Catalogue of Analytical Solutions for Consolidation of Unsaturated Soils Subjected to Various Initial Conditions and Time-Dependent Loadings

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LIEM HUU HO, BEng (1st class Hons, UTS)

School of Civil and Environmental Engineering,
Faculty of Engineering and Information Technology
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CERTIFICATE OF ORIGINAL AUTHORSHIP

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledge within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Liem Ho
February 2016

ABSTRACT

Soil consolidation has been a primary geotechnical interest for decades. Such phenomenon involves the gradual dissipation of excess pore pressures from the soil deposit subjected to an external applied load, resulting in a considerable reduction of soil volume. Majority of industrial and residential areas have been vigorously developed in arid and semi-arid climatic regions, where the underground water table is relatively deep. In these regions, construction activities can significantly influence the upper unsaturated zone. In particular, the earthworks, such as excavation and compaction, and changes in the climate and surface vegetation may result in further creation of unsaturated soils, whose properties are much more complicated than those of saturated soils. Past decades have witnessed the significant growth of engineering interests in unsaturated soils and that has motivated researchers to conduct more insightful research. A great attention has been given to the unsaturated consolidation theory due to many foundation-related problems particularly relate to time-dependent soil volume change and settlement. However, a typical unsaturated soil usually has nonlinear properties and intricate phase relationships, which result in theoretical difficulties in formulating a reliable model for the consolidation prediction.

This thesis presents a systematic catalogue of analytical solutions for the consolidation of unsaturated soils subjected to various loading and initial conditions. Particularly, eigenfunction expansions and standard Laplace transformation techniques are used to solve the consolidation equations. This research provides rigorous solutions to estimate the rates of excess pore-air and pore-water pressure dissipation and consolidation settlement under the one-dimensional (1D), two-dimensional (2D) plane strain and axisymmetric consolidation conditions. For the mathematical derivation, uniform and linearly depth-dependent initial conditions are adopted along with homogeneous boundary conditions, including one-way and two-way drainage boundary conditions. In addition, effects of time-dependent loadings are also captured in this study. Four primary types of external loads, namely ramping, asymptotic, sinusoid and damped sine wave, are simulated and then incorporated in the proposed solutions. On the other hand, the 1D consolidation of unsaturated soils under non-isothermal conditions is sufficiently discussed. This study also demonstrates that the proposed

analytical solutions can change back to the traditional equations for saturated soils. Most results are graphically presented in the semi-logarithmic plots. Changes in excess pore pressures and settlement are investigated against the air to water permeability ratio ($\mathrm{k_a/k_w}$). Moreover, pore pressure isochrones along the flow domains are also highlighted in each consolidation field. Verification exercises are conducted by comparing the predicted results with other solutions obtained from existing literature. The proposed equations can be used by practicing engineers. Programmable methods such as Microsoft Excel or MATLAB can be simply adopted to generate results from proposed equations to predict the time-dependent settlement of unsaturated soils.

For all consolidation cases, it is predicted that variations in the permeability ratio $\mathrm{k_a/k_w}$ result in double inverse S curves for the excess pore-water pressure and settlement, while forming a single S curve for the excess pore-air pressure. The study shows that the 1D consolidation process in the two-way drainage soil stratum tends to proceed more quickly than that in the one-way drainage system. However, the consolidation rates under these boundary conditions are almost comparable when drain wells (for the 2D plane strain and axisymmetric cases) are installed in the soil profile. In the 2D plane strain consolidation system, if the horizontal permeability is greater than the vertical permeability (i.e. $\mathrm{k_x/k_z > 1}$), the horizontal flow will govern the dissipation rate and the effects of vertical flow is much attenuated. This point is also supported in the axisymmetric analysis.

Additionally, the time-dependent loadings and temperature variations have significant impacts on changes in excess pore pressures and settlement. For the loading effects, it can be predicted that excess pore-water pressures and settlement are considerably influenced by the loading patterns irrespective of $\mathrm{k_a/k_w}$ values. However, in most loading cases, effects of the applied loads on the excess pore-air pressure are less pronounced as $\mathrm{k_a/k_w}$ increases. On the other hand, variations in soil temperature are substantially attributed to the air temperature and the heat from solar radiation. It is predicted that, for time-dependent linear temperature variations, the excess pore-air pressure initially increases dramatically and then attains a constant value, while the excess pore-water pressure diminishes a very long time after the heat begins to increase. Besides, excess pore-air and pore-water pressures near the ground surface increase faster than those at lower depths when the temperature increases exponentially. Both pressures then are fully dissipated as the temperature approaches the maximum value. For the case of diurnal temperature wave, the excess pore pressure curves would oscillate capturing damping and retarding effects. Development of analytical solutions for the unsaturated consolidation incorporating the above influencing factors would provide fundamental understandings of deformation of unsaturated soils.

To my father, *Tuan Huu Ho*, my mother, *Thu Thi Le*, my brothers, *Hieu Huu Ho* and *Tai Huu Ho*, and my sister, *Vy Khanh Ho*, who shared love and strength with me throughout this marvellous journey.

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LIST OF NOTATIONS

English letters

$\mathrm{C_a}$	interactive constant associated with the air phase;
$\mathrm{C_w}$	interactive constant associated with the water phase;
$\mathrm{c^a_{v_r}}$	coefficient of consolidation with respect to the air phase in the radial domain;
$\mathrm{c^a_{v_z}}$	coefficient of consolidation with respect to the air phase in the vertical domain;
$\mathrm{c^a_{v_x}}$	coefficient of volume change with respect to the air phase in x-direction;
$\mathrm{c^a_\sigma}$	coefficient of consolidation due to total stress with respect to the air phase;
$\mathrm{c^w_{v_r}}$	coefficient of consolidation with respect to the water phase in the radial domain;
$\mathrm{c^w_{v_z}}$	coefficient of consolidation with respect to the water phase in the vertical domain;
$\mathrm{c^w_{v_x}}$	coefficient of volume change with respect to the water phase in x-direction;
$\mathrm{c^w_\sigma}$	coefficient of consolidation due to total stress with respect to the water phase;
g	gravitational constant;
H	thickness of the soil stratum;
i	integer for the Fourier sine series (for 2D plane strain consolidation) Fourier Bessel series (for axisymmetric consolidation);
j	integer for the Fourier sine series (for both plane strain and axisymmetric conditions);
K	eigenvalue for vertical boundary condition (for 1D consolidation);
K	ratio of the changes in horizontal (or radial) and vertical stresses at a point;
k	integer for the Fourier sine series (for 1D consolidation);
$\mathrm{k_{a_r}}$	coefficient of permeability for the air phase in the radial domain;

k_{a_x}	coefficient of permeability for air in x-direction;
k_{a_z}	coefficient of permeability for the air phase in the vertical domain;
k_{w_r}	coefficient of permeability for the water phase in the radial domain;
k_{w_x}	coefficient of permeability for water in x-direction;
k_{w_z}	coefficient of permeability for the water phase in the vertical domain;
L	width of soil layer;
M	molecular mass of the air phase;
m_v	coefficient of volume change in saturated soils;
m_1^a	coefficient of volume change of the air phase with respect to the change of net stress;
m_2^a	coefficient of volume change of the air phase with respect to the change of suction;
m_1^s	coefficient of volume change of the soil with respect to the change of net stress;
m_2^s	coefficient of volume change of the soil with respect to the change of suction;
m_1^w	coefficient of volume change of the water phase with respect to the change of net stress;
m_2^w	coefficient of volume change of the water phase with respect to the change of suction;
n	porosity;
q_z	external loading;
R	universal air constant;
$R_a(r)$	eigenfunction with respect to the excess pore-air pressure for domain r;
$R_w(r)$	eigenfunction with respect to the excess pore-water pressure for domain r;
r	investigated radius;
r_e	radius of zone of influence;
r_p	pore radius;
r_w	radius of the drain well;
S	matric suction;
S^*	normalized settlement;

$\mathrm{S(t)}$	consolidation settlement;
$\mathrm{S_{max}}$	maximum settlement;
$\mathrm{S_r}$	degree of saturation;
$\mathrm{S_\infty}$	final ground surface settlement;
$\mathrm{s_f}$	final matric suction;
$\mathrm{s_0}$	initial matric suction;
T	time factor;
$\mathrm{T_a(t)}$	generalized Fourier coefficients varying with time with respect to the air phase;
$\mathrm{T_w(t)}$	generalized Fourier coefficients varying with time with respect to the water phase;
t	elapsed time;
U	degree of consolidation;
$\mathrm{\overline{U}}$	average degree of consolidation;
$\mathrm{U_r}$	degree of consolidation under axisymmetric conditions;
u	displacement in x-domain;
$\mathrm{u_a}$	excess pore-air pressure;
$\mathrm{u_{a,t}}$	first order of partial differential equation (PDE) of air with respect to time;
$\mathrm{u_{a,r}}$	first order of partial differential equation (PDE) of air with respect to radius;
$\mathrm{u_{a,rr}}$	second order of partial differential equation (PDE) of air with respect to radius;
$\mathrm{u_{a,xx}}$	second order of partial differential equation (PDE) of air with respect to width;
$\mathrm{u_{a,zz}}$	second order of partial differential equation (PDE) of air with respect to depth;
$\mathrm{u_{atm}}$	atmospheric pressure;
$\mathrm{u_a^0}$	maximum initial excess pore-air pressure;
$\mathrm{u_w}$	pore-water pressure;
$\mathrm{u_{w,t}}$	first order of partial differential equation (PDE) of water with respect to time;

$u_{w,r}$	first order of partial differential equation (PDE) of water with respect to radius;
$u_{w,rr}$	second order of partial differential equation (PDE) of water with respect to radius;
$u_{w,xx}$	second order of partial differential equation (PDE) of water with respect to width;
$u_{w,zz}$	second order of partial differential equation (PDE) of water with respect to depth;
u_w^0	maximum initial excess pore-water pressure;
V_0	initial volume of the soil element;
V_a	volume of air within soil element;
V_w	volume of water within soil element;
v	displacement in y-domain;
$v(r_p, s)$	predicted volume fraction of pores at suction s;
w	displacement in z-domain;
$X_a(x)$	eigenfunction with respect to pore-air pressure for domain x;
$X_w(x)$	eigenfunction with respect to pore-water pressure for domain x;
x	investigated width;
$Z_a(z)$	eigenfunction with respect to the excess pore-air pressure for domain z;
$Z_w(z)$	eigenfunction with respect to the excess pore-water pressure for domain z; and
z	investigated depth;

Greek letters

γ_w	water unit weight;
Δs	suction change;
$\Delta\overline{\sigma}$	change in net stress;
ε_v	total volumetric strain;
Θ	absolute temperature in Kelvin;
θ	angular coordinate;
θ°	temperature in degree Celsius;
λ_a^{ij}	separation constant with respect to the air phase;

λ_w^{ij}	separation constant with respect to the water phase;
μ^j	eigenvalue for vertical boundary condition (i.e., PTIB and PTPB);
ξ^i	eigenvalue for radial boundary condition;
σ_r	total stress in r-domain;
σ_x	total stress in x-direction;
σ_z	total stress in z-domain;
σ_θ	total stress in θ-domain; and
$\overline{\sigma}$	net stress.