Commodity Markets, Price Limiters and Speculative Price Dynamics

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Abstract

We develop a behavioral commodity market model with consumers, producers and heterogeneous speculators to characterize the nature of commodity price fluctuations and to explore the effectiveness of price stabilization schemes. Within our model, nonlinear interactions between market participants can create either bull or bear markets, or irregular price fluctuations between bull and bear markets. Both the imposition of a bottoming price level (to support producers) or a topping price level (to protect consumers) can reduce market price volatility. However, simple policy rules, such as price limiters, may have unexpected consequences in a complex environment: a minimum price level decreases the average price while a maximum price limit increases the average price. In addition, price limiters influence the price dynamics in an intricate way and may cause volatility clustering.

Keywords

commodity markets, price stabilization, simple limiters, technical and fundamental analysis, bifurcation analysis, chaos control

JEL classification

D84, G18, Q11

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1 Introduction

Commodity prices are, by any standard, extremely volatile. After inspecting thirteen primary commodities over the period 1900-1987 (deflated annual data), Deaton and Laroque (1992) found price variation coefficients, defined as the standard variation over the mean, ranging from 0.17 (bananas) to 0.60 (sugar). In addition, one often observes dramatic boom and bust episodes. For instance, the decline in prices from the highest level reached in the period from 1974 to August 1975 was 67 percent for sugar, 58 percent for sisal, more than 40 percent for cotton and rubber, and more than 25 percent for cocoa and jute (Newbery and Stiglitz 1981). In a recent study, Osborne (2003) reported that in Ethiopia the price of maize has more than doubled three times over the last fifteen years.

Not only many developing countries, but also the United States and the European Union, have thus experimented with some form of commodity price stabilization scheme in the past. In particular, attempts have been made to stabilize agricultural commodity markets by means of a commodity buffer stock scheme. The idea of such schemes is to put a certain amount of output into storage in years in which there is a good harvest, thus increasing the price from what it would have been, and to sell output from the storage in years in which there is a small harvest, thus reducing the price from what it would have been. Another prominent example is the oil market. Following the oil crises in the 1970´s, many countries built up huge oil reserves in order to influence the market.

Demand and supply schedules, storage and fully rational speculators are the key elements in neo-classical commodity market models (Waugh 1944, Brennan 1958, Williams and Wright 1991, Deaton and Laroque 1992, 1996, Chambers and Bailey 1996, Osborne 2003). While these models undoubtedly capture some important aspects of commodity markets, their ability to mimic features such as bubbles and crashes is, however, limited. Supporters of these models – in which the markets are efficient by nature – judge commodity price stabilization schemes as unlikely to have a significant beneficial effect (Newbery and Stiglitz 1981).

Contrary to the efficient market hypothesis, however, there is not only widespread populist feeling that speculators are a major cause of price instability, but also theoretical papers have started to explore this aspect. The chartist-fundamentalist approach, developed in the last decade, offers a new and promising alternative behavioral perspective of financial market dynamics. The main feature of this approach is that interactions between heterogeneous agents, so-called chartists and fundamentalists, may generate an endogenous nonlinear law of motion of asset prices. In Day and Huang (1990), Chiarella (1992), and Farmer and Joshi (2002) the nonlinearity originates from nonlinear technical and fundamental trading rules whereas in Kirman (1991), Brock and Hommes (1998), Lux and Marchesi (2000), the nonlinearity is caused by the agents switching between a given set of predictors.[1] More recent refinements and applications include Chiarella and He (2001), Chiarella, Dieci and Gardini (2002) and Westerhoff (2003). Since these models have demonstrated their ability to match the stylized facts of financial markets quite well one may conclude that this framework is suitable to conduct some policy evaluation experiments.

This paper aims at developing a commodity market model along the lines of the chartist-fundamentalist approach to characterize price fluctuations and to unravel the potential effects of price limiters. Its main ingredients are as follows: For simplicity,

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[1] A closely related branch of research studies complex dynamics in cobweb markets, e.g. Hommes (1994), Brock and Hommes (1997), Goeree and Hommes (2000) and Chiarella and He (2003).

demand and supply schedules are expressed in a reduced log-linear form. Fundamental to the model is the behavior of the speculators who switch between technical and fundamental trading rules to determine their positions in the market. Prices adjust via a log-linear price impact function: Excess supply (demand) decreases (increases) the price. Our model shows that: (i) the chartists are a source of market instability, as commonly believed, (ii) weak reaction of the speculators (either the fundamentalists or the chartists) can push the market to be either a bull or a bear market (through pitchfork bifurcations); and (iii) strong reaction of the speculators causes market prices to switch irregularly between bull and bear markets (through flip bifurcations). Since prices fluctuate in a complex way between bull and bear markets, the model is capable of replicating some features of commodity price motion.

The paper then focuses on the impact of simple price limiters as a potential stabilizing mechanism to reduce price fluctuations. Simulations reveal that if a central authority guarantees a minimum price, e.g. to support the producers, volatility declines. Although the price is backed up from below, the average price of the commodity surprisingly decreases, too. Setting up an upper price limit, e.g. to protect consumers from excessive prices, again yields a drop in price variability. However, the average price the consumers have to pay increases. At least at first sight, this result appears to be counterintuitive and should give policy-makers a warning. Simple measures to control prices may have surprising consequences in a nonlinear world.

This puzzling outcome is caused by a dynamic lock-in effect. Consider the case of a crash without a price limiter mechanism. Within our model, a bull market turns into a bear market after the price has crossed a critical upper level. A central authority that intervenes successfully against high prices obviously destroys the necessary condition for such a regime shift. As a result, the average price is higher than without an upper price restriction. Moreover, since the price fluctuates at a high level, it reaches the upper price boundary repeatedly so that the buffer stock is likely to run empty rather quickly. We show that one way to counter this problem is to alternate temporarily between an upper and a lower price boundary. The price volatility then decreases, yet the market remains distorted. However, on-off switching of the stabilization mechanism as well as changing the level of price limiters interferes with the price discovery process and may cause severe bubbles and crashes or volatility clustering.

As it turns out, price limiters as applied in our model are identical to a recently developed chaos control method. The development of chaos control algorithms was initiated by Ott, Grebogi and Yorke (1990) (henceforth OGY). Other popular suggestions include, for instance, the delayed feedback control method of Pyragas (1992) or the constant feedback method of Parthasarathy and Sinha (1995). The OGY control scheme and its descendants have been applied in various fields such as mechanics, electronics or chemistry. Economic applications include Kopel (1997), Kaas (1998) or Westerhoff and Wieland (2003). The feasibility of using chaos controllers in reality depends on the complexity and efficiency of the control algorithm. The chaos control process requires measurement of the system´s state, generation of a control signal, and the application of the control signal to an accessible system parameter. For instance, the original OGY control scheme requires knowledge of the map and its fixed point. While such information may be identified from observations in natural science applications, chaos control in an economic context is often seen as rather critical.

However, Corron, Pethel and Hopper (2000) present experimental evidence that chaos control can be accomplished using simple limiters and argue that chaos control can be practically applied to a much wider array of important problems than thought possible until recently. This method, which has been analytically and numerically explored by Wagner and Stoop (2000) and Stoop and Wagner (2003), simply restricts the phase space that can be explored. Suppose that a variable fluctuates between $0<x<1$. A limiter from below resets all values $x<h$ to h. As a result, the new system may replace previously chaotic behavior with periodic behavior. One advantage of the limiter method is that it does not add complexity to the system by increasing the size of the system´s state space. Another advantage is that stabilization may already be achieved by infrequent interventions. As far as we are aware, this paper contains the first economic study of limiters. And indeed, the method is able to decrease price fluctuations quite easily, yet with the (economic) disadvantage of a lock-in effect as stressed above.

The remainder of this paper is organized as follows. Section 2 presents a simple commodity market model with heterogeneous interacting agents and, by using stability and bifurcation analysis, section 3 examines the price dynamics of the model without price limiter mechanisms. In section 4, we discuss the consequences of single-price limiters for the price dynamics, and in section 5, we introduce conditional price limiters. The final section concludes the paper.

2 The Model

This section aims at developing a behavioral commodity market model. The model consists of well-established building blocks often used in the literature on chartist and fundamentalist interactions. Only the model´s ability to capture some commodity market aspects is novel. Our main goal is to analyze and evaluate the effectiveness of price limiters, for this reason, we strive to design a model that is as simple as possible. To be precise, we consider a market with three types of agents: consumers, producers and speculators. Speculators are heterogeneous in the sense that they are aware of both technical and fundamental trading strategies, and, at the beginning of each trading period, they choose one of the two strategies as their trading strategy for the trading period. Their behavior may be regarded as boundedly rational since the selection of a strategy depends on market circumstances.

Following Farmer and Joshi (2002), we assume that the price adjustment on the commodity market may be approximated by a log-linear price impact function. Hence, the log of price S at time t+1 is

$$S_{t+1} = S_t + a(D_t^M + W_t^C D_t^C + W_t^F D_t^F), \quad (1)$$

where a is a positive scaling coefficient to calibrate the price adjustment speed, D_t^M, D_t^C and D_t^F stand for the excess demand of the real economy, the chartists and the fundamentalists respectively at time t. The weight of the chartists at time t is given as W_t^C, whereas the weight of the fundamentalists is given as W_t^F. According to (1), the price of the commodity increases when there is an excess demand, and vice versa.

We use a reduced form to describe the demand and supply decisions of the real economy. Suppose that the demand and supply schedules of consumers and producers are log-linear. Then the excess demand may be expressed as

$$D_t^M = m(F - S_t), \quad (2)$$

where m reflects the slopes of the demand and supply curves. The log of the long-run equilibrium price, which we also call the fundamental value, is denoted by F. Since we assume that the structure of the economy is stable, i.e. there are no (permanent) demand and supply shocks, F is constant over time.

The excess demand of the real economy is zero when the price of the commodity is equal to its fundamental value. Such a state is obviously efficient. Note that in the absence of speculators ($W_t^C = W_t^F = 0$), the law of motion of the commodity price has a unique fixed point at $S_t = F$, which is stable for $0 < am < 2$.

Speculators are familiar with both technical and fundamental analysis. Indeed, the use of destabilizing trend extrapolation and stabilizing mean reversion trading strategies has been confirmed in survey studies among professional traders (Taylor and Allen 1992) as well as in laboratory experiments (Smith 1991, Sonnemans et al. 2003). To model the excess demand generated by technical analysis we adopt a formulation of Day and Huang (1990)

$$D_t^C = b(S_t - F), \tag{3}$$

where b is a positive reaction coefficient. So-called chartists typically believe in bear and bull markets. As long as the price is above its fundamental value, chartists regard the market as bullish. Since a further price increase is expected, chartists tend to buy the commodity. However, if the price drops below its fundamental value then the chartists become pessimistic. In a bear market, chartists sell the commodity.

Fundamental analysis presumes that prices revert toward their fundamental value. If the price is below (above) its equilibrium value, higher (lower) prices are expected and fundamental analysis favors buying (selling) the commodity. The excess demand generated by fundamental analysis may be formalized as

$$D_t^F = c(F - S_t). \tag{4}$$

The reaction coefficient c is positive.

The switching mechanism is based on an argument put forward by Hommes (2001). Speculators try to exploit bull and bear market situations. However, the more the price deviates from its fundamental value, the greater the speculators perceive the risk that the bull or bear market might collapse. As a result, an increasing number of speculators opt for fundamental trading strategies.[2] The market share of speculators who follow technical analysis may thus be defined as

$$W_t^C = \frac{1}{1+d(F-S_t)^2}, \tag{5}$$

where $d > 0$ is a switching parameter. The higher d is, the faster speculators switch to fundamental analysis as the mispricing increases. The weight of the fundamentalists is, of course, $W_t^F = 1 - W_t^C$.

The solution of the model is derived by combining (1)-(5)

$$S_{t+1} = S_t + a[m(F-S_t) - b\frac{(F-S_t)}{1+d(F-S_t)^2} + c\frac{d(F-S_t)^3}{1+d(F-S_t)^2}], \tag{6}$$

which is a one-dimensional nonlinear map.

3 Price Dynamics without Price Limiters

The model (6) can be written as:

$$S_{t+1} = f(S_t), \tag{7}$$

where

$$f(x) = x + a(x-F)\left[-m + \frac{b - cd(F-x)^2}{1+d(F-x)^2}\right], \tag{8}$$

and *a, b, c, d, m* are positive constants. To understand the impact of price boundaries,

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[2] An alternative justification is advocated by de Grauwe, Dewachter and Embrechts (1993). They argue that fundamentalists are heterogeneous with respect to their perception of the fundamental value. If the price is equal to its fundamental value, half of the fundamentalists underestimate the fundamental value and the other half overestimate the fundamental value. The net demand of fundamentalists is therefore zero. As the distance between *S* and *F* grows, the net position of fundamentalists becomes increasingly higher, implying that the market impact of chartists decreases.

this section focuses on the price dynamics of the deterministic model (6) without imposing any price limiter. Our analysis includes the existence of multiple steady states, their stability and bifurcation routes to complex price dynamics. The result is summarized as follows.

Proposition 1: For model (6),

- if $b \le m$, then F is the unique steady state. In addition,
  - it is locally asymptotically stable (LAS) if $a(m-b)<2$;
  - a flop bifurcation (i.e. the eigenvalue is 1) occurs when $m=b$;
  - a flip bifurcation (i.e. the eigenvalue is -1) occurs when $(m-b)a=2$.
- if $b>m$, then there are three steady states $F, S_\pm$ with

  $$S_\pm = F \pm \sqrt{\frac{b-m}{(c+m)d}},$$

  and F is always unstable, while $S_\pm$ is LAS if

  $$a^2 \frac{(b-m)(c+m)}{b+c} < 1.$$

  In addition,
  - a flop bifurcation occurs when $m=b$;
  - a flip bifurcation occurs when $a^2 \frac{(b-m)(c+m)}{b+c} = 1$.

Proof: See Appendix.

Denote

$$b_1 = \frac{am-2}{a}, \tag{9}$$

$$b_2 = \frac{a^2 m(c+m)+c}{a^2(c+m)-1}. \tag{10}$$

Then $b = b_1 \left(b = b_2\right)$ defines the flip bifurcation boundary of the local stability region of the fundamental (nonfundamental) steady-state price, while $b=m$ defines the common flop bifurcation boundary of the local stability regions of both fundamental and non-fundamental steady-state prices. It can be verified that, for $b>m$, $b_2 = b_2\left(c\right)$ decreases as c increases. In addition, $b_2 \to \left(a^2 m+1\right)/a^2$ as $c \to \infty$ and $b_2 \to a^2 m^2 / \left(a^2 m^2 - 1\right)$ as $c \to 0$. The local stability regions of both fundamental and nonfundamental steady-state prices and their bifurcation boundaries are plotted in figure 1.

Figure 1 goes about here

The implications of Proposition 1 are discussed with respect to the effects of reactions from the chartists and fundamentalists, respectively.

3.1. Effect of Price Dynamics Under the Chartists

We first examine the effect of the chartists' extrapolation, which is measured by the reaction coefficient b. It follows from Proposition 1 that for $0<b_1<b<m$ the fundamental price F is locally asymptotically stable; while for $m<b<b_2$, the fundamental price F becomes unstable but the two nonfundamental prices $S_\pm$ become locally stable. Hence, $b=m$ leads to a pitchfork bifurcation with respect to the extrapolation coefficient b of the chartists. In addition, the nonfundamental steady-state price $S_+ \left(S_-\right)$ increases (decreases) as the extrapolation of the chartists increases. This reflects a double-edged effect of the speculators. On the one hand, chartist activity to some extent improves the stability of the fundamental steady-state price to otherwise unstable commodity prices,

and hence improves market efficiency.[3] On the other hand, a strong extrapolation from the chartists leads to market instability, which is a common belief that chartists are a source of market instability.[4]

Figure 2 goes about here

For fixed *c*=1.5*, d*=1*, m*=1*, a*=1 and *F=0*, the upper panel of figure 2 illustrates the bifurcation plot of the price in terms of the extrapolation coefficient *b* of the chartists over $b \in (0,5)$. Following from Proposition 1, the fundamental price *F=0* is locally asymptotically stable for $b \in (0,1)$, unstable for *b*>1, and *b*=1 leads to a pitchfork bifurcation. For *b*>1, depending on the initial values, the two nonfundamental steady-state prices are locally stable for 1<*b*<2.66 and *b*=2.66 leads to a flip bifurcation from each of the two nonfundamental steady-state prices. The bifurcation plot in the upper panel of figure 2 has verified those analytical results. As *b* increases further, period-doubling type of bifurcation appears, leading to periodic cycles, quasi-periodic cycles and even more complicated price dynamics. For example, for *b*=4, 4.5, 5.5 and 6, figure 3 illustrates the price series and figure 5 gives the corresponding phase plots, that is the price in period t+1 is plotted against the price in period t, for *b*=4 and *b*=4.5.

Overall, weak extrapolations from the chartists can generate either a bull or a bear market, while strong extrapolations can make the market price fluctuate between

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[3] In fact, in the absence of speculators $\left(W_t^C = W_t^F = 0\right)$, the fundamental price *F* is stable for *0<am<2*. However, in the presence of speculators (in particular, of chartists), this stability region of the fundamental price *F* is enlarged to *0<am<2+ab* with *b<m*.

[4] An increase in the extrapolation (measured by parameter *b*) of the chartists results in two non-fundamental steady-state prices $S_{\pm}$. One is above and one is below the fundamental steady-state price, leading to a pitchfork-type bifurcation.

bull and bear markets. In addition, both time series and bifurcation plots indicate that the market price becomes more volatile as the extrapolation from the chartists increases.

Figures 3 and 4 go about here

3.2. Effect of Price Dynamics Under the Fundamentalists

We now examine the effect of the fundamentalists. It is interesting to see that the reaction coefficient *c* from the fundamentalists plays no role in the stability of the fundamental price *F* (since coefficient *c* is associated with a higher order term $(F - S_t)^3$ in equation (6)). However, it affects the stability of the two nonfundamental steady-state prices $S_\pm$. Note that $S_+ (S_-)$ decreases (increases) as parameter *c* increases. Both S_+ and S_- reflect the average price levels of bull and bear markets when the fundamental price is unstable. Hence, an increase in *c* brings these average price levels close to the fundamental price level, implying a stabilizing role of the fundamentalists.

When $b=b_2$, the two nonfundamental steady states become unstable through flip bifurcation, leading to period cycles and strange attractors. For example, for fixed *b*=4.5, when *c* is near 0, prices converge to either one of the steady states, depending on the initial prices. For *c*=0.5 up to 0.8, two-period cycles bifurcate from the two non-fundamental steady states and prices converge to either one of the two 2-period cycles. As *c* increases further, period cycles, quasi-periodic cycles and even complicated price dynamics induced from period-doubling bifurcation occur. For example, for *c*=0.85, prices converge to either one of two 4-period cycles, one being above the fundamental price and one being below the fundamental price, depending on the initial price. For *c*=0.93, prices converge to either one of the two 4-piece attractors. For *c*=0.94 up to 2, the two 4-piece attractors become two 2-piece attractors. As *c* increases further, it has the same effect on the price dynamics as the strong extrapolation from the chartists, discussed in the previous subsection. Such complicated price dynamics generated through period-doubling (flip) bifurcations are illustrated by the bifurcation plot for parameter c in the lower panel of figure 2.

Based on the above analysis, one can see that weak reaction from the fundamentalists has a stabilizing effect on the price dynamics. However, because of the instability of the fundamental price, such stabilizing efforts from the fundamentalists result in pushing the price away from the nonfundamental steady-state prices. This leads prices to converge to periodic and quasi-periodic cycles, which oscillate either above or below the fundamental price with low volatility. On the other hand, strong reaction from the fundamentalists results in irregular price fluctuations on both sides of the fundamental price with high volatility.

Both analytical and numerical analyses have shown that both fundamentalists and chartists affect the market price in a complicated way. Chartists are the source of market instability. When the fundamental steady-state price becomes unstable, there are two nonfundamental steady-state prices, one being above and one being below the fundamental steady-state price. Weak reaction from either fundamentalists or chartists can generate either bull or bear markets, while strong reaction from both speculators can make the market price fluctuate between bull and bear markets. In general, strong reaction from the chartists leads to high volatility, while strong reaction from the fundamentalists leads to lower volatility (as long as the price does not explode).

4 Simple Price Limiters

Next, we study the consequences of the limiter method – as suggested by Corron, Pethel and Hopper (2000) – on commodity price dynamics. Price limiters may easily be implemented in our framework.[5] In the case of a minimum price $S^{\min}$, (1) becomes

$$S_{t+1} = Max\,[f(S_t), S^{\min}], \tag{11}$$

in the case of a maximum price $S^{\max}$, the price adjustment modifies according to

$$S_{t+1} = Min\,[f(S_t), S^{\max}], \tag{12}$$

and in the case of a minimum and a maximum price restriction, one obtains

$$S_{t+1} = Min\,[Max\,[f(S_t), S^{\min}], S^{\max}]. \tag{13}$$

In order to avoid black markets, the central authority has, of course, to intervene in the market. For instance, to prevent the price from dropping below the minimum price the central authority has to buy a fraction of the supplied commodity.

Let us start the analysis by comparing the two time series displayed in figure 5, where we assume $a = 1$, $b = 4.5$, $c = 1.5$, $d = 1$, $m = 1$ and $F = 0$. The top panel of figure 5 shows the evolution of the commodity price without a price limiter in the time domain (200 observations). Visual inspection reveals again the model´s ability to produce bubbles and crashes, as observed in many commodity markets.

Concerned with the turbulent dynamics, a central authority may try to support the producers by guaranteeing them a minimum price. The bottom panel of figure 5 presents a simulation run in which $S^{\min} = -1.6$. The impact of such a price stabilization scheme may be quite dramatic. Note first that the price fluctuations appear to be much lower. And in fact, the variance of the price drops from around 1.11 (no

[5] For the sake of convenience, we refer to *S* as the price instead of the log of the price in this section.

price restriction) to 0.18. But the average price is also affected by this policy. Now, the price always fluctuates below its fundamental value. To be precise, the average price decreases from about 0 to –1.07 (all statistics are based on simulation runs with 10,000 observations). At least at first sight, this is surprising. The central authority aims to protect the producers from too low prices, yet the average price drops.

Figure 5 goes about here

What causes this puzzling outcome? Figure 6 presents the dynamics in phase space. The left panel shows the unrestricted dynamics while the right panel contains the restricted dynamics. The parameter setting is the same as in figure 5. The smooth lines indicate the two one-dimensional maps. From the left panel it becomes clear that a change from a bear market into a bull market requires the price to drop to a rather low value. But, due to the minimum price, the map in the right panel is flat at $S^{\min} = -1.6$. This prevents the system from switching from a bear market to a bull market. Clearly, the price is locked-in below its fundamental value. Furthermore, the right plot reveals that the commodity price is not longer chaotic but follows a period 14 cycle. Since the price hits the lower price limit every 14 periods, the central authority has to buy a fraction of the supplied commodity every 14 periods. By regularly taking out a fraction of the excess supply, the central authority builds up a huge buffer stock or even has to destroy its purchases. Both options are costly and unsustainable over a longer time.

Figure 6 goes about here

Figure 7 demonstrates the robustness of the dynamic lock-in effect. The top left panel shows a bifurcation diagram in which the minimum price is increased in 500 steps from –2 to 0. To allow the system to settle on its attractor, the dynamics is plotted for the prices between the periods 500 and 600. As can be seen, the chaotic behavior turns into periodic behavior as the minimum price increases. At around $p^{\min} = -1.18$, even a fixed point emerges. The top right panel reveals symmetrical results for an upper price boundary, which decreases from 2 to 0.

The second and third panels in figure 7 present how the mean and the variance of the price react to a change in the price limit. To obtain reasonable statistics, IID shocks with white noise N (0, 0.1) are added to the system at every time step. The limiter method indeed proves its power in stabilizing the dynamics. Already minor price restrictions may eliminate the larger part of the price variability. But the average price is simultaneously affected. Restricting the price from below (above) increases (decreases) the mean of the price in an adverse manner. Only if the restrictions are very sharp may a lower (higher) price boundary lead to an increase (decrease) in the average price.

Figure 7 goes about here

To sum up, it turns out that the price dynamics under price limiters when both speculators react strongly is similar to the price behavior without imposing price limiters when either of the speculators react weakly. Consequently, one may conclude that the central authority eliminates the strong reactions of the speculators that push for the market to crash, leading to a bull market with high average price

5 Conditional Price Limiters

Comparing the dynamics with and without price limiters, one can see that by imposing certain price limiters a central authority can effectively limit strong reactions of the speculators and stabilize the market price. However, such a policy may lead to substantial costs for the central authority. For example, to prevent the price from dropping below (going above) the minimum (maximum) price, the central authority permanently has to buy (sell) a fraction of the supplied (stored) commodity. Apart from the cost issue (e.g. for maintaining a buffer stock), non-negative storage of the commodity prevents unlimited selling at high price limiters. To implement such interventions more successfully, we introduce a conditional price limiter mechanism in this section.

The top panel of figure 8 shows the price dynamics when the central authority switches between two price limiters: $S^{\min} = -1.3$ and $S^{\max} = 1.3$. If the buffer stock exceeds a level of about ±15 a regime shift occurs. As can be seen, the price is thus stabilized either in the bull market or in the bear market. The duration of a regime is around 80 periods. Between the two regimes we observe a brief transient phase of around 20 periods in which the price evolves uncontrolled. Although the price is still distorted, conditional price limiters decrease the price volatility. The bottom panel of figure 8 presents the corresponding development of the buffer stock which now neither runs empty nor becomes infinitely large. Note that by buying low and selling high the interventions may be profitable in the long run.

Figure 8 goes about here

It is quite interesting to see how the market prices are influenced by such a policy. Speculators may discover and respond to fixed price limiters. To prevent arbitrage opportunities, central authorities may thus use more flexible price limiters. Figure 9 aims to demonstrate that the imposition of price limiters may create dramatic price changes such as bubbles and crashes. In the top panel of figure 9, it is assumed that the central authority stabilizes the price in the bear market with a price limiter of -1.3. However, from time to time interventions are briefly interrupted. For instance, the policy-makers may become afraid of the costs associated with their policy. But seeing that without interventions the price increases dramatically, the policy-makers may change their opinion once more and reactivate their old policy. Then the price is again bounded in the lower region. As a result, price patterns that resemble bubbles and crashes may simply emerge due to the activity of policy-makers.

Figure 9 goes about here

In the second panel of figure 9, the central authority applies a price limiter of $S^{min} = -1.3$, buffeted with dynamic noise $N(0,\ 0.3)$. Overall, the dynamics is still stabilized. However, now and then prices run away from bear markets. The reason is that the price limiter may be set too low to achieve the lock-in effect. In the third panel of figure 9, the central authority regulates the market with $S^{min} = -1.3$ and $S^{max}=1.3$, both buffeted with dynamic noise $N(0,\ 0.3)$. Again, temporary stabilizations either in the bull or in the bear market set in.

Finally, in the fourth panel of figure 9, the price limiters are modeled as first-order auto-regressive processes around ± 1.3 with AR coefficients of 0.975 and noise $N(0,\ 0.1)$. Note that the price dynamics become increasingly realistic when the limiters are varied in a stochastic matter. Although the behaviors of consumers, producers and speculators are still deterministic, the price behavior is quite intricate. Visual inspection reveals bubbles and crashes, alternating periods of low and high volatility and also larger jumps, which may yield fat tail behavior of the distribution of the returns.[6] What causes the increase in the complexity of the dynamics? The answer is quite simple. Figure 7 already reveals that different price limiters result in different dynamic outcomes, e.g. a fixed point may be transformed into a limit cycle. In addition, perturbations of the price limiters work as shocks to the system and thus transient behavior may occur.

[6] Indeed, applying the Hill tail index estimator procedure we find that the model is able to produce tail indices of around 3.5. Estimation of the Hurst coefficient for absolute returns reveals values of around 0.75, indicating strong volatility clustering.

6 Conclusions

This paper is concerned with commodity price dynamics. Actual commodity prices fluctuate strongly: Not only is the price volatility high, also severe bubbles and crashes regularly emerge. Hence, this topic is of great practical importance, particularly for the formulation of economic policy. Although producers and consumers are two primary participants in commodity markets, there are also other participants, such as speculators, who may have a marked effect both on the degree of price variability and on the success of any commodity price stabilization scheme.

Within our model, interactions between heterogeneous agents create complex bull and bear market fluctuations, which resemble the cyclical price dynamics of many commodity markets. Our model shows that: (i) the chartists are a source of market instability, as commonly believed; (ii) weak reaction of the speculators (either the fundamentalists or the chartists) can push the market to be either a bull or a bear market (through pitchfork bifurcations); and (iii) strong reaction of the speculators causes market prices to fluctuate irregularly between bull and bear markets (through flip bifurcations). Furthermore, we investigate how price boundaries, which function identically to a recently suggested chaos control method, affect the price dynamics. We find that simple price limits (i) reduce the variability of prices quite strongly, (ii) are likely to shift the price in an adverse direction, (iii) and may lead to an unsustainable buffer stock. The results are caused by a dynamic lock-in effect. By restricting the evolution of the price, the dynamics may become stuck in either the bull or the bear market. However, jumping between bottoming and topping price limiters allows a central authority to manage the evolution of the buffer stock. Prices are then temporarily stabilized in the bull market or the bear market. But it should not be overlooked that whenever a central authority introduces a price stabilization scheme it changes the price discovery process. For instance, price limiters may trigger marked bubbles and crashes or volatility clustering.

The study of heterogeneous interacting agents has yielded a number of quite sophisticated models which have proven to be quite successful in explaining financial market dynamics. Our simple commodity market model is inspired by this approach and we would finally like to point out some interesting extensions. First of all, one may consider some other popular technical trading rules. For example, agents are often reported to extrapolate the most recent price trend. Moreover, as argued in Chiarella (1992) or Farmer and Joshi (2002), technical analysis may be nonlinear. Secondly, agents may involve some adaptive learning processes when choosing a particular trading strategy. For example, the behavior of chartists and fundamentalists may not be constant over time with respect to their reaction coefficients, and, although agents are boundedly rational, they may try to learn those coefficients. Alternatively, agents' expectations may follow some adaptive learning processes. Thirdly, agents may incorporate other switching mechanisms. As argued in Brock and Hommes (1998), one may assign each forecast rule a fitness function (which may depend on the historical performance of the rules) and then let the agent select a rule according to its fitness. Higher complexity may also be achieved by switching from a two-speculator type analysis to a real multi-agent market model (Lux and Marchesi 2000). Of course, the behavior of both the producers and consumers may also be modeled in more detail. For instance, the producers may base their production decision on expected future prices and thus select between different kinds of forecast rules, as modeled in Brock and Hommes (1997). Finally, the working of different price limiter schemes may also be tested in a laboratory setting. Promising work on experimental asset pricing markets has been done by Smith (1991) or Sonnemans et al. (2003).

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Appendix. Proof of Proposition 1

Let $\overline{x}$ be the steady state of the map $x_{t+1} = f(x_t)$. Then it satisfies

$$(\overline{x} - F)\frac{(b-m)-(c+m)d(F-\overline{x})^2}{1+d(F-\overline{x})^2}.$$

Obviously, the fundamental steady-state price $\overline{x} = F$ is always a steady state. For *b*>*m*, the map has two more nonfundamental steady-state prices $\overline{x} = S_{\pm}$ with

$S_{\pm} = F \pm \sqrt{\frac{b-m}{(c+m)d}}$. In other words, when the chartists extrapolate strongly (*b*>*m* is satisfied), the model generates two nonfundamental steady-state prices $S_{\pm}$ with one (S_+) above and one (S_-) below the fundamental steady-state price *F*. Note that

$$f'(x) = 1 + a\left[\frac{(b-m)-(c+m)d(F-\overline{x})^2}{1+d(F-\overline{x})^2} - 2ad(b+c)\frac{(x-F)^2}{\left[1+d(F-x)^2\right]^2}\right].$$

At the fundamental steady-state price *F*, $f'(F) = 1 - a(m-b) \equiv \lambda$. Hence *F* is local asymptotically stable if $0 < a(m-b) < 2$. In addition, the eigenvalue *λ=1* when *m=b* and *λ=-1* when *(m-b)a=2*. At $S_{\pm}$,

$$f'(S_{\pm}) = 1 - 2a^2\frac{(b-m)(c+m)}{b+c} \equiv \lambda.$$

Hence, $S_{\pm}$ is local asymptotically stable (LAS) if

$$0 < a^2\frac{(b-m)(c+m)}{b+c} < 1.$$

Furthermore, *b*=*m* leads to *λ=1*, and $a^2\frac{(b-m)(c+m)}{b+c} = 1$ leads to *λ*=-1.

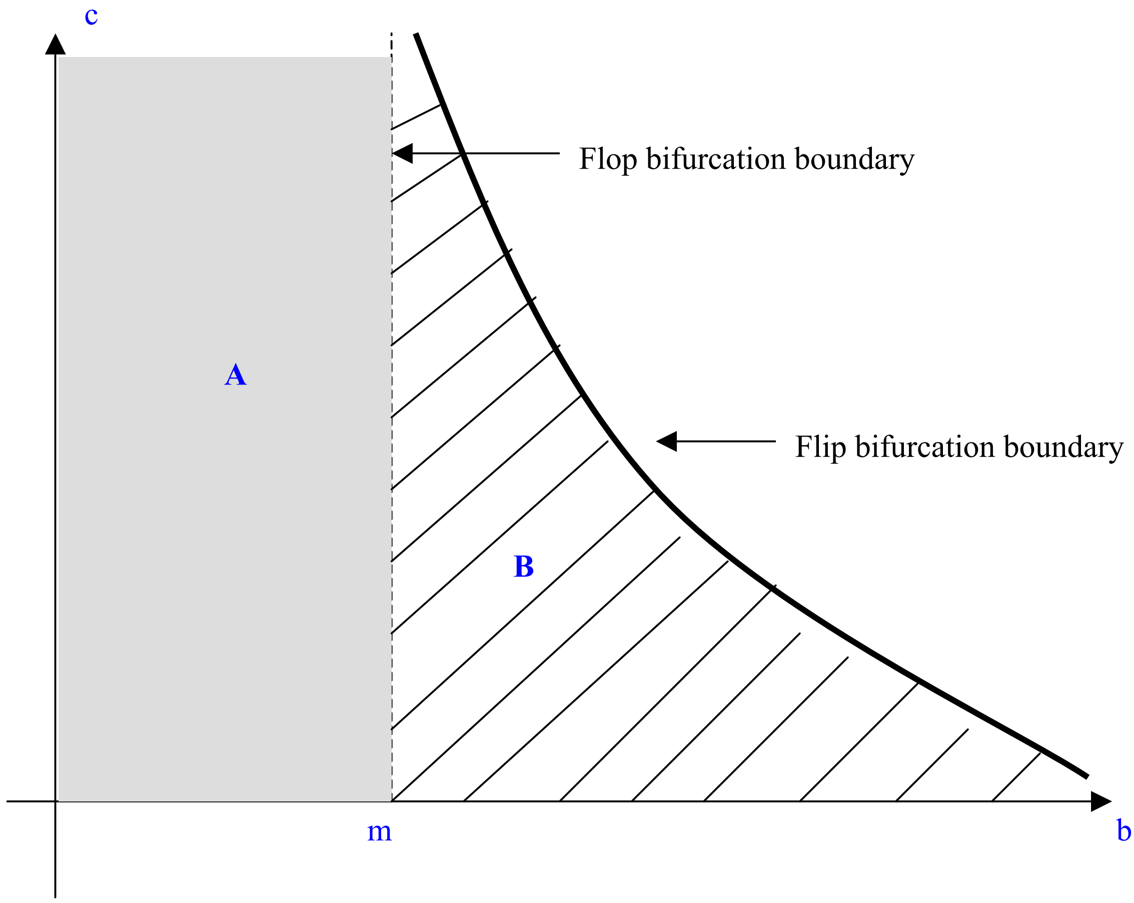

Figure 1: Local stability region A (B) of the fundamental (nonfundamental) steady-state price.

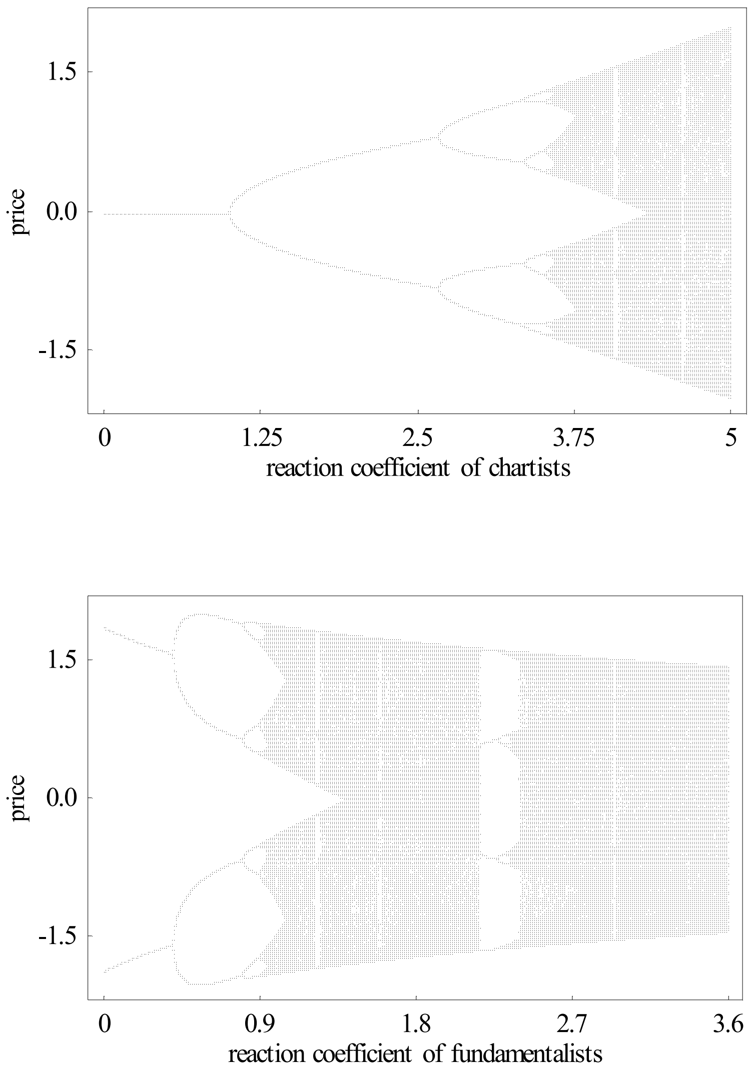

Figure 2: Bifurcation diagrams for parameters *b* and *c*. The parameters are increased in 500 steps as indicated on the axis. The prices are plotted from t=500-600. The other parameters are $a=1$, $b=4.5$, $c=1.5$, $d=1$, $m=1$ and $F=0$.

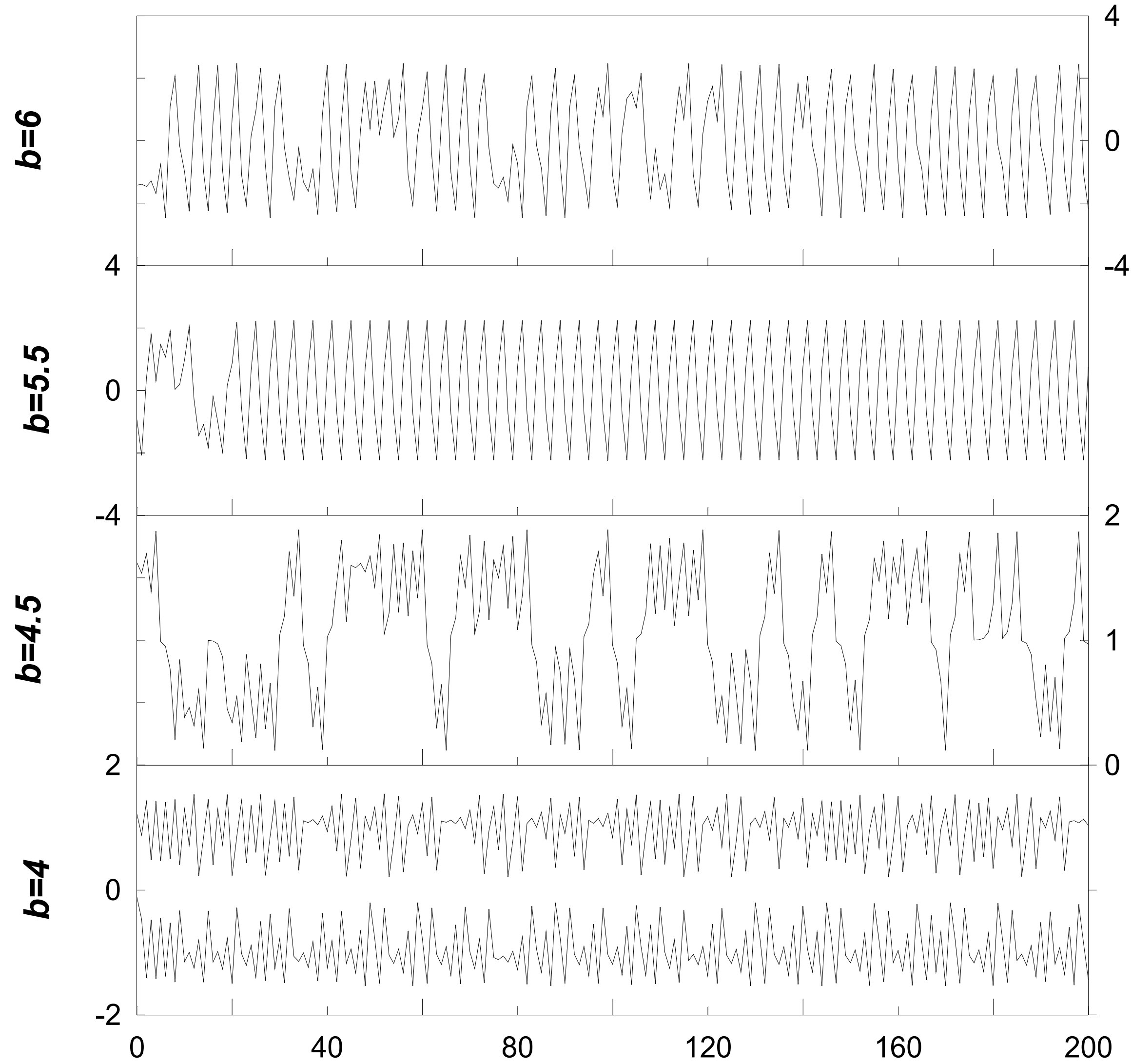

Figure 3: Commodity prices for fixed parameters $a=1$, $c=1.5$, $d=1$, $m=1, F=0$ and different values of parameter *b*.

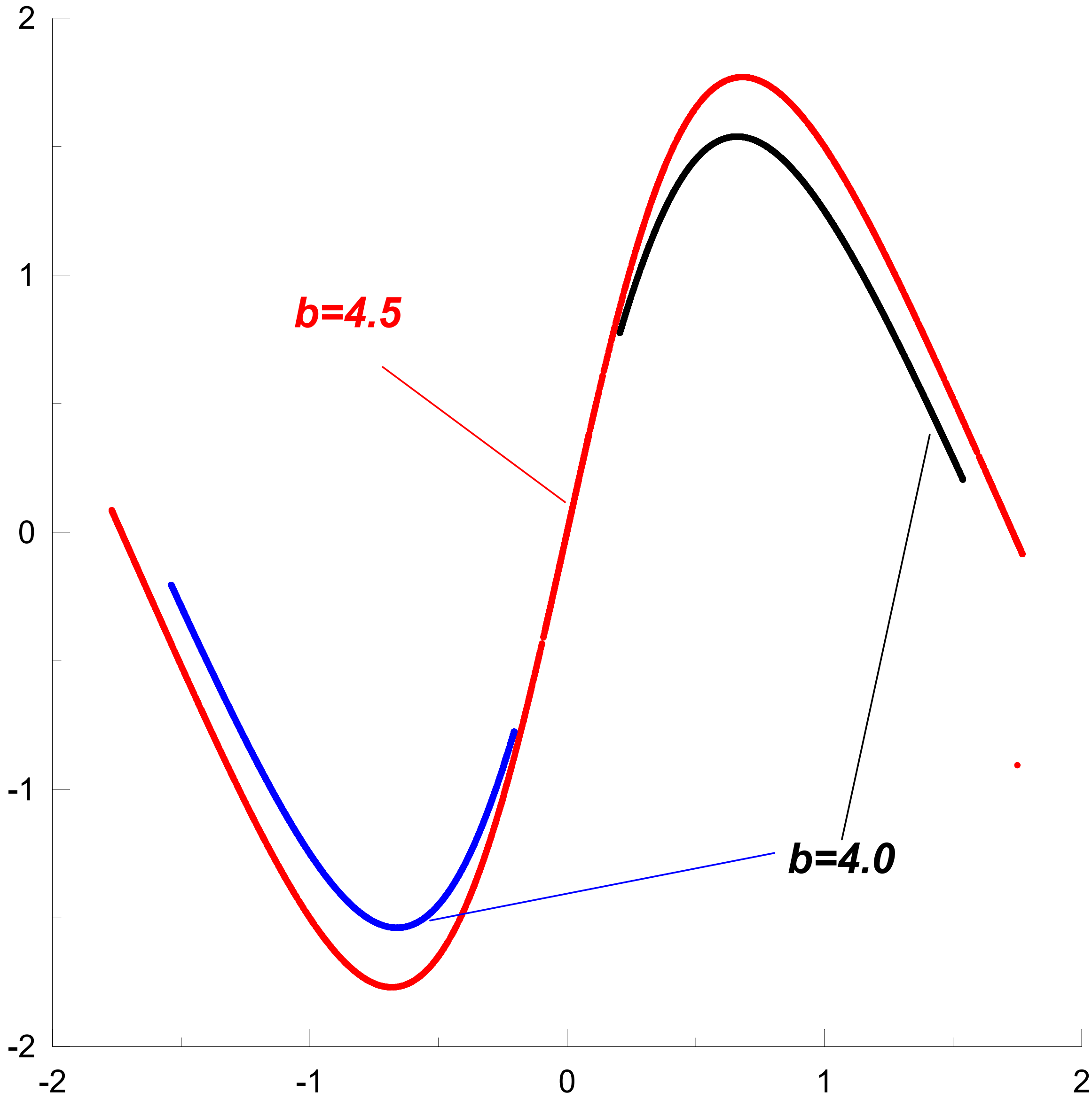

Figure 4: Commodity prices phase plots for b=4.0 and 4.5 and fixed parameters $a=1$, c=1.5, $d=1$, $m=1$ and $F=0$.

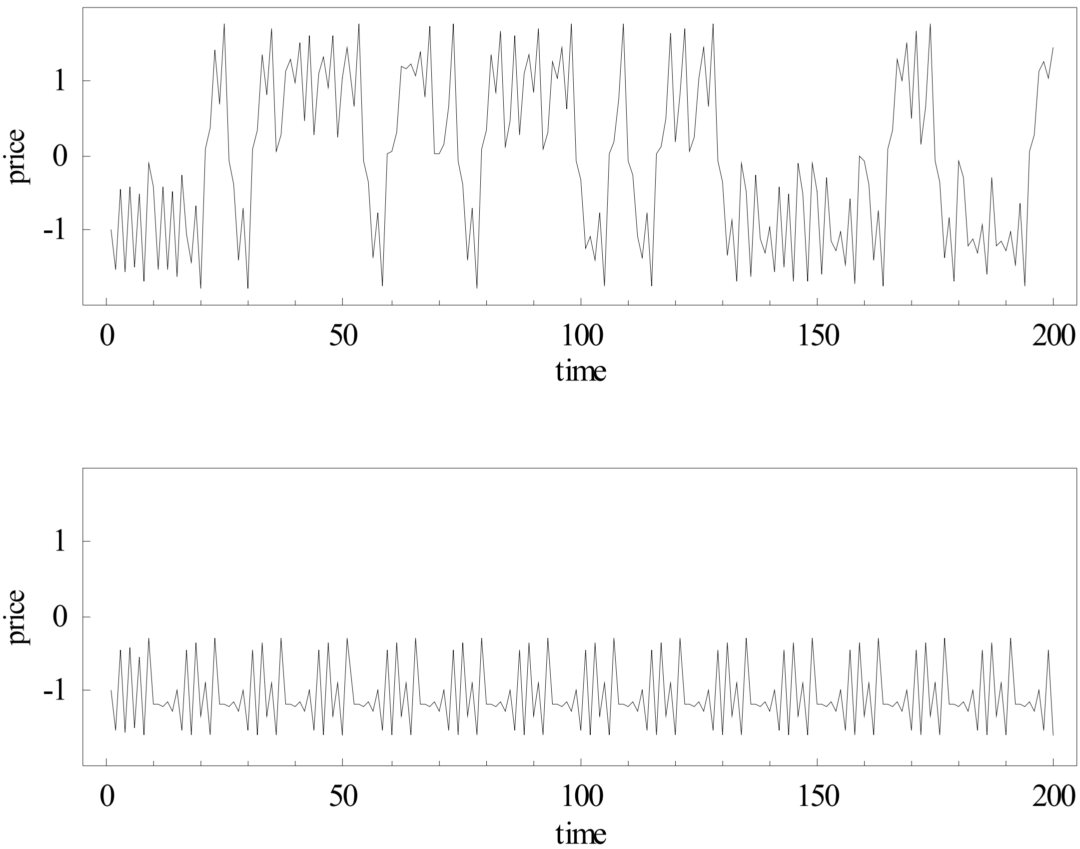

Figure 5: The top panel shows the unrestricted evolution of the commodity price in the time domain. The bottom panel shows the same, but with a lower price boundary of $S^{\min} = -1.6$. The other parameters are $a = 1$, $b = 4.5$, $c = 1.5$, $d = 1$, $m = 1$ and $F = 0$.

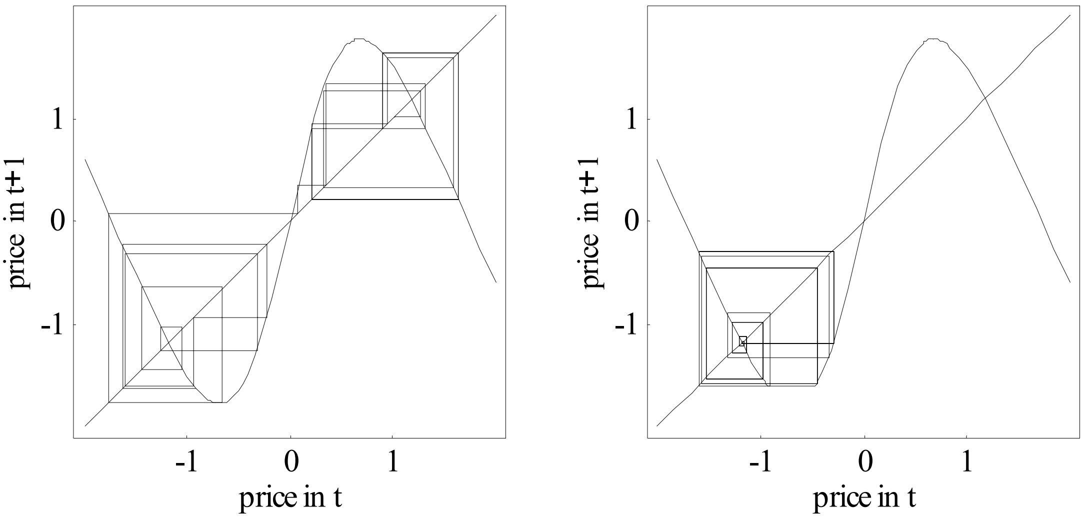

Figure 6: The left (right) panel presents the unrestricted (restricted) evolution of the commodity price in phase space. The smooth lines indicate the one-dimensional map. Parameter setting as in figure 1.

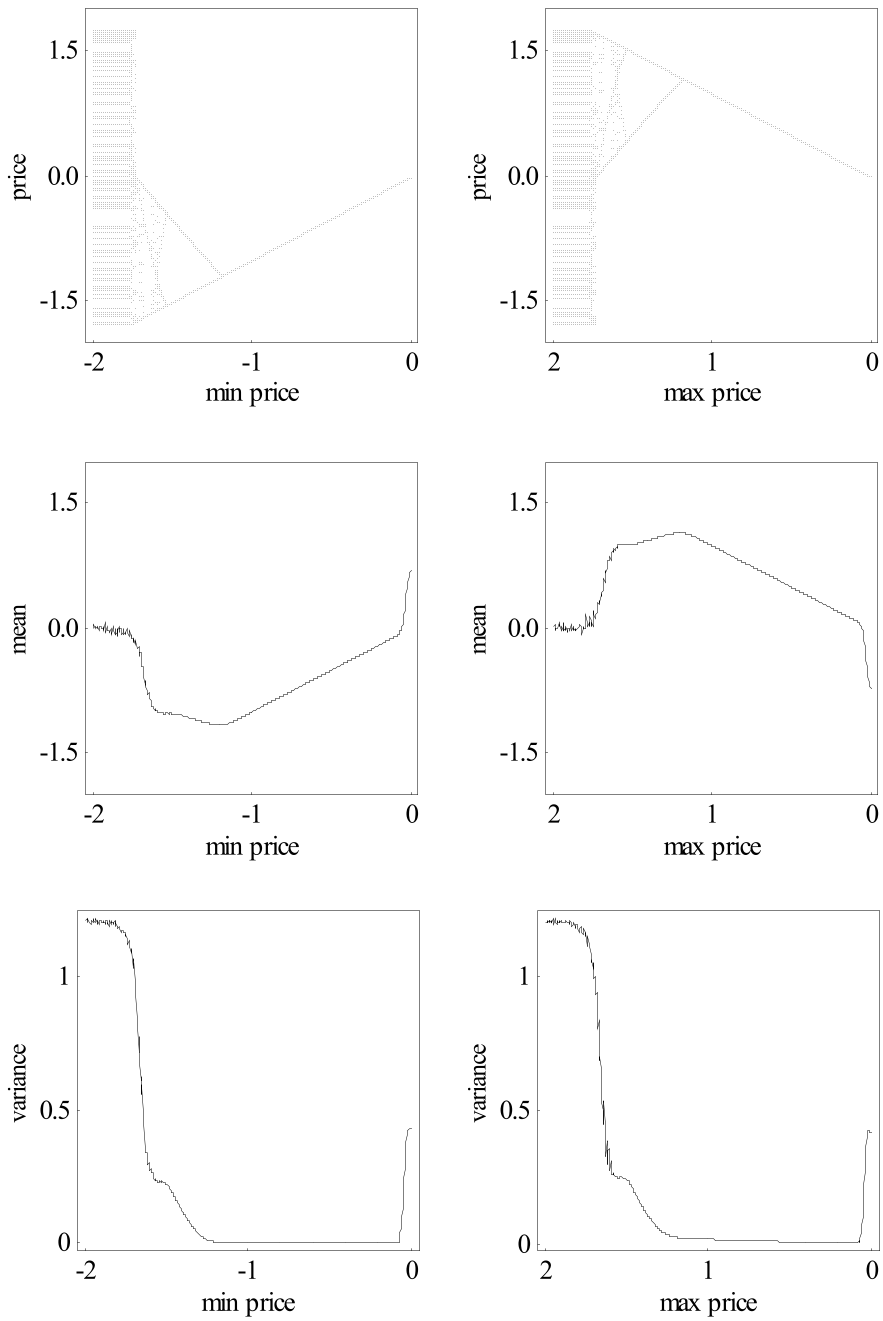

Figure 7: The bifurcation diagrams in the first line of panels show how the price reacts to more restrictive price boundaries. The price limits are varied in 500 steps and the prices are plotted from t=500-600. The parameter setting is as in figure 1. The second and third lines of panels show the mean and the variance of the price process (buffeted with dynamic noise N (0, 0.1). All statistics are based on 10,000 observations.

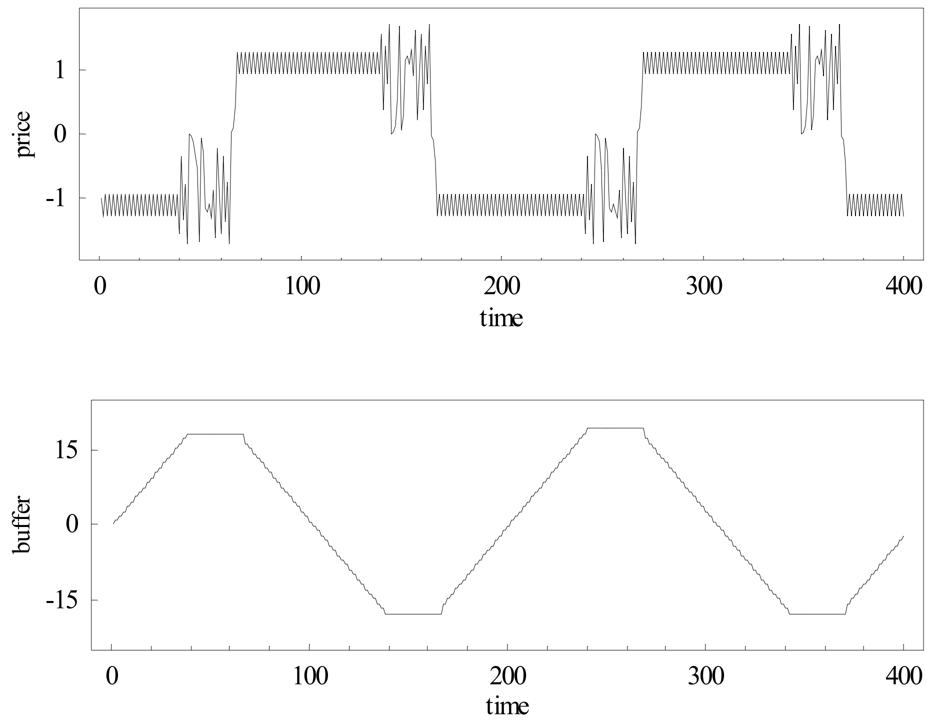

Figure 8: The top panel shows the price dynamics when the central authority switches between the price limiters $S^{\min} = -1.3$ and $S^{\max} = 1.3$. A change in regime occurs if the buffer stock exceeds a level of about ±15. The bottom panel shows the corresponding evolution of the buffer stock. The other parameters are $a = 1$, $b = 4.5$, $c = 1.5$, $d = 1$, $m = 1$ and $F = 0$.

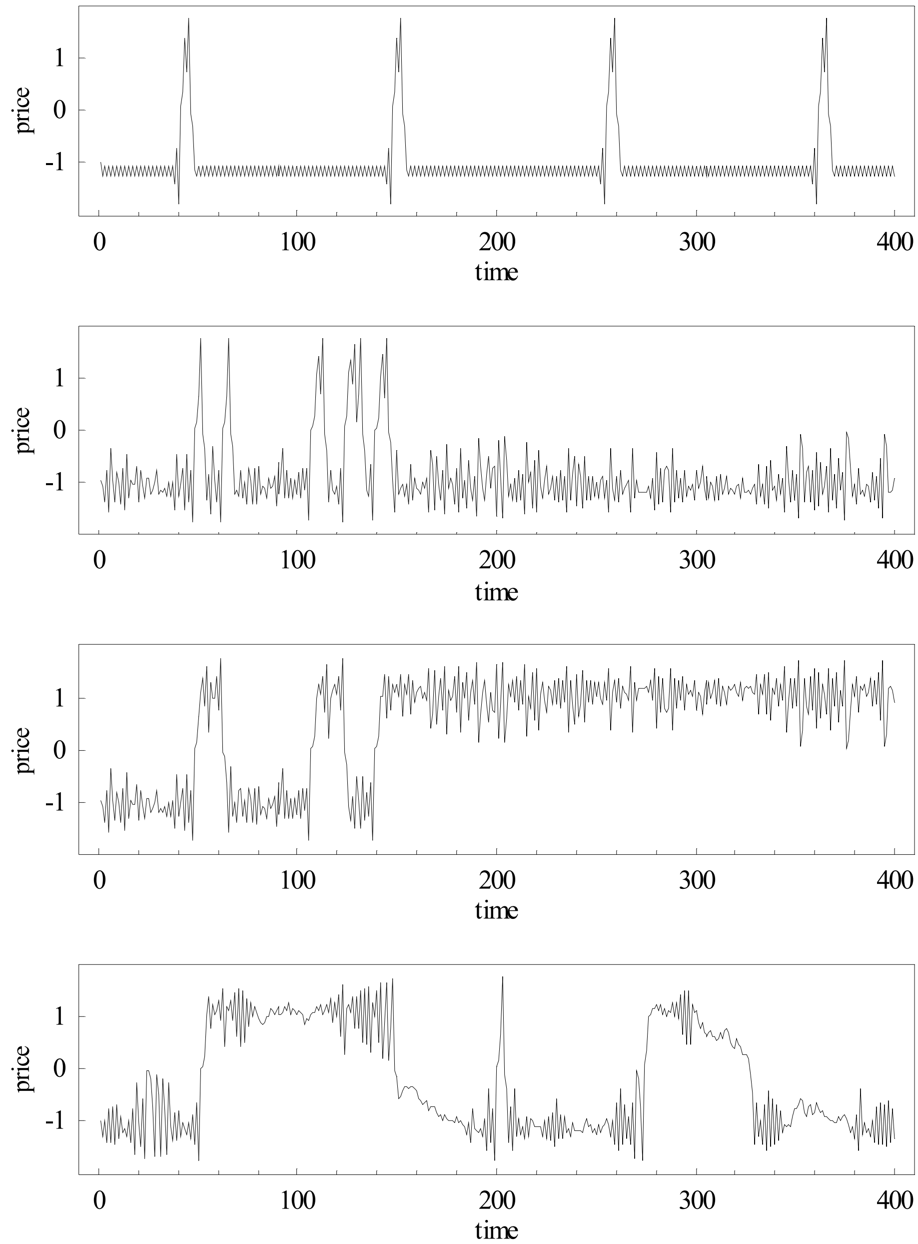

Figure 9: Price dynamics under different regimes. First panel: Price limiter $S^{\min} = -1.25$; interupted every 100 periods. Second panel: Price limiter $S^{\min} = -1.3$, buffeted with dynamic noise *N* (0, 0.3). Third panel: Price limiters $S^{\min} = -1.3$ and $S^{\max}$=1.3, both buffeted with dynamic noise *N* (0, 0.3). Fourth panel: Price limiters as first-order auto-regressive processes around ± 1.3 with AR coefficients of 0.975 and noise *N* (0, 0.1). The other parameters are $a = 1$, $b = 4.5$, $c = 1.5$, $d = 1$, $m = 1$ and $F = 0$.