th>Liability</th>
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<tbody>
<tr>
<td>Loan +</td>
<td></td>
<td>Deposit +</td>
</tr>
</tbody>
</table>

Table 3.1: Loans create deposits

In the discussion of commercial bank’s lending activities, MMT maintains that loans create deposits and refutes the notion of constant money multiplier, as all Post-Keynesians agree. The money creation process can be postulated by the following equation according to Taylor (2004):\(^2\)

\[ L = R\lambda(\lambda), \]

where

- \( L \) is the quantity of loan created by commercial bank,
- \( R \) is the quantity of unborrowed reserves,

\(^2\)Note that this ratio is a measure of endogenous money, which is in stark contrast with a commonly-known constant money multiplier. The loan-making decision of banks is unrelated to the size of unborrowed reserves, yet it is relevant in the discussion of endogenous dynamics over the course of credit cycle: during the upturn it would rise since banks desire to make more loans and hold less excess reserves and during the downturn it would fall since banks desire to make less loans and hold more excess reserves.
• λ(.) is a function that describes bank’s leverage in terms of loan-to-reserve ratio: the ratio between loans and unborrowed reserves.

We maintain that λ(.) is determined either passively by the real sector (the commercial banks lend in order to accommodate the real expansion)\(^3\), or actively by banker’s lending attitude. Therefore

\[
\dot{\lambda} = \lambda(\dot{y}, x),
\]

where \(\dot{y}\) is the growth in real sector and \(x\) is the lending attitude of banks. Following Lux (1995), we may specify that \(x \in [-1, 1]\): \(x < 0\) indicates pessimism while \(x > 0\) indicates optimism; \(x = 0\) indicates a state of neutrality.

<table>
<thead>
<tr>
<th>Bank A</th>
<th>Liability</th>
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<tbody>
<tr>
<td>Asset</td>
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<tr>
<td>Reserve +</td>
<td>Interbank Loan +</td>
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<table>
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<tr>
<th>Bank B</th>
<th>Liability</th>
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</thead>
<tbody>
<tr>
<td>Asset</td>
<td></td>
</tr>
<tr>
<td>Reserve -</td>
<td></td>
</tr>
<tr>
<td>Interbank Loan +</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: The interbank lending and borrowing

Despite the commercial bank’s ability to create money out of thin air, banks still face the constraint of reserve requirement\(^4\) as well as the desire to hold excess reserves to meet the uncertainty of deposit withdrawal. Banks may resort to interbank borrowing to fulfill the excess demand for reserves. Suppose that \(\lambda^*(.)\) is a function that captures the ratio between the amount of loans and the bank’s desired amount of reserves. Some banks would borrow additional reserves if \(\lambda_+(.) > \lambda^*(.)\), while other banks would lend out their unwanted reserves if \(\lambda_-(.) < \lambda^*(.)\). More precisely, it can be defined in the following two equations:

\[
L_+ = R\lambda_+(.) = (R_+ + BR^d\lambda^*(.) \text{  (3.3)})
\]
\[
L_- = R\lambda_-(.) = (R_- - BR^s\lambda^*(.) \text{  (3.4)})
\]

where

• \(L_+ (L_-)\) is the loans created by over-lending (under-lending) banks,

• \(R_+ (R_-)\) is the unborrowed reserves of the over-lending (under-lending) banks,

\(^3\)See Lavoie (2003) for detailed discussion.

\(^4\)This only applies to countries where reserve requirement is strictly imposed by regulatory authority.
• $\lambda_+$ ($\lambda_-$) is the loan-to-reserve ratio of over-lending (under-lending) banks,

• $BR^d$ ($BR^s$) is the demand (supply) of borrowed reserves.

It is easy to derive the aggregate excess demand for reserves from the central bank ($ER^d$) via equation (3.3-3.4):

\[
BR^d = \frac{L_+}{\lambda^*} - R_+ = R_+\left(\frac{\lambda_+}{\lambda^*} - 1\right),
\]

(3.5)

\[
BR^s = R_- - \frac{L_-}{\lambda^*} = R_-\left(1 - \frac{\lambda_-}{\lambda^*}\right),
\]

(3.6)

\[
ER^d = BR^d - BR^s
\]

(3.7)

\[
= R_+\left(\frac{\lambda_+}{\lambda^*} - 1\right) - R_-\left(1 - \frac{\lambda_-}{\lambda^*}\right)
\]

(3.8)

\[
= \frac{1}{\lambda^*}(\lambda_+R_+ + \lambda_-R_-) - (R_+ + R_-)
\]

(3.9)

\[
= \frac{L}{\lambda^*} - R.
\]

(3.10)

\[\leftrightarrow \lambda^* = \frac{L}{RR^d} \text{ (} RR^d = R + ER^d\text{).} \]

(3.11)

It is worthy to note that the new loan-to-reserve ratio $\lambda^*$ reflects the condition of the aggregate banking system regardless of heterogeneity that exists within the banking sector, which distinguishes itself from $\lambda_+$ and $\lambda_-$ that capture the endogenous monetary dynamics of separate clusters of banks that either over-lend or under-lend. Furthermore, we argue that $\lambda^*(.)$ is primarily determined by the lending attitude of banks ($x$). During the period of optimism, the aggregate banking sector may desire to hold less reserves ($\lambda^* \uparrow$); during the period of pessimism, it may desire to hold more reserves ($\lambda^* \downarrow$). We also postulate that the value of $\lambda^*(x)$ is bounded between $\lambda_0 - \theta_0$ and $\lambda_0 + \theta_0$. Hence

\[
\lambda^*(.) \in [\lambda_-^*, \lambda_+^*],
\]

(3.12)

\[
\frac{\partial \lambda^*(.)}{\partial x} > 0,
\]

(3.13)

\[
\lambda^*(x = 0) = \lambda_0.
\]

(3.14)

\[\rightarrow \lambda^*(x) = \lambda_0 + \theta_0\tanh(\theta_1x) \text{ (a special case).} \]

(3.15)

If the parameter $\theta_1$ is sufficiently large, then equation (3.15) will be approximated by the following piecewise-defined function:

\[
\lambda^*(x) = \begin{cases} 
\lambda_0 + \theta_0 & \text{if } x > 0, \\
\lambda_0 - \theta_0 & \text{if } x < 0, 
\end{cases}
\]

(3.16)

\[\text{if } \theta_1 \to \infty. \]

(3.17)
Note that there will be no additional demand for reserves from the central bank \((ER^d = 0)\) if the reserves are sufficient to accommodate \(\lambda^*\) in the aggregate banking system \((\lambda^* = \frac{L}{R})\). However, the banks will need additional reserves from the central bank if \(ER^d > 0 \rightarrow \lambda^* < \frac{L}{R}\), which may be acquired either through central bank’s discount lending facility or through selling government bonds (normally in the form of repurchase agreement) in the Open Market Operation. This will be discussed in details in the following section.

### 3.3 The disequilibrium dynamics of Open Market Operation (OMO)

<table>
<thead>
<tr>
<th>Central Bank</th>
<th>Liability</th>
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</thead>
<tbody>
<tr>
<td>Asset</td>
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<tr>
<td>Government Bond +</td>
<td>Reserve +</td>
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<td>Reserve +</td>
</tr>
</tbody>
</table>

Table 3.3: The effect of Open Market Operation

In the discussion over the conduct of monetary policy, MMT holds that the central bank is not able to control the quantity of bank reserves. The interest rate is exogenously determined in a control sense. In order to maintain the target short-term interest rates, the central bank has to constantly accommodate the excess demand of reserves from commercial banks. The excess demand for reserves leads to an upward pressure on (interbank) money market rate, thus the central bank has to buy government bonds from commercial banks via Open Market Operation in order to counterbalance the excess demand, until the rate reverts back to the target level. This process is dynamic in nature: the interbank market is constantly in disequilibrium, and the central bank has to constantly adjust its position. The dynamic interaction between the interbank market and the central bank can thus be captured by the following differential equation:

\[
\dot{r} = f(ER^d, ER^s(r^*))
\]

(3.18)

where

- \(ER^d\) is the excess demand for reserves,
- \(ER^s\) is the excess supply for reserves,
• \( r^* \) is the targeted interest rate set by the central bank.

In a parsimonious linear form, we specify that

\[
\dot{r} = \beta_1 ER^d + \beta_2 (r^* - r).
\]  
(3.19)

It is also possible to present equation (3.19) in a stochastic form, extending from a standard Ornstein–Uhlenbeck (O-U) process to capture the uncertainty of deposit withdrawals:

\[
dr = [\beta_1 ER^d + \beta_2 (r^* - r)]dt + \sigma(.)dW,
\]  
(3.20)

where

• \( \dot{r} \) is the time derivative of \( r \),
• \( ER^d \) is the excess demand for borrowed reserves,
• \( r^* \) is the exogenous target rate set by the central bank,
• \( \beta_1 \) is the adjustment speed of interbank market,
• \( \beta_2 \) is the adjustment speed of the central bank,
• \( \sigma(.) \) is the diffusion term of a Wiener process (\( dW \)).

The setting of equation (3.19) is similar to that of the Fundamentalist-Chartist interaction in the classical HAM literature\(^5\). The central bank here represents a stabilizing “fundamentalist” whereas the commercial banks are similar to the “chartists” in the sense that they constantly destabilize the market. It is important to stress that this simple equation is just one of the many forms to capture the dynamic interaction between the central bank and the commercial banks. The function \( ER^d \) may be highly complex when other dynamic variables such as the expectations over future rate are taken into account. The function \( ER^d \) maybe self-stabilizing if there exists a fixed point \( \bar{r} \) that equates \( BR^d \) and \( BR^s \) and \( \frac{\partial ER^d}{\partial r} < 0 \). In this case the interbank rate will converge to \( \bar{r} \) without the central bank’s intervention. It may also contain components that are outside the central bank’s control in terms of conventional monetary policies (at least under this linear setting), thus disabling central bank to maintain its interest rate target.

We can also rearrange equation (3.19) in the following form:

\[
\dot{r} = \beta_1 (ER^d - ER^s),
\]  
(3.21)

\(^5\)HAM stands for Heterogeneous Agent Model. For example, Beja and Goldman (1980) and Chiarella (1992) proposed a model of endogenous asset pricing with Fundamentalist-Chartist interaction. See chapter 2 for detailed discussions.
where

- \( ER^d = BR^d - BR^s = \frac{L}{\lambda} - R, \)
- \( ER^s = \frac{\beta_2}{\beta_1}(r - r^*). \)

Equation (3.21) gives the quantity of excess reserves that the central bank ought to inject \( (ER^s = \frac{\beta_2}{\beta_1}(r - r^*)) \) in order to achieve the desired target rate \( (r^*) \), assuming that the relationship between \( ER^d \) and \( \dot{r} \) is linear. Apparently this framework can be easily adopted to a scenario where a country pegs its currency to a foreign currency. The equation then becomes:

\[
\dot{e} = \gamma_1(EE^d) + \gamma_2(e^* - e) = \gamma_1(EE^d - EE^s), \quad (3.22)
\]

\[
EE^s = \frac{\gamma_2}{\gamma_1}(e - e^*), \quad (3.23)
\]

where

- \( e \) is the exchange rate,
- \( EE^d \) is the excess demand for foreign exchange,
- \( EE^s \) is the excess supply due to the central bank’s intervention.

The detailed discussion of an open economy case is beyond the scope of this chapter, hence we leave it for future research. It is clear that in this scenario, if the central bank’s foreign exchange reserves do not exceed the amount \( \frac{\beta_2}{\beta_1}(e - e^*) \), it will be unable to peg its currency.

### 3.4 The monetary effect of fiscal policy

Perhaps one of the most controversial claim made by MMT is on the effect of government spending over the short-term interest rates. Contrary to the conventional IS-LM view that the government spending “crowds out” the private spending and leads to an upward pressure on \( (\text{real}) \) interest rate, MMT maintains that the government deficit actually provides a downward pressure on the \( (\text{nominal}) \) short-term interest rate, since deficit spending increases the amount of reserves in the banking system. Taxation, on the other hand, drains the reserves from the banking system, thus raising short-term interest rate. This is illustrated in Table 3.4.
Table 3.4: The monetary effect of government spending (left) and taxation (right)

Suppose that the quantity of government spending ($\delta$) at a single time period now enters the excess supply of reserves ($ER_{\text{new}}^s = ER^s + \delta$), equation (3.21) then becomes:

$$\dot{r} = \beta_1 (ER^d - ER^s - \delta),$$

(3.24)

where $\delta$ is the government deficit and $\delta = G - T$.

In order to see the effect of government spending on the equilibrium level of interest rate, Equation (3.24) is then rearranged in the following form:

$$\dot{r} = \beta_1 ER^d + \beta_2 [r^* - \frac{\beta_1}{\beta_2} \delta] - r].$$

(3.25)

From equation (3.25), we notice that the new level of interest rate is permanently lower than the target rate $r^*$ by the amount $\frac{\beta_1}{\beta_2} \delta$ after this one-period transaction, as MMT argues. However, one has to address the possibility during the phase of credit-expansion, since more deposits are created and more reserves are demanded from the banking sector. Mathematically, according to equation (3.11):

$$\frac{\dot{\lambda}^*}{\lambda^*} = \frac{\dot{L}/L - (RR^d)/(RR^d)}{(RR^d)(RR^d)} = 0 \quad (\theta_1 \to \infty),$$

(3.26)

$$\frac{\dot{L}}{L} = (RR^d)/(RR^d).$$

(3.27)
Clearly, if the growth of private sector credit expansion outpaces the relative growth of government’s deficit expansion, it will lead to a positive $ER^d$ in the absence of the central bank’s intervention, thus pushing up interbank rate. If the growth of deficit expansion outpaces the relative growth of credit expansion, the reversed effect would happen, which leads to the central bank conducting reversed repo that drains the excess supply of reserves. In either way the interest rate is unaffected from this dynamic perspective. In a more extreme circumstance such as a severe financial crisis, $ER^d$ will rise dramatically due to the loss of banker’s confidence, and $\lambda^*$ will decrease significantly (liquidity hoarding), as we have witnessed in the aftermath of the 2007 GFC. The sudden increase of $ER^d$ will lead to dramatic interest rate hikes until the central bank takes unconventional intervention to inject new liquidity ($ER^e \uparrow\uparrow\uparrow$).

3.5 The long-run dynamics of monetary growth in a fiat money system

3.5.1 Genesis: from a special case to a general case

In previous sections, we have discussed the short run implications of monetary and fiscal policies. This section continues with the previous discussion from a long run perspective. Consider a special scenario in a hypothetical economy where, at the beginning of time, the government had just introduced fiat money. At $t = 0$, the government procures a commodity (bomb) from the private sector with the value $\delta$ by issuing perpetuity bond, and the bond eventually reaches the central bank’s balance sheet. The balance sheet effect is illustrated in Table 3.5-I. Note that there is a simultaneous increase of reserves and private deposits in the commercial bank’s balance sheet.

Upon receiving the additional reserves, the commercial bank decides to keep a portion of it as a basis $\alpha \delta$ for private lending. Given a loan-to-reserve ratio of $\lambda$, it leads to an increase of deposits by the amount $\alpha \delta \lambda$. The other portion is intended to be lent out to the (interbank) money market by the amount $(1 - \alpha) \delta$. This is illustrated in Table 3.5-II.

Since this intended amount of excess supply of reserves $(1 - \alpha) \delta$ from the aggregate banking sector poses a downward pressure toward the money market rate, the central

---

6This is purely a simplification, since in reality the central bank cannot purchase government bond directly in the primary market. Yet it is possible that the central bank purchases the full amount later on from the secondary market via Open Market Operation.
bank decides to pre-emptively sell part of the bond by equal amount in order to counterbalance this excess supply of reserves and maintain the original money market rate.\footnote{This simplification is made so that we don’t have to worry about the balance sheet of money market dealer.}

Now the commercial bank holds government bonds by the amount \((1-\alpha)\delta\). This is illustrated in Table 3.5-III. It is also important to note from the final position that the quantity of one-period government’s deficit spending equals the quantity of the private sector’s net worth, regardless of the intermediate financial transactions. In other words: “private debt is debt, but government debt is financial wealth to the private sector. (Wray, 2012)”

Suppose that the three variables being discussed: the deficit increment, the bank’s lending attitude, and the endogenous leverage ratio, are expressed as implicit functions: \(\delta(.)\), \(\alpha(.)\), and \(\lambda(.)\). The term \(\cdot\) reflects an inclusion of various economic factors that determine the net leverage of the private sector. The one-period increment of money supply \(\dot{M}\), defined as the total amount of deposits created in this process, is therefore
given by the following equation:

\[ \dot{M} = \delta(.) + \alpha(.)\delta(.)\lambda(.) \]

\[ = \delta(.)[1 + \alpha(.)\lambda(.)]. \] (3.29)

Note that in equation (3.39), \( \delta(.) \) captures the simultaneous increase of the private deposit and the increase of High Powered Money (HPM) due to government’s deficit spending; \( \alpha(.)\delta(.)\lambda(.) \) is the quantity of deposit created by banks through private lending where \( \alpha(.)\delta(.) \) is the quantity of reserves held by the aggregate banking sector.\(^8\)

We integrate \( \dot{M} \) in order to derive the stock of money over a period of time up to \( t = T \):

\[ M = \int_{0}^{T} \delta(.)[1 + \alpha(.)\lambda(.)]dt, \] (3.30)

with \( M(t = 0) = 0 \).

Equation (3.30) reflects that the aggregate dynamics of monetary growth is driven by the exogenously determined government’s fiscal stance, as well as the endogenously determined private sector’s net leverage. In this setting, the government’s deficit spending provides the only source of unborrowed reserves in the banking sector, which can further be drained or released via Open Market Operation as determined by the central bank’s interest rate policy. If the net injection of reserves via the government’s deficit spending can not meet the excess demand of reserves by the private banking sector, the excess part has to be borrowed from the central bank via discount lending facility, or certain unconventional asset swaps that involves the purchase of private assets by the central bank. The net leverage of private sector in this special case \([1 + \alpha(.)\lambda(.)]\) is determined by economic factors that drive the complex heterogeneous interactions amongst private firms/households, the commercial bank’s lending attitude, as well as the central bank’s policy stance.

3.5.1.1 A digression on the hedging assumption: a new perspective on the government’s budget constraint

One may question the balance sheet effects of the government’s paying interest on the perpetuity as well as the effect of raising revenue through taxation over the course of time. Apparently, taxation reduces the total amount of reserves in the banking system whereas interest payment to sovereign debt holders has the reversed effect. Here we set

\(^8\)For simplicity, we assume that there are no private withdrawal, hence there is no currency in circulation.
a condition to simplify the analysis by setting the government’s fiscal role as a control variable: all the tax revenue collected by the government is distributed to serve the interest payments to the sovereign bond holders, thus the aggregate monetary effect (in terms of the total quantity of reserves in the banking sector) is not affected in this process of wealth redistribution. More specifically, the single period hedging constraint is formulated as:

\[ \tau_Y Y(.) + \tau_S S(.) = (r^* + \Delta r) D_G = r_G D_G. \]  (3.31)

We may also loosen the perpetuity assumption by introducing a variety of the term structure of interest rates:

\[ \tau_Y Y(.) + \tau_S S(.) = \sum_{i=n}^{N} (r^* + \Delta r_i) D_G(i). \]  (3.32)

If the government can not meet the hedging constraint during single time periods, it should do so over the course of time before the critical point \( T_0 \) that triggers Ponzi inflation\(^9\). Hence the continuous time hedging constraint is formulated as:

\[ \int_0^{T_0} \tau_Y Y(.) dt + \int_0^{T_0} \tau_S S(.) dt = \int_0^{T_0} \sum_{i=n}^{N} (r^* + \Delta r_i) D_G(i) dt. \]  (3.33)

If the government can not even meet the hedging constraint over a sustained period of time, it will accumulate Ponzi debt that is issued solely for financing interest rate payment, which sums up to

\[ P_G(T_0) = \int_0^{T_0} \sum_{i=n}^{N} (r^* + \Delta r_i) D_G(i) dt - \int_0^{T_0} \tau_Y Y(.) dt - \int_0^{T_0} \tau_S S(.) dt, \]  (3.34)

where

- \( r_G = r^* + \Delta r \),
- \( \tau_Y \) is the G&S tax collected from the real sector \((Y(.))\),
- \( \tau_S \) is the Tobin type tax collected from the speculative sector that penalizes capital gains \((S(.))\).

\(^9\)Here we coin the term “Ponzi inflation” to describe the inflationary consequence due to the government’s failure to meet the interest payment through taxation, which would potentially result in an excess and exponential accumulation of monetary base in the private sector. It would occur not only through the exponential compounding of interest payment but also through the term structure of interest rate that directly influences inflationary expectations. The modelling of Ponzi debt in relation to its real and inflationary consequence could be an interesting aspect for future research.
Chapter 3. The dynamics of endogenous money, banking, and public finance: a critical look at Modern Money Theory

- $i$ is the term structure of interest rate,
- $r^\star$ is the benchmark interbank rate determined by the central bank and $\Delta r_i$ is the mark-up spread varied by the term structure,
- $D_G(i)$ is the size of government debt in different terms to maturity,
- $P_G(T_0)$ is the size of public sector Ponzi debt.

We may investigate the economic impact of Ponzi debt in our future research. It is also worthy to note that the interest rate spread is endogenously determined by the sovereign bond market. When the supply of government bond exceeds the demand, it would result in a lower bond price and a higher spread and \textit{vice versa}. We formulate that

$$\dot{\Delta r} = \theta (D^s_G - D^d_G) = \theta ED^s_G,$$

where

- $ED^s_G = D^s_G - D^d_G$ is the excess supply of government bond,
- $\theta$ is the adjustment parameter in the bond market.

Interestingly, we note that the determinant of long-term interest rate in the sovereign bond market is in stark contrast with the determinant of short-term interbank rate, since an increase of excess supply for government bond would lower the bond price and \textit{raise} the bond yield, while an increase of excess supply for reserves would \textit{lower} the interbank rate. Furthermore, the short-term money market rate is under the firm grip of the central bank while the longer-term bond rate is primarily driven by the market forces. One may argue that the central bank can also lower the long-term rate by purchasing the longer-term government security, yet it may not affect the slope of the yield curve since it would result in a simultaneous decrease of short-term rate as well due to the the increase of $ER^\delta$ when the central bank purchases the long-term bond, unless the interbank rate has already reached zero bound. The discrepancy between the short-term and long-term rate would potentially drive the central bank to adjust its short-term rate due to the market force in order to flatten the yield curve and to stabilize inflation expectations. The detailed quantitative discussion regarding this aspect is beyond the scope of this chapter, hence we leave it for future discussions.

Although the hedging constraint here is merely a simplification in the context of our current discussion, it alludes to a very important role of taxation that neo-chartalists generally ignore: \textit{tax serves government debt}. The revenue from taxation may help hedging the interest payment to the sovereign debt holders so that
the deficit growth is controllable in the long run. Otherwise, in the absence of taxation, the deficit would eventually grow and evolve into an uncontrollable Ponzi scheme, since the government is obliged to increase its deficit in the next period to serve the interest payment, which adds further to the money base in an excessive and undesirable way that potentially poses an upward pressure on inflation and the long-term bond yield. Eventually an increasing deficit growth and a rising inflationary expectation will interact and form an uncontrollable positive feedback loop, which would cause an undesirable and catastrophic inflation in the long run. Furthermore, if the government penalizes the speculative sector by levying a higher Tobin-type tax $\tau_S$, it would not only curb speculative inflation effectively, but also potentially benefit the real sector since now we require a lower $\tau_Y$ in order to fulfill this constraint. As the simplest case of single period constraint, in the absence of a speculative sector with the perpetuity assumption ($i \to \infty$), we have:

$$\tau_Y Y = D_G r_G, \quad (3.36)$$

$$\to \tau_Y = \frac{D_G Y r_G}{\psi} = \psi r_G \quad (\psi = \frac{D_G Y}{Y}) \quad (3.37)$$

$$\to \dot{\tau}_Y \tau_Y = \frac{\dot{\psi}}{\psi} + \frac{\dot{r}_G}{r_G}. \quad (3.38)$$

In this case, the tax rate equals the perpetuity yield adjusted from the Debt-to-GDP ratio in order to fulfill the hedging constraint. An increase of government debt will raise the tax rate to fulfill this constraint if the government fails to stimulate the economy in the long run.

We take one step further to postulate a generalized scenario:

$$\dot{M} = \delta(.) \Lambda(.), \quad (3.39)$$

$$M = \int_0^T \delta(.) \Lambda(.) dt, \quad (3.40)$$

$$\text{s.t.} \quad \int_0^T \tau_Y Y(.) dt + \int_0^T \tau_S S(.) dt = \int_0^T \sum_{i=n}^N (r^* + \Delta r_i) D_{G(i)} dt, \quad (3.41)$$

where

- $\delta(.)$ captures the fiscal stance of the public sector,
- $\Lambda(.)$ captures the endogenous dynamics of private sector’s net leverage.
We maintain that $\Lambda(\cdot) \in [0, \Lambda_+]$. The upper boundary parameter $\Lambda_+$ reflects the “Minsky moment”, whereas the zero boundary reflects a “doomsday scenario” when everything ceases to exist. We further discuss four different regimes of $\Lambda(\cdot)$ below:

1. $\Lambda(\cdot) = \bar{\Lambda} = 1$ (**fiat money system with banks acting as non-leveraged intermediaries**): without endogenous monetary dynamics, the monetary growth is solely determined by the government’s fiscal stance. Banks in this hypothetical scenario act purely as full-reserve, non-leveraged intermediaries. Hence the net government debt equals net private sector’s wealth ($D_G = W_e$).

2. $\Lambda(\cdot) = \bar{\Lambda} > 1$ (**fractional reserve banking system with constant money multiplier**): in this case, $\bar{\Lambda}$ represents fractional reserve banking system where banks are reserve-constrained financial intermediary with constant leverage position. The commonly received textbook money multiplier only holds true in this special case. The net debt position of the private sector is the difference between total money stock and net government debt $D_G$: $\bar{D}_e = D_G(\bar{\Lambda} - 1)$.

3. $\Lambda(\cdot) > 1$ (**endogenous money**): this scenario represents a more generalized case of endogenous money, where banks act purely as profiteers without reserve constraint, since they can always obtain reserves from the central bank upon request anytime and everywhere. The net debt position of the private sector now becomes $D_e(\cdot) = \int_0^T \delta(\cdot)\Lambda(\cdot) - D_G$.

4. $\Lambda(\cdot) < 1$ (**severe austerity and the “doomsday” scenario**): the fiscal-driven money stock is eroded through taxation and there is a lack of private sector endogenous expansion. Further taxation destroys wealth from private sector until $\Lambda(\cdot) \to 0$ and the economy reaches the “doomsday” scenario.

Now we examine equation (3.40) from a dynamic perspective. As McLeay et al. (2014) argues, despite the commercial banks’ ability to create credit out of thin air, the endogenous monetary expansion in the private sector is still bounded by factors such as the market forces, the bank’s perceived risks over lending, as well as the regulatory controls set by the authority in the short run. In the long run, the inherent self-reinforcing and self-destructive dynamics of private sector leverage is well depicted by the “Financial Instability Hypothesis” of Minsky (1975). The mathematical modelling of Financial Instability Hypothesis is well documented in the literature of non-linear economic dynamics and heterogeneous agent modelling (Chiarella and Di Guilmi, 2011, Keen, 1995, Taylor and O’Connell, 1985). The detailed discussion is beyond the scope of this chapter. On the other hand, the growth of fiscal-driven monetary expansion by the public sector may lead to an exogenous, controllable monetary growth, given that the government acts at
least as a “speculative” borrower in the sense that the inflows of tax revenue is able to meet the interest outflows. Apparently, in an economy where the financial system is less developed and strict regulation is imposed on the banking sector, the fiscal expansion will play a much more important role in determining the long-term monetary growth. The mismanagement of government deficit will lead to a more pronounced economic consequence in the long run. Yet in an economy where financial markets are full-fledged and the banks are subject to more loosened regulations, the role of a controllable, fiscal-driven monetary expansion will be hampered and will become more trivial in size, since private sector credit expansion plays a much more significant role in determining the monetary growth and real activities in the short-to-medium run. However, it is still inherently bounded by the Minskyan dynamics if the growth of private sector monetary expansion is unregulated and follows an exponential path, since \( \Lambda(.) \) cannot grow unlimitedly in a realistic sense. Clearly it poses a significant regulatory challenge since the endogenous monetary dynamics is more under the grip of the private sector. In this scenario, it is more crucial to implement policies that aim to curb speculative activity in order to avoid the Minskyan dynamics and to promote long-term financial stability.

### 3.5.2 The time-dependent cyclical dynamics of \( \Lambda(.) \): a special case

We further formulate some specific functional forms to support the discussions in the previous subsections. We first simplify the private sector endogenous dynamics \( \Lambda(.) \) by specifying a special time-dependent case:

\[
\Lambda(t) = \bar{\Lambda} \exp[\exp(\sin(k_1 t)) \cos(k_2 t)].
\]

(3.42)

Setting \( t_1 = t_2 \) initially, equation (3.42) essentially captures a regular set of boom-bust cycles, with a gradual surge of \( \Lambda(t) \) during the boom period and a sudden slump during the bust period. It becomes less regular if \( t_1 \neq t_2 \). We apply it to characterize the cyclical dynamics of \( \Lambda(.) \) in the subsequent discussions.

The public sector’s deficit growth \( \delta(t) \), on the other hand, is assumed to expand at an exogenous rate \( s_G \) that ultimately leads to the growth of money base \( \text{MB}_G \). Hence we have\(^{10}\):

\[
\begin{align*}
\delta(t) &= \delta_0 \exp(s_G t), \\
\rightarrow \text{MB}_G(T) &= \int_0^T \delta_0 e^{s_G t} \, dt = \frac{\delta_0}{s_G} (e^{s_G T} - 1), \\
&= D_G(T).
\end{align*}
\]

Here we only limit our attention to the deterministic component. It is also possible to assume that \( \Lambda(t) \) and \( \delta(t) \) follow a stochastic path. We leave this for future discussions.
Overall, the monetary dynamics is given by the following equation:

\[ M(T) = \delta_0 \bar{\Lambda} \int_0^T \{\exp[s_G t + \exp(\sin(k_1 t)) \cos(k_2 t)]\} dt. \]  

(3.46)

We numerically solve this integral and illustrate equation (3.42-3.46) in Figure 3.1. The parameters are set as follows: \( s_G = 0.05, k_1 = 0.7, k_2 = 0.7/0.68/0.5/0.1, \delta_0 = 1, \) and \( \bar{\Lambda} = 10. \) The left panel of double y-axis plot shows the cyclical dynamics of \( \Lambda(t) \) in relation to the growth of government deficit \( \delta(t) \), whereas the right panel of double y-axis plot shows the long run accumulation of government debt \( D_G(t) \) in relation to the dynamics of aggregate money stock \( M(t) \). The top panel illustrates the regular case \((k_1 = k_2)\) whereas the lower panels illustrate the less regular cases \((k_1 \neq k_2)\). It is worthy to note that the fiscal-driven monetary growth rate \( s_G \) still governs the long-term trend rate of aggregate monetary growth despite the variations of the private sector net leverage \( \Lambda(t) \).

In order to examine the relative importance of public sector fiscal-driven money stock over private sector credit-driven money stock, we further calculate the weight of \( D_G \) in
Chapter 3. The dynamics of endogenous money, banking, and public finance: a critical look at Modern Money Theory

Figure 3.2: $\omega_e$ vs. $\omega_G$

Figure 3.3: $\omega_e$ with varying $\Lambda$
comparison to the weight of $D_e$ in Figure 3.2, which are defined as:

$$\omega_G(.) = \frac{D_G}{D_G + D_e(\cdot)} = 1 - \omega_e(\cdot),$$  \hspace{1cm} (3.47)  

$$\omega_e(.) = \frac{D_e(\cdot)}{D_G + D_e(\cdot)}.$$ \hspace{1cm} (3.48)

The parameters in Figure 3.2 are set the same as our previous discussions. We observe that $\omega_G(.)$ follows a regular cyclical patterns when $k_1 = k_2 = 0.7$. Yet it becomes less regular when we set $k_1 \neq k_2$ in the subsequent three plots. In Figure 3.3, we set $k_1 = k_2 = 0.7$ while varying the value of $\bar{\Lambda}$ ($\bar{\Lambda} = 1/2/10$). We observe that when $\bar{\Lambda}$ is relatively small, the fiscal-driven money stock plays relatively a more important role in determining the aggregate money stock. As $\bar{\Lambda}$ increases over time, the credit-driven endogenous money in the private sector starts to play a much more significant role in determining the aggregate money stock.

### 3.5.3 The endogenous dynamics of monetary growth: a generalized case

We may also assume an one-off increase of government deficit $\bar{\delta} = D_G$ to capture the autonomous, endogenous monetary dynamics of the private sector in three different forms:

$$M_e(T) = D_G \int_0^T \Lambda(.) dt \quad \text{(flow form)} \hspace{1cm} (3.49)$$

$$= D_G \Lambda s(.) \quad \text{(stock form: } \dot{\Lambda} s(.) = \Lambda(.) \text{)}$$ \hspace{1cm} (3.50)

$$= D_G e^{s(.)T} - 1 \quad \text{(endogenous rate form)} \hspace{1cm} (3.51)$$

$$= D_e(.) \quad \text{(total stock of private debt).}$$ \hspace{1cm} (3.52)

where $s(.)$ is the endogenous monetary growth determined by other autonomous (time-independent) economic factors.

Overall, we postulate that the aggregate dynamics of monetary growth can be approximated by the weighted growth rate of both fiscal-driven public sector and credit-driven private sector. Hence:

$$M(T) = M_0 \exp[s_T(.)T],$$ \hspace{1cm} (3.53)

$$= M_0 \exp[(\omega_G(.)s_G + \omega_e(.)s(.)T],$$ \hspace{1cm} (3.54)

$$= M_0 \exp[(s'_G + s'(\cdot))T],$$ \hspace{1cm} (3.55)
Chapter 3. The dynamics of endogenous money, banking, and public finance: a critical look at Modern Money Theory

where

- $M(0) = M_0$,
- $s_T(.) = \omega_G(.)s_G + \omega_e(.)s(.)$,
- $\omega_G(.) = \frac{D_G}{D_G + D_e(.)} = \frac{s'_G}{s_G} = 1 - \omega_e(.)$, $\omega_e(.) = \frac{D_e(.)}{D_G + D_e(.)} = \frac{s(.)}{s'}$,
- $s'_G + s'(.)$ is the weight-adjusted rate. $s'_G = \omega_G(.)s_G$ and $s'(.) = \omega_e(.)s(.)$.

3.6 The long-run effect of monetary expansion: speculative vs. fiscal inflation

We have now reached a point to argue that under a fiat money system, the growth of government deficit may provide an exogenous, controllable source of money supply, which is also the main determinant of long-term trend rate monetary growth, given that the government acts at least as a “speculative” borrower. Yet in the short-to-medium run, the overall monetary dynamics would be perturbed by the endogenous dynamics of private sector leverage with Minskyan character. A more important question is yet to be answered: what is the long run economic effects of fiscal/credit-driven monetary growth in nominal and real terms?

Let’s start with the inflationary effect. One may resort to the commodity-money-barter-economy view derived from the Quantity Theory of Money (QTM), which asserts that in the long run, the growth of money supply will lead to nothing but a proportional increase of inflation. Clearly, under the fiat money system with endogenous monetary dynamics of the private sector, the boundary between nominal effect and real effect of long-term monetary growth is much less clear-cut, since real sector growth leads to a simultaneous increase in monetary growth under endogenous money framework. Furthermore, the QTM view is generally rejected by academics of Post-Keynesian strand due to its poor empirical evidence, as DeGrauwe and Polan (2005) claims:

"The quantity theory of money is based on two propositions. First, in the long run, there is proportionality between money growth and inflation, i.e., when money growth increases by x% inflation also rises by x% .... We subjected these statements to empirical tests using a sample which covers most countries in the world during the last 30 years. Our findings can be summarised as follows. First, when analysing the full sample of countries, we find a strong positive relation between the long-run growth rate of money and inflation. However, this relation is not proportional. Our second finding is that
this strong link between inflation and money growth is almost wholly due to the presence of high-inflation or hyperinflation countries in the sample. The relation between inflation and money growth for low-inflation countries (on average less than 10% per year over 30 years) is weak, if not absent.”

In the context of this chapter, we do not intend to draw a clear line between asset price inflation and G&S inflation, given the complex nature of aggregate price and the vague boundary between speculative markets and G&S markets. The speculative activities that we are about to discuss may stem from either commodity markets, which would directly affect the aggregate price level through cost channel and then possibly transmit to the asset market, or initially from the asset markets, which would later on transmit back to the G&S market and affect aggregate price due to wealth effect. We define inflation \( \pi \) in this context as

\[
\pi = \xi_Y \pi_Y + \xi_S \pi_S, \tag{3.56}
\]

where \( \xi_Y \) and \( \xi_S \) measures the proportion of aggregate price increase due to G&S markets and asset markets respectively (\( \xi_Y + \xi_S = 1 \)).

As a starting point, we argue that the monetary growth in the long run will contribute to either real growth or inflation. On one hand, real growth will lead to monetary growth given its endogenous nature; on the other hand, the pro-active lending activities in the banking sector will stimulate the real sector growth. It implies that the Debt-to-GDP ratio, defined as the ratio between monetary aggregate and nominal GDP \( (M(t)/Y(t)) \), ought to be mean-reverting in the long run. Suppose \( M(t) \) and \( Y(t) \) both grow at variable rates \( s \) and \( g + \pi \) respectively, namely \( M(t) = M_0 e^{st} \) and \( Y(t) = Y_0 e^{(g+\pi)t} \). We postulate the long-term adjustment equation as follows:

\[
s - g - \pi = 0 \leftrightarrow \pi = s - g, \tag{3.57}
\]

\[
\frac{M(t)}{Y(t)} = \frac{M_0}{Y_0} e^{(s-g-\pi)t}. \tag{3.58}
\]

**Proposition 3.1 (Speculative Inflation):** The private sector endogenous monetary growth is non-inflationary if it is utilized in productive sectors. *Speculative inflation* would arise and persist through a positive feedback mechanism if credit is further extended for speculative purpose beyond a certain critical point of maximum capacity growth rate in the real sector.

As is illustrated in Figure 3.4, we denote \( \bar{g} \) as the maximum capacity growth rate; \( g_c \in (0, \bar{g}) \) as the below-capacity growth rate of the real sector; \( s_c \) as the endogenously

\(^{11}\)Goods and Services.
determined monetary growth that serves productive purpose. It is clear that $s_e$ is non-inflationary since it is driven by the proportionate increase of real growth, if banks are able to accommodate real growth always and everywhere at any time. Hence $\pi = s_e - g_e = 0$. Beyond $(\bar{s}, \bar{g})$, an external push of monetary growth will eventually contribute to a proportionate increase of inflation ($\Delta \pi$).

Now let us put this scenario in a dynamic context, as illustrated in Figure 3.5. Suppose the real sector expands with an accelerating rate: in the course of time the economy transits from a low-growth regime to a high-growth regime, which approaches a certain
critical point near maximum capacity growth rate. There is less and less room for further growth opportunity from the real sector. Yet, given the short-term profit seeking nature of the private sector, the economy would therefore be vulnerable to any potential inflationary shocks that would encourage leveraged speculative activities without contributing to real growth. In the presence of certain demand-pull/cost-push shocks, more credit may be extended to gain from such speculative opportunities. Inflation will arise eventually from a single price to aggregate price over the course of time due to speculative-fuelled growth of endogenous money \( (s_p) \), which is self-reinforcing in nature \( (p \uparrow \rightarrow s_p \uparrow \rightarrow \pi \uparrow \rightarrow s_p \uparrow \rightarrow \pi \uparrow \) \). Furthermore, if more economic resources are directed from productive sectors to speculative sectors, it would potentially “crowd out” and hamper the growth capacity \( \tilde{g} \downarrow \) of the real sector in the long run, which leads to even further upward pressure on inflation \( (s_p \uparrow & \tilde{g} \downarrow \rightarrow \pi \uparrow \) \). Similarly, the “Ponzi finance” scenario of public sector that we have discussed in section 3.5.1.1 would not only lead to an unproductive accumulation of base money beyond \( (\tilde{s}, \tilde{g}) \), but also accelerate this process of triggering speculative inflation.

**Proposition 3.2 (Fiscal Inflation):** Excess government spending leads to persistent, exogenous inflation. In a low-growth regime, the exogenous monetary expansion due to government’s fiscal expansion is non-inflationary. In a high-growth regime, the government’s fiscal stimulus that aims to further capacity growth is more-often-than-not inflationary.

![Figure 3.6: The inflationary effect of excess government spending](image)

We illustrate this proposition in Figure 3.6. Clearly, the exogenous, fiscal-driven monetary expansion \( s_G \uparrow \) below maximum capacity would lead to a proportionate increase of fiscal-driven real growth \( g_G \), therefore \( \pi = s_G - g_G = 0 \). Beyond the maximum capacity,
the government may continue to use fiscal instrument that aims to expand the maximum capacity \( \bar{g} \uparrow \) such as building more infrastructures or funding R&D. Yet the effect is less likely to be proportionate in the presence of diminishing return to scale (\( \frac{\Delta \bar{g}}{\Delta G} < 1 \)). In an extreme case when the intended fiscal stimulus is rendered ineffective, it may even in the long run “crowd out” the productive resources that leads to a decrease of maximum capacity (\( \bar{g} \downarrow \)), which would potentially lead to further inflationary pressure before the economy adjusts back to a non-inflationary state with lower growth. Another danger of fiscal stimulus in a high growth regime is that it may encourage speculative activities near the critical point of maximum capacity growth rate, as we have discussed previously. It implies that in a low growth regime, the government’s fiscal stimulus is not only necessary but also potentially effective. Yet in a high growth regime, further fiscal stimulus, accompanied by loosening monetary policy, may push the economy to a critical point that is vulnerable to any potential inflationary shocks due to the emergence of leveraged speculative behaviour. Indeed, in the absence of any tangible fiscal constraint under the fiat money system, the government is more likely to over-spend rather than under-spend even if we preclude the possibility of the self-fulfilling speculative inflation in the private sector. It is the excess government spending that causes persistent, exogenous inflation, as Arthur Burns claims in Alan Greenspan’s classroom during his formative years at graduate school.

Formally, we postulate a 3D non-linear dynamic system that synthesizes the two propositions above. We assume in this case that the real sector always grows at its variable maximum capacity, whereas the government’s excess spending causes an exogenous level of inflation \( \pi^* \):

\[
\dot{\pi} = \alpha_1 s_p - \alpha_2 (g - \bar{g}) + \alpha_3 (\pi^* - \pi), \quad (3.59)
\]

\[
\dot{s}_p = \theta_1 (\pi - \pi^*)^{\theta_2} - \theta_3 (g - \bar{g})^{\theta_4}, \quad (3.60)
\]

\[
\dot{\bar{g}} = -\gamma_1 s_p^{\gamma_2} + \gamma_3 (\bar{g} - g), \quad (3.61)
\]

where

- \( \pi \) is inflation rate (\( \pi^* \) is the rate of exogenous inflation determined by deficit-driven monetary growth),
- \( s_p \) is the growth rate of speculative-driven endogenous money,
- \( g \) is maximum capacity growth that deviates from the long-term level (\( \bar{g} \)) due to “crowd-out” effect of speculation,
- \( \alpha_x, \theta_x, \) and \( \gamma_x \) are adjustment parameters.
Equation (3.59) captures a positive relationship between speculative endogenous money and inflation. Furthermore, a rise of maximum productive capacity will relieve the inflationary pressure and re-direct the financial activities from speculative sectors to productive sectors. We assume that the level of fiscal inflation as determined by the government’s fiscal stance ($\pi^*$) provides a stabilizing benchmark to this law of motion.

The subsequent two equations capture the speculative and productive dynamics respectively. We attribute the non-linearity of this sub-dynamic system solely to speculative factors, as is reflected in the power components that are associated with $s_p$ ($\theta_2$, $\theta_4$, and $\gamma_2$). We maintain that $\theta_2 > 1$, since higher inflation further motivates speculative activities.

**Proposition 3.3:** The system (3.59-3.61) has a unique steady state solution ($\pi = \pi^*, s_p = 0, g = \bar{g}$). It is locally unstable in the presence of demand-pull/cost-push inflationary shocks.

By setting $LHS = 0$ we can easily derive this steady state solution. The Jacobian at this equilibrium is derived as

$$
\begin{pmatrix}
-\alpha_3 & \alpha_1 & -\alpha_2 \\
0 & 0 & 0 \\
0 & 0 & -\gamma_3
\end{pmatrix}.
$$

Clearly, this equilibrium is always unstable since $Det(J) = 0$, which violates the Routh-Hurwitz condition\(^{12}\). It is also worthy to note that the 2D sub-dynamics of (3.59) & (3.61) (excluding speculative dynamics) is always stable, as $Tr(J_2) = -\alpha_3 - \gamma_3 (< 0)$ and $Det(J_2) = \alpha_3 \gamma_3 (> 0)$.

We do not go beyond further analysis of the system (3.59-3.61), but rather to describe the global characteristics of the system by running two representative simulations. We also simulate the Debt/GDP ratio discussed previously. Recall that

$$\frac{M(t)}{Y(t)} = \frac{M_0}{Y_0} e^{(s-g-\pi)t}. \quad (3.62)$$

\(^{12}\)The Routh-Hurwitz necessary and sufficient condition for the stability is given by: $Tr(J) < 0$, $J_1 + J_2 + J_3 > 0$, $Det(J) < 0$, and $-Tr(J)(J_1 + J_2 + J_3) > 0$ (Chiarella and Flaschel, 2000).
Since we have now introduced speculative money \( s_p \), we would modify the long run adjustment equation as follows:

\[
\begin{align*}
\dot{s} &= s_e + s_G + s_p = g + s_G + s_p, \\
\dot{s} - g - (\pi + \pi^*) &= g + s_p + s_G - g - \pi - \pi^* \\
\dot{s} &= s_p, \\
\frac{M(t)}{Y(t)} &= \frac{M_0}{Y_0} e^{(s_p - \pi)t}.
\end{align*}
\] (3.63) (3.64) (3.65) (3.66)

The following parameters are assigned in the simulation: \( \alpha_1 = 0.19, \alpha_2 = -0.03, \alpha_3 = 0.01, \theta_1 = 0.01, \theta_2 = 4(I)/1(II), \theta_3 = 0.03, \theta_4 = 0.4(I)/1(II), \gamma_1 = 0.06, \gamma_2 = 0.08(I)/1(II), \gamma_3 = 4.3, \pi^* = 0.05, \bar{g} = 0.1, M_0/Y_0 = 1. \) The initial value of the simulation is assumed to be on the equilibrium except \( s_p \), where an initial shock of \( s_p(1) = 0.01 \) is assigned to generate the subsequent dynamics. In Figure 3.7, we observe irregular fluctuations and seemingly complex dynamics of the three variables which eventually converges toward a strange attractor. Albeit locally unstable near

\[\text{Figure 3.7: The bounded fluctuations of } (\pi, s_p, g)\]
the fixed point, the global dynamics is stable due to the stabilizing components of the system. From the top-left panel we observe that real growth capacity generally moves in tandem with inflationary dynamics, yet it is persistently pushed below its long-term value $\bar{g} = 0.1$ due to the “crowd-out” effect of speculative activities. This is further illustrated in the mid-right plot where we observe the negative relationship between $s_p$ and $g$. The top-right panel shows that $s_p$ would not only give rise to endogenous inflation, but also lead to more volatility of $\pi$ as $s_p$ increases. More interestingly, we observe the gradual surge and sudden slump of the Debt/GDP dynamics on the bottom-left panel, as well as a similar dynamics in the long run adjustment equation $s_p - \pi$ on the bottom-right panel, which resembles a real world case of exponential development of the Debt/GDP ratio, from a simulative perspective.

Figure 3.8: The unbounded growth of speculative inflation

In the second scenario, as shown in Figure 3.8, the endogenous dynamics of speculative-driven inflation becomes unbounded globally and rises through an exponential path, which eventually leads to the subsequent decline and destruction of real sector ($g \downarrow$) due to “crowd-out” effect of speculative activity. It can be seen from the bottom-left
panel that the Debt-to-GDP ratio also grows out of bound as $s_p$ consistently rises faster than $\pi$ (bottom-right) in this scenario (the long run becomes infinity). It is clear that in the absence of regulatory controls over inflation such as a strict mandate of inflation targeting imposed by the central bank that aims to curb inflation in both real market and speculative financial market, the private sector endogenous inflationary dynamics may potentially grow unboundedly over time, which would eventually erode real growth in the long run.

3.7 Conclusion

Nearly 110 years ago, Georg Friedrich Knapp, driven by his enquiry over the mechanics of using modern fiat money by sovereign governments, wrote “The State Theory of Money” that established the chartalist school of monetary theory. In recent years, there is a surge of “neo-chartalists” movements that strives to bring more realism to the analysis of monetary and fiscal policy through an analysis of the operational details of monetary transactions. Despite its high relevance in today’s policy arena that demands a thorough understanding over the mechanism of fiat money system, MMT is generally not well-received by mainstream as well as Post-Keynesian academics due to its controversial claims derived from over-simplified static analysis, especially in its audacious support for the government’s deficit expansion with a minimal consideration of its potential long run inflationary effects.

This chapter proposes a set of models that aims to take a further investigation over the static MMT analysis from a dynamic perspective. It is, to a certain extent, extended from earlier works of disequilibrium asset pricing and macro-dynamical models (Beja and Goldman, 1980, Chiarella, 1992, Keen, 1995). We contend that some of the claims made by Neo-chartalists are questionable due to its omission of dynamic and behavioural analysis. We make a brief summary of this chapter in the following four aspects:

1. The monetary effect of fiscal policy: the claim made by MMT that government’s deficit expansion lowers (nominal) short-term interest rate due to an increase of reserves in the aggregate banking system is only true in a static sense. Yet when we put dynamics into perspective, this claim is questionable, since the rising deficit may accommodate the rising excess demand for reserves during the expansionary phase of credit cycle. Even if the growth of government deficit outpaces the growth of excess demand for reserves, resulting in a negative aggregate demand for excess reserves in the banking system, the central bank will intervene constantly to maintain the target rates, thus leaving the overall dynamics
unaffected. Furthermore, the government’s excess issuing of long-term bond may potentially push up long-term interest rate and affect the term structure, thus raising the slope of the yield curve of the sovereign bond.

2. **The hedging purpose of taxation**: Neo-chartalists generally ignore the counter-balancing effect of tax inflows against interest outflows. The tax collected by the government may serve as a hedging payment against interest payable to sovereign debt holders, so that in the long run, deficit will remain in balance or grow in a *controllable* manner. If the government is fully financed by Ponzi-type borrowing, the obligation to increase deficit in the next period and the mounting pressure of inflation in the long run will develop into a positive feedback loop that would lead to an *uncontrollable* explosion of debt accumulation of the public sector, which would eventually cause catastrophic inflation.

3. **The long-term determinant of monetary growth**: the long run monetary dynamics is determined by a complex interaction between the public sector’s fiscal stance and the Minskyan dynamics of the private sector leverage. If the government acts at least as a “speculative borrower” in Minskyan terms, the growth of deficit would lead to an exogenous and controllable source of long-term monetary growth. The private sector leverage on the other hand, is subject to cyclical fluctuations of Minskyan dynamics in the absence of a pro-active intervention that aims to curb excessive speculation.

4. **The inflationary and real effect of monetary growth**: we argue that monetary growth, whether endogenously driven by the credit growth of private sector, or exogenously driven by the deficit growth of the public sector, is not inflationary if it serves real growth below its maximum capacity rate. Persistent inflation arises when credit is extended to speculative sectors beyond the maximum capacity growth rate. The potential “crowd-out” effect due to speculation would temporarily lower the capacity growth rate and further lead to higher inflation, as productive resources and activities may be destroyed and diverted to speculative activities due to higher nominal return. Furthermore, we maintain that the government’s proactive fiscal policy that aims to stimulate growth is non-inflationary if the economy is in a low-growth regime. Yet it may be potentially inflationary in a high-growth regime not only by accelerating speculative inflation of the private sector, but also causes persistent, fiscal inflation even if we preclude speculative factors when the government spends excessively and unwisely to unproductive sectors beyond the maximum capacity. Hence it is important for the government to maintain the hedging constraint as well as to maintain its fiscal stance in a counter-cyclical manner in order to maximize growth and to minimize inflation.
More importantly, the modelling framework proposed in this chapter opens several potential paths for future research. Here we list a few: (i), a detailed discussion of an open economy with pegged exchange rate is one area worthy to pursue in future research from this modelling framework, since it is highly relevant in the context of long run Sino-US economic relationship; (ii), we may examine the economic effects of Ponzi debt issued by public sector in real and inflationary terms; (iii), it is worthwhile to model the term structure of interest rate driven by the discrepancy between short-term interbank rate and long-term bond yield and its implications for the conduct of monetary and fiscal policy; (iv), we may further extend the dynamic model of speculative inflation proposed in section 3.6 by adding the short-to-medium run factors over the course of credit cycle; (v), in our current discussion, we have excluded the financial markets with private equity and bonds for the sake of emphasizing the role of banks in determining endogenous monetary expansion. We may extend this aspect in our future discussions with a detailed portfolio approach, similar to Asada et al. (2009) and Charpe et al. (2011b); (vi), we may examine and model the heterogeneity of banking sector that propagates credit cycles from a boundedly-rationality perspective, extending from the modelling framework proposed at the beginning this chapter. There is already numerous literature that models Financial Instability Hypothesis from the perspective of heterogeneous interactions of investors with hedging, speculative, and Ponzi characters. Yet there is a gap in the current literature regarding the modelling of bounded rationality and heterogeneity in the banking sector, given its pivotal role in propagating credit cycles and macroeconomics fluctuations. We leave these areas for future discussions.
Chapter 4

Modelling the “animal spirits” of bank’s lending behaviour

4.1 Introduction

“Banks are much more social creature than most people think. Understanding banks is not about brute facts. The values on a balance sheet are dependent upon confidence, and when an institution is in trouble those values are quite different from the figures when the institution is thought to be doing well. So value is dependent on confidence, which is a social fact rather than a material property.”

- Rethel and Sinclair (2012), “The Problem with Banks”

The recent financial crisis has revived the interest in studying the role of financial markets and institutions in amplifying business cycles in the real sector. While models in the Keynesian tradition have always featured an integrated financial sector, mainstream analysis has only recently been applied to study the role of the financial sector beyond its role of generating frictions for the adjustment process. Recent history has shown once again that the financial and the real sector are interconnected, and the boom-bust cycle in the asset market plays an important role in influencing aggregate demand and amplifying the business cycle. It is therefore crucial to build macroeconomic models that take into account of the factors such as the balance sheet composition of economic agents, in particular, their credit, debt, and leverage position.

In the traditional banking literature, the commercial bank is often modelled as a passive intermediary that channels funds from the ultimate borrower to the ultimate lender.
In reality however, the role of banks goes well beyond the intermediation between supply and demand of savings. A bank functions as an active credit creator (Ryan-Collins et al., 2012, Taylor, 2004). As recently remarked by McLeay et al. (2014), the creation of a loan simultaneously creates a deposit, which endogenously enlarges the money stock, since deposit is part of the broad money (M3). In other words, the bank’s behaviour is not a passive reflection of the conditions of the economy, but is in itself an important factor that influences the economy through the creation of credit. Minsky (1975) puts the role of credit creation of banks at the centre of his framework: it is the relaxing of credit conditions that drives the expansionary phase of the cycle and it is the contraction in credit that exacerbates the downturn.

Another important aspect, which is often overlooked in the traditional banking literature, is the role of the bank’s lending attitude. An optimistic attitude in the banking sector collectively lowers the lending standard and prompts banks to lend excessively, which potentially leads to the development of a credit bubble. Eventually the bubble bursts due to an unsustainable level of debt (Kindleberger, 1989). On the contrary, a collectively pessimistic banking system not only hinders economic growth but also renders expansionary monetary policy ineffective, as we have observed in the recent financial crisis. As it is shown in Figure 4.1, in the aftermath of the crisis, the money base in the US has grown nearly threefold due to three rounds of Quantitative Easing (QE), however, it has virtually no effect on the growth of broad money due to the negative outlook (Koo, 2011).

The pessimism or optimism, which can potentially influence behaviour of economic agents such as bankers, is referred to as “animal spirits” by Keynes (1936) in chapter 12 of the General Theory, in which he claims that: most probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits: of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities. This fact brings two relevant corollaries. First, expectations may be self-fulfilling: an optimistic/pessimistic sentiment will bring forth a positive/negative outcome to the market, which further reinforces the optimistic/pessimistic sentiment. Second, market sentiment may be contagious: sentiment spreads and it eventually leads to herding amongst agents. The herding behaviour in financial market is well-documented in empirical literature (Haiss, 2005, Sharma and Bikhchandani, 2000). There is also notable amount of literature that finds empirical

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1Exceptions are Asanuma (2013) and Berger and Udell (2004).

2The lending attitude also involves the composition of the credit portfolio and the decision of lending in increasing proportion to a particular sector, as the latest crisis dramatically showed. On the pros and cons of diversification see for example All and Battiston et al. (2007).
A number of finance and macroeconomic studies in the last decades models the “animal spirits” as herding behaviour. This modelling philosophy is rooted in the approach initially proposed by Weidlich and Haag (1983). The basic idea is to model heterogeneous agents that choose and switch between two attitudes in probabilistic terms. A reduced-form Master equation that captures the “average opinion” is applied to simplify the analysis of the stochastic system. Lux (1995) proposes a seminal work that examines the relationship between investors’ sentiment and asset price bubble/crash. Franke (2012) terms this approach “Weidlich-Haag-Lux” and extends the Lux model to the context of macroeconomic dynamics. He studies the interplay between firm’s sentiment, inflation and output gap, which establishes an alternative microfoundation for macroeconomics in the Keynesian tradition. This model is further extended by Charpe et al. (2012), which proposes a “Dynamic Stochastic General Disequilibrium (DSGD)” model. The DSGD model examines the real-financial interaction by incorporating a speculative financial market populated by heterogeneous investors and it takes a “disequilibrium” approach that models the dynamical adjustment process, instead of assuming immediate
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

equilibrium adjustment.\(^3\)

This chapter follows this strand of literature and examines the role of “animal spirits” as the determinant of banks’ lending decisions. The aim is to assess how the contagious waves of optimism and pessimism contributes to the boom-bust of the credit cycle, via a modification of banks’ balance sheet positions and how it amplifies business cycle in the real sector. From this perspective our analysis integrates related contributions in the Minskyan tradition\(^4\) by focusing on the role of banking sector rather than on borrowers.

We present an aggregative model in which banks follow heterogeneous lending strategies and, given their cognitive limits, are assumed to follow herd behaviour. The banks’ opinion formation dynamics is modelled in the spirit of Weidlich-Haag-Lux approach. The joint evolution of banks’ behaviour, credit supply and aggregate output are analysed in a dynamical system.

In our take of this approach, herding among banks is characterized by two different behaviours. The first is the switching of banks between the two categories of optimistic and pessimistic, which is more intense the bigger is the size of the majority group. The second regards the herding behaviour of banks for what concerns their decision about their loans-to-reserve ratio, which is larger during optimistic phases\(^5\). In this way we further extend the Weidlich-Haag-Lux approach by including a further dimension of herding. In particular we study two different scenarios. In the first banks are assumed to have a constant loan-to-reserve ratio (assumed to be larger for optimistic banks) and, as a consequence, the variations in the supply of credit depends only on the switching of banks from one group to another. In the second scenario, the loan-to-reserve ratio for each group of banks follows the market mood, expanding and contracting. Within this second scenario we study separately the special case of a constant ratio for pessimistic banks.

The chapter involves two main novelties. First, while several other contributions have already treated the effects of herding and bounded rationality of firms and households on the business cycle, to the best of our knowledge this study represents one of the first attempt to model banking behaviour as influenced by animal spirits in a dynamic setting. The second original aspect concerns the introduction of heterogeneity in the credit sector, which represents a novelty in this stream of aggregative dynamical models.

\(^3\)In addition to this strand of literature that models the “animal spirits” in disequilibrium process, there are numerous papers that take a more standard equilibrium approach. For example, DeGrauwe (2011) develops a DSGE model that features waves of optimism and pessimism by incorporating agents’ cognitive limitations. For a study of the banks lending attitude in an agent-based model see Asanuma (2013).

\(^4\)See Charpe et al. (2011a), Chiarella and Di Guilmi (2011) and Gatti et al. (1993).

\(^5\)For an analytical investigation of the separate effects of switching and herding on market volatility see Di Guilmi et al. (2013).
In contrast to the traditional banking literature, we stress the role of the mechanism of credit-creation by banks as a potentially destabilising factor.

Anticipating some of the results, the analytical and numerical study of the system reveals that contagion and collective behaviour amongst bankers play an important role in destabilizing the system and propagating boom-bust of credit cycle and business cycle in the real sector.

The remainder of the chapter is organized as follows. In the next section, we propose a 2D baseline model, where banks are categorized in optimistic and pessimistic. The two-dimensional dynamical system consists of the average opinion and the output dynamics. In section 4.3, the loan-to-reserve ratio of the optimistic banks follows the market mood while the pessimistic banks’ one is held constant. The three-dimensional dynamical system consists of the optimistic banks’ loan-to-reserve ratio, the average opinion, and the output dynamics. In section 4.4 we extend the model by endogenizing the behavioural rule for pessimistic banks and analyse the role of heterogeneous lending strategies in destabilizing the real sector in a four dimensional dynamical system. In section 4.5, we modify the baseline 4D model by introducing a nonlinear I-S disequilibrium dynamics in a Kaldorian manner, instead of using the Blanchard (1981) AS-AD dynamic multiplier in the baseline 4D model. In such a way we introduce a credit-driven investment function as well as an income-driven saving function. In section 4.6 we further extend the model from an alternative perspective by incorporating a speculative financial sector based on the framework of Charpe et al. (2012). Finally, section 4.7 offers some concluding remarks.

4.2 The 2D CDGZ Model

In this first section we present the main behavioural hypotheses of the model. Banks are classified into the two categories of optimistic and pessimistic. Both types of banks are assumed to keep constant their loan-to-reserve ratio. The fluctuations in the supply of credit (and consequently in the real output) are therefore an effect of the switching of banks between the optimistic and the pessimistic state.

Table 4.1 illustrates the structure of a typical balance sheet of a commercial bank. On the asset side it consists of bank reserves, loans, and other assets such as treasury bonds; on the liability side there are deposits, bank borrowing, and bank equity. When a bank makes loans, it simultaneously creates deposits.
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

4.2.1 The basic set-up

Following Taylor (2004), we focus on the loan-to-reserve ratio ($\lambda^s$), which is defined as the ratio between loans and unborrowed reserves. The variable $\lambda^s$ reflects not only the banks’ lending attitude, but also the level of debt accumulation due to banks’ credit creation. Specifically, we have that

$$L^s = \lambda^s T_c,$$  \hspace{1cm} (4.1)

where

- $L^s$ is the level of aggregate credit supply,
- $\lambda^s$ is the loan-to-reserve ratio of banks,
- $T_c$ is the total amount of unborrowed reserves, which is assumed to be exogenous.

The total number of banks is $2N$, while $n_+$ is number of optimistic banks and $n_-$ is the number of pessimistic banks ($2N = n_+ + n_- )$. The optimistic banks lend at a loan-to-reserve ratio $\lambda_+$ assumed to be larger than the pessimistic banks’ one $\lambda_-$. We assume that each bank holds the same amount of reserves, and the two loan-to-reserve ratios are initially set as constant as $\bar{\lambda}_+$ and $\bar{\lambda}_-$. Hence equation (4.1) becomes

$$L^s = R(n_+ \bar{\lambda}_+ + n_- \bar{\lambda}_-),$$ \hspace{1cm} (4.2)

$$T_c = 2NR,$$ \hspace{1cm} (4.3)

where $R$ indicates the reserves. Following Lux (1995), the difference in the size of the two groups is quantified by the index $x$

$$x = (n_+ - n_-)/2N.$$ \hspace{1cm} (4.4)

---

6The modelling of an inter-banks market is beyond the scope of the present paper and consequently we assume it away. Reserves therefore can only be unborrowed.
As $2N = n_+ + n_-$, it is easy to derive that
\begin{align}
    n_+ &= N(1 + x), \\
    n_- &= N(1 - x).
\end{align}
(4.5) (4.6)

The index $x$ describes the "average opinion" or, in the context of this chapter, the general lending attitude of banks. When $x = 0$, there are equal number of optimistic and pessimistic banks. When $x = \pm 1$, it implies that all the banks are either optimistic or pessimistic. Since $T_c = 2NR$, equation (4.2) is therefore modified in terms of $x$ as
\begin{equation}
    L^s = \frac{T_c}{2} [(1 + x)\bar{\lambda}_+ + (1 - x)\bar{\lambda}_-].
\end{equation}
(4.7)

We postulate that the availability of credit $L^s$ determines the aggregate demand in the real sector. Furthermore, we assume that output ($y$) is demand-driven and it follows Blanchard (1981) as a stylized AS-AD dynamic multiplier process. That is,
\begin{align}
    y^d &= y^d_0 + kL^s, \\
    \dot{y} &= \sigma(y^d - y),
\end{align}
(4.8) (4.9)

where $y^d_0$ is the autonomous component of the aggregate demand.

Substituting (4.7) and (4.8) into (4.9) we get
\begin{equation}
    \dot{y} = \sigma \{ y^d_0 + k \frac{T_c}{2} [(1 + x)\bar{\lambda}_+ + (1 - x)\bar{\lambda}_-] - y \}.
\end{equation}
(4.10)

### 4.2.2 Opinion dynamics

In this sub-section we introduce the law of motion for the average opinion $x$. Let $p_{+-}$ be the transition probability for a pessimistic bank to become optimistic, while $p_{-+}$ is the probability of the opposite transition. Accordingly, the change in the level of $x$ depends on the size of each group multiplied by their transition probability. Thus we have
\begin{equation}
    \dot{x} = (1 - x)p_{+-} - (1 + x)p_{-+}.
\end{equation}
(4.11)

Two factors affect the probability of transition of banks from one group to another: the bankers average opinion $x$, which captures the contagion effect; the output gap in the real sector $y^d - y$ and a general financial index $d$, representing the propensity of banks to switch. We assume $d$ to be constant as it is related to institutional factors. Accordingly,
we compose a switching index $s$ as a linear combination of the three factors

$$s(x, y, d) = a_1 x + a_2 (y^d - y) + d. \quad (4.12)$$

The parameter $a_1$ quantifies the effect of herding and plays an important role in our story. Assuming that the relative changes of $p_{+-}$ and $p_{-+}$ in response to changes in $s$ are symmetric, the probabilities can be written as

$$p_{+-} = v \cdot \exp(s), \quad (4.13)$$
$$p_{-+} = v \cdot \exp(-s). \quad (4.14)$$

Hence

$$\dot{x} = v[(1 - x) \exp(s) - (1 + x) \exp(-s)]. \quad (4.15)$$

Equations (4.12) and (4.15) show that the emergence of a lending behaviour as the most popular is self-strengthening: more and more banks are assumed to follow the strategy adopted by the largest number of banks. An increase in $s$ increases the probability that the pessimistic banks will become optimistic ones. Equation (4.15) models the lending behaviour of banks in fashion similar to the famous Keynes' beauty contest metaphor\(^7\). In a situation of uncertainty and less than perfect information, an anchor to the expectations is provided by the behaviour of other agents.

It is therefore possible to represent the dynamics by means of the two-dimensional system composed by equations (4.9) and (4.15).

$$\dot{y} = \sigma(y^d - y), \quad (4.16)$$
$$\dot{x} = v[(1 - x) \exp(s) - (1 + x) \exp(-s)], \quad (4.17)$$

where

$$y^d = y_0^d + kL^s = y_0^d + \frac{k^2}{2} [(1 + x)\bar{\lambda}_+ + (1 - x)\bar{\lambda}_-], \quad (4.18)$$
$$s = a_1 x + a_2 (y^d - y) + d. \quad (4.19)$$

\(^7\)“It is not a case of choosing those [faces] that, to the best of one’s judgement, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.” (Keynes, 1936).
4.2.3 Analysis of the two-dimensional system

In order to study the properties of the system (4.16), (4.17) we first set LHS = 0 on both equations and derive the following isoclines

\[
\begin{align*}
  y &= y_0^d + k \frac{T_c}{2} [(1 + x) \bar{\lambda}_+ + (1 - x) \bar{\lambda}_-], \\
  y &= \frac{a_1}{a_2} x - \frac{1}{2a_2} \ln \frac{1 + x}{1 - x} + y^d + d.
\end{align*}
\]

(4.20) (4.21)

It is difficult to obtain the close-form solution of equation (4.20-4.21). Yet in a special case when we set \(d = 0\), we can easily obtain a neutral opinion equilibrium \((x^*, y^*) = (0, y^d)\). Furthermore, there is potentially an emergence of other equilibria \((x^*_+, y^*_+)\) and \((x^*_-, y^*_-)\), depending on the value of contagion parameter \(a_1\) - as shown in Figure 4.2, which plots the two isoclines. We analyse the local stability of the neutral opinion equilibrium by deriving the Jacobian of the system (4.16-4.17)

\[
J = \begin{pmatrix}
-\sigma & \frac{\sigma k T_c}{2} (\bar{\lambda}_+ - \bar{\lambda}_-) \\
-2v a_2 & 2v[a_1 + a_2 k T_c \frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-)]
\end{pmatrix}.
\]

(4.22)

The trace and determinant of the Jacobian at neutral opinion equilibrium \((x = 0)\) is calculated as

\[
\begin{align*}
  Tr(J) &= 2v[a_1 + a_2 k T_c \frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-)] - \sigma, \quad (4.23) \\
  Det(J) &= 2v[a_1 + a_2 k T_c \frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-)] - \sigma[a_1 + a_2 k T_c \frac{T_c}{2}(\bar{\lambda}_+ - \bar{\lambda}_-) - 1]. \quad (4.24)
\end{align*}
\]

The necessary and sufficient condition for local stability for this equilibrium is that \(Tr(J) < 0\) and \(Det(J) > 0\). Otherwise it would become locally unstable in the form of repelling cycle or saddle node. The contagion parameter \(a_1\) plays an important role in determining the local stability: this neutral opinion equilibrium is more likely to be stable when \(a_1\) is relatively small. The results of a representative simulation, together with the bifurcation analysis for the contagion parameter \(a_1\), are provided in Figure 4.3. The parameters are set as follows: \(a_1 = 0.7, a_2 = 2.1, \sigma = 0.8, k = 0.1, T_c = 1, y^d = 10, \bar{\lambda}_- = 5, \bar{\lambda}_+ = 20, v = 0.4, d = 0.5\). Note that in the numerical simulation the parameter \(d\) takes a non-zero value. As we can see, the neutral opinion equilibrium becomes unstable and a limit cycle emerges as the contagion parameter increases and passes through \(a_1 \approx 0.3\). As \(a_1\) further increases, the system becomes stable again, yet it converges to another equilibrium with higher value of \(x\).

---

\(8\)The following parameters are set for the isoclines: \(a_1 = 0.9\) (left), \(a_1 = 1.4\) (right), \(a_2 = 0.4, T_c = 1, k = 0.1, d = 0, v = 0.7, y^d = 10, \bar{\lambda}_- = 5, \bar{\lambda}_+ = 20, \sigma = 0.8.\)
Figure 4.2: The isocline of 2D model with $\dot{y} = 0$ (straight line) and $\dot{x} = 0$

Figure 4.3: Simulations of the baseline model

4.3 The 3D CDGZ model: herding amongst optimistic banks

This section adds one additional dimension by introducing the behavioural rule of $\lambda$. In this setting the pessimistic banks are supposed to be inactive, in the sense that they keep a constant loan-to-reserve ratio, while the optimistic banks become active since they adjust their ratio. By keeping one group of banks inactive in terms of a constant loan-to-reserve ratio, we are able to analytically assess the properties of the dynamical system with the varying loan-to-reserve ratio at least in one group.
which is reflected in the constant loan-to-reserve ratio $\lambda_-$. We focus the analysis on the endogenous variables $x$ and $\lambda_+$. Given the bank’s role as a credit creator, the bank’s lending behaviour is not constrained by the availability of deposit, but determined by its lending attitude. For simplicity, we assume that the relationship between the rate of change of $\lambda_+$ and $x$ is linear. By hypothesis, when $x > 0$, the active banks expand their balance sheet at a constant speed $\gamma_1$; when $x < 0$, they contract their balance sheet position at the same speed; when $x = 0$, the balance sheet position of active banks remains unchanged. We assume that $\lambda_+$ adjusts also by the change of real output. Hence the law of motion for $\lambda_+$ takes the following form:

$$\dot{\lambda}_+ = \gamma_1 x + \gamma_2 \dot{y}. \quad (4.25)$$

From this perspective the index $x$ is interpreted by the banks as a proxy of the state of the economy and in particular of the capacity of the productive sector to repay loans in the future. In equation $(4.25)$, $\gamma_1$ captures the speed of adjustment regarding average opinion $x$ and $\gamma_2$ captures the speed of adjustment regarding changes to $\dot{y}$. The first term can be interpreted as the stimulative role of bank’s credit creation driving by banker’s sentiment, whereas the second term represents a passive, accommodative role of banks driven by the growth in the real sector\(^{10}\). Given the above assumptions, the three-dimensional system with variable $\lambda_+$ can be written as

$$\dot{\lambda}_+ = \gamma_1 x + \gamma_2 \dot{y}, \quad (4.26)$$
$$\dot{y} = \sigma(y_d - y), \quad (4.27)$$
$$\dot{x} = v[(1 - x) \exp(s) - (1 + x) \exp(-s)]. \quad (4.28)$$

Figure 4.4 illustrates the feedback loop of the 3D system.

---

\(^{10}\) This is consistent with the literature regarding horizontalist & structuralist views of banking (Goodhart, 1989, Moore, 1988), as well as literature of financial fragility and business cycle: a positive correlation between aggregate output and debt is assumed here.
thus increasing the credit supply. This is accompanied by an increase of aggregate
demand, thus raising output. When bankers’ sentiment becomes more optimistic, and
output gap increases, thus raising $s$. This forms a positive feedback loop. In addition, we
have another positive feedback loop via the herding channel, where the bankers’ decision
making is affected by majority opinion. However, this does not last forever. The increase
of bankers’ leverage leads to an accumulation of debt, which has a negative impact on
the opinion formation index $s$. At the beginning, the positive feedback outweighs the
negative feedback. Up to a certain point when debt accumulation becomes significantly
large, the system turns in the reverse direction.

We provide in the following an assessment of the properties of the system.

**Proposition 4.1:** The steady state of the dynamical system (4.26-4.28) is unique and
given by $\lambda^* = -\frac{d}{a_2}, \ y^* = y^*_0 + k\frac{T_c}{\lambda} (\lambda_+ - \frac{d}{a_2}), \ x^* = 0$.

**Proof:** By setting equations (4.26), (4.27), (4.28) equal to zero, we have

\[
\begin{align*}
0 &= \gamma_1 x + \gamma_2 \dot{y}, \\
0 &= \sigma (y^d - y), \\
0 &= v [(1 - x) \exp(s) - (1 + x) \exp(-s)].
\end{align*}
\] (4.29) (4.30) (4.31)

Since $\dot{y} = 0$, we derive that $y^* = y^d$, and $x^* = 0$ in equation (4.27) and (4.29).
Substituting to equation (4.28) we obtain $v [\exp(s) - \exp(-s)] = 0$, or $\exp(2s) = 1$. Hence
$s = 0$. Since $s = a_1 x + a_2 \lambda_+ + a_3 (y^d - y) + d$, we derive that $\lambda_+^* = -\frac{d}{a_2}$. Finally, we
derive that $y^* = y^d = y^*_0 + k\frac{T_c}{\lambda} (\lambda_+ - \frac{d}{a_2})$.

Next we analyse the local stability of the system (4.26), (4.27), (4.28). The Jacobian of
this system at equilibrium is derived as

\[
\begin{pmatrix}
\gamma_2 \sigma k T_c / 2 & -\gamma_2 \sigma & \gamma_1 + \gamma_2 \sigma k (T_c / 2) (-\frac{d}{a_2} - \lambda_+) \\
\sigma k (T_c / 2) & -\sigma & \sigma k (T_c / 2) (-\frac{d}{a_2} - \lambda_+) \\
2v(a_2 + a_3 k (T_c / 2)) & -2va_3 & 2v(a_1 + a_3 k (T_c / 2) (-\frac{d}{a_2} - \lambda_-) - 1)
\end{pmatrix},
\]

which has the sign structure

\[
\begin{pmatrix}
+ & - & + \\
+ & - & + \\
? & - & ?
\end{pmatrix}.
\]
Proposition 4.2: The system (4.26-4.28) is more likely to be locally stable near equilibrium if \( a_1, a_3, \gamma_1, \gamma_2 \) are sufficiently small, and \( |a_2| \) is sufficiently large.

Proof: The trace (\( Tr \)), determinant (\( Det \)), and the principle minors (\( J_i \)) are derived as follows:

\[
Tr(J) = \gamma_2 \sigma k T_e / 2 - \sigma + 2v(a_1 + a_3 k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) - 1),
\]

\[
Det(J) = \gamma_2 \sigma k T_e / 2 \begin{vmatrix} -\sigma & \sigma k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) \\ -2va_3 & 2v(a_1 + a_3 k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) - 1) \end{vmatrix} + \gamma_2 \sigma \begin{vmatrix} \sigma k(T_e/2) & -\sigma \\ 2v(a_2 + a_3 k(T_e/2) & 2v(a_1 + a_3 k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) - 1) \end{vmatrix} + \begin{vmatrix} \gamma_1 + \gamma_2 \sigma k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) & \sigma k(T_e/2) \\ 2v(a_2 + a_3 k(T_e/2) & 2v(a_1 + a_3 k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) - 1) \end{vmatrix},
\]

\[
J_1 = \begin{vmatrix} -\sigma & \sigma k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) \\ -2va_3 & 2v(a_1 + a_3 k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) - 1) \end{vmatrix},
\]

\[
J_2 = \begin{vmatrix} \gamma_2 \sigma k T_e / 2 & \gamma_1 + \gamma_2 \sigma k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) \\ 2v(a_2 + a_3 k(T_e/2) & 2v(a_1 + a_3 k(T_e/2)(-\frac{d}{a_2} - \bar{\lambda}_-) - 1) \end{vmatrix},
\]

\[
J_3 = \begin{vmatrix} \gamma_2 \sigma k T_e / 2 & -\gamma_2 \sigma \\ \sigma k(T_e/2) & -\sigma \end{vmatrix}.
\]

According to the Routh-Hurwitz theorem, the necessary condition for the stability of the 3D sub-dynamics is that \( tr(J) < 0, J_1 + J_2 + J_3 > 0 \), and \( det(J) < 0 \). In order to satisfy these conditions we need to have sufficiently small \( a_1, a_3, \gamma_1, \gamma_2 \), as well as sufficiently large \( |a_2| \).

The system (4.26-4.28) is more likely to be stable when contagion amongst banks is weak (herding and switching are small), bankers are less tolerant toward debt, and banks’ opinion is less sensitive toward changes in the real economy.

In order to investigate the global features of the nonlinear 3D dynamics, we reformulate the baseline model in discrete time by using a standard Euler method\(^{12}\). The simulations involve two different scenarios: \( a_1 = 0.3 \) (stable scenario) and \( a_1 = 1.5 \) (unstable scenario). The other parameters are set as follows: \( a_2 = -0.02, a_3 = 1.3, \sigma = 0.8, k = 0.1, \)

\(^{11}\)The necessary and sufficient condition for local stability requires an addition of \(-tr(J)(J_1 + J_2 + J_3) + det(J) > 0\) (Chiarella and Flaschel, 2000).

\(^{12}\)The Euler method has the advantage of preserving the economic content of the discretised equations (Chiarella et al., 2005).
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

Figure 4.5: The 3D CDGZ model: representative simulation of a stable (left) and an unstable (right) scenario.

$T_c = 1, y_0^d = 10, d = 0.5, v = 0.4, \gamma_1 = 0.5, \gamma_2 = 2, \lambda_+ = 5$. As is shown in Figure 4.5, we simulate the stable scenario where herding effect is relatively weak ($a_1 = 0.3$) on the left-hand panel, as well as an unstable scenario where the herding effect is relatively strong on the right-hand panel ($a_1 = 1.5$). We observe that, when the herding effect is relatively weak, the system tends toward a stable equilibrium ($x = 0, y^d = y, and \lambda_+ = d/(-a_2)$). In the second case where the contagion parameter $a_1$ is relatively large, the equilibrium becomes unstable. It is possible to observe relatively regular sequences of fluctuations of the three variables. The upper right panel captures waves of optimism and pessimism amongst bankers, accompanied by expansion and contraction of the bank’s balance sheet. The mid-right panel describes the interplay between $x$, $y$, and $\lambda_+$. Most interestingly, the lower right panel shows a three-stage cycle in the real sector: a gradual expansion, a sudden slump, and a prolonged period of recession. The unstable scenario is in line with anatomy of a typical credit cycle as described by Kindleberger (1989) and Minsky (1963): the optimism at the initial stage triggers banks to lend more, which ultimately leads to the growth in the real sector. However, due to an over-accumulation of debt that modifies lenders’ expectations, the cycle eventually reverses until it reaches the state where a majority of banks become pessimistic. The representative simulation also shows the crucial role of coordination failure, reflected in herding behaviour amongst banks, in propagating the credit cycle and destabilizing the real sector.
We further examine the dynamical interplay between output and debt by analysing the debt/GDP ratio, which is defined as

\[(Debt/GDP) \text{ ratio} = \frac{L^s}{y}.\]  

(4.37)

The different patterns of the Debt/GDP ratio with different values of the contagion parameter \(a_1\) are shown in Figure 4.6. The other parameters are unchanged with respect to the previous section. The simulation shows that a higher value of \(a_1\) tends to destabilize the system, which is reflected in a larger magnitude of boom-bust cycle both in the credit sector and the real sector. For the highest value of the parameter, also the shape of the orbit appears to change toward a 3-stage cycle.

Both fluctuations and limit cycle are of larger amplitude in this scenario compared to the setting with only switching. It is possible to conclude that the herding within the group of optimistic banks further destabilises the economy.

### 4.3.1 Sensitivity and bifurcation analysis of the behavioural parameters

In this section we further analyse the effect of changes in the cognitive and behavioural parameters. We confine our attention to the three cognitive parameters in the opinion

![Figure 4.6: The dynamics of Debt/GDP ratio in the 3D system: \(a_1 = 0.5\) (dashed limit cycle), \(a_1 = 1.2\) (solid rectangular), \(a_1 = 1.7\) (dashed triangular).](image)
formation index \( (a_1, a_2, \text{and } a_3) \), as well as the two behavioural parameters \( (\gamma_1 \text{ and } \gamma_2) \) in equation (4.26), since they are the most relevant in determining bankers’ behaviour. The scope of the sensitivity and bifurcation analysis is two-fold: first, it serves as a robustness test; second, it provides important insight on how bankers’ behaviour stabilises or destabilises the system.

We first simulate the dynamics of output \( (y) \) showing its dynamics for values of contagion parameter \( (a_1) \) and keeping fixed the other parameters. As is shown in Figure 4.7, when \( a_1 = 0.3 \), output tends toward the stable equilibrium; when \( a_1 = 0.7 \), a period of growth and tranquillity is observable, followed by a persistent period of instability. In this setting, “stability breeds instability”, as Minsky famously wrote, as this pattern seems to replicate the one in many of the recent crises. When \( a_1 = 1.5 \), the dynamics of output becomes a three-stage cycle, which is in line with the previous representative simulation.

We further draw the bifurcation diagram for \( a_1 \) in Figure 4.8. It can be seen that Hopf bifurcation occurs when \( a_1 \) increases and passes through \( a_1 \approx 0.65 \), ceteris paribus. The system transits from a stable equilibrium to persistent fluctuations. The destabilizing effect further increases when \( a_1 \) increases and passes through approximately 1.2.

Figure 4.9 provides the bifurcation diagrams for the other non-contagion parameters \( (a_2, a_3, \gamma_1, \text{and } \gamma_2) \). The Hopf bifurcation points turn out to be \( a_2 \approx -0.025 \) and \( a_3 \approx 1.4 \). The two bifurcation diagrams show that banker’s degree of debt-tolerance and their opinion over the change of real sector also play important roles in destabilizing the system. The system tends to be unstable if the bankers are more tolerant toward debt
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

Figure 4.8: The 3D CDGZ model: bifurcation diagram for $a_1$.

Figure 4.9: The 3D CDGZ model: bifurcation for non-herding parameters.
and hold more sensitive opinion toward a change in the real sector. On the other hand, the two behavioural parameters tend to be less sensitive in this particular scenario since we do not observe bifurcation, which implies that behavioural parameters play relatively a less significant role in destabilizing the system compared to cognitive parameters.

### 4.4 The 4D CDGZ model: the convergence and divergence of heterogeneous lending strategies

In this section the model is further extended by incorporating heterogeneous lending strategies between optimistic and pessimistic banks. The dynamics of loan-to-reserve ratio of pessimistic banks ($\lambda_-$) is now modelled as an endogenous variable and added to the new system. We assume that

\[
\dot{\lambda}_+ = \gamma_1(x + g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+),
\]

(4.38)

\[
\dot{\lambda}_- = \gamma_1(x - g(.)) + \gamma_2\dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-).
\]

(4.39)

There are two additional terms in the extended dynamics of $\lambda_+$ and $\lambda_-$. 

**First**, following DeGrauwe (2011), we introduce a function $g(.)$ that captures the reaction gap between optimists and pessimists over the average opinion, which is quantified by

\[
g(.) = \xi_0 \exp(-\xi_1 x^2).
\]

(4.40)

The optimistic banks react to $x + g(.)$ while the pessimistic banks react to $x - g(.)$. The parameter $\xi_0$ determines the magnitude of opinion gap and $\xi_1$ captures the sensitivity of $g(.)$ relative to the change of average opinion ($x$). Equation (4.40) captures a scenario where there is a convergence or divergence of lending strategies during the exuberant/calm period, with agents behaving in a more coordinated manner during a period of increasing optimism or pessimism. On the contrary, the lending strategies diverge during a relative calm period. However, the gap $g(.)$ narrows and the lending strategies converge when the optimistic or pessimistic sentiment grows over time.

**Second**, we add two mean-reverting terms $\gamma_3(\bar{\lambda}_+ - \lambda_+)$ and $\gamma_3(\bar{\lambda}_- - \lambda_-)$ to capture the long run adjustments: we assume that optimistic banks tend toward a higher loan-to-reserve ratio while the pessimistic banks tend toward a lower one ($\bar{\lambda}_+ > \bar{\lambda}_-$). The mean reverting process serves the purpose to provide a ceiling (floor) to the growth (decrease) of the lending ratio of optimistic (pessimistic) banks, which is necessary with endogenous $\lambda_+$ and $\lambda_-$. 
Recalling that \( L^s = R(n_+ \lambda_+ + n_- \lambda_-) \), \( n_+ = (1 + x)N \), \( n_- = (1 - x)N \) and \( T_c = 2NR \), the quantities \( L^s \) and \( y^d \) are given by

\[
\begin{align*}
L^s &= T_c/2((1 + x)\lambda_+ + (1 - x)\lambda_-), \\
y^d &= y_0^d + k[T_c/2((1 + x)\lambda_+ + (1 - x)\lambda_-)].
\end{align*}
\]

(4.41) (4.42)

Hence the new system is written as

\[
\begin{align*}
\dot{\lambda}_+ &= \gamma_1(x + g(\cdot)) + \gamma_2 \dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+), \\
\dot{\lambda}_- &= \gamma_1(x - g(\cdot)) + \gamma_2 \dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-), \\
\dot{y} &= \sigma(y^d - y), \\
\dot{x} &= \nu[(1 - x) \exp(s) - (1 + x) \exp(-s)],
\end{align*}
\]

(4.43) (4.44) (4.45) (4.46)

where

\[
\begin{align*}
y^d &= y_0^d + kL^s = y_0^d + k(T_c/2)[(1 + x)\lambda_+ + (1 - x)\lambda_-], \\
g(\cdot) &= \xi_0 \exp(-\xi_1 x^2), \\
s &= a_1 x + a_2+ \lambda_+ + a_2- \lambda_- + a_3(y^d - y) + d.
\end{align*}
\]

(4.47) (4.48) (4.49)

We first analyse the steady state and local stability of the above 4D system. We notice that there is no closed form solution for the steady state condition. Therefore, we consider a special case where the average opinion is neutral at equilibrium \((x^* = 0)\). In this case, the general financial condition index \(d\) becomes \(d = -a_2+ \lambda_+^* - a_2- \lambda_-^*\). In this special case, it is easy to derive the close form solution for \(\lambda_+^*, \lambda_-^*, \) and \(y^*\).

**Proposition 4.3:** In the special case where \(x^* = 0\), the steady state of the system (4.43-4.46) is unique and given by

\[
\begin{align*}
\lambda_+^* &= \bar{\lambda}_+ + \frac{\gamma_1}{\gamma_3} \xi_0, \\
\lambda_-^* &= \bar{\lambda}_- - \frac{\gamma_1}{\gamma_3} \xi_0, \\
y^* &= y^{d*} = y_0^d + k(T_c/2)[(\lambda_+^* + \lambda_-^*)], \\
x^* &= 0.
\end{align*}
\]

(4.50) (4.51) (4.52) (4.53)

\[\text{By setting the LHS } = 0, \text{ we derive that } a_2+ \frac{\gamma_1}{\gamma_3}(x + \xi_0 e^{-\xi_1 x^2}) + \bar{\lambda}_+ + a_2- \frac{\gamma_1}{\gamma_3}(x - \xi_0 e^{-\xi_1 x^2}) + \bar{\lambda}_- = \frac{1}{2} \ln \left[ \frac{1 + e^{-2(a^1 + d)}}{1 - e^{-2(a^1 + d)}} \right]. \text{This equation has no closed form solution.} \]

\[\text{Setting the LHS } = 0 \text{ in equation } (42), \text{ we find that } s = 0 \text{ when } x^* = 0, \text{ hence } d = -a_2+ \lambda_+^* - a_2- \lambda_-^*. \]
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

Proof: By setting $LHS = 0$, we have

$$0 = \gamma_1(x + \xi_0 e^{(-\xi_1 x^2)}) + \gamma_2 \hat{y} + \gamma_3 (\bar{\lambda}_+ - \lambda_+) \quad (4.54)$$

$$0 = \gamma_1(x - \xi_0 e^{(-\xi_1 x^2)}) + \gamma_2 \hat{y} + \gamma_3 (\bar{\lambda}_- - \lambda_-) \quad (4.55)$$

$$0 = \sigma(y^d - y) \quad (4.56)$$

$$0 = v[(1 - x) \exp(s) - (1 + x) \exp(-s)] \quad (4.57)$$

Since $x^* = 0$ and $\hat{y} = 0$, from equation (4.43) and (4.44) we derive that $\lambda^*_+ = \bar{\lambda}_+ + \frac{\gamma_1}{\gamma_3} \xi_0$ and $\lambda^*_- = \bar{\lambda}_- - \frac{\gamma_1}{\gamma_3} \xi_0$. Substituting this result to equation (4.47) we have $y^* = y^{d*} = y^d_0 + k(T_c/2)[(\lambda^*_+ + \lambda^*_-)].$

We then analyse the local stability of the system. To make the system analytically tractable, we consider a special case without the real sector by setting $\gamma_2 = 0$, $\sigma = 0$, and $a_3 = 0$. The Jacobians of equation (4.43), (4.44), and (4.46) sub-dynamics at neutral opinion equilibrium without the real sector is derived as

$$\begin{bmatrix}
-\gamma_3 & 0 & \gamma_1 \\
0 & -\gamma_3 & \gamma_1 \\
2\nu a_2+ & 2\nu a_2- & 2\nu(a_1 - 1)
\end{bmatrix},$$

which has the sign structure

$$\begin{pmatrix}
-0+ \\
0-+ \\
-+?
\end{pmatrix}.$$

Proposition 4.4: The sub-dynamical system (4.43), (4.44), (4.46) without the real sector is locally asymptotically stable if $a_1$, $a_3$, $\gamma_1$, and $\gamma_2$ are sufficiently small, and $a_2$, $\gamma_3$ are sufficiently large.

Proof: The trace $Tr(J)$, determinant $Det(J)$, and the three principle minors $J_1$, $J_2$, and $J_3$ are derived as follows:

$$Tr(J) = 2[\nu(a_1 - 1) - \gamma_3], \quad (4.58)$$

$$Det(J) = 2\nu[\gamma_3^2(a_1 - 1) - \gamma_1 \gamma_3 (-a_2+ - a_2-)], \quad (4.59)$$

$$J_1 = -2\nu[\gamma_3(a_1 - 1) + \gamma_1 a_2-], \quad (4.60)$$

$$J_2 = -2\nu[\gamma_3(a_1 - 1) + \gamma_1 a_2+], \quad (4.61)$$

$$J_3 = \gamma_3^2. \quad (4.62)$$
According to the already cited Routh-Hurwitz theorem, the necessary and sufficient condition for the stability of the 3D sub-dynamics is that $\text{tr}(J) < 0$, $J_1 + J_2 + J_3 > 0$, $\text{det}(J) < 0$, and $-\text{tr}(J)(J_1 + J_2 + J_3) + \text{det}(J) > 0$.

The parameter $\gamma_3$, which captures the long-run adjustment of $\lambda_+$ and $\lambda_-$, plays an important role in stabilizing/destabilizing the system. The system tends to be stable if $\gamma_3$ is relatively large, as the reversion toward an average loan-to-reserve ratio is more pronounced.

As for the investigation of the global features of the complete 4D system, we turn to numerical simulations. Figure 4.10 provides a representative simulation of the extended model with the following parameter set: $a_1 = 1.5$, $a_{2+} = -0.3$, $a_{2-} = -0.5$, $a_3 = 1.3$, $\sigma = 0.8$, $k = 0.1$, $T_c = 1$, $y_0^d = 11$, $d = 10$, $v = 0.4$, $\gamma_1 = 0.3$, $\gamma_2 = 0.4$, $\gamma_3 = 0.03$, $\xi_0 = 3.4$, $\xi_1 = 5$. In the top panel we observe that the optimistic/pessimistic banks become increasingly optimistic/pessimistic over time, until they settle down to two distinct and irregular limit cycles with higher value of $\lambda_+$ and lower value of $\lambda_-$. The mid-left panel shows the dynamics of $x$ characterized by a transition between a sustained period of optimism and a sudden switch to a period of pessimism. Similar dynamics is observed in output $y$ at the bottom-left panel. The mid-right panel shows the cyclical dynamics of $g(\_)$, which indicates a constant convergence and divergence of lending strategies between two groups of banks. In this scenario the convergence of the strategies smooths down the cycle, as shown by the milder swings in $x$ compared with the previous settings. Also, the variable $g$ displays a pattern with a “double-peak”. Possibly, the first downswing in $g$ during the cycle is not strong enough to revert the pattern of $\lambda_+$ (and therefore of $y$). The bottom-right panel shows a three-stage dynamics of the Debt-to-GDP ratio, which is similar to the dynamics observed in the 3D system discussed in previous section.

Figures 4.11 provides the bifurcation diagrams of $a_1$, $\gamma_1$, $\gamma_2$, and $\gamma_3$ of the extended 4D model. We observe a similar bifurcation of $a_1$ compared to the 3D model, indicating that a higher contagion value tends to destabilize the system. However, contrary to the 3D model, we observe bifurcation occurring for the two behavioural parameters $\gamma_1$ and $\gamma_3$ at $\gamma_1 \approx 0.2$ and $\gamma_3 \approx 0.04$ respectively. It indicates that these two behavioural parameters now play a destabilizing role in the 4D system. We do not observe a clear bifurcation range for $\gamma_2$. Yet as $\gamma_2$ increases, the cycle diminishes in magnitude. This may be interpreted as a stabilizing factor if lending activity is more directed toward the real sector.
4.5 The Kaldorian Investment-Saving disequilibrium dynamics with credit-driven investment sector

The 4D CDGZ model in the previous section is formulated from a simple AS-AD setting in terms of Blanchard (1981). One possible extension that incorporates a more Keynesian insight is by incorporating a detailed treatment of investment sector with a Kaldorian-Keynesian dynamic multiplier where output is driven by the Investment-Saving disequilibrium (Chang and Smyth, 1971, Kaldor, 1940, Tu, 1992):

\[ \dot{y} = \sigma_K (I - S). \]  

(4.63)
4.5.1 The credit-driven Kaldorian investment function

Kaldor (1940) discusses the non-linearity of investment sector by postulating that $\frac{dI}{dy}$ will be small both for low and for high levels of $y$ relatively to its “normal” level, which can simply be characterized by the hyperbolic function: $I = a +\tanh(ay)$ (Tu, 1992). In the context of this chapter, we postulate that the investment function is credit-driven:

\begin{align*}
I = \dot{K} &= \theta L^s, \\
&= \theta IT_{c}\left[(1 + x)\lambda_+ + (1 - x)\lambda_-ight], \\
&= \theta IT_{c}\left[(\lambda_+ + \lambda_-) + (\lambda_+ - \lambda_-)x\right].
\end{align*}

(4.64) \hspace{1cm} (4.65) \hspace{1cm} (4.66)

Hence

\begin{align*}
x &= AI - B, \\
\text{where} \quad A &= \frac{1}{\theta IT_{c}/2(\lambda_+ - \lambda_-)}, \\
B &= \frac{\lambda_+ + \lambda_-}{\lambda_+ - \lambda_-}.
\end{align*}

(4.67) \hspace{1cm} (4.68) \hspace{1cm} (4.69)
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

It is important to relate this investment function to the value of $x$. We argue that in the extended model, the opinion formation index $s$ is determined by

$$s = a_1 x + a_2 + \lambda_+ + a_2 - \lambda_- + a_K (y - y^*) + d. \quad (4.70)$$

Now the parameter $a_K$ enters the opinion formation index, which essentially characterizes the non-linear Kaldorian investment function. In a static sense, if we ignore the dynamics of $\lambda_+ - \lambda_-^*$ and $x$ ($\lambda_+ = \bar{\lambda}_+, \lambda_- = \bar{\lambda}_-, a_1 = a_2 + a_2 = d = 0$) then the static $I-y$ relationship becomes Kaldorian hyperbolic, since

$$0 = v[(1 - x)\exp(s) - (1 + x)\exp(-s)], \quad (4.71)$$

$$\rightarrow 2a_K (y - y^*) = \ln(\frac{1+x}{1-x}). \quad (4.72)$$

Substituting equation (4.67) into equation (4.72) we have

$$2a_K (y - y^*) = \ln(\frac{1 + AI - B}{1 - AI + B}). \quad (4.73)$$

Equation (4.73) essentially captures the non-linear Kaldorian relationship between $y$ and $I$, which resembles the $\tanh(.)$ specification of Tu (1992). We note that the level of investment $I$ depends crucially on $x$, which measures not only the average opinion but also the relative number of optimistic/pessimistic banks. During the period of extreme pessimism where $x = -1$, the aggregate level of investment $I_- = (B - 1)/A$. During the period of extreme optimism, $I_+ = (B + 1)/A$. At the equilibrium of neutrality when $x = 0$, $I_0 = B/A$.

On the other hand, the Kaldorian saving function can simply be adopted from Tu (1992). As Keynes (1936) argues, the level of saving is primarily determined by the level of aggregate income:

$$S = S^* + \theta S (y - y^*)^3. \quad (4.74)$$

In Equation (4.74), $S^*$ captures the level of saving at neutral opinion equilibrium where $x = 0$. The value can be derived by equating $S^*$ and $I^*$. Hence $S^* = I^* = \theta I_T (\bar{\lambda}_+ + \bar{\lambda}_-)$. We plot this Kaldorian I-S relationship in Figure 4.12.
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

Figure 4.12: The Kaldorian I-S disequilibrium

Figure 4.13: The 4D CDGZ model with Kaldorian I-S Disequilibrium
4.5.2 The 4D credit-driven investment sector with Kaldorian I-S disequilibrium

The 4D CDGZ dynamics with non-linear Kaldorian I-S disequilibrium is therefore written as

\[
\begin{align*}
\dot{\lambda}_+ &= \gamma_1(x + g(,)) + \gamma_2 \dot{y} + \gamma_3(\bar{\lambda}_+ - \lambda_+), \\
\dot{\lambda}_- &= \gamma_1(x - g(,)) + \gamma_2 \dot{y} + \gamma_3(\bar{\lambda}_- - \lambda_-), \\
\dot{y} &= \sigma_K(I - S), \\
\dot{x} &= \nu[(1 - x) \exp(s) - (1 + x) \exp(-s)],
\end{align*}
\]

where

\[
\begin{align*}
I &= \theta_I L^s, \\
S &= S^* + \theta_S(y - y^*)^3, \\
s &= a_1 x + a_2 + \lambda_+ + a_2 - \lambda_- + a_K(y - y^*) + d.
\end{align*}
\]

By analyzing the 2D sub-dynamics ($\lambda_+ = \bar{\lambda}_+, \lambda_- = \bar{\lambda}_-, a_1 = a_2 = a_2 = d = 0$), we can derive the Jacobian of the 2D sub-dynamics ($x = 0, y = y^*$):

\[
J = \begin{pmatrix}
0 & \sigma_K \theta_I \frac{T_c}{2} (\bar{\lambda}_+ - \bar{\lambda}_-) \\
2va_K & 2v(a_1 - 1)
\end{pmatrix}.
\]

The Trace and Determinant are calculated as

\[
\begin{align*}
Tr(J) &= 2v(a_1 - 1), \\
Det(J) &= -2va_K \sigma_K \theta_I \frac{T_c}{2} (\bar{\lambda}_+ - \bar{\lambda}_-).
\end{align*}
\]

Clearly, the Determinant $Det(J)$ is always negative, implying that the neutral opinion equilibrium is always locally unstable.

As for the discussion of the global characteristics of the full 4D Kaldorian dynamics with credit-driven investment sector, we resort to numerical simulation. The parameters are set as follows: $a_1 = 2.5, \theta_I = 0.3, \theta_S = 0.4, y^* = 10$, ceteris paribus for other parameters from the representative simulation of the baseline 4D CDGZ model discussed in previous section. As is shown in Figure 4.13, this simulation takes one step closer to a realistic scenario in the sense that the leverage cycle becomes asymmetric: during the boom period the leverage rises gradually for both optimistic and pessimistic banking sectors,
which is then followed by a sudden slump during the period of recession (top-left). The real sector, on the other hand, moves in tandem with the banking sector (bottom-left): we observe the endogenous switching between the regime of stability and instability (characterized by the sudden falls of output). We also observe the S-shaped saving function and a nonlinear credit-driven investment sector characterized by a regular, 4-stage limit cycle (bottom-right).

4.6 The 7D CDGZ model: the interaction between banks and a speculative financial sector

In order to capture more insights of real-financial market interaction in Minskyan terms, one has to consider also the destabilizing role of a speculative financial market in relation to the real sector. This section extends the 4D baseline model from an alternative perspective to address this issue by adopting the framework of Charpe et al. (2012). Specifically, we now postulate the following law of motion for output ($y$), replacing Equation (4.9):

$$\dot{y} = \beta_y[(a_y - 1)y + a_k(p_k - p_{k0})K + \bar{A}], \quad (4.85)$$

where $\bar{A}$ is autonomous expenditure; $K$ is the total capital stock; $a_y \in (0, 1)$ is the propensity to spend; $a_k > 0$ captures the sensitivity of investment and consumption demand to deviations between the actual and the equilibrium level of capital stock.

As is pointed out by Charpe et al. (2012), the market for $K$ is imperfect due to information asymmetries, adjustment costs, or institutional constraints, hence price do not move instantaneously to clear markets. We assume that the price of $K$ moves according to the expected rate of return on the capital stock, $\rho_k^e$. The law of motion for capital price ($\hat{p}_k$) and the expected rate of return ($\dot{\pi}_k^e$) is given by

$$\dot{\hat{p}}_k = \theta_S L^e(\rho_k^e - \rho_{k0}), \quad (4.86)$$

$$\dot{\pi}_k^e = \beta_\pi \frac{1 + x_p}{2}[\hat{p}_k - \pi_k^e], \quad (4.87)$$

$$\rho_k^e = \frac{b y}{p_k K} + \pi_k^e, \quad (4.88)$$

where $b$ is the profit share; $\rho_{k0}$ denotes the equilibrium level of expected rate of profit. Here we assume that the profits are completely distributed as dividends. Furthermore, the aggregate level of loan issued by banking sector ($L^e$) enters equation (4.86) in the

\footnote{See Charpe et al. (2012) for detailed discussion of this real sector dynamics.}
form of an adjustment parameter $(\theta_{S}L^s)$, since credit expansion will accelerate the rise or fall of asset prices.

We further adopt the Weidlich-Haag-Lux approach to model the opinion formation dynamics of speculative investors. Following Charpe et al. (2012), we assume that there are $2M$ number of investors. Of these, $M_c$ are chartists and $M_f$ are fundamentalists so that $M_c + M_f = 2M$. Let $m = \frac{M_c - M_f}{2}$ and $x = \frac{m}{M}$. We focus on the difference in the size of the two groups (normalised by $M$). Let $p^{f+c}$ be the transition probability that a fundamentalist becomes a chartist, and similarly for $p^{c+f}$. The change in $x_p$ depends on the relative size of each population multiplied by the relevant transition probability ($\exp(s_p)$ and $\exp(-s_p)$). To be precise, we define

$$
\dot{x}_p = v_p[(1 - x_p)\exp(s_p) - (1 + x_p)\exp(-s_p)],
$$

$$
s_p = s_{xp}x_p + s_xx - s_{pk}(p_k - p_{k0})^2 - s_{\pi_k}(\pi_k^e)^2,
$$

$$
s = a_1x + a_2\lambda_+ + a_2\lambda_-
$$

$$
+ a_3((a_y - 1)y + a_k(p_k - p_{k0})K + \hat{A}) + a_{sp}x_p + d,
$$

where $s_{xp}$, $s_{pk}$, and $s_{\pi_k}$ are the cognitive parameters that determine the average opinion of investors, $s_x$ and $a_{sp}$ capture the cognitive interactions between banks and the speculative financial sector.

The full 7D dynamics with a speculative financial sector thus becomes:

$$
\dot{\lambda}_+ = \gamma_1(x + g(\cdot)) + \gamma_2\dot{y} + \gamma_3(\dot{\lambda}_+ - \lambda_+),
$$

$$
\dot{\lambda}_- = \gamma_1(x - g(\cdot)) + \gamma_2\dot{y} + \gamma_3(\dot{\lambda}_- - \lambda_-),
$$

$$
\dot{y} = \beta_y[(a_y - 1)y + a_k(p_k - p_{k0})K + \hat{A}],
$$

$$
\dot{x} = v[(1 - x)\exp(s) - (1 + x)\exp(-s)],
$$

$$
\dot{p}_k = \theta_{S}L^s(p_k^e - p_{k0}),
$$

$$
\dot{\pi}_k^e = \beta_{\pi_k^e}\left[\frac{1}{2} + \frac{x}{p_k - \pi_k^e}\right],
$$

$$
\dot{x}_p = v_p[(1 - x_p)\exp(s_p) - (1 + x_p)\exp(-s_p)].
$$

We conduct numerical simulations to investigate the global properties of system (4.93-4.99). The parameters are set as follows: $a_1 = 1$, $a_{2+} = -0.3$, $a_{2-} = -0.5$, $a_3 = 1$, $d = 10$, $\sigma = 0.8$, $k = 0.1$, $T_c = 1$ $v = 0.4$, $\gamma_1 = 0.3$, $\gamma_2 = 0.4$, $\gamma_3 = 0.03$, $\hat{\lambda}_+ = 15$, $\hat{\lambda}_- = 5$, $\xi_0 = 0.2$, $\xi_1 = 3$, $s_{xp} = 0.9$, $s_{pk} = 2$, $s_{\pi_k^e} = 2$, $b = 0.1$, $K = 1$, $\beta_y = 0.4$, $a_x = 0.3$, $a_k = 0.5$, $p_{k0} = 0.8$, $A = 2$, $\theta = 0.1$, $\rho_{k0} = 0.7$, $\beta_{\pi_k^e} = 0.2$, $v_p = 0.3$, $s_x = 0.2$, $a_{xp} = 0.4$. Figure 4.14 provides the simulation results. Again we stress that this is just one particular example, providing results that are interesting from the
perspective of the 7D full dynamics of real-financial interaction. The result would be highly sensitive with different parameter settings. Yet it generates some interesting results that capture the real-financial market interaction in a stylized manner. The top-left panel shows a turbulent, chaotic swings of asset price in the financial sector generated from this deterministic framework. The top-mid/right panel shows the dynamics of “animal spirits” in both the banking sector ($x$) and the speculative financial sector ($x_p$).

We observe that $x$ and $x_p$ generally move in tandem. Furthermore, we observe that the volatility of the mood swing in the financial market increases when the average opinion of the banking sector becomes more optimistic. This is consistent with the observed positive relationship between $x$ and the volatility of $p_k$ in the mid-right panel. In other words, as the sentiment in banking sector becomes increasingly optimistic, more speculative behaviours start to emerge in the stock market in the form of larger swings of both $x_p$ and $p_k$. The mid-left panel shows the irregular dynamics of real sector, as well as the positive relationship between output and stock price (mid-mid), which is similar to the simulation results of Charpe et al. (2012). Finally, the bottom-mid/right panel shows the dynamics of $\lambda_+$ and $\lambda_-$. It indicates that the loan-to-reserve ratio of the pessimistic bank tends to be more volatile in the long run. Overall, this set of simulations captures the crucial role of banks in propagating financial instability, which eventually transmits into the macroeconomic fluctuations in the real sector.

We also consider the efficacy of policy interventions in the form of Tobin-type tax (at the rate $\tau$) in line with Charpe et al. (2012), where the law of motion for capital gain expectations ($\dot{\pi}_k$) is modified as

$$
\dot{\pi}_k = \beta \pi_k \left[ \frac{1 + x_p}{2} (1 - \tau) \hat{p}_k - \pi_k \right].
$$

(4.100)

Figure 4.15 provides the simulation results augmented by a Tobin-type tax parameter$^{16}$. From this bifurcation plot we observe that the Tobin tax parameter $\tau$ has a stabilizing effect over the system. When $\tau$ increases and passes through $\tau \approx 0.24$, the system switches from a regime of instability to stability with the disappearance of limit cycle. It is arguable that Tobin tax is able to stabilize the system by altering the adjustment speed of capital gain expectations.

$^{16}$Here the Tobin-type tax is referred to as a tax over the capital gains, which is different from Tobin tax that is levied on financial transactions in currency market.
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

Figure 4.14: The extended 7D CDGZ model with a speculative financial sector

Figure 4.15: Bifurcation diagram: the effect of Tobin-type tax
Chapter 4. Modelling the “animal spirits” of bank’s lending behaviour

4.7 Conclusion

The inherent instability of the credit cycle lies at the centre of financial crises (Kindleberger, 1989, Minsky, 1975). In particular, the commercial banks’ coordination failure, driven by waves of optimism and pessimism, ultimately leads to sub-optimal credit provision and amplifies the credit cycle. Keynes (1936) points out that “A sound banker, alas, is not one who foresees danger and avoids it, but one who, when he is ruined, is ruined in a conventional and orthodox way with his fellows, so that no one can really blame him”. More recently, Chuck Prince, the CEO of Citibank, vividly depicts the collective behaviour of banks with his analogy: “as long as the music is playing, you’ve got to get up and dance.”. Better policy actions in the aftermath of a credit crunch require a sound conceptual understanding of the dynamical behaviour of banks (Aikman D., 2011).

This chapter presents a simple model to investigate this issue. The analysis of the model shows that the self-fulfilling and contagious waves of optimism and pessimism amongst bankers lead to a boom-bust pattern for the credit cycle.

Summarising the results, we find that the switching and the herding behaviours of banks are destabilising for the economy. The study of the two scenarios with a constant or variable loan-to-reserve ratio for banks highlights the different destabilizing effects of herding and switching. Switching of banks between the two clusters with different levels in the supply of loan is able by itself to generate fluctuations in the aggregate output. Introducing herding in the banks’ decisions about their loan-to-reserve adds to the instability and amplifies the fluctuations. Instability and amplitude of fluctuations grow with the intensity of herding. A larger value in the herding parameter also causes higher values of the aggregate leverage ratio. The study of the stability and the numerical analysis demonstrate that the parameters that measure the interaction of banks prove to be more relevant than other behavioural parameters, such as the targeted loan-to-reserve ratio.

The baseline 4D model is further extended in two different scenarios. The first extension involves a Kaldorian I-S disequilibrium with credit-driven investment sector. We keep the system in 4 dimensions, yet the nonlinear credit-driven investment function can be naturally derived from the isocline of $x$. The local stability analysis indicates that the neutral equilibrium is always unstable, yet the global stability of the 4D system is insured due to the self-stabilizing long run adjustment of loan-to-reserve ratio ($\lambda_+$ and $\lambda_-$) both in cognitive (through influencing the average opinion index) and in behavioural terms (through influencing the behavioural rules of bank’s lending activity). The representative simulation is characterized by asymmetric leverage cycles with gradual boom and sudden
bust, as well as an endogenous switching from stability to instability in the real sector. The next extension of the present framework involves a properly modelled speculative financial sector, adopting from the DSGD\textsuperscript{17} framework proposed by Charpe et al. (2012). The simulation captures a positive correlation between bank’s sentiment and the degree of exuberance in the speculative market, in the form of volatility of asset prices and mood swing of speculative investors. Besides providing a better assessment of the destabilising effects of the credit cycle on the real sector, the interaction between the banking sector and a speculative financial sector can open interesting perspective for policy analysis in the present setting. The complete model is useful to study the feedback effect on the credit cycle of the phenomena of over-investment and over-indebtedness.

\textsuperscript{17}Dynamic Stochastic General Disequilibrium.
Chapter 5

Credit bubble, “monetary famine”, and stock market crash: a disequilibrium dynamic model of interbank market

5.1 Introduction

“Any sudden event which creates a great demand for actual cash may cause, and will tend to cause, a panic in a country where cash is much economised, and where debts payable on demand are large. In such a country an immense credit rests on a small cash reserves, and an unexpected and large diminution of that reserve may easily break up and shatter very much, if not the whole, of that credit.”

- Bagehot (1873), Lombard Street - Chapter VI. Why Lombard Street is often very dull, and sometimes extremely excited

In the mid-2013, China’s interbank market had experienced an unprecedented liquidity shortage, which had sparked off a fear of “monetary famine”\(^1\) (Ma and Shu, 2013, Sheng, 2013). The benchmark overnight and seven-day repo rates reached above 10% in mid-June, and soared to a record high of nearly 30% on 20/June. At the end of December it rose again from 4.5% on 17/Dec to an intra-day high of nearly 10% on 23/Dec.

\(^1\)The term “monetary famine” is coined by Andrew Sheng, the President of Fung Global Institute Hong Kong, in his article “China’s inter-bank liquidity shortage: Feast or Famine?” (Sheng, 2013).
Consequently the interbank activity contracted significantly in the aftermath of the “monetary famine”.

Nearly two years after the “monetary famine”, China had witnessed another colossal crash in the stock market. On 12/June/2015, almost a third of the value of A-shares on the Shanghai Stock Exchange was lost within one month. It once again triggered panics in investors’ sentiments, which had prompted a drastic intervention from both regulatory and monetary authorities.

While the term “monetary famine” is relatively new, the phenomenon of money market liquidity shortage in the burst of credit bubble is recurrent throughout history. As Bagehot (1873) describes, the money market is “often very dull, and sometimes extremely excited”. One of the earliest “monetary famine” can be dated back to “the panic of 1893” in the US, during which time the bank credit was extended to overbuilding and shaky finance of railway in the preceding period. When the sudden collapse of confidence triggered bank runs, the banks did not have enough reserves and were forced to scramble for cash. Subsequently the short-term interested rates soared (Akerlof and Shiller, 2009).

Interbank market, a major subset of money market, is where banks lend or borrow reserves. Given that some banks will be short of reserves while others will be in surplus of reserves, they will trade with each other, paying or receiving interests on these transactions (Ryan-Collins et al., 2012). The interbank market plays at least two important roles in the modern financial system: first, it facilitates the implementation of monetary policy - it is a market that the central bank constantly intervene in order to control and stabilize the short-term interest rate; second, a well-functioning interbank market facilitates the efficient allocation of resources from institutions to institutions (Furfine, 2001).

Surprisingly, there is no widely accepted theoretical framework in financial literature that analyses how interbank market operates, despite its apparent importance in the financial system (Allen et al., 2009). This is possibly due to the common perception that interbank market is amongst the most liquid during normal times, as the interbank loans are short-term in nature (Iori, 2012). Yet the world-wide malfunction of interbank markets since the 2007 GFC has triggered a new interest in the study of this field. In the mainstream theoretical literature, numerous studies had attempted to model the interbank market by adopting the two-period framework proposed by the seminal paper of Diamond and Dybvig (1983) (Acharya and Skeie, 2011, Allen and Gale, 2000, 2004, Allen et al., 2009, Diamond and Rajan, 2005, Freixas and Jorge, 2008). In this strand of literature, the bank is treated merely as a financial intermediary: it acts as a central planner that takes deposits on one hand (at period “0”) and invests in both short and
long assets on the other hand, which is then redistributed to early and late consumers (at period “1” and “2”). The model is static in the sense that the bank allocates deposit only at the initial period. The bank is not able to hedge the risk of liquidity shock completely at the beginning, since the information about early/late consumers is not revealed until period “1”, when the bank buys or sells the long asset in the interbank market in order to meet the liquidity demand.

Yet this two-period framework fails to provide a realistic picture of how interbank market works, since it overlooks the bank’s role in creating credit, nor does it capture the dynamic interaction between the central bank and commercial banks. In reality, the bank extends loans in the first place, which simultaneously creates deposits. It will borrow reserves later on in order to fulfill the reserve requirement, as well as to meet the uncertainty of deposit withdrawal. It has an important implication over the credit cycle. As Minsky (1975) and Kindleberger (1989) depict: during the boom phase, the initial optimism in the banking sector leads to a credit expansion. Loosening regulatory environment, coupled with financial innovation, allow banks to circumvent regulation even further. Newly created “financial service product” arises and shadow banking activity emerges, which further reinforces the self-fulfilling credit expansion process. At certain point, the vast amount of credit created in the preceding period rests upon small cash reserves as Bagehot describes, which makes the banking system extremely fragile to any external shocks. Subsequently, the initial optimism is suddenly replaced by pessimism. Regulation eventually tightens and bad loans start to emerge. Banks are forced to hoard more cash reserves in the presence of increasing uncertainty, which causes panics.
Chapter 5. Credit bubble, “monetary famine”, and stock market crash: a disequilibrium dynamic model of interbank market

in interbank market and reinforces the downturn. In the case of China, as we can see from Figure 5.1, from 1999 to 2010, the ratio between \( M_2 \) and \( M_0 \) in China had almost doubled. This indicates a drastic expansion of credit, which poses a warning sign prior to the “monetary famine” in the year 2013 and the subsequent stock market crash two years later. From a Minskyan perspective, these two seemingly unrelated events essentially reveal the same mechanism underneath: the preceding period of moderation encourages the growth of debt-financed speculative behaviour, which increases financial fragility. It eventually causes financial collapse in the subsequent bust period that leads to macroeconomic instability and debt deflation. Apart from the speculative factors that triggered the crash, a lack of pre-emptive policy intervention against speculation is another factor to blame.

The aim of this chapter is twofold: first, we build an interbank market model in the spirit of disequilibrium, non-market-clearing approach. The model is extended from the previous chapter of Chiarella et al. (2015a). It proposes a number of departures from the traditional Diamond and Dybvig (1983) two-periods framework. We start with constructing a 5D dynamical system that incorporates the feedbacks between banks’ lending attitudes, the interbank lending rate, and a credit-augmented, demand-driven dynamic multiplier in the real sector. We analyze the effect of monetary policy both in conventional and unconventional terms. Second, to gain a deeper understanding of the real-financial interaction from a Minskyan perspective, we extend the model by incorporating a speculative financial sector from the 7D CDGZ model of Chiarella et al. (2015a) proposed in the previous chapter. The chapter contributes to the interbank market literature in two aspects of novelties. First, it represents one of the first attempt to model the interbank market by applying a dynamical system approach, to the best of our knowledge. The second originality involves the modelling of dynamic interactions between the central bank and commercial banks both in conventional (OMO) and unconventional (QE) manner, which sheds light on conditions over the effectiveness of monetary policy. The numerical simulations successfully capture some stylized dynamics of credit cycles that involves over-lending during the boom period and the liquidity shortage (“monetary famine”) during the bust period in terms of interest rate spikes during the phases of panics and economic downturns.

The rest of the chapter is organized as follows: in the next section, we propose a baseline 5D model extended from the 4D CDGZ framework. We analyze the local stability of the 2D/3D sub-dynamics and numerically simulate the global dynamics of both the 3D sub-system and the 5D system in section 5.3 and 5.4. We further add a new dynamic law that captures the effect of Unconventional Monetary Policy in section 5.5. Finally, section 5.6 presents a full 8D system that incorporates a speculative financial sector, extending from the 7D CDGZ model. As a policy prescription, we maintain that it is necessary
for the monetary authority to conduct *ex-post* unconventional policy intervention of Quantitative Easing (QE) type to stabilize the interbank market during the period of crisis, and it is equally crucial to consider the timing of exiting the QE scheme. Yet it is more important to implement *ex-ante* policy measures such as Tobin-type tax over capital gains in order to curb excess speculative behaviour in the financial market prior to the crisis.

5.2 The baseline model

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<th>Liability</th>
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<tr>
<td><strong>Asset</strong></td>
<td><strong>Liability</strong></td>
</tr>
<tr>
<td>Reserve (-)</td>
<td>Interbank Loan (+)</td>
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<th>Borrowing Bank</th>
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<tr>
<td><strong>Asset</strong></td>
</tr>
<tr>
<td>Reserve (+)</td>
</tr>
</tbody>
</table>

Table 5.1: The change of balance sheet after interbank transaction

Table 5.1 illustrates the change of commercial banks’ balance sheets after an interbank transaction. For the lending bank, we see that the change happens on the asset side: there is a simultaneous decrease of reserves and an increase of interbank loans. While for the borrowing bank, the increase of reserves on the asset side is accompanied by a simultaneous increase of an interbank loan on the liability side.

5.2.1 The baseline 4D CDGZ model

Our starting point of the analysis begins with the baseline 4D CDGZ model of Chiarella et al. (2015a):

- There are $2N$ banks. Each bank has an equal amount of unborrowed reserves ($R$) on the balance sheet, with a total reserve ($T_c = 2NR$) in the aggregate banking system;

- The banks are categorized into two groups, i.e. optimistic and pessimistic banks. Each follows their own law of motion of the loan-to-reserve ratios ($\lambda_+$ and $\lambda_-$), which is defined as the ratio between loans and unborrowed reserves. The loan-to-reserve ratio is assumed to be influenced by the average opinion ($x$), the change in the real sector ($\dot{y}$), and a long run adjustment factor ($\bar{\lambda}_+$ and $\bar{\lambda}_-$);
The real sector follows a Blanchard (1981) demand-driven dynamic multiplier where the aggregate demand is credit-driven;

- The law of motion of the average opinion follows Lux (1995) and Franke (2012);
- There is a convergence or divergence of lending strategies when the average opinion is optimistic/pessimistic or neutral. This is captured by a function \( g(x) = \xi_0 \exp(-\xi_1 x^2) \).

The dynamical system is given by equation (5.1-5.4):

\[
\begin{align*}
\dot{\lambda}_+ &= \gamma_1(x + g(\cdot)) + \gamma_2 \dot{y} + \gamma_3 (\bar{\lambda}_+ - \lambda_+), \\
\dot{\lambda}_- &= \gamma_1(x - g(\cdot)) + \gamma_2 \dot{y} + \gamma_3 (\bar{\lambda}_- - \lambda_-), \\
\dot{\gamma} &= \sigma(y^d - y), \\
\dot{x} &= v[(1 - x) \exp(s) - (1 + x) \exp(-s)]\end{align*}
\]

where

\[
\begin{align*}
g^d &= y^d + kL^* = y^d + k(T_c/2)[(1 + x)\lambda_+ + (1 - x)\lambda_-], \\
g(x) &= \xi_0 \exp(-\xi_1 x^2), \\
s &= a_1 x + a_2 + \lambda_+ + a_2 - \lambda_- + a_3 (y^d - y) + d, \\
\bar{\lambda}_+ &> \bar{\lambda}_-.
\end{align*}
\]

Equation (5.1-5.2) defines the law of motion for \( \lambda_+ \) and \( \lambda_- \). The first term \( \gamma_1(x + g(\cdot)) \) and \( \gamma_1(x - g(\cdot)) \) describes the optimistic/pessimistic banks’ reactions toward the average opinion; the second term \( \gamma_2 \dot{y} \) captures the accommodative role of banks toward a change in GDP \( \dot{y} \); The last term \( \gamma_3(\bar{\lambda}_+ - \lambda_+) \) and \( \gamma_3(\bar{\lambda}_- - \lambda_-) \) describes the long run tendency that optimistic banks over-lend while the pessimistic banks under-lend.

5.2.2 The loan-to-reserve ratio after interbank borrowing/lending

We define a new loan-to-reserve ratio \( \lambda^* \), which is the ratio between loans and reserve balances after interbank borrowing (lending) activities:

\[
\lambda^* = \frac{L^*_+}{n_+R + BR^d} = \frac{L^*_+}{n_-R - BR^d},
\]

where

- \( L^*_+ \) and \( L^*_- \) are the aggregate loans for over-lending banks and under-lending banks,
• $n_+$ and $n_-$ are the number of over-lending banks and under-lending banks ($n_+ + n_- = 2N$),

• $R$ is the unit reserve of each bank (total reserves $T_c = 2NR$),

• $BR^d$ and $BR^s$ are the demand and supply of borrowed reserves.

As discussed in chapter 3, we assume that $\lambda^*$ is bounded between $[\lambda_0 - \theta_0, \lambda_0 + \theta_0]$, and it is influenced by the average lending attitude ($x$). When the average lending attitude is optimistic, the banks would hold less reserves (a higher $\lambda^*$). On the other hand, when the average lending attitude is pessimistic, the banks would desire to hold more reserves (a lower $\lambda^*$). We also assume that the value of $\lambda^*$ is bounded between $\lambda_0 - \theta_0$ and $\lambda_0 + \theta_0$. Thus

$$\lambda^* = \lambda_0 + \theta_0 \text{tanh}(\theta_1 x)^2. \quad (5.10)$$

### 5.2.3 The excess demand of reserve in the banking system

We then derive the demand equation for borrowed reserve ($BR^d$) from the over-lending banks by rearranging Equation (5.9). Note that $L^S_+ = T_c/2(1 + x)\lambda_+$ and $n_+ R = T_c/2(1 + x)$. Thus

$$\lambda^* = \frac{T_c}{2} (1 + x)\lambda_+ + BR^d, \quad (5.11)$$

$$\rightarrow BR^d = \frac{T_c}{2} (1 + x)(\frac{\lambda_+}{\lambda^*} - 1). \quad (5.12)$$

Similarly, the supply of borrowed reserves ($BR^s$) from the under-lending banks is derived as

$$\lambda^* = \frac{T_c}{2} (1 - x)\lambda_-, \quad (5.13)$$

$$\rightarrow BR^s = \frac{T_c}{2} (1 - x)(1 - \frac{\lambda_-}{\lambda^*}). \quad (5.14)$$

The excess demand for borrowed reserves ($ER^d$) is the difference between $BR^d$ and $BR^s$:

$$ER^d = BR^d - BR^s = \frac{T_c}{2} [(1 + x)(\frac{\lambda_+}{\lambda^*} - 1) - (1 - x)(1 - \frac{\lambda_-}{\lambda^*})]. \quad (5.15)$$

\(^2\)For mathematical convenience, we assume that $\lambda_0 = \frac{\lambda_+ + \lambda_-}{2}$. 
5.2.4 The dynamics of interbank borrowing rate

We assume that the interbank borrowing rate is driven by two forces: the excess demand for borrowed reserves and the central banker’s intervention through Open Market Operation that provides the excess supply of reserves:

\[ \dot{r} = \beta_1(ER^d) + \beta_2(r^* - r), \quad (5.16) \]

where \( ER^d = BR^d - BR^s = \frac{\gamma_1}{\gamma_2}[(1 + x)(\frac{\lambda_+}{\lambda_-} - 1) - (1 - x)(1 - \frac{\lambda_+}{\lambda_-})]. \)

In equation (5.16), the first term \( \beta_1(ER^d) \) captures the adjustment speed of excess demand of reserves over the interbank rate. A positive excess demand for reserves in the aggregate banking system would pose an upward pressure over the interbank rate, at the rate \( \beta_1 \). The second term \( \beta_2(r^* - r) \) describes central bank’s policy action by providing the counterbalancing excess supply in aggregative terms. When the actual interest rate \( r \) deviates from the target rate \( r^* \), the central bank will conduct repo or reversed repo until the rate goes back to the target.

Alternatively, we would re-arrange equation (5.16) in terms of supply of excess reserves \( ER^s \) from the central bank. That is

\[ \dot{r} = \beta_1(ER^d - ER^s) = \beta_1(ER^d - \frac{\beta_2}{\beta_1}(r - r^*)), \quad (5.17) \]

where \( ER^s = \frac{\beta_2}{\beta_1}(r - r^*). \)

5.2.5 The extended 5D CDGZ model with an interbank lending market

Combining the law of motion for interbank market rate with the 4D CDGZ model, the 5D system with an interbank market is written as follows:

\[ \dot{\lambda}_+ = \gamma_1(x + g(\cdot)) + \gamma_2\dot{y} + \gamma_3(\dot{\lambda}_+ - \lambda_+) \quad (5.18) \]
\[ \dot{\lambda}_- = \gamma_1(x - g(\cdot)) + \gamma_2\dot{y} + \gamma_3(\dot{\lambda}_- - \lambda_-) \quad (5.19) \]
\[ \dot{y} = \sigma(y^d - y) \quad (5.20) \]
\[ \dot{x} = v[(1 - x)\exp(s) - (1 + x)\exp(-s)] \quad (5.21) \]
\[ \dot{r} = \beta_1(BR^d - BR^s) + \beta_2(r^* - r) \quad (5.22) \]
where

\[ y^d = y_0^d + kL^s = y_0^d + k(T_c/2)[(1 + x)\lambda_+ + (1 - x)\lambda_-], \quad (5.23) \]
\[ g(.) = \xi_0 \exp(-\xi_1 x^2), \quad (5.24) \]
\[ s = a_1 x + a_2 + \lambda_+ + a_3 (y^d - y) + a_4 (r - r^*) + d, \quad (5.25) \]
\[ BR^d - BR^s = ER^d = \frac{T_c}{2}[(1 + x)(\lambda_+ - 1) - (1 - x)(1 - \frac{\lambda_-}{\lambda^*})]. \quad (5.26) \]

The excess interbank rate \( (r - r^*) \) now enters the opinion formation index \( (s) \). We could possibly interpret this term as a “Wicksellian Differential”. Suppose the target rate \( (r^*) \) is the natural rate of interest that leads to price stability and \( r \) is the money rate in the Wicksellian sense. A negative Wicksellian differential will lead to an expansion of credit, thus boosting the real sector \( (r < r^* \rightarrow s \uparrow \rightarrow L^s \uparrow \rightarrow y \uparrow) \), and vice versa.

Figure 5.2 provides a graphical illustration of system (5.18-5.22). The two blue rectangles \( (n_+ R \text{ and } L^+_s) \) show the reserves and loans of the over-lending banks, while the two red rectangles \( (n_- R \text{ and } L^-_s) \) show the reserves and loans of the under-lending banks. The banks expand or contract their balance sheets according to the law of motion (5.18-5.19). The overlapping area \( (BR) \) is the interbank market where banks lend and borrow reserves to each other. The disequilibrium between \( BR^d \) and \( BR^s \) would eventually result in fluctuations in the interbank market rate, in the absence of policy interventions from the central bank. The black rectangular \( (ER^s) \) shows the central bank’s intervention by injecting excess supply of reserve \( (ER^s) \) in order to counterbalance the excess demand of reserves in the aggregate banking sector and maintain the target rate \( (r^*) \).
5.3 Analysis of sub-dynamical system

5.3.1 The 2D sub-dynamics with \( x \) and \( r \)

Now we analyze the equilibrium and local stability of system (5.18-5.22). Since the definitive local stability condition is difficult to obtain in the 5D system, we firstly focus our attention on the reduced system (5.21-5.22). The reasons are two fold: first, the destabilizing effect of heterogeneous lending strategy, captured by equation (5.18-5.19), has been extensively analysed in Chiarella et al. (2015a); second, the main focus of this chapter is to study the interplay between banker’s sentiment contagion and the interbank market rate. Hence we assume that the loan-to-reserve ratio will converge to the long-run adjustment factor \( \bar{\lambda}_+ \) and \( \bar{\lambda}_- \) respectively. We set \( \gamma_1, \gamma_2, \gamma_3, \sigma, a_2+, a_2-, a_3, d \) to zero, in order to obtain the following 2D sub-dynamics:

\[
\begin{align*}
\dot{x} &= v[(1-x)\exp(s) - (1+x)\exp(-s)], \quad (5.27) \\
\dot{r} &= \beta_1(BR^d - BR^s) + \beta_2(r^* - r), \quad (5.28)
\end{align*}
\]

where

\[
\begin{align*}
s &= a_1x + a_4(r - r^*), \quad (5.29) \\
BR^d - BR^s &= ER^d = T_c^2 \left[(1+x)\left(\frac{\bar{\lambda}_+}{\lambda^*} - 1\right) - (1-x)(1-\frac{\bar{\lambda}_-}{\lambda^*})\right]. \quad (5.30)
\end{align*}
\]

By setting the \( LSH = 0 \), we can derive two isoclines of the system (5.27-5.28) below, which is illustrated in Figure 5.3.

\[
\begin{align*}
r - r^* &= \frac{1}{2a_4} \ln\left(\frac{1+x}{1-x}\right) - \frac{a_1}{a_4}x, \quad (5.31) \\
r - r^* &= \frac{\beta_1 T_c}{\beta_2} \frac{1}{2} [(1+x)(\frac{\bar{\lambda}_+}{\lambda^*} - 1) - (1-x)(1-\frac{\bar{\lambda}_-}{\lambda^*})]. \quad (5.32)
\end{align*}
\]

**Proposition 5.1:** There always exists a neutral opinion equilibrium \( E_0 = (0, r^*) \) for system (5.27-5.28) if \( \frac{\bar{\lambda}_+ + \bar{\lambda}_-}{2} = \lambda_0 \). \( E_0 \) is locally stable when (i) \( \beta_2 > 2v(a_1 - 1) \) and (ii) \( (a_1 - 1)\beta_2 > \frac{\alpha_0 T_c}{2\lambda_0} (2\theta_0 \bar{\lambda}_1 - \bar{\lambda}_+ - \bar{\lambda}_-) \). Otherwise, the equilibrium becomes unstable and it is possible that multiple non-neutral equilibria would emerge, depending on the shape of isocline (5.31-5.32).

**Proof:** By setting \( LSH = 0 \) for equation (5.27) we derive that \( 2s = \ln\left(\frac{1+x}{1-x}\right) \). It is easy to see that when \( x = 0 \) and \( r = r^* \) this equation will be satisfied. Substituting these to equation (5.28) with \( LSH = 0 \) we have \( \beta_1 \frac{T_c}{2} \left[\frac{\bar{\lambda}_+ + \bar{\lambda}_-}{\lambda_0} - 2\right] = 0 \Rightarrow \frac{\bar{\lambda}_+ + \bar{\lambda}_-}{\lambda_0} = 2 \).
we derive the Jacobian of system (5.27-5.28) at $E_0 = (0, r^*)$ as follows:

$$\begin{pmatrix}
2v(a_1 - 1) & 2va_4 \\
\frac{\beta_1 T_c}{2\lambda_0}(\bar{\lambda}_+ - \bar{\lambda}_- - 2\theta_0 \theta_1) & -\beta_2
\end{pmatrix},$$

with Trace and Determinant:

$$
Tr(J) = 2v(a_1 - 1) - \beta_2,
$$

$$
Det(J) = -2v[(a_1 - 1)\beta_2 + a_4 \frac{\beta_1 T_c}{2\lambda_0}(\bar{\lambda}_+ - \bar{\lambda}_- - 2\theta_0 \theta_1)],
$$

The necessary and sufficient condition for local stability is that $Tr(J) < 0$ and $Det(J) > 0$, from which we derive that (i) $\beta_2 > 2v(a_1 - 1)$ and (ii) $(a_1 - 1)\beta_2 > \frac{a_4 \beta_1 T_c}{2\lambda_0}(2\theta_0 \theta_1 - \bar{\lambda}_+ - \bar{\lambda}_-)$.

It is difficult to obtain other possible non-neutral equilibria ($x \neq 0$ and $r \neq r^*$), since both isoclines (5.31-5.32) are highly nonlinear. Yet it is easy to see from Figure 5.3 that there exist other equilibria when the two isoclines intersect, depending on the parameter setting.

The two conditions proposed above essentially shed light on the effectiveness of monetary policy. It implies that in order to maintain the target interest rate $r = r^*$, as well as a neutral sentiment amongst banking sector ($x = 0$), the central bank has to act quick enough so that it outweighs (i) the herding/contagion effect amongst banks (captured
by the parameter \( a_1 \), (ii) the money market adjustment speed (captured by \( \beta_1 \)) as well as banks’ sensitivity of aggregate reserve demand in relation to the average opinion (captured by the switching parameter \( \theta_0 \theta_1 \)).

From system (5.27-5.28) we can also see the positive feedback loop of interbank contagion through interest rate channel: in the absence of effect policy intervention, an initial rise in interbank rate leads to lower opinion formation index \( (s) \), thus leads to a decrease in lending attitude \( (x) \). Consequently the aggregate loan-to-reserve ratio \( (\lambda^\star) \) decreases and the aggregate demand for excess reserve \( (ER^d) \) increases, which leads to higher rate \( (r) \). That is, \( r \uparrow \rightarrow s \downarrow \rightarrow \lambda^\star \downarrow \rightarrow ER^d \uparrow \rightarrow r \uparrow \).

### 5.3.2 The 3D sub-dynamics of \( x, r, \) and \( y \)

Next, we now consider the 3D dynamics with the inclusion of equation (5.20). The system now becomes:

\[
\dot{y} = \sigma (y^d - y) \\
\dot{x} = v[(1 - x) \exp(s) - (1 + x) \exp(-s)] \\
\dot{r} = \beta_1 (BR^d - BR^s) + \beta_2 (r^* - r)
\]

**Proposition 5.2:** the system (5.35-5.37) possesses a neutral opinion equilibrium \((y^d_0 + k(T_c/2)[\bar{\lambda}_+ + \lambda_-], 0, r^*)\) if \( \bar{\lambda}_+ + \lambda_- - \frac{\lambda_0}{2} = 0 \). The neutral opinion equilibrium is more likely to be stable if (i) \((0 < a_1 < 1)\); (ii) \(a_3 \text{ and } a_4\) are sufficiently close to 0.

**Proof:** As a first step, it is easy to note that the dynamical system (5.35-5.37) has the following neutral opinion equilibrium:

\[
y = y^d = y^d_0 + k(T_c/2)[\bar{\lambda}_+ + \lambda_-], \\
x = 0, \\
r = r^*.
\]

The Jacobian at neutral opinion equilibrium is derived as:

\[
\begin{pmatrix}
-\sigma & \sigma k \frac{T_c}{2} (\bar{\lambda}_+ - \lambda_-) & 0 \\
-2v a_3 & 2v (a_1 - 1) & 2v a_4 \\
0 & \frac{\beta_1 T_c}{2\lambda_0} (\bar{\lambda}_+ - \lambda_- - 2\theta_0 \theta_1) & -\beta_2
\end{pmatrix}
\]
Suppose that $a_3$ and $a_4$ is sufficiently close to 0. the Trace, Determinant, and the 3 principle minors are derived as:

\[\text{Tr}(J) = 2v(a_1 - 1) - \sigma - \beta_2,\]  
\[\text{Det}(J) = 2v\sigma\beta_2(a_1 - 1),\]  
\[J_1 = -2\beta_2v(a_1 - 1),\]  
\[J_2 = \beta_2\sigma,\]  
\[J_3 = -2\sigma v(a_1 - 1).\]

It is obvious that as long as $a_1 < 1$, three of the necessary Routh-Hurwitz conditions are easily satisfied. That is: $\text{tr}(J) < 0, J_1 + J_2 + J_3 > 0, \text{det}(J) < 0$.\(^3\)

### 5.4 Numerical simulations

#### 5.4.1 Simulations of the 3D sub-dynamics

In order to examine the global features of the non-linear 3D dynamics, we apply the standard Euler method to reformulate the model in discrete time in order to simulate the system under different parameter sets. We start with the 3D sub-system (5.35-5.37). It is important to note that they only provide a representative simulation, and the dynamics can be very sensitive with respect to variations in parameters. The initial parameters are set as follows: $a_1 = 0.05$ (stable scenario), $a_1 = 0.3$ (unstable scenario), $a_3 = 2.4$, $a_4 = -1.1$, $T_c = 1$, $k = 0.1$, $v = 0.5$, $\beta_1 = 0.5$, $\beta_2 = 5.8$, $\theta_0 = 5$, $\theta_1 = 4$, $y_0 = 11$, $r^* = 0.05$, $\lambda_+ = 15$, $\lambda_- = 5$, $\sigma = 0.5$, and $\lambda_0 = 10$.

We first run a set of two representative simulations. The first simulation describes a stable scenario while the second simulation describes an unstable scenario. The stable scenario is shown on the left panel of Figure 5.4. The upper-left panel shows that the opinion dynamics and interbank rate fluctuate counter-cyclically until they converge to the point attractor over time, where $x = 0$ (neutral-opinion equilibrium) and $r = r^* = 0.05$. The mid-left panel provides the phase plot of $x$ and $r$, which shows the inverse relationship between lending attitude and interbank rate - a state of optimism corresponds to lower interbank rate while a state of pessimism corresponds to a higher one. The lower-left panel plots the relationship between $r$ and $y$, which can be interpreted in Wicksellian terms in the absence of sufficient policy intervention from the central bank: a negative deviation of money rate from the natural rate (the ideal policy rate $r^*$) leads

\[^3\text{Local stability is insured if } -\text{tr}(J)(J_1 + J_2 + J_3) + \text{det}(J) > 0.\]
to a credit expansion, which ultimately brings a positive disequilibrium in the real sector, while a positive deviation of money rate from the natural rate leads to a reduction of the credit and real sector. The unstable scenario in the right panel shows that when the contagion parameter $a_1$ increases from $a_1 = 0.05$ to $a_1 = 0.3$, the fixed point becomes locally unstable.

We further draw the bifurcation diagram of $a_1$, $a_3$, and $a_4$, in order to see the effect of varying opinion formation parameters on the stability of system (5.35-5.37), as is shown in Figure 5.5. The interpretation of upper (bifurcation for $a_1$) and mid (bifurcation for $a_3$) panel is straightforward: it shows that the system tends to be stable if the two parameters are sufficiently small. Hopf bifurcation occurs when $a_1$ and $a_3$ increase and pass through $a_1 \approx 0.14$ and $a_3 \approx 2.1$ respectively, holding other parameters constant. The bifurcation for $a_4$ seems more complex with four different regimes of stability/instability, corresponding to different fixed points or limit cycles.
5.4.2 Simulations of the 5D dynamics

We then proceed to simulate the complete 5D system (5.18-5.22) with heterogeneous lending strategies. The initial parameters are set as follows: $a_1 = 1.4$, $a_{2+} = -0.3$, $a_{2-} = -0.5$, $a_3 = 1.1$, $a_4 = -2.1$, $\sigma = 0.5$, $T_c = 1$, $k = 0.1$, $y_0^d = 12$, $d = 6$, $v = 0.5$, $\gamma_1 = 0.3$, $\gamma_2 = 0.01$, $\gamma_3 = 0.07$, $\beta_1 = 0.5$, $\beta_2 = 9.8$, $\theta_0 = 8$, $\theta_1 = 10$, $\xi_0 = 0.5$, $\xi_1 = 3$, $\bar{r}^* = 0.05$, $\bar{\lambda}_+ = 15$, $\bar{\lambda}_- = 5$, $\lambda_0 = 9$. Figure 5.6 provides a representative simulation of system (5.18-5.22). We can observe a sequence of persistent, yet not so complex fluctuations of the 5 dynamic variables. The upper-left panel shows the dynamics of $\lambda_+$ and $\lambda_-$. The under-lending banks have a lower loan-to-reserve ratio compared to the over-lending banks, which have a higher loan-to-reserve ratio. The lower-right graph shows the constant switch of $\lambda^*$ in relation to the lending attitude $x$. It implies that
the aggregate banking system desires to hold less reserves (higher $\lambda^*$) when the average opinion is optimistic, and vice versa.

One of the key tasks in this chapter is to simulate the link between the boom period on the real side that leads to over-lending in the banking sector, which ultimately causes the interbank turmoil during the bust period. Hence we look at the dynamics of interbank rate in relation to the dynamics of the real sector. The lower-left graphs show the dynamics of $r$ in relation to $y$. We observe that the interbank rate is generally stable during a boom period. Yet a sequence of interest rate spikes occurs during the phase of economic downturn. There are two spikes in each phase. The first one is possibly due to the sudden increase of reserve holdings when the lending attitude suddenly switches from optimism to pessimism, in the presence of economic downturn; the second interest spike is possibly due to the ongoing demand pressure for reserves during the down cycle on
Chapter 5. Credit bubble, “monetary famine”, and stock market crash: a disequilibrium dynamic model of interbank market

The real side, since more and more banks becomes pessimistic, which further reinforces the uprise of interest rate in the interbank market.

We further analyze the sensitivity of switching parameter $\theta_0$ that determines the bank’s aggregate reserve demand in relation to average opinion. Figure 5.9 shows the phase plot of interbank rate with varying $\theta_0$. We first look at a scenario where bank’s aggregate reserve demand is insensitive toward the average opinion ($\theta_0 = 0$). Then we increase $\theta_0$ in a *ceteris paribus* manner, with two scenarios ($\theta_0 = 4$ and $\theta_0 = 8$). With an
increasing value of $\theta_0$, the complex limit cycle in the representative simulation starts to emerge, which captures tranquility during the period of optimism and chaos during the period of pessimism. It can be seen that banks’ general attitude toward reserve demand in aggregative terms is crucial in generating the endogenous regime switching between tranquility and chaos in the interbank market.
5.4.3 The uncertainty of deposit withdrawal

So far we have not yet considered the uncertainty of deposit withdrawal. We postulate a scenario where the depositor’s demand for cash is stochastic \((ER^d_w = ER^d + \epsilon)\). Furthermore, the variance of \(\epsilon\) is relatively small during the period of positive sentiment, yet relatively large during the period of panic. The dynamics of \(r\) thus becomes:

\[
dr = [\beta_1 ER^d + \beta_2 (r^* - r)]dt + \eta dW,
\]

where

- \(dW\) is a standard Wiener process,
- \(\eta\) is a drift defined below:

\[
\eta = \begin{cases} 
\eta_- &: x \geq 0 \\
\eta_+ &: x < 0
\end{cases}
\]

The detailed analytical discussion of this SDE is beyond the scope of this chapter. We only conduct numerical experiment on equation (5.46) combined with equation (5.18-5.21). Figure 5.10-5.11 provides the numerical experiments in four scenarios with different values of \(\eta_-\) and \(\eta_+\). We observe similar patterns compared to the deterministic simulation, with extra uncertainty due to the presence of the newly-introduced stochastic term.

5.5 The effect of Unconventional Monetary Policy

This section discusses the effect of Unconventional Monetary Policy. We propose a counter-cyclical policy measure that aims to smooth the cycle during the period of panics and economic downturns. We assume that the central bank will provide an additional liquidity injection \((q)\) if \(x < 0\). However, the central bank will gradually exit this scheme if \(x \geq 0\). Hence we postulate that

\[
ER^s = \frac{\beta_2}{\beta_1} (r - r^*) + q,
\]

\[
\dot{q} = \frac{\alpha_1}{1 + exp(\alpha_2 x)} + \alpha_3 q.
\]

Equation (5.48) is the law of motion for the proposed counter-cyclical policy measure \(q\). As is illustrated in Figure 5.12, the first term \(\frac{\alpha_1}{1 + exp(\alpha_2 x)}\) provides a triggering mechanism:
Figure 5.10: The uncertainty of deposit withdrawal I

\[
\frac{\alpha_1}{1 + \exp(\alpha_2 x)} \quad \text{will approach zero during periods of optimism} \quad (x > 0) \quad \text{and it will approach} \quad \alpha_1 \quad \text{during periods of pessimism}, \quad \text{implying additional liquidity will be injected at such rate during this period.} \quad \text{We postulate such a functional form so that the volume of UMP will not exceed the maximum value} \quad \alpha_1, \quad \text{given the sensitivity parameter} \quad \alpha_2 \quad \text{in relation to sentiment.} \quad \text{The second term captures the exit mechanism.} \quad \text{Assuming that} \quad \alpha_3 < 0, \quad \text{it will allow} \quad q \quad \text{to revert over time.}
\]

We first examine the 4D system with constant loan-to-reserve ratios (\(\lambda_+ = \bar{\lambda}_+\) and \(\lambda_- = \bar{\lambda}_-\)) augmented by UMP:

\[
\dot{y} = \sigma(y^d - y), \quad \text{(5.49)}
\]

\[
\dot{x} = v[(1 - x) \exp(s) - (1 + x) \exp(-s)], \quad \text{(5.50)}
\]

\[
\dot{r} = \beta_1 (ER^d - ER^s), \quad \text{(5.51)}
\]

\[
\dot{q} = \frac{\alpha_1}{1 + \exp(\alpha_2 x)} + \alpha_3 q, \quad \text{(5.52)}
\]
Chapter 5. Credit bubble, “monetary famine”, and stock market crash: a disequilibrium dynamic model of interbank market

Figure 5.11: The uncertainty of deposit withdrawal II

\[ \eta_0.01 \text{ and } \eta_0.2 \]

\[ \eta_0.05 \text{ and } \eta_0.2 \]

\[ \eta_0.05 \text{ and } \eta_0.05 \]

Figure 5.12: \( \dot{q} = \frac{\alpha_1}{1 + \exp(\alpha_2 x)} \)

where

\[ s = a_1 x + a_4 (r - r^*) + a_5 q + d, \]  \hspace{1cm} (5.53)

\[ ER^d = ER^d = \frac{T_e}{2} [ (1 + x)(\bar{\lambda}_x - 1) - (1 - x)(1 - \frac{\bar{\lambda}_x}{\lambda^*})], \]  \hspace{1cm} (5.54)

\[ ER^s = \frac{\beta_2}{\beta_1} (r - r^*) + q. \]  \hspace{1cm} (5.55)
We illustrate the counter-cyclical effects of the proposed unconventional monetary policy by conducting a bifurcation analysis over the three policy parameters ($\alpha_1$, $\alpha_2$, and $\alpha_3$) in the 4D sub-dynamics augmented by UMP, as is shown Figure 5.13. The top and mid panels show that under this particular parameter setting, bifurcation occurs when $\alpha_1 \approx 0.48$ and $\alpha_2 \approx 8$ respectively. It implies that $\alpha_1$ and $\alpha_2$ have to be large enough in order to render the policy effective. The bottom panel shows the bifurcation of $\alpha_3$. We observe that as $|\alpha_3|$ becomes smaller, $x^*$ also decreases. During the interval near $\alpha_1 \in [-0.031, -0.016]$, bifurcation occurs and the system becomes unstable. As $|\alpha_3|$ passes through $-0.016$ and further increases, the system becomes stable again, yet $x$ declines further until it reaches nearly $-1$: the pessimistic equilibrium. In other words, if the mean-reverting parameter $\alpha_3$ is too small, it will drag the system toward a
permanently pessimistic equilibrium. It implies that the proposed counter-cyclical policy would potentially bring undesirable outcomes if it fails to quit in a timely manner.

We then simulate the full 6D system by incorporating $\lambda_+$ and $\lambda_-$ with equation (5.18), (5.19), and (5.49-5.52). The parameters are set the same as previous simulations. The simulation result is self-explanatory: we observe from Figure 5.14 that the interbank rate is soothed and permanently pushed down to the zero-rate bound since the introduction
of UMP, despite the unpreventable interest rate hike at the start of economic downturns. We further show the consequences of varying policy parameter \( \alpha_1 \) (\( \alpha_1 = [0, 0.5, 0.9] \)) in Figure 5.15. We observe that a higher value of \( \alpha_1 \) induces a higher stabilizing effect over the period of economic downturn.

### 5.6 Extension: the 8D CDGZ model with a speculative financial sector

To gain a better understanding of the linkage between the interbank market, the speculative financial market, and the real sector, we further extend our model by adopting Charpe et al. (2012) and Chiarella et al. (2015a), where output is driven by a speculative financial market. The banking sector now plays an intermediate role that amplifies financial/macroeconomic instability through credit creation.

Specifically, we now adopt the following law of motion for output \((y)\), replacing equation (5.20):

\[
\dot{y} = \beta_y[(a_y - 1)y + ak(p_k - p_{k0})K + \bar{A}], \tag{5.56}
\]

where \(\bar{A}\) is autonomous expenditure; \(K\) is the total capital stock; \(a_y \in (0, 1)\) is the propensity to spend; \(a_k > 0\) captures the sensitivity of investment and consumption demand to deviations between the actual and the equilibrium level of capital stock.

Due to imperfection for capital market \(K\), the price does not move immediately to clear the market. Following Charpe et al. (2012), we assume that the price of \(K\) moves according to the expected rate of return on the capital stock, \(\rho_k^e\). The law of motion for capital price \((p_k)\) and the expected rate of return \((\dot{\pi}_k^e)\) is given by:

\[
\dot{p}_k = \theta L^e(p_k^e - \rho_{k0}), \tag{5.57}
\]

\[
\dot{\pi}_k^e = \beta\pi_k^e[1 + x_p\dot{p}_k - \pi_k^e], \tag{5.58}
\]

\[
\rho_k^e = \frac{by}{p_kK} + \pi_k^e - r, \tag{5.59}
\]

where \(b\) is the profit share; \(\rho_{k0}\) denotes the equilibrium level of expected rate of profit. Here we assume that the profits are totally distributed as dividends. The aggregate level of loan issued by banking sector \((L^e)\) enters equation (5.57) in the form of adjustment parameter \((\theta L^e)\), since credit expansion will exacerbate volatility of asset prices. In addition, the money market rate \(r\) now enters equation (5.59). The rationale is that it

---

\(^4\)See Charpe et al. (2012) for detailed discussion of this real sector dynamics.
measures the opportunity cost of investing in stocks, since interbank rate can be used as a benchmark for alternative investment in debt market.

We further adopt the Weidlich-Haag-Lux approach to model the opinion formation dynamics of speculative investors. Following Charpe et al. (2012), we assume that there are $2M$ number of investors. Of these, $M_c$ are chartists and $M_f$ are fundamentalists so that $M_c + M_f = 2M$. Let $m = \frac{M_c - M_f}{M}$ and $x = \frac{m}{M}$. We focus on the difference in the size of the two groups (normalised by $M$). Let $p^{f \rightarrow c}$ be the transition probability that a fundamentalist becomes a chartist, and similarly for $p^{c \rightarrow f}$. The change in $x_p$ depends on the relative size of each population multiplied by the relevant transition probability ($\exp(s_p)$ and $\exp(-s_p)$). We further introduce an interest rate component to the opinion formation index of the speculative sector ($s_p$). To be precise, we define

\begin{align*}
\dot{x}_p &= v_p[(1 - x_p) \exp(s_p) - (1 + x_p) \exp(-s_p)], \\
\dot{s}_p &= s_{xp} x_p + s_x x - s_{pk}(p_k - p_{k0})^2 - s_{\pi_k} \pi_k^2 - s_{rp}(r - r^*). 
\end{align*}

The full 8D system with a speculative financial sector thus becomes

\begin{align*}
\lambda_+ &= \gamma_1(x + g(\cdot)) + \gamma_2 \dot{y} + \gamma_3(\lambda_+ - \lambda_-), \\
\lambda_- &= \gamma_1(x - g(\cdot)) + \gamma_2 \dot{y} + \gamma_3(\lambda_- - \lambda_+), \\
\dot{y} &= \beta_y[(a_y - 1)y + a_k(p_k - p_{k0})K + A], \\
\dot{x} &= v[(1 - x) \exp(s) - (1 + x) \exp(-s)], \\
\dot{r} &= \beta_1(BR^d - BR_s) + \beta_2(r^* - r), \\
p_k &= \theta L^s(\rho_k - p_{k0}), \\
\pi_k &= \beta_{\pi_k}[\frac{1}{2} x_p \rho_k - \pi_k], \\
\dot{x}_p &= v_p[(1 - x_p) \exp(s_p) - (1 + x_p) \exp(-s_p)], 
\end{align*}

where

\begin{align*}
s &= a_1 x + a_{2+} \lambda_+ + a_{2-} \lambda_- + a_3 (r - r^*) + a_4 (y^d - y) + a_5 x_p + d, \\
\dot{s}_p &= s_{xp} x_p + s_x x - s_{pk}(p_k - p_{k0})^2 - s_{\pi_k} \pi_k^2 - s_{rp}(r - r^*). 
\end{align*}

Given its complexity in the full 8D system, we resort to numerical simulations to gain an understanding of its global dynamics. The parameters are set as follows: $a_1 = 1, a_{2+} = -0.3, a_{2-} = -0.5, a_3 = -0.3, a_4 = 1, a_5 = 0.01, \sigma = 0.6, T_c = 1, k = 0.1, y_{0d}^d = 9, d = 5, v = 0.5, \gamma_1 = 1.2, \gamma_2 = 0.9, \gamma_3 = 1.7, \beta_1 = 0.1, \beta_2 = 18, \theta_0 = 9, \theta_1 = 10, \xi_0 = 0.5, \xi_1 = 6, r^* = 0.05, \lambda_+ = 15, \lambda_- = 5, \lambda_0 = 10, s_{xp} = 4.2, s_{pk} = 1.7, s_{\pi_k} = 5, b = 0.8,
Chapter 5. Credit bubble, “monetary famine”, and stock market crash: a disequilibrium dynamic model of interbank market

\[ K = 1.1, \beta_y = 0.6, a_y = 0.2, a_k = 3.7, p_{k0} = 0.4, A = 2, \theta = 0.3, \rho_{k0} = 18, \beta_{p_k} = 0.5, \]
\[ v_p = 0.3, s_{spk} = 0.9, a_{sp} = 0.5, \alpha_1 = 0.3, \alpha_2 = 1, \alpha_3 = -0.05. \]

We stress that the system dynamics is highly sensitive to changes in parameter settings. This is just one highly-stylized simulation, providing outcomes that we consider interesting in terms of the qualitative dynamics of the full 8D system.\(^5\) It is worth to note that \( \lambda^* \) essentially captures the aggregate leverage ratio in the financial system. Hence it is interesting to examine how changes in \( \lambda^* \) are related to the dynamics of output \( y \) and asset price \( p_k \) in a Minskyan sense, as is illustrated in Figure 5.16. On the top panel, it shows a constant switching between regimes of tranquility and volatility in the financial market, accompanied by ups and downs in the leverage ratio of the system.

\(^5\)We have re-scaled some of the variables to make the simulation more visible. It is justifiable since here we only consider the qualitative dynamics of the system. The re-scaled variables are: \( y = (y - 0.58) \times 10^3, p_k = (p_k - 0.02) \times 10^2, \lambda^* = (\lambda^* - 1) \times 10^7. \)
We pay a closer attention with shorter time span in the mid panel, where the simulation starts with a relative long period of tranquility while $\lambda^*$ gradually accumulates during this period. Gradually we observe an emergence of financial instability once $\lambda^*$ reaches a plateau: $p_k$ becomes more volatile and $\lambda^*$ falls and rises during this period of readjustments. On the bottom panel, we observe lagged reactions of $y$ in relation to $p_k$, implying that financial instability precedes macroeconomic fluctuations. Overall, this set of simulation results is broadly in line with Minsky (1990) and Fisher (1933): the increase in leverage during the period of tranquility essentially breeds instability in the subsequent period. Over the course of time financial instability emerges due to over-leveraging, which eventually leads to macroeconomic instability and debt deflation.

More interestingly, in Figure 5.17, we observe a “Leverage Tornado” in the 3D plot that vividly captures the heterogeneous interactions of the system: as $\lambda^*$ rises over time, instability increases in both financial and real sector, which ultimately leads to its own defeat.

As Minsky (1975) points out: a capitalist economy is prone to instability due to its speculative nature. The period of moderation fosters reckless risk-taking in financial speculation, which eventually leads to the subsequent chaos. Minsky also stresses that the main function of capital market is to facilitate the efficient allocation of financial resources that serves the real economy, hence gambling activity should be discouraged to prevent financial collapse. A more fundamental question, therefore, is what should the government do to stabilize the unstable economy? While ex-post monetary interventions (i.e. UMP/QE type) or regulatory constraints (such as banning short-selling) are proved
necessary during the period of panics and crisis, it is more crucial to implement \textit{ex-ante} regulatory control that curbs and eliminates reckless speculative behaviours prior to the crisis. Hence, following Charpe et al. (2012) and Chiarella et al. (2015a), we introduce a Tobin-type Tax on capital gains and examine how it affects the overall dynamics of the 8D system. We augment equation (5.68) with the Tobin-type tax parameter ($\tau$):

$$\pi_k^c = \beta \pi_k^c \left[ \frac{1 + x_p}{2} (1 - \tau) \hat{p}_k - \pi_k^c \right].$$

(5.72)

Figure 5.18 provides the bifurcation analysis of $\tau$ under the same parameter setting. We observe that, as $\tau$ increases and passes through $\tau \approx 7.5\%$, the system transits from instability to stability. From a policy perspective, we argue that, while it is important to implement \textit{ex-post} policy intervention of UMP/QE type, it is more crucial to impose
ex-ante regulatory control such as Tobin-type tax on capital gains to penalize excessive speculations in the financial market prior to the crisis, for the purpose of stabilizing the unstable economy.

5.7 Conclusion

This chapter presents a disequilibrium dynamic model of interbank market as an alternative modelling strategy to the more received Diamond and Dybvig (1983) and Allen et al. (2009) framework. The model is built upon earlier work of Chiarella et al. (2015a), which applies Weidlich-Haag-Lux approach\(^6\) to model bank’s lending attitude. Inspired by Minsky (1975) and Kindleberger (1989), as well as earlier work of Bagehot (1873), we emphasis on bank’s role of credit creation. During the boom phase, the general sense of optimism leads to a self-reinforcing credit expansion in the banking sector. As an increasing amount of credit sits on relatively small amount of reserves over time, the banking system becomes fragile. When optimism is suddenly replaced by pessimism at certain point, the banks are forced to hold more reserves, which leads to a drastic liquidity shortage, or “monetary famine”, in the interbank lending market. Subsequently, the conventional monetary policy becomes ineffective during the crisis.

For a general characterization of the dynamic feedbacks of our model, the following three features are worth being mentioned. (i) We emphasis the role of heterogeneity in the banking sector, where some banks are short of reserves while others are in surplus. It gives banks the incentive to lend or borrow reserves to and from each other. (ii) Our model features a constant interaction between the central bank and the commercial banking sector in a dynamic adjustment process. The excess demand for reserve arises due to insufficient reserves in the aggregate banking system, hence the central bank will provide excess supply of reserves in the Open Market Operation (OMO) in order to reach the interest rate target. The new liquidity will then be redistributed in the interbank system. Furthermore, an increase of excess demand for reserves leads to a rise in interbank rate, which is counterbalanced by central bank’s OMO intervention. Yet OMO is rendered ineffective during the panic, which calls for an Unconventional Monetary Policy (UMP) of Quantitative Easing (QE) type. From the bifurcation analysis of the UMP parameters we find that the UMP is necessary during the period of crisis. Yet it will somehow bring undesirable outcome if the policy maker fails to quit the UMP scheme in a timely manner. We further extend the model by incorporating a speculative financial market. The simulation of the final 8D model captures some stylized Minskyan

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dynamics, in the form of a “Leverage Tornado”. It also shows the efficacy of *ex-ante* Tobin-type tax that penalizes speculative gains and its role in stabilizing the economy.

In future research, we would consider incorporating a more comprehensive set of financial assets and adopting a detailed portfolio approach to study the dynamic interactions between the banking and financial sectors in relation to the real economy, following Asada et al. (2009) and Charpe et al. (2011b). It is also viable to study the system proposed in this chapter in an empirical context in order to gain a better understanding of the dynamic interactions between speculative and real sectors from a more realistic perspective.
Chapter 6

Conclusion

The recent financial crisis and global economic downturn since 2007 have provided a fertile ground for rethinking macroeconomics. The micro-founded, homogeneous assumptions of economic agents, combined with rational expectations has been the dominant paradigm for modern-day macroeconomics prior to the crisis. However, despite its simplicity and analytical tractability, this approach had failed to confront the macroeconomic reality that is intrinsically characterized by its heterogeneity, bounded-rationality, non-linearity, and complexity. Traditional Keynesian macroeconomics on the other hand, becomes highly relevant not only in explaining the current crisis but also in recommending vital policy interventions in the aftermath of the economic turmoils.

In recent years, there is an emergence of studies that strives to find alternative paradigm in modelling financial and macroeconomic instability, due to the ongoing development of non-linear economic dynamics and heterogeneous agent model, following insights of Keynes and Minsky. This strand of literature studies almost exclusively on the role of irrational behaviour of investors and firms. Yet there is another crucial aspect that deserves further investigation, i.e. the financial and macroeconomic instability due to the capricious behaviour of financial institutions, since they play a pivotal role in credit creation that potentially propagates and magnifies financial and macroeconomic instability.

As an experimental and preliminary step, this thesis has attempted to fill this gap. We begin our journey with a critical literature review in chapter 2, which examines the relationship between the origin of disequilibrium macroeconomic thinking by John Maynard Keynes, and the development of Keynesian disequilibrium macroeconomic models. Keynes alludes the idea of disequilibrium macroeconomics in his General Theory, which consists of four broad aspects: (i) the independently determined saving and investment; (ii) the price and wage rigidity; (iii) the self-fulfilling “animal spirits” dynamics and (iv),
the intrinsic connection and interaction between real and financial sector. It has inspired the ongoing development of disequilibrium macroeconomic literature that strives to formalize Keynes’s disequilibrium thinking with the use of advanced tools in nonlinear dynamic systems. We have discussed two particular strands of modelling approaches, i.e. (i) the Keynes-Metzler-Goodwin approach, which systematically investigate the first three aspects of disequilibrium thinking derived from its predecessor models; and (ii) the Weidlich-Haag-Lux approach, which formalizes the modelling of “animal spirits”.

Chapter 3 takes a critical look at the controversial “Modern Money Theory” (a.k.a Neo-Chartalism) from a dynamic perspective, which leads to the first original contribution of the thesis. The Chartalism school of monetary theory is established by Knapp (1905), who examines the causes and consequences of using fiat money by sovereign governments. On one hand, the descriptive component of MMT strives to examine the operational details in the complex interactions amongst the government, the central bank, the commercial banks, as well as the non-banking private sector; on the other hand, the normative component advocates a pro-active fiscal stance in maintaining full employment based on the idea of functional finance proposed by Abba Lerner. Despite a high relevance in today’s policy arena that demands a thorough understanding over the mechanism of fiat money system, MMT is generally rejected by not only mainstream but also many Post-Keynesian academics due to some of its controversial claims derived from over-simplified static analysis. In this chapter, we propose a set of dynamic models that aims to take a further investigation over the operational details proposed by MMT. We contend that some of the MMT’s claims are questionable due to a general lack of formal quantitative analysis that justifies and measures its policy effects and consequences, especially the long run inflationary consequence of the fiscal/credit-driven monetary expansion. This chapter also provides a modelling framework that is further adopted in the subsequent two chapters of the thesis.

Perhaps the main original contribution of this thesis is the systematic development of the CDGZ model that aims to examine how heterogeneity in the banking sector induces credit cycles due to “animal spirits”-driven bounded rationality of financial institutions and how it propagates macroeconomic instability. The idea of “animal spirits” has been widely treated in the literature with particular reference to investment in the productive sector. Chapter 4 takes a different view and analyses from a theoretical perspective the role of banks’ collective behaviour in the creation of credit that, ultimately, determines the credit cycle. In particular, we propose a dynamic model to analyse how the transmission of waves of optimism and pessimism in the supply side of the credit market interacts with the business cycle. We adopt the Weidlich-Haag-Lux approach to model the opinion contagion of bankers. We test different assumptions on banks’ behaviour and find that opinion contagion and herding amongst banks play an important role in
propagating the credit cycle and destabilizing the real economy. The boom phases trigger banks’ optimism that collectively lead the banks to lend excessively, thus reinforcing the credit bubble. Eventually the bubbles collapse due to an over-accumulation of debt, leading to a restrictive phase in the credit cycle.

Chapter 5 investigates the mechanisms of interbank market and the policy implications over the course of credit cycles. Interbank market, a major subset of money market, is a market where banks lend or borrow reserves to each other. The interbank market plays at least two important roles in the modern financial system: first, it facilitates the implementation of monetary policy - it is a market that the central bank constantly intervenes in order to control and stabilize the short-term interest rate; second, a well functioning interbank market facilitates the efficient allocation of resources from institution to institution (Furfine, 2001). Despite its apparent importance in the financial system, there is surprisingly no widely accepted theoretical framework that formalizes the dynamic study of interbank market. Yet the world-wide malfunction of interbank markets and the subsequent liquidity crises since the 2007 GFC has triggered a new interest in this field. This chapter proposes a number of departures from the traditional Diamond and Dybvig (1983) two-periods framework. We begin with constructing the 5D baseline CDGZ model of interbank market that incorporates the dynamics of interbank rate. Second, we propose a dynamic law of Unconventional Monetary Policy (UMP) and analyze the 4D(sub-dynamics)/6D UMP dynamics under this setting. Third, we extend this 5D baseline model by incorporating a speculative financial sector, which leads to the final 8D CDGZ model. From the study we conclude that, while ex-post monetary interventions (i.e. UMP/QE type) or regulatory constraints are proven to be necessary during the period of panics and crisis, it is more crucial to implement ex-ante regulatory control such as Tobin-type tax that curbs and eliminates reckless speculative behaviours prior to the crisis.

With this, we conclude the summary of this thesis. We have enlisted numerous paths for potential future research at the end of each of the individual chapters. We hope future researchers in the area will find it worthwhile to pursue as we do.
### Appendix A

**Name of variables and parameters in CDGZ model**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Name of the variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>the average opinion dynamics in the banking sector</td>
</tr>
<tr>
<td>$y$</td>
<td>the output</td>
</tr>
<tr>
<td>$\lambda_+$</td>
<td>the loan-to-reserve ratio of optimistic (over-lending) banks</td>
</tr>
<tr>
<td>$\lambda_-$</td>
<td>the loan-to-reserve ratio of pessimistic (under-lending) banks</td>
</tr>
<tr>
<td>$r$</td>
<td>the interbank rate</td>
</tr>
<tr>
<td>$p_k$</td>
<td>the stock price</td>
</tr>
<tr>
<td>$\pi_k^c$</td>
<td>the expected rate of profit</td>
</tr>
<tr>
<td>$x_p$</td>
<td>the average opinion dynamics in speculative stock market</td>
</tr>
<tr>
<td>$g(.)$</td>
<td>the function of convergent/divergent lending strategy</td>
</tr>
<tr>
<td>$BR^{d(s)}$</td>
<td>the demand(supply) of borrowed reserves from over-lending banks</td>
</tr>
<tr>
<td>$ER^{d(s)}$</td>
<td>the aggregate demand(supply) of borrowed reserves</td>
</tr>
<tr>
<td>$\lambda^{*}(.)$</td>
<td>the loan-to-reserve ratio after interbank activity</td>
</tr>
<tr>
<td>$s(.)$</td>
<td>the opinion formation index</td>
</tr>
<tr>
<td>$q(.)$</td>
<td>the function of UMP</td>
</tr>
</tbody>
</table>
### Table A.2: Name of parameters in the CDGZ model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Name of the variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$</td>
<td>the aggregate quantity of unborrowed reserve</td>
</tr>
<tr>
<td>$\gamma_x$</td>
<td>the behavioural parameters of the banking sector</td>
</tr>
<tr>
<td>$a_x$</td>
<td>the cognitive parameters of the banking sector</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>the cognitive parameters of the speculative financial sector</td>
</tr>
<tr>
<td>$\zeta \in [s_{xp}, s_x, s_{pk}, s_{pk}^e, s_{rp}]$</td>
<td></td>
</tr>
<tr>
<td>$\xi_x$</td>
<td>the parameters of convergent/divergent lending strategy</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>the adjustment parameter of dynamic multiplier in the real sector</td>
</tr>
<tr>
<td>$\beta_x$</td>
<td>the conventional policy parameters for central bank</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>the unconventional policy parameters for central bank</td>
</tr>
<tr>
<td>$r^*$</td>
<td>the exogenously-determined policy rate target</td>
</tr>
<tr>
<td>$\lambda_+$</td>
<td>the long run adjustment parameter for optimistic banks</td>
</tr>
<tr>
<td>$\lambda_-$</td>
<td>the long run adjustment parameter for pessimistic banks</td>
</tr>
<tr>
<td>$\eta$</td>
<td>the diffusion term that captures the uncertainty of deposit withdraw</td>
</tr>
<tr>
<td>$a_y$</td>
<td>the propensity to spend</td>
</tr>
<tr>
<td>$a_k$</td>
<td>the sensitivity of investment and consumption demand</td>
</tr>
<tr>
<td>$b$</td>
<td>the profit share</td>
</tr>
<tr>
<td>$\theta$</td>
<td>the parameter of credit multiplier in the financial market</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>the sensitivity of aggregate reserve demand over average opinion</td>
</tr>
<tr>
<td>$\rho_{K}$</td>
<td>the expected rate of profit</td>
</tr>
</tbody>
</table>
Appendix B

The mathematics of nonlinear dynamical system

B.1 Definition of nonlinear dynamical system

For much of the thesis I will consider dynamical systems which can be expressed in terms of the nonlinear, autonomous differential system

\[ \dot{x} = f(x) \]  

(B.1)

Here \( \dot{\cdot} \) denotes differentiation with respect to time. \( x \) is an n-dimensional vector and \( f \) is an n-dimensional vector function. More concisely, we may write \( x \in \mathbb{R}^n \) and \( f : \mathbb{R}^n \to \mathbb{R}^n \):

\[ \dot{x} = f(x) : U \subset \mathbb{R}^n \to \mathbb{R}^n \]  

(B.2)

Typically we place some initial values \( x(0) \in \mathbb{R}^n \) and we are interested in the subsequent solution curve from this initial point. The following theorem ensures the existence and uniqueness of such solution, which is crucial for our subsequent discussions.

Theorem A.1. (Existence and Uniqueness of Solution) Let \( \dot{x} = f(x) : U \subset \mathbb{R}^n \to \mathbb{R}^n \) be differentiable, with \( x_0 \in U \) and the Lipschitz condition\(^2\) be satisfied, then the solution to (B.2) exists and is unique.


\(^1\)The Definitions and Theorems below are primarily adopted and cited from Tu (1992) and Chiarella and Flaschel (2000).

\(^2\)A function satisfies Lipschitz condition at \( x_0 \) if \( |f(x) - f(x_0)| \leq K(x - x_0) \), assuming the Lipschitz constant \( K \) exists.
Appendix B. *The mathematics of nonlinear dynamical system*

## B.2 Linearization of a nonlinear ODE system

**Definition A.1.** Given the system of \( n \) nonlinear ordinary differential equations (B.2), a point \( x^* \) at which \( f(x^*) = 0 \) is called a fixed point or equilibrium.

**Definition A.2.** Consider the nonlinear system \( \dot{x} = f(x) \) in B.2. Using Taylor expansion at some point \( x^* \), we have

\[
\dot{x} = Ax + \psi(x) \quad (B.3)
\]

Where

\[
A \equiv Df(x^*) = \begin{pmatrix}
\partial f_1 / \partial x_1 & \ldots & \partial f_1 / \partial x_n \\
\ldots & \ldots & \ldots \\
\partial f_n / \partial x_1 & \ldots & \partial f_n / \partial x_n 
\end{pmatrix} \quad (B.4)
\]

and \( \psi(x) \) is such that \( \lim_{x \to 0} \psi(x) = 0 \). \( Ax \) in (B.4) is called the linearization of (B.3).

**Definition A.3.** A simple fixed point is called hyperbolic if \( A \) in its linearization \( Ax \) has no eigenvalues with zero real parts.

**Theorem A.2.** *(Linearization Theorem of Hartman & Grobman).* Let the nonlinear dynamic system \( \dot{x} = f(x) \) in (B.2) have a simple hyperbolic fixed point \( \bar{x} \) set at the origin for simplicity. Then in the neighborhood \( U \) of \( \bar{x} \in \mathbb{R}^n \) of this equilibrium, the phase portraits of the nonlinear system and its linearization \( L \) are equivalent.

**Proof.** See Hartman (1964).

**Definition A.4.** Given \( \dot{x} = Ax \), the equilibrium position \( x = 0 \) is said to be locally asymptotically stable if \( \lim_{x \to \infty} x(t) = 0 \)

**Corollary A.1.** In a 2D dynamical system \( \dot{x} = Ax : U \subset \mathbb{R}^2 \to \mathbb{R}^2 \), the necessary and sufficient condition to insure local stability of a fixed point \( x^* \) at which \( f(x^*) = 0 \) is that \( Tr(A) < 0 \) and \( Det(A) > 0 \).

**Corollary A.2.** *(The Routh-Hurwitz Conditions for 3D System (Chiarella and Flaschel, 2000))* In a 3D dynamical system \( \dot{x} = Ax : U \subset \mathbb{R}^3 \to \mathbb{R}^3 \), the necessary and sufficient condition to insure local stability of a fixed point \( x^* \) at which \( f(x^*) = 0 \) is that

\[
Tr(A) < 0, \quad (B.5)
\]

\[
A_1 + A_2 + A_3 > 0, \quad (B.6)
\]

\[
Det(A) < 0, \quad (B.7)
\]

\[
-Tr(A)(A_1 + A_2 + A_3) + |A| > 0 \quad (B.8)
\]
B.3 Hopf bifurcation

Theorem A.3. (Hopf Bifurcation Theorem) Let \( \dot{x} = f(x, \mu) : \mathbb{R}^{n+1} \to \mathbb{R}^n \) have a critical point \((x^*, \mu_0)\) at the origin and the Jacobian matrix \(A(\mu) \equiv D_x f(x^*, \mu_0)\) have one and only one pair of pure eigenvalue \(\lambda(\mu_0) = \pm i\beta(\mu_0)\) such that \(\alpha(\mu_0) = 0 < d\alpha(\mu_0)/d\mu, \beta(\mu_0) \neq 0\) and \(\mu_1 < \mu_0 < \mu_2\), then

1. \(\mu = \mu_0\) is a bifurcation point,
2. for \(\mu \in (\mu_1, \mu_0)\) the origin is a stable focus,
3. for \(\mu \in (\mu_0, \mu_2)\) the origin is an unstable focus surrounded by a stable limit cycle whose size grows with \(\mu\).

B.4 Chaos in ODE

Theorem A.4. Ruelle (1979) Given \( \dot{x} = f(x, \mu) : \mathbb{R}^{n+1} \to \mathbb{R}^n \), a bounded set \(A\) of \(\mathbb{R}^n\) is a strange attractor if

1. \(A\) is invariant under the flow of the system,
2. There exists an open neighbourhood \(U\) of \(A\) such that all points \(x^t \in U\) tend to \(A\) as \(t \to \infty\),
3. The trajectory is highly sensitive to initial conditions,
4. \(A\) is indecomposable i.e. cannot be divided into pieces.
Appendix C

Matlab codes for representative simulations

1

C.1 Matlab code for Figure 3.7

alpha1=0.01;
alpha2=-0.02;
alpha3=0.01;
theta1=0.01;
theta2=4;
theta3=0.03;
theta4=0.4;
gamma1=0.06;
gamma2=0.08;
gamma3=2.3;
pistar=0.05;
gbar=0.1;
h=0.01;
t=0:h:3000;
gg(1:length(t))=pistar;
pi=zeros(length(t),1);
sp=zeros(length(t),1);
g=zeros(length(t),1);
spg=zeros(length(t),1);
dg=zeros(length(t),1);
pi(1)=pistar;
sp(1)=0.01;
g(1)=gbar;
for i=1:length(t)-1;
    pi(i+1)=pi(i)+(alpha1*sp(i)+alpha2*(g(i)-gbar)+alpha3*(pistar-pi(i)))*h;

1This appendix lists a series of Matlab codes that are compiled for the use of numerical simulations in each chapters. Other codes may be provided by the author upon request.
Appendix C. *Matlab codes for representative simulations*

C.2 Matlab code for Figure 4.10

```matlab
sp(i+1)=sp(i)+(theta1*(pi(i)-pistar)^theta2-theta3*(g(i)-gbar)^theta4)*h;
g(i+1)=g(i)+(-gamma1*sp(i)^gamma2+gamma3*(gbar-g(i)))*h;
spg(i+1)=sp(i+1)-pi(i+1);
dg(i+1)=exp(sp(i+1)-pi(i+1));
end
figure(1)
subplot(2,3,1)
[hAx,hLine1 , hLine2] = plotyy(t,g,t,pi);
%title('Multiple Decay Rates ')
xlabel('Time ')
ylabel(hAx(1),'g') % left y-axis
ylabel(hAx(2),'\pi') % right y-axis
subplot(2,3,4)
plot(sp(5000:300000),g(5000:300000))
xlabel('s_p ')
ylabel('g ')
subplot(2,3,2)
plot(sp,pi)
xlabel('s_p ')
subplot(2,3,3)
plot(t,sp)
xlabel('time ')
ylabel('s_p ')
subplot(2,3,5)
plot(t(50000:300000),dg(50000:300000))
xlabel('time ')
ylabel('M(t)/Y(t)')
axis([500 3000 -1 8])
subplot(2,3,6)
plot(t(50000:300000),spg(50000:300000))
xlabel('time ')
ylabel('s_p -\pi')
axis([500 3000 -1 3])
```

---

```matlab
ai=1.5;
a2_opt=-0.3;
a2_pes=-0.5;
a3=1.3;
sigma=0.8;
k=0.1;
Tc=1;
Tc2=Tc/2;
h=0.01;
N=15;
yd0=11;
t=0:h:600;
v=0.4;
gamma1=0.3;
gamma2=0.4;
gamma3=0.03;
```

---
l_bar_opt=15;
l_bar_pes=5;
x10=0.2;
x11=3;
%d=-a2_opt*(l_bar_opt+gamma1/gamma3*x10)-a2_pes*(l_bar_pes -gamma1/gamma3*x10);
d=10;
L=zeros(length(t),1);
L_rate=zeros(length(t),1);
lambda_opt=zeros(length(t),1);
lambda_pes=zeros(length(t),1);
z=randn(length(t),1);
x=zeros(length(t),1);
y=zeros(length(t),1);
yd=zeros(length(t),1);
debtgdp=zeros(length(t),1);
gg=zeros(length(t),1);
L(1)=30;
x(1)=0.1;
y(1)=10;
yd(1)=yd0;
for i=1:length(t)-1
    y_d=yd0+k*Tc2*((1+x(i))*lambda_opt(i)+(1-x(i))*lambda_pes(i));
    yd(i+1)=y_d;
    s=a1*x(i)+a2_opt*lambda_opt(i)+a2_pes*lambda_pes(i)+a3*(y_d-y(i))+d;
    g=x10*exp(-x11*x(i)^2);
    gg(i+1)=g;
    x(i+1)=x(i)+v*((1-x(i))*exp(s)-(1+x(i))*exp(-s))*h;
    debtgdp(i)=Tc2*((1+x(i))*lambda_opt(i)+(1-x(i))*lambda_pes(i))/y(i);
    dydt=sigma*(y_d-y(i));
    lambda_opt(i+1)=
        lambda_opt(i)+(gamma1*((x(i)+g))+gamma2*dydt+gamma3*(l_bar_opt -lambda_opt(i)))*h;
    lambda_pes(i+1)=
        lambda_pes(i)+(gamma1*((x(i)-g))+gamma2*dydt+gamma3*(l_bar_pes -lambda_pes(i)))*h;
end
gap=yd-y;
n_opt=(1+x)*N;
n_pes=(1-x)*N;
figure(1)
subplot(3,2,1)
plotyy(t,lambda_opt(t),lambda_pes(t))
title('\lambda_+ (blue) and \lambda_- (green) I')
subplot(3,2,3)
plot(t(55000:60000),x(55000:60000))
title('x')
subplot(3,2,2)
plotyy(t(55000:60000),lambda_opt(t(55000:60000)),lambda_pes(t(55000:60000)))
title('\lambda_+ (blue) and \lambda_- (green) II')
subplot(3,2,4)
plot(t(55000:60000),gg(55000:60000))
title('g(.)')
### Appendix C. Matlab codes for representative simulations

```matlab
subplot(3,2,5)
plotyy(t(55000:60000),y(55000:60000),t (55000:60000),debtgdp(55000:60000))
title('the dynamics of output and debt/GDP ratio')
subplot(3,2,6)
plot(y(500:50000),debtgdp(500:50000))
xlabel('output')
ylabel('debt/GDP ratio')
title('debt/GDP ratio vs. output')
```

### C.3 Matlab code for Figure 5.16-5.17

```matlab
ai=1;
a2_opt=-0.3;
a2_pes=-0.5;
a3=-0.3;
a4=1;
a5=0.01;
sigma=0.6;
Tc=1;
Tc2=Tc/2;
h=0.01;
k=0.1;
yd0=9;
d=5;
t=0:h:300;
v=0.5;
gamma1=1.2;
gamma2=0.9;
gamma3=1.7;
beta1=0.11;
beta2=18;
theta0=9;
theta1=10;
x10=0.5;
x1i=6;
r_star=0.05;
lbar_opt=15;
lbar_pes=5;
lambda0=10;
sxp=4.2;
spk=1.7;
nspike=5;
b=0.8;
K=1.1;
beta_y=0.6;
ay=0.2;
ak=3.7;
pk0=0.4;
y0=0.5;
A=2;
theta=0.3;
rho_k0=18;
```
beta_pke=0.5;
v_p=0.3;
sxp_x=0.9;
srp=6;
as_p=0.5;
tau=0;
alfa_1=0.3;
alfa_2=1;
alfa_3=-0.05;
q=zeros(length(t),1);
lambda_opt=zeros(length(t),1);
lambda_pes=zeros(length(t),1);
x=zeros(length(t),1);
r=zeros(length(t),1);
q_g=zeros(length(t),1);
y=zeros(length(t),1);
yd=zeros(length(t),1);
dqdt=zeros(length(t),1);
l_star=zeros(length(t),1);
ERDD=zeros(length(t),1);
ERSS=zeros(length(t),1);
yd=zeros(length(t),1);
gg=zeros(length(t),1);
pk=zeros(length(t),1);
pike=zeros(length(t),1);
xp=zeros(length(t),1);
x(1)=0.2;
ap(1)=0.5;
r(1)=0.1;
y(1)=10;
yd(1)=yd0;
pk(1)=0.4;
pk(1)=0.5;
lambda_opt(1)=10;
lambda_pes(1)=3;
for i=1:length(t)-1
  s=a_1*x(i)+a_2_opt*lambda_opt(i)+a_2_pes*lambda_pes(i)+a_3*(r(i)-r_star)+a_4*(((ay-1)*y(i)+ak*(pk(i)-pk0)+K+A)+asp*xp(i)+d;
  sp=sxp_x*xp(i)+sxpx*x(i)+spk*(pk(i)-pk0)^2-spike*pike(i)^2-srp*(r(i)-r_star);
  rho_ke=b*y(i)/pk(i)/K+pike(i)-r(i);
  g=xi0*exp(-xi1*x(i)^2);
  gg(i+1)=g;
  x(i+1)=x(i)+(1-x(i))*v*exp(s)-(1+x(i))*v*exp(-s))*h;
  y(i+1)=y(i)+(beta_y*((ay-1)*y(i)+ak*(pk(i)-pk0)*K+A))*h;
  dydt=beta_y*(((ay-1)*y(i)+ak*(pk(i)-pk0)*K+A));
  lambda_star=lambda0+theta0*tanh(theta1*x(i));
  l_star(i)=lambda_star;%rescaling to make simulation more clear
  lambda_opt(i+1)=lambda_opt(i)+(gamma_1((x(i)+g))+gamma_2*(dydt)+gamma_3*(lbar_opt -lambda_opt(i)))*h;
  lambda_pes(i+1)=lambda_pes(i)+(gamma_1((x(i)-g))+gamma_2*(dydt)+gamma_3*(lbar_pes -lambda_pes(i)))*h;
  Ls=Tc2*((1+x(i))*lambda_opt(i)+(1-x(i))*lambda_pes(i));
  L(i)=Ls;
  BR_d=Tc2*(1+x(i))*(lambda_opt(i)/lambda_star-1);
  BRd(i+1)=BR_d;
Appendix C. Matlab codes for representative simulations

\[ BR_s = Tc2 \times (1-x(i)) \times (1-lambdaPes(i)/lambdaStar); \]
\[ BRs(i+1) = BR_s; \]
\[ q(i+1) = q(i) + (alfa1/(1+exp(alfa2*(x(i))))) + alfa3*q(i)*h; \]
\[ r(i+1) = r(i) * (beta1*(BR_d-BR_s) + beta2*(rStar-r(i))) \times h; \]
% \[ ERD = BR_d - BR_s; \]
% \[ ERD(i) = ERD; \]
% \[ ERS = beta2/beta1 * (r(i) - rStar); \]
\[ ERS(i) = ERS; \]
\[ pk(i+1) = pk(i) + theta*Ls*pk(i) * (rhoKe-rhoK0) \times h; \]
\[ pkh = theta*Ls * (rhoKe-rhoK0); \]
\[ pike(i+1) = pike(i) + betaPke * (0.5*(1-tao)*(1+xp(i)))*pkh - pike(i); \]
\[ xp(i+1) = xp(i) + vp * ((1-xp(i))*exp(sp) - (1+xp(i))*exp(-sp)) \times h; \]
\[ e(i) = L(i)/y(i); \]
end
% rescaling variables in order to make simulation more visible
% \[ y = (y-0.58) \times 10000; \]
% \[ pk = (pk-0.02) \times 10000; \]
% \[ lStar = (lStar-1) \times 100000000; \]
figure(1)
subplot(3,1,1)
plotyy(t(25000:30000), pk(25000:30000), t(25000:30000), lStar(25000:30000));
title('pk (blue) and \lambda^* (red) I')

subplot(3,1,2)
plotyy(t(29000:30000), pk(29000:30000), t(29000:30000), lStar(29000:30000));
title('pk (blue) and \lambda^* (red) II')

subplot(3,1,3)
[AX,H1,H2]=plotyy(t(29000:30000), pk(29000:30000), t(29000:30000), y(29000:30000));
title('pk (blue) and y (red)')
figure(2)
plot3(y(20000:30000), pk(20000:30000), lStar(20000:30000))
xlabel('y')
ylabel('pk')
ylabel('\lambda^*')
box on
title('The 3D plot of y, pk, and \lambda^*')
Appendix D

John Law and the advent of modern banking, financial bubbles, and crisis

John Law and the Mississippi Bubble: 1718-1720

- by Jon Moen

“In the early 18th century the economy of France was depressed. The government was deeply in debt and taxes were high. In addition, the French controlled the colony of Louisiana, a vast settlement in the interior of North America. The Louisiana Colony included the Natchez district and the area along the Mississippi Gulf Coast in present-day Mississippi. France was the first European country to settle this area of North America (1699-1763).

The American land was much larger than France and the French knew little about it. Many did not even know where it was. But many had heard the rumor that this land

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1. This appendix quotes Jon Moen’s article “John Law and the Mississippi Bubble: 1718-1720”. It might not seem relevant to the central theme of the thesis at the first glance. Yet one could not refer to a better historical event than the Mississippi Bubble - the advent of modern financial bubble and crisis. In order to understand other financial crises in modern days that involves the interconnection between credit-creation banking sector and speculative financial sector. Besides mathematical modelling of this interaction, our thesis has also peeked into the current crises both in the US and China. It can be seen that the underlying boom-bust mechanism works the same way in different countries across different time periods since the birth of modern banking system. As the French idiom goes: “the more things change, the more they stay the same”.

2. Jon Moen is professor of business administration, School of Business Administration, University of Mississippi.
was rich in silver and gold, the French currency. The depressed French economic environment was fertile ground for some of the monetary and economic ideas of John Law (1671-1729). Law was a Scottish financier born in Edinburgh. He was a colorful character who has been described as tall, handsome, and vain. He had a passion for women and gambling. When Law came to France in 1714, he renewed his acquaintance with the nephew of King Louis XIV, the Duke of Orleans. The duke became Regent of France after the king’s death in 1715. The regent served as ruler while the heir to the throne, five-year-old Louis XV, was still a minor. The duke recalled Law’s financial prowess and sought his advice and assistance in straightening out France’s financial mess left over from years of reckless spending under Louis XIV. This association with the Duke of Orleans would ensure Law’s place in history. Not only would Law advance the use of paper money, the French word millionaire would come into use as a result of his most famous scheme — the Mississippi Company. In 1716 Law convinced the French government to let him open a bank, the Bank Generale, that could issue paper money, or bank notes. The paper notes would be supported by the bank’s assets of gold and silver and would circulate as a medium of exchange. Paper money was a new concept for the French; money to them was silver and gold. Law believed that paper notes would increase the money in circulation, which, in turn, would increase commerce. These conditions would help revitalized and rehabilitate the finances of the French government.

In August 1717, he organized the Compagnie d’Occident (Company of the West) to which the French government gave the control of trade between France and its Louisiana and Canadian colonies. In Canada, the French would trade in beaver skins. In the Louisiana colony they would trade in precious metals. The colony stretched for 3,000 miles from the mouth of the Mississippi River to parts of Canada. It included the present-day states that hug the river: Louisiana, Mississippi, Arkansas, Missouri, Illinois, Iowa, Wisconsin, and Minnesota. The colony of Louisiana’s connection to the Mississippi River gave rise to the company’s more popular name, The Mississippi Company. Law’s company had exclusive trading privileges in the territory for twenty-five years; it could appoint its own governor and officers in the colony and make land grants to potential developers. In turn, the company accepted the responsibility of transporting 6,000 settlers and 3,000 slaves to the colony before expiration of its charter. The scheme to finance the initial operations of the Mississippi Company was simple. Law would raise the money by selling shares in the company for cash and, more importantly, for state bonds. Law accepted a low interest rate on the bonds which helped French finances while promising the company a more secure cash flow. Simply put, Law came up with a way to finance a big business scheme. The lure of gold and silver brought out many eager investors in the Mississippi Company. Later Law would create cash flow from new economic activity. It turns out that the Mississippi Company was a small part of a much grander empire he was about to create. In September 1718 the company acquired the monopoly in tobacco...
trading with Africa. Law’s Bank Generale was taken over by the French government in January 1719 and was renamed the Bank Royale. Law remained in charge, however, and the crown further guaranteed the bank’s note issue. In May he obtained control of the companies trading with China and the East Indies. He renamed his entire business interest the Compagnie des Indes, but most people still called it the Mississippi Company. In effect, Law now controlled all trade with France and the rest of the world outside of Europe. The company next purchased the right to mint new coins for France, and by October it had purchased the right to collect most French taxes. In January 1720, Law became the Controller General and Superintendent General of Finance. Law now controlled all of France’s finance and money creation. He also controlled the company that handled all of France’s foreign trade and colonial development. Furthermore, by holding much of the French government’s debt, he had created a stable source of income for future business ventures. Law had created Europe’s most successful conglomerate. Law paid for these activities and privileges by issuing additional shares in the company. These shares could be paid for with bank notes (from his bank) or with government debt. The value of shares in the Mississippi Company rose dramatically as Law’s empire expanded. Investors from across France and Europe eagerly played in this new market. The financial district in Paris became so agitated at times with investors that soldiers would be sent in at night to maintain order. Shares in the Mississippi Company started at around 500 livres tournois (the French unit of account at the time) per share in January 1719. By December 1719, share prices had reached 10,000 livres, an increase of 1900 percent in just under a year. The market became so seductive that people from the working class began investing whatever small sums they could scrape together. New millionaires were commonplace. The weak spot in Law’s scheme was his willingness to issue more bank notes to fund purchases of shares in the company. Stock prices began falling in January 1720 as some investors sold shares to turn capital gains into gold coin. To stop the sell-off, Law restricted any payment in gold that was more than 100 livres. The paper notes of the Bank Royale were made legal tender, which meant that they could be used to pay taxes and settle most debts. The company was trying to get people to accept the paper notes rather than gold. The bank subsequently promised to exchange its notes for shares in the company at the going market price of 10,000 livres. This attempt to turn stock shares into money resulted in a sudden doubling of the money supply in France. It is not surprising then that inflation started to take off. Inflation reached a monthly rate of 23 percent in January 1720. Law devalued shares in the company in several stages during 1720, and the value of bank notes was reduced to 50 percent of their face value. By September 1720 the price of shares in the company had fallen to 2,000 livres and to 1,000 by December. The fall in the price of stock allowed Law’s enemies to take control of the company by confiscating the shares of investors who could not prove they had actually paid for their shares with real assets rather than credit. This reduced
investor shares, or shares outstanding, by two-thirds. By September 1721 share prices had dropped to 500 livres, where they had been at the beginning. The rise and fall of the Mississippi Company became known as the Mississippi Bubble. Indeed, Law is most famous, or perhaps infamous, for his involvement in this prominent financial disaster. A “bubble” in the world of finance is a term applied to an unusually rapid increase in stock prices or the value of some other asset such as real estate. The increase is then followed by an equally rapid collapse in prices. The wild fluctuations in prices are usually viewed as irrational and the product of uncontrolled speculation rather than sensible investment practices. The dramatic increase in the NASDAQ stock index, primarily technology stocks, in 1999-2000 and its subsequent collapse in 2000-2004 is sometimes presented as a recent example of a bubble. Economists are divided on how to interpret Law’s scheme. Charles Kindleberger, an economic historian at Yale University, believes Law’s intentions were legitimate and that the Mississippi Company was intended to be a real enterprise. Law’s financial arrangements, however, were misguided. Others have noted that Law did help straighten out the convoluted system of French taxation and finance. And, the economist Peter Garber believes that Law’s system had more potential than is often believed. The story of John Law and the Mississippi Company is as intriguing as any modern financial disaster. In the end, many of the new millionaires were financially destroyed. So was France. It would be eighty years before France would again introduce paper money into its economy. Meanwhile, France maintained control of the Louisiana colony until 1763 when it lost the Seven Years’ War to the United Kingdom of Great Britain. At the Paris Conference in 1763, all of Louisiana east of the Mississippi River, except the Isle of Orleans, went to Great Britain. Louisiana west of the Mississippi and the Isle of Orleans went to Spain. Spain returned its territory to France in 1800 through a secret deal in which the French, under Napoleon Bonaparte, promised to set up Spanish rule in Italy. In 1803, the United States, under Thomas Jefferson, purchased the territory from France in a deal known as The Louisiana Purchase. 

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3Source: http://mshistory.k12.ms.us/articles/70/john-law-and-the-mississippi-bubble-1718-1720
Bibliography


N. B. Aikman D., Haldane A. Curbing the credit cycle. internet article, 2011. viewed on 17/Sep./2013.


R. E. Backhouse and M. Boianovsky. Disequilibrium macroeconomics: An episode in the transformation of modern macroeconomics. *Anais do xxxiii encontro nacional


O. J. Blanchard and J. A. Simon. The long and large decline in u.s. output volatility. Mit dept. of economics working paper no. 01-29, MIT, 2001.


C. Liu. Herding behavior in bank lending: Evidence from u.s. commercial banks. European finance association (efa) working paper, Queen’s School of Business. Queen’s University, 2012.


R. Werner. The quantity theory of credit and some of its applications. Presentation slides, Centre for Banking, Finance, and Sustainable Development, University of Southampton, 2012.

