PRICING SWAPTIONS AND CREDIT DEFAULT SWAPTIONS IN THE QUADRATIC GAUSSIAN FACTOR MODEL

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

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CONTENTS

Acknowledgments . . . iii

Abstract . . . vi

List of Figures . . . vii

List of Tables . . . viii

Abbreviations and Notation . . . ix

Introduction . . . 1

1. *Quadratic Gaussian Factor Models* . . . 5
   - 1.1 A Quadratic Gaussian Model to include a Foreign Economy . . . 24
   - 1.2 Explicit Solutions for Piecewise Constant Case . . . 43

2. *Swaptions in the Quadratic Gaussian Model* . . . 46
   - 2.1 Introduction . . . 46
   - 2.2 Pricing Swaptions under the Forward Measure . . . 49
   - 2.3 Pricing Swaptions under the Swap Measure . . . 68
   - 2.4 Method of Moments Swaption Pricing . . . 81

3. *Credit Default Swaps and Credit Default Swaptions* . . . 90
   - 3.1 Pricing of Credit Default Swaps . . . 91
   - 3.2 Pricing Credit Default Swaptions . . . 111

4. *A Two-Country Reduced Form Model* . . . 132
   - 4.1 The Framework . . . 133
   - 4.2 Valuation of Quanto Default Swaps . . . 149
   - 4.3 A Two-Country Contagion-Type Reduced Form Model . . . 153
   - 4.4 Pricing Quanto Default Swaps in a Contagion-Type Model . . . 160

Appendix 182

A. *Matrix Riccati Equations* . . . 183

B. Some Results for Multi-factor Quadratic Gaussian Models 186

ABSTRACT

In this thesis we show how the multi-factor quadratic Gaussian model can be used to price default free and defaultable securities. The mathematical tools used include the theory of stochastic processes, the theory of matrix Riccati equations, the change of measure technique, Itô's formula, use of Fourier Transforms in swaption valuation and approximation methods based on replacing the values of some stochastic processes by their time zero values.

The first chapter of the thesis deals with the derivation of efficient closed form formulas for the price of zero coupon bonds in the multi-factor quadratic Gaussian model and the calibration of the multi-factor quadratic Gaussian model to the domestic and foreign forward rate term structures through closed form formulas.

In the second chapter of the thesis, we derive approximations for the price of default free swaptions which are based on log-quadratic Gaussian processes. Using numerical experiments, we show the limitations of these approximations. We also give some numerical results for the pricing of a default free swaption using moment-based density approximants of the probability density function of the swaption's payoff.

The third chapter of the thesis deals with the calibration of a quadratic Gaussian reduced form model of credit risk to the default free forward rate curve and to the survival probability of an obligor. We also consider different approximations for the price of credit default swaptions. Using numerical experiments, we show the limitations of the approximations.

The final chapter of this thesis considers a two country reduced form model of credit risk. We examine the relationship between the domestic forward credit spread and the foreign forward credit spread of an obligor and provide quanto adjustment formulas for the probability of survival of an obligor. In the final part of this chapter, we show that the valuation of a quanto default swap is tractable in a contagion type reduced form model of credit risk which assumes that underlying processes are modelled by quadratic Gaussian processes.

LIST OF FIGURES

2.1 interest rate swap over $[T_\alpha, T_\beta]$ 48
2.2 Abs. Err. in 1×10 Swaption Price 88
2.3 Abs. Err. in 1×10 Swaption GC 88
2.4 Rel. Err. in 1×10 Swaption Price 89
2.5 Rel. Err. in 1×10 Swaption Price 89

3.1 CDS over $[T_\alpha, T_\beta]$ 92
3.2 Discount curve 109
3.3 CDS quotes given as basis points $* 10^{-4}$ 110
3.4 Extracted Survival Probability Under Correlation $\Delta = 0.0625$ . . . 110
3.5 Calibration results to CDS quotes 111
3.6 Extracted Survival Probability with Extrapolation for years 10 to 15 126

LIST OF TABLES

2.1 Cap and Floor Data(price in bp) 64
2.2 Discount Curve 65
2.3 Relative Error(in %) for ATM Swaptions(price in bp) 67
2.4 Relative Error(in %) for ITM Swaptions(price in bp) 67
2.5 Relative Error(in %) for OTM Swaptions(price in bp) 67
2.6 Relative Error(in %) for ATM Swaptions 78
2.7 Relative Error(in %) for ITM Swaptions 78
2.8 Relative Error(in %) OTM Swaptions 78
2.9 Relative Error(in %) for ATM Swaptions using (2.81) 79
2.10 Relative Error(in %) for ITM Swaptions using (2.81) 79
2.11 Relative Error(in %) OTM Swaptions using (2.81) 80
2.12 Relative Error(in %) for ATM Swaptions using (2.82) 81
2.13 Relative Error(in %) for ITM Swaptions using (2.82) 81
2.14 Relative Error(in %) OTM Swaptions using (2.82) 81
2.15 Swaption price and error using 3 moments 87
2.16 Swaption price and error using 5 moments 87
2.17 Swaption price and error using 7 moments 88

3.1 Zero Rates 109
3.2 CDS quotes 109
3.3 Calibration results to CDS quotes 110
3.4 Zero Rates for years 10 to 15 126
3.5 Relative Error of Approximation given by (3.80) for $K = K_{DATM}$. 128
3.6 Relative Error of Approximation given by (3.80) for $K = 0.85 \times K_{DITM}$ 129
3.7 Relative Error of Approximation given by (3.80) for $K = 1.15 \times K_{DOTM}$ 129
3.8 Relative Error of Approximation given by (3.88) for $K = K_{DATM}$. 129
3.9 Relative Error of Approximation given by (3.88) for $K = 0.85 \times K_{DITM}$ 130
3.10 Relative Error of Approximation given by (3.88) for $K = 1.15 \times K_{DOTM}$ 130
3.11 Relative Error of Approximation based on a Gram Charlier series(based on 3 moments) for $K = K_{DATM}$ 130
3.12 Relative Error of Approximation based on a Gram Charlier series(based on 3 moments)for $K = 0.85 \times K_{DITM}$ 131
3.13 Relative Error of Approximation based on a Gram Charlier series(based on 3 moments) for $K = 1.15 \times K_{DOTM}$ 131

ABBREVIATIONS AND NOTATION

Notation for Chapter 1:

- $\log_e(x)$=Natural Logarithm of x i.e. logarithm to the base e.
- $Tr[A]$=Trace of the rectangular matrix A.
- I_n=The identity matrix of dimension n.
- $0_{n\times n}$=The zero matrix which is the square matrix of dimension $n \times n$ which has all its elements equal to zero.
- 0_n=The zero column vector which is the column vector of length n such that all its elements are equal to zero.
- $\mathbf{1}_n := (1, \ldots, 1)^\top$=Column vector of dimension n which has all its elements equal to the number one.
- SDE=Stochastic differential equation;
- $\dot{F}$=first order derivative of the time dependent (matrix) function F.
- RDE=Matrix Riccati differential equation;
- $\mathbb{Q}$: (Domestic) risk neutral measure for default free and defaultable economy.
- $\mathbb{Q}^f$: Foreign risk neutral measure for default free and defaultable economy.
- $\mathbb{E}^{\mathbb{Q}}$: Expectation under the probability measure $\mathbb{Q}$.
- $\mathbb{E}^{\mathbb{Q}^f}$: Expectation under the probability measure $\mathbb{Q}^f$.
- W_t: Standard multi-dimensional Brownian motion under the Risk Neutral Measure.
- $\mathbb{F} = (\mathcal{F}_t)_{(0\leqslant t\leqslant T^*)}$: Filtration generated by W_t representing default free market information.
- τ: Default time of an obligor or a corporation.
- H_t: Indicator function for default time τ.
- $\mathcal{H}_t = \sigma(H_u : u \leqslant t)$: Filtration generated by H_t.

- $\mathcal{G}_t = \sigma(G_u : u \leqslant t) = \sigma(F_u \vee H_u : u \leqslant t)$: Filtration generated by default free market information and whether default has taken place or not.
- r_t: The (domestic) default free instantaneous rate of interest rate.
- r_t^f: The foreign default free instantaneous rate of interest rate.
- I^d: Diagonal matrix with a one for row i if the ith factor is used to model r_t and a zero otherwise.
- I^f: Diagonal matrix with a one for row i if the ith factor is used to model r_t^f and a zero otherwise.
- Y_t: Gaussian Ornstein Uhlenbeck process with zero drift.
- Z_t: Gaussian Ornstein Uhlenbeck process used for modeling state variables.
- A: Constant diagonal matrix used to denote the speed of mean reversion matrix in the SDE of Y_t.
- Σ: Constant matrix used to denote the instantaneous volatility in the SDE of Y_t.
- $\alpha(t)$: A time dependent deterministic vector function used to calibrate Z_t to the term structure of default free zero coupon bonds .
- $\alpha^f(t)$: A time dependent deterministic vector function used to calibrate Z_t to the foreign term structure of default free zero coupon bonds.
- $\widehat{C}$: Constant symmetric matrix used to model the quadratic part of r_t in the quadratic Gaussian multifactor model.
- $\widehat{B}$: Time dependent deterministic vector function used to model the linear part of r_t in the quadratic Gaussian multifactor model.
- $\widehat{A}$: Time dependent deterministic scalar function used to model the scalar part of r_t in the quadratic Gaussian multifactor model.
- $D(t)$: The default free savings account.
- λ_t: The intensity of default in a reduced form model.
- $\widetilde{C}$: Constant symmetric matrix used to model the quadratic part of λ_t in the quadratic Gaussian multifactor model.
- $\widetilde{B}$: Time dependent deterministic vector function used to model the linear part of λ_t in the quadratic Gaussian multifactor model.
- $\widetilde{A}$: Time dependent deterministic scalar function used to model the scalar part of λ_t in the quadratic Gaussian multifactor model.

- $P(t,T)$: Price of the domestic default free zero coupon bond at time t for maturity T.
- $P^f(t,T)$: Price of the foreign default free zero coupon bond at time t for maturity T.
- S_t: The price of one unit of foreign currency in terms of domestic currency.
- $F(t,T)$: The (domestic) default free instantaneous forward rate.
- $F^f(t,T)$:The foreign default free instantaneous forward rate.
- $\mathbf{1}_{\tau>t}\bar{P}(t,T)$: Price of defaultable zero coupon bond at time t for maturity T.
- $C(t,T)$: Symmetric positive definite matrix used to express the quadratic part of $\log(P(t,T))$ in the quadratic Gaussian multifactor model.
- $B(t,T)$: Time dependent vector used to express the linear part of $\log(P(t,T))$ in the quadratic Gaussian multifactor model.
- $A(t,T)$: Time dependent scalar function used to express the scalar part of $\log(P(t,T))$ in the quadratic Gaussian multifactor model.
- $\bar{C}(t,T)$: Symmetric positive definite matrix used to express the quadratic part of $\log(\bar{P}(t,T))$ in the quadratic Gaussian multifactor model.
- $\bar{B}(t,T)$: Time dependent vector used to express the linear part of $\log(\bar{P}(t,T))$ in the quadratic Gaussian multifactor model.
- $\bar{A}(t,T)$: Time dependent scalar function used to express the scalar part of $\log(\bar{P}(t,T))$ in the quadratic Gaussian multifactor model.
- $C^S(t)$: Symmetric positive definite matrix used to express the quadratic part of $\log(S_t)$ in the quadratic Gaussian multifactor model.
- $B^S(t)$: Time dependent vector used to express the linear part of $\log(S_t)$ in the quadratic Gaussian multifactor model.
- $A^S(t)$: Time dependent scalar function used to express the scalar part of $\log(S_t)$ in the quadratic Gaussian multifactor model.
- $A^f(T) := A + 2\Sigma\Sigma^\top C^S(T)$: Time dependent speed of mean reversion matrix for the dynamics of Y_t under the foreign risk neutral measure.
- $\mathbb{T}$: Default free forward measure for maturity T corresponding to using $P(t,T)$ as the numeraire.
- $\mathbb{E}^{\mathbb{T}}$: Expectation under the (domestic) default free forward measure.

- $\mathbb{T}^f$: Foreign default free forward measure for maturity T corresponding to using $P^f(t,T)$ as the numeraire.
- $\mathbb{E}^{\mathbb{T}^f}$: Expectation under the foreign default free forward measure.
- $\Phi(\Omega, z)$: Characteristic function of the quadratic form Ω in Gaussian random variables.
- $M(t,T)$: The mean of Y_T under $\mathbb{T}$ conditional on $\mathcal{F}_t$.
- $V(t,T)$: The variance-covariance matrix of Y_T under $\mathbb{T}$ conditional on $\mathcal{F}_t$.
- $M^f(t,T)$: The mean of Y_T under $\mathbb{T}^f$ conditional on $\mathcal{F}_t$.
- $V^f(t,T)$: The variance-covariance matrix of Y_T under $\mathbb{T}^f$ conditional on $\mathcal{F}_t$.

Notation for Chapter 2:

- FFT: Fast Fourier Transform;
- DFT: Discrete Fourier Transform;
- ATM: At the money strike rate;
- ITM: In the money strike rate;
- OTM: Out of the money strike rate;
- bp: Basis points;
- CMS: Constant Maturity Swap;
- LLM: Lognormal Libor Market Model;
- PVBP01: Present Value of a Basis Point;
- $Re[c]$: Real part of the complex number c.
- $\mathbb{T}_\alpha$: The default free forward measure corresponding to using $P(t,T_\alpha)$ as the numeraire.
- $\mathbb{E}^{\mathbb{T}_\alpha}$: Expectation under $\mathbb{T}_\alpha$.
- $\mathcal{T} = \{T_{\alpha+1},\ldots,T_\beta\}$: Payment dates for a forward swap starting on initial day T_α and ending on final day T_β.
- K: Fixed rate payed in a default free interest rate swap by the receiver.
- τ_i: Year fraction between T_{i-1} and T_i.

- $Swap_{\alpha,\beta}(t)$: The swap rate at time t which is the value of the fixed rate K that will make the value of a swap with starting date T_α and ending date T_β equal to zero.
- $Swaptn_{\alpha,\beta}(t)$: The exact price of a European payer swaption with maturity date T_α to enter a swap starting on date T_α and ending T_β.
- $\widetilde{P}_{\alpha,\beta}(t)$: The log-quadratic Gaussian process that is used to derive approximation to swaption prices.
- $w_i(t)$: The ith weight obtained by dividing $K\tau P(t,T_i)$ by $\widetilde{P}_{\alpha,\beta}(t)$ for $i = \alpha+1,\ldots,\beta-1$ and $(1+K\tau_\beta)P(t,T_\beta)$ by $\widetilde{P}_{\alpha,\beta}(t)$ for $i=\beta$.
- $\Phi(Q_1,\ldots,Q_N,z_1,\ldots,z_N)$: Joint characteristic function of N quadratic forms in Gaussian random variables dented by $Q_1,\ldots,Q_N$.
- $G_i(x,k)$: Type of payoff function which varies with i.
- $\hat{C}_{G_i}(z)$: Fourier transform with respect to strike of an option with payoff given by $G_i(x,k)$
- $\widetilde{Swaptn}_{\alpha,\beta}(t)$: The approximation of a swaption price that is obtained by approximating the exercise region through $\widetilde{P}_{\alpha,\beta}(t)$.
- $Swaptn1_{\alpha,\beta}(t)$: The approximation of a swaption price that is obtained by approximating the exercise region and the payoff through $\widetilde{P}_{\alpha,\beta}(t)$.
- $P_{\alpha,\beta}(t)$: The present value of a basis point.
- $\mathbb{Q}_{\alpha,\beta}$: The swap measure corresponding to using $P_{\alpha,\beta}(t)$ as the numeraire.
- $\mathbb{E}^{\mathbb{Q}_{\alpha,\beta}}$: Expectation under $\mathbb{Q}_{\alpha,\beta}$.
- $L(t,T_i)$: Forward libor rate for period $[T_i,T_{i+1}]$;
- K_{ATM}: The at the money strike rate of a default free swaption.
- K_{ITM}: An in the money strike rate of a default free swaption.
- K_{OTM}: An out of the money strike rate of a default free swaption.

Notation for Chapter 3:

- CIR: Cox-Ingersoll-Ross;
- CDS: Credit Default Swap;
- DPVBP: defaultable present value of a basis point;

- $\mathcal{T} = \{T_{n+1}, \ldots, T_N\}$: Payment dates for the premium leg of a credit default at time $T_n < T_{n+1}$.
- β_i: Year fraction between T_i and T_{i-1}.
- $\zeta(\tau)$: The last premium payment before default or the premium date on which default occurred if default coincided with the premium date.
- K: The premium rate of a CDS
- Z: Deterministic amount payed as default protection in case of default in CDS contract.
- δ: The recovery rate which is used to determine the default protection payment amount.
- $CDS(t, \mathcal{T}, T, K, Z)$: The value at time t of a payer forward credit default swap starting at time T with premium payment schedule $\mathcal{T}$, premium rate K and default protection payment Z.
- $CDS_{OP}(t, T_n, \mathcal{T}_{n,N}, T, K, Z)$: The value of a credit default swaption at time t which gives the owner of the swaption to enter into a CDS at time T paying a premium rate of K to get a default protection of Z.
- $R_f(T)$: The market CDS rate which is the value of the premium rate that would make the value of a CDS equal to zero.
- $G(0, T)$: The probability of survival of an obligor under the risk neutral measure.
- $\bar{G}(0, T)$: The probability of survival of an obligor under the default free forward measure.
- g_j: The conditional probability of default over (T_j, T_{j+1}).
- $H(0, T)$: The probability of default of an obligor under the risk neutral measure.
- $d(g_j, g_{j+1})$: Distance between the conditional probabilities of default.
- ν: Parameter used to determine smoothness of probability of survival.
- σ_k: Estimate of Gaussian error in market CDS quotes.
- $\overline{M}(t, T)$: The mean of Y_T under the defaultable forward measure.
- $\bar{V}(t, T)$: The variance-covariance matrix of Y_T under the defaultable forward measure.
- $U_{n,N}(T_n)$: The defaultable present value of a basis point.

- $\bar{w}_j(T_n)$: The ratio of the defaultable bond $\bar{P}(T_n, T_j)$ and the defaultable present value of a basis point.
- $\mathbb{U}$: The measure which is absolutely continuous to the risk neutral measure corresponding to using the defaultable present value of a basis point as the numeraire.
- $\mathbb{E}^{\mathbb{U}}$: Expectation under the measure $\mathbb{U}$.
- K_{DATM}: The at the money strike rate of a credit default swaption.
- K_{DITM}: An in the money strike rate of a credit default swaption.
- K_{DOTM}: An out of the money strike rate of a credit default swaption.

Notation for Chapter 4:

- $\mathbb{Q}^d$: The domestic risk neutral measure for default free and defaultable securities.
- $\mathbb{Q}^f$: The foreign risk neutral measure for default free and defaultable securities.
- $\mathbb{E}^{\mathbb{Q}^d}$: Expectation under the domestic risk neutral measure denoted by $\mathbb{Q}^d$.
- $\mathbb{E}^{\mathbb{Q}^f}$: Expectation under the foreign risk neutral measure denoted by $\mathbb{Q}^f$.
- W_t^d: Standard Brownian motion under the domestic risk neutral measure denoted by $\mathbb{Q}^f$.
- W_t^f: Standard Brownian motion under the foreign risk neutral measure denoted by $\mathbb{Q}^f$.
- τ: Default time of a reference entity(corporation or obligor).
- τ^d: Default time of a reference entity(corporation or obligor) in the domestic economy.
- τ^f: Default time of a reference entity(corporation or obligor) in the foreign economy.
- λ: Intensity of default for the default time τ.
- λ^d: Intensity of default for the default time τ^d.
- λ^f: Intensity of default for the default time τ^f.
- $H_t = \mathbf{1}_{\tau \leqslant t}$: Indicator function for default time τ.
- $H_t^d = \mathbf{1}_{\tau \leqslant t}$: Indicator function for default time τ^d.

- $H_t^f = \mathbf{1}_{\tau \leqslant t}$: Indicator function for default time τ^f.
- $\mathcal{F}_t$: Filtration generated by W_t^d.
- $\mathcal{H}_t$: Filtration generated by H_t.
- $\mathcal{G}_t$: Filtration generated by $\mathcal{F}_t \vee \mathcal{H}_t^d \vee \mathcal{H}_t^f$.
- Λ^d: The $(\mathcal{F}_t, \mathbb{Q})$-martingale hazard process of τ^d.
- Λ^f: The $(\mathcal{F}_t, \mathbb{Q})$-martingale hazard process of τ^f.
- Γ^d: The $\mathcal{F}_t$-hazard process of τ^d.
- Γ^f: The $\mathcal{F}_t$-hazard process of τ^f.
- $P^d(t,T)$: The price of a domestic default free zero coupon bond.
- $P^f(t,T)$: The price of a foreign default free zero coupon bond.
- $\mathbf{1}_{\tau>t}\bar{P}^d(t,T)$: The price of a domestic defaultable zero coupon bond when there is cross default of the entity(corporation or obligor).
- $\mathbf{1}_{\tau>t}\bar{P}^f(t,T)$: The price of a foreign defaultable zero coupon bond when there is cross default of the entity(corporation or obligor).
- $\mathbf{1}_{\tau^d>t}\bar{P}^d(t,T)$: The price of a domestic defaultable zero coupon bond when there is no cross default of the entity(corporation or obligor).
- $\mathbf{1}_{\tau^f>t}\bar{P}^f(t,T)$: The price of a foreign defaultable zero coupon bond when there is no cross default of the entity(corporation or obligor).
- S_t: The foreign exchange representing the price of one unit of currency in terms of domestic currency.
- σ_S: The instantaneous volatility of the foreign exchange rate.
- $X(t,T)$: The forward exchange rate.
- $\sigma_M(t,T)$: The instantaneous volatility of the forward exchange rate.
- $\bar{X}(t,T)$: The defaultable forward exchange rate.
- $\mathbb{T}^d$: The domestic forward measure corresponding to using $P^d(t,T)$ as the numeraire.
- $\mathbb{T}^d$: The foreign forward measure corresponding to using $P^f(t,T)$ as the numeraire.
- $\overline{\mathbb{T}}^d$: The defaultable forward measure for the domestic economy corresponding to using the domestic defaultable zero coupon bond as the numeraire.

- $\overline{\mathbb{T}}^f$: The defaultable forward measure for the foreign economy corresponding to using the foreign defaultable zero coupon bond as the numeraire.
- $f^d(t,T)$: The domestic continuously compounded default free instantaneous forward rates.
- $f^f(t,T)$: The foreign continuously compounded default free instantaneous forward rates.
- $\bar{f}^d(t,T)$: The domestic continuously compounded defaultable instantaneous forward rates.
- $f^f(t,T)$: The foreign continuously compounded defaultable instantaneous forward rates.
- $s^d(t,T)$: The domestic continuously compounded instantaneous forward rate spread.
- $s^f(t,T)$: The foreign continuously compounded instantaneous forward rate spread.
- $\sigma^d(t,T)$: The volatility of domestic default free instantaneous forward rates.
- $\sigma^d_s(t,T)$: The volatility of the domestic continuously compounded instantaneous forward rate spread.
- $\sigma^f_s(t,T)$: The volatility of foreign continuously compounded instantaneous forward rate spread.
- $\bar{G}^d(t,T)$: The probability of survival of the reference entity(corporation or obligor) under the domestic forward measure.
- $\bar{G}^f(t,T)$: The probability of survival of the reference entity(corporation or obligor) under the foreign forward measure.
- σ^d_g: The instantaneous volatility of the probability of survival of the reference entity(corporation or obligor) under the domestic forward measure.
- σ^f_g: The instantaneous volatility of the probability of survival of the reference entity(corporation or obligor) under the foreign forward measure.
- $\eta^d(t,T)$: The instantaneous volatility of $P^d(t,T)$.
- $\eta^f(t,T)$: The instantaneous volatility of $P^f(t,T)$.
- $\bar{\eta}^d(t,T)$ The instantaneous volatility of $\bar{P}^d(t,T)$.
- $\bar{\eta}^f(t,T)$ The instantaneous volatility of $\bar{P}^f(t,T)$.
- η^i_t: Diffusion part of the intensity of default λ^i_t in a contagion type model.

- α^d: The jump in the intensity of default in the domestic market due to the default of the obligor in the foreign market.
- α^f: The jump in the intensity of default in the foreign market due to the default of the obligor in the domestic market.
- $s^i(t,T)$: Forward credit spread process for the obligor in the ith economy.
- η_{ij}: Deterministic function representing the jump in the forward credit spread $s^i(t,T)$ due to default of obligor in the jth market.