Maximizing Expected Achievable Rates for Block-Fading Buffer-Aided Relay Channels

Journal:	*IEEE Transactions on Wireless Communications*
Manuscript ID	Paper-TW-Oct-15-1357
Manuscript Type:	Original Transactions Paper
Date Submitted by the Author:	09-Oct-2015
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Keyword:	

Maximizing Expected Achievable Rates for Block-Fading Buffer-Aided Relay Channels

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Abstract

In this paper, the long-term average achievable rate over block-fading buffer-aided relay channels is maximized by using a hybrid scheme that combines three essential transmission strategies, which are decode-and-forward, compress-and-forward and direct transmission. The proposed hybrid scheme is dynamically adapted based on the channel state information. The integration and optimization of these three strategies provide a more generic and fundamental solution and give better achievable rates than the known schemes in the literature. Despite the large number of optimization variables, the proposed hybrid scheme can be optimized using simple closed-form formulas that are easy to apply in practical relay systems. This includes adjusting the transmission rate and compression when compress-and-forward is the selected strategy based on the channel conditions. Furthermore, in this paper, the hybrid scheme is applied to three different models of the Gaussian block-fading buffer-aided relay channels, depending on whether the relay is half or full duplex and whether the source and the relay have orthogonal or non-orthogonal channel access. Several numerical examples are provided to demonstrate the achievable rate results and compare them to the upper-bounds of the ergodic capacity for each one of the three channel models under consideration.

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This paper was made possible by NPRP grant # 5-401-2-161 from the Qatar National Research Fund (a member of Qatar Foundation). Furthermore, KAUST funded the efforts of A. Zafar partially and the efforts of M.-S. Alouini. The statements made herein are solely the responsibility of the authors.

Keywords

Relay channel; block-fading; decode-and-forward; compress-and-forward; buffer-aided relaying

I. Introduction

The relay channel is a three-node network that consists of a source (S) – destination (D) pair that is aided by a third node; the relay (R). The relay channel was first introduced to the information theory literature by Van Der Meulen [1], and important capacity theorems were established for the physically degraded and reversely degraded discrete memoryless full-duplex relay channel by Cover and El-Gamal [2]. Due to the potential advantages of relaying in improving the transmission capacity and reliability of wireless systems, this topic has emerged as an important research area in the wireless communication field as well [3], [4]. In this context, the communication channel between a transmitter and a receiver is commonly modeled as additive white Gaussian noise (AWGN) channel, and the fading effects of the wireless channels can be modeled as block-fading (i.e. quasi-static) channels, or equivalently as parallel fixed-gain Gaussian channels, where each fixed-gain channel represents a fading state. Optimal resource (such as bandwidth sub-carriers and transmission rates) allocation based on the channel state information (CSI) can produce significant capacity (achievable rates) gains when we average the capacity over long-term usage of the fading channel. Therefore, combining the best achievable coding strategies that are used for constant (non-fading) channels and adapting them using dynamic resource allocation over the fading states of the channels is the core for approaching the ergodic capacity bounds. Consequently, this is really an important and fundamental research area. In this paper, we are interested in this specific topic and we apply it to block-fading relay channels. We distinguish between three channel models depending on whether the relay is half or full duplex and whether the source and the relay share the same bandwidth or transmit using orthogonal channels.

Achievable rates and capacity upper-bound results for half-duplex relays in fixed-gain Gaussian channels were provided in the literature assuming non-orthogonal channel access of the source and relay [5], and also assuming orthogonal channel access [6], [7]. More recent results were provided in [8]. We know from these references that, similar to the full-duplex case [2], [9], the best known upper bounds on the capacity are the max-flow min-cut bounds, and that there are three different coding strategies that maximize the achievable rates, which are decode-and-forward (DF), compress-and-forward (CF) and direct transmission (DT) from the source to the destination. These coding strategies were also named cooperation (for DF), observation (for CF) and facilitation (for DT) [2], [4]. None of these three strategies is globally dominant over the other two, but rather each one of them can achieve higher rates that the others in specific scenarios depending on the qualities of the $\mathsf{S}-\mathsf{R}$, $\mathsf{S}-\mathsf{D}$ and $\mathsf{R}-\mathsf{D}$ channels. Furthermore, the exact capacity of the Gaussian relay channel is not known in general except for a restricted range of the channel qualities and fixed channel allocations [6], [7].

Furthermore, there are some contributions in the literature that consider fading relay channels. For example, the quasi-static (block-fading) half-duplex relay channel was studied, and it was shown that dynamic adaptation of the transmission strategies using DF and DT is needed in order to maximize the expected achievable rates [10]. However, CF was not considered and channel allocation was fixed beforehand and not subject to optimization therein. It is obvious that making channel allocation dynamic and subject to optimization would add to the degrees of freedom in the system design and enable achieving higher rates. Optimal channel allocation for Gaussian (non-fading) orthogonal and non-orthogonal relay channels was considered in a number of papers, and the obtained results for the best achievable schemes were based on DF only [5], [6], [11].

One important observation when extending the best relaying strategies, such as DF, from the fixed-gain channel case into the block-fading channel case is that the relay does not necessarily have to forward a source message that is received in a given channel-block to the destination in the same (or in the next) channel block if there are no delay constraints and the objective is to maximize the expected achievable rate. Having the ability to adapt the relay transmission based on the channel conditions gives more degrees-of-freedom in the system design and enables achieving higher expected rates than in the cases when a given source message is restricted to be completely delivered to the destination in the same channel block. Of course, the relay (and the destination) should have buffering capabilities in order to enable this dynamic relaying scheme. This concept was not taken into consideration in the aforementioned papers, and it was introduced in the literature recently under the name of “buffer-aided relaying”, and it was studied for the cases when there is no direct link from the source to the destination [12], [13], and also when the direct link is available and utilized [14]. Of course, the latter case is more general and more important, and we are interested in it in this work.

Having gone through many of the most important works in the literature that considered block-fading relay channels, we still believe that there is still room for improvement since they all focus on dynamic adaption of decode-and-forward relaying strategies and they do not consider compress-and-forward as well, although there are certain scenarios over which CF can be better than DF as we know from the case of fixed-gain channels. So, in this work, we consider a buffer-aided hybrid scheme that combines DF, CF and DT and switches among them dynamically based on the channel conditions, and we consider optimizing the resource allocation for this hybrid scheme to maximize the long-term average achievable rates. We believe that this is an important contribution to the literature since it is more generic than the known schemes and, hence, it can achieve higher rates when optimized properly. To the best of our knowledge, this was not discussed before in the literature. The solution of our problem involves the optimization of the transmission rate and compression when CF is selected. In the literature, optimizing CF was done in a different context than our work [15]. Furthermore, we characterize upper bounds on the ergodic capacity of the block-fading relay channels, and provide several numerical examples to compare the best achievable scheme to the upper-bound. One of the most favorable aspects of our work is that we show that optimal resource allocation is based on simple closed-form formulas that can be applied in practical relay-aided communication networks. Notice that in our work we assume that the source and the relay nodes are constrained by maximum power (per bandwidth sub-carrier) constraints rather than average power constraints. Therefore, power is assumed to be fixed beforehand at a given value. Such an assumption is favorable for practical implementations. Furthermore, as known in the literature, the prospected gains of adaptive power allocation is usually minimal, e.g. [16].

Before we end this section, we want to mention that the concept of "buffer-aided relaying" was also considered for dual-hop broadcast channels and it was called "joint user-and-hop scheduling" since the buffering capabilities are actually needed to enable dynamic and flexible scheduling (i.e. channel allocation) among multiple users (destination nodes) and the relay [16]. Also, it was applied to other channel models that involve relaying such as the bi-directional relay channel [17], [18], the shared relay channel [19] and overlay cognitive radio networks [20]. Moreover, the buffers can improve the performance of relay selection as discussed in [21]–[23]. The list of references on buffer-aided relaying provided here is not exhaustive.

The remainder of this paper is organized as follows. In Section II, we describe the three models for the Gaussian block-fading relay channel that are considered in this work. After that, we define the main optimization variables for the considered hybrid (DF, CF, DT) scheme, list their relevant constraints, and formulate the main optimization problem in Section III. Then, in Section IV, we go through the solution steps of the main optimization problem and list some of the important characteristics of the optimal solution. Next, we discuss in Section V the upper bounds for each one of the three channel models that are considered in this work. After that, we demonstrate our findings via several numerical results and give comments on these results in Section VI. Finally, we summarize the main conclusions in Section VII.

II. Channel Models and Comments on the Motivation

A. Three Models for the Gaussian Block-Fading Relay Channel

We consider a three-node network that consists of a source (S) that wants to send information to a destination (D) with the assistance of a relay (R). We assume a Gaussian block-fading model for the channels between the nodes. We also assume that all channel blocks have the same duration (T in seconds) and bandwidth (W in Hz) and that they are large enough to achieve the instantaneous capacity[1]. Furthermore, we assume that the source and relay transmit

[1]As well-known from the information theory, achieving the capacity of AWGN channels requires using very large codes with infinite code length. Otherwise, error-free transmission cannot be guaranteed. However, with sufficiently long codewords, we can transmit at channel capacity with very small and negligible probability of error.

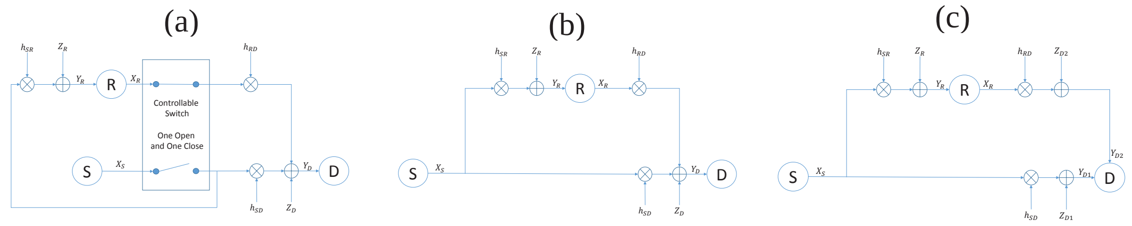

Fig. 1: Channel Models; (a) Half-duplex orthogonal access, (b) Full-duplex non-orthogonal access, (c) Orthogonal access.

using a constant (maximum) power per unit bandwidth (in Jouls/sec/Hz). We also assume that all nodes are equipped with a single antenna.

We investigate three different models for the relay channel that are shown in Fig 1. We call them; (a) half-duplex – orthogonal access, (b) full-duplex – non-orthogonal access, and (c) orthogonal access. A half-duplex relay is a relay that cannot transmit and receive simultaneously in the same channel block, while a full-duplex channel can do that. Orthogonal access means that the source and relay do not transmit simultaneously on the same bandwidth, while non-orthogonal access means that they do so, and hence they share the same bandwidth to transmit to the destination forming a multiple-access channel. In a channel block k, the input-output relationships for channel model (a) in Fig 1 are given by

$$Y_{\mathsf{R}}[k] = \delta_{\mathsf{S}}[k]h_{\mathsf{SR}}[k]X_{\mathsf{S}}[k] + Z_{\mathsf{R}}[k], \tag{1}$$

$$Y_{\mathsf{D}}[k] = \delta_{\mathsf{S}}[k]h_{\mathsf{SD}}[k]X_{\mathsf{S}}[k] + \delta_{\mathsf{R}}[k]h_{\mathsf{RD}}[k]X_{\mathsf{R}}[k] + Z_{\mathsf{D}}[k], \tag{2}$$

where $X_{\mathsf{S}}[k]$ and $X_{\mathsf{R}}[k]$ are the transmitted (complex field) source signal and relay signal, respectively, in channel block k. They have power density $\bar{P}_{\mathsf{S}}$ and $\bar{P}_{\mathsf{R}}$, respectively. Similarly, $Y_{\mathsf{R}}[k]$ and $Y_{\mathsf{D}}[k]$ are the received signals at the relay and destination, respectively, and $Z_{\mathsf{R}}[k]$ and $Z_{\mathsf{D}}[k]$ are the added Gaussian noise at these two nodes, which are mutually independent and have circularly symmetric, complex Gaussian distribution with unit variance. Furthermore, $h_{\mathsf{SD}}[k]$, $h_{\mathsf{SR}}[k]$ and $h_{\mathsf{RD}}[k]$ are the channel complex coefficients, which stay constant during one channel block k and change randomly afterwards, of the source-destination, source-relay, relay-destination links respectively. The corresponding signal-to-noise-ratio (SNR) of these channels, in a given channel block k, are given by $\gamma_{\mathsf{SR}}[k] = |h_{\mathsf{SR}}[k]|^2 \bar{P}_{\mathsf{S}}$, $\gamma_{\mathsf{SD}}[k] = |h_{\mathsf{SD}}[k]|^2 \bar{P}_{\mathsf{S}}$ and $\gamma_{\mathsf{RD}}[k] = |h_{\mathsf{RD}}[k]|^2 \bar{P}_{\mathsf{R}}$, respectively. The probability density function (PDF) of the channel gain ($|h|^2$) over each one of the three links is a continuous[2] function over $[0, \infty)$. Over each link, the receiver knows the channel complex coefficient $h[k]$ perfectly, but the corresponding *transmitter*[3] *knows only the channel gain* $|h|^2$.

The controllable switch in channel model (a) is presented by two signals $\delta_{\mathsf{S}}[k]$ and $\delta_{\mathsf{R}}[k]$, which can have either zero or one value, and the sum of the two signals equals one all the time, $\delta_{\mathsf{S}}[k] + \delta_{\mathsf{R}}[k] = 1$. The input-output relationships for channel model (b) in Fig 1 are given by (1) and (2) with the exception that $\delta_{\mathsf{S}}[k] = \delta_{\mathsf{R}}[k] = 1$. Finally, the input-output relationships for channel model (c) in Fig 1 are given by (1), with $\delta_{\mathsf{S}}[k] = 1$, and the following two equations

$$Y_{\mathsf{D}_1}[k] = h_{\mathsf{SD}}[k] X_{\mathsf{S}}[k] + Z_{\mathsf{D}_1}[k], \tag{3a}$$

$$Y_{\mathsf{D}_2}[k] = h_{\mathsf{RD}}[k] X_{\mathsf{R}}[k] + Z_{\mathsf{D}_2}[k], \tag{3b}$$

where $Y_{\mathsf{D}_1}[k]$ and $Y_{\mathsf{D}_2}[k]$ are the received signals from the source and the relay, respectively, over orthogonal channels. Both $Z_{\mathsf{D}_1}[k]$ and $Z_{\mathsf{D}_2}[k]$ are added Gaussian noise with unit variance. We assume that the two orthogonal channels have the same size (TW).

B. Instantaneous Channel Capacities

The instantaneous (i.e. in a given channel block k) channel capacities are denoted be $C_{\mathsf{SD}}[k]$, $C_{\mathsf{SR}}[k]$ and $C_{\mathsf{RD}}[k]$ for the source-destination, source-relay and relay-destination links, respectively. For channel models (a) and (c) in Fig 1, where we have orthogonal access, the channel

[2] The continuity of the PDF functions means that the probability that the channel gain of a particular link equals a certain value is zero, i.e. $\Pr(|h_x[k]| = c) = 0$, where $x \in \{\mathsf{SD}, \mathsf{SR}, \mathsf{RD}\}$ and $c > 0$ is any arbitrary constant. This assumption will be used in the solution of the optimization problem.

[3] This assumption is stemmed from practical system design considerations. As a consequence of it, beamforming of the source and relay signals towards the destination is not feasible, and, hence, β in formulas (5) and (7) in [5] equals zero under our assumptions.

capacities (per unit bandwidth) follow the well-known capacity of AWGN channels:

$$C_x[k] = \log\left(1 + \gamma_x[k]\right), \qquad \forall x \in \{\mathsf{SD}, \mathsf{SR}, \mathsf{RD}\} \tag{4}$$

For channel model (b) in Fig 1, where we have non-orthogonal access, the source-relay link will still be an AWGN channel and its instantaneous capacity follows (4). On the other hand, the source-destination and relay-destination links form a multiple-access channel (MAC), and hence we have a two-dimensional capacity region, where the instantaneous capacity of the pair $(C_{\mathsf{SD}}[k], C_{\mathsf{RD}}[k])$ can have infinite number of possibilities that satisfy

$$C_{\mathsf{SD}}[k] + C_{\mathsf{RD}}[k] = \log\left(1 + \gamma_{\mathsf{SD}}[k] + \gamma_{\mathsf{RD}}[k]\right), \tag{5a}$$

$$C_{\mathsf{SD}}[k] \leq \log\left(1 + \gamma_{\mathsf{SD}}[k]\right), \tag{5b}$$

$$C_{\mathsf{RD}}[k] \leq \log\left(1 + \gamma_{\mathsf{RD}}[k]\right) \tag{5c}$$

For notation, $C_{\mathsf{MAC}}[k]$ denotes the sum capacity of the MAC channel in (5a). Furthermore, $\gamma'_{\mathsf{SD}}[k]$ and $\gamma'_{\mathsf{RD}}[k]$ denote the signal-to-noise-and-interference-ratio (SINR) of the SD and RD channels, respectively. Thus, $\gamma'_{\mathsf{SD}}[k] = \exp\left(C_{\mathsf{SD}}[k]\right) - 1$ and $\gamma'_{\mathsf{RD}}[k] = \exp\left(C_{\mathsf{RD}}[k]\right) - 1$, where the instantaneous capacities are measured in nats/sec/Hz. It is straightforward to show that $\frac{\gamma_{\mathsf{SD}}[k]}{1+\gamma_{\mathsf{RD}}[k]} \leq \gamma'_{\mathsf{SD}}[k] \leq \gamma_{\mathsf{SD}}[k]$ and $\frac{\gamma_{\mathsf{RD}}[k]}{1+\gamma_{\mathsf{SD}}[k]} \leq \gamma'_{\mathsf{RD}}[k] \leq \gamma_{\mathsf{RD}}[k]$ depending on the specific operating point on the boundary of the MAC channel.

III. Communication System Description and Optimization

We investigate a hybrid communication scheme that combines three different strategies; direct transmission (DT), decode-and-forward (DF) and compress-and-forward (CF). These schemes are adapted dynamically and optimally based on the channel conditions in order to maximize the expected achievable rate.

A. *Optimization Variables for Adaptive System*

We would like first to emphasize that the main objective of the investigated communication scheme is to maximize the long-term average (ergodic) achievable rate of the relay channel assuming that the source node does always have information bits to transmit and that there are no delay constraints on the communication between the source and destination. This objective is common in the literature and has been adopted for different channels, e.g. [24]–[27]. Furthermore, we assume that the channel gain information over the three links is perfectly known and exploited in order to maximize the long-term average achievable rate. Consequently, the used communication scheme is adaptive based on the instantaneous (i.e. in a given channel block) condition of the block-fading channels. The adaptivity of the communication scheme includes

- The dynamic selection of the proper coding strategy (DF, CF or DT). This may include orthogonal time-sharing of different coding strategies in the same channel block. The time sharing ratios are subject to optimization. For notation, $\theta_{\mathsf{DT}}[k]$, $\theta_{\mathsf{DF}}[k]$ and $\theta_{\mathsf{CF}}[k]$ denote the time sharing ratio in a given channel block k for the DT, DF and CF transmission strategies, respectively. They refer to the source transmission phase of all of these strategies. Therefore, in channel models (b) and (c) in Fig. 1, we have (for every channel block k)

$$\theta_{\mathsf{DT}}[k] + \theta_{\mathsf{DF}}[k] + \theta_{\mathsf{CF}}[k] \leq 1 \tag{6}$$

- The adjustment of the transmission rate based on the channel condition. For notation, $R_{\mathsf{DT}}[k]$, $R_{\mathsf{DF}}[k]$ and $R_{\mathsf{CF}}[k]$ denote the normalized[4] data rate of the source codeword in a given channel block k for the DT, DF and CF transmission strategies, respectively. Furthermore, $R^*_{\mathsf{DF}}[k]$ and $R^*_{\mathsf{CF}}[k]$ denote the normalized information rates that are generated and stored by the relay at the end of channel block k, which corresponds to the DF and CF transmission strategies, respectively. Moreover, $R_{\mathsf{RD}}[k]$ denotes the normalized data rate for the relay transmission in channel block k including when it forwards both decoded or compressed messages. The specific ordering of what the relay forwards (among decoded and compressed messages) does not affect the expected achievable rate, and hence it is not subject to optimization in our problem formulation. Notice that we assume that the nodes operate at their maximum power (per channel sub-carrier) and hence power allocation is

[4]The data rates are normalized to the size of one channel block TW.

not subject to optimization.

- The orthogonal multiplexing of the source and the relay in case of channel model (a) in Fig. 1. In this case, $\theta_{\mathsf{RD}}[k]$ denotes the time sharing ratio for the relay transmission, and the constraint in (6) should be replaced by the following one,

$$\theta_{\mathsf{DT}}[k] + \theta_{\mathsf{DF}}[k] + \theta_{\mathsf{CF}}[k] + \theta_{\mathsf{RD}}[k] \leq 1 \tag{7}$$

- The selection of the operating point on the MAC channel in model (b) in Fig. 1. For notation, we define $\omega[k]$ as an optimization variable to select the specific operating point in this case. The definition of $\omega[k]$ is given in Appendix A.

B. System Requirements

In addition to the availability of the channel state information, another important requirement to support the adaptivity of the system is having unlimited buffering capability at the relay and the destination. This is because when the source transmits a new codeword and the relay decodes or compresses it, it does not forward it directly to the destination in the same or the following channel block, but it rather stores it and it adjusts its transmission rate based on the relay-destination channel quality. This means that the relay might send the information bits that corresponds to one codeword of the source over multiple channel blocks (if the transmission rate over the relay-destination link is low) or combine the information bits that corresponds to more than one codeword of the source (if the transmission rate over the relay-destination link is high). This was explained properly in [14].

C. Transmission Strategies and Rate Constraints

When the source transmits a new codeword, it decides (subject to optimization) if the codeword will be used for DT, DF or CF, and it adjusts the data rate of the codeword accordingly.

1) Direct Transmission: In this case, the relay does not need to do anything. The data rate of the source codeword should be bounded by the direct channel capacity.

$$R_{\mathsf{DT}}[k] \leq \theta_{\mathsf{DT}}[k] C_{\mathsf{SD}}[k] \tag{8}$$

2) Decode-and-Forward: In this case, the relay fully decodes the source message and it generates and stores an amount of information that would be sufficient for the destination to decode the source message reliably (given that the destination utilizes both the source and relay signals to decode the source codeword). For example, the relay can store a bin index (in the sense of Slepian-Wolf coding [28]) of the source message that indicates the partition at which the source codeword lies. The data rate of the source codeword must be bounded by the capacity of the source-relay link in order for the relay to be able to decode the source message.

$$R_{\mathsf{DF}}[k] \leq \theta_{\mathsf{DF}}[k] C_{\mathsf{SR}}[k] \tag{9}$$

Furthermore, the corresponding amount of information to be generated and stored by the relay (normalized to the size of one channel block TW) is given by:

$$R^*_{\mathsf{DF}}[k] \geq \Big(R_{\mathsf{DF}}[k] - \theta_{\mathsf{DF}}[k] C_{\mathsf{SD}}[k] \Big)^+, \tag{10}$$

where $(x)^+ = \max(x, 0)$. Notice that if $R_{\mathsf{DF}}[k] \leq \theta_{\mathsf{DF}}[k] C_{\mathsf{SD}}[k]$, then the destination can decode the source message via direct transmission and the relay does not need to forward anything.

3) Compress-and-Forward: The most important element of our work that makes it distinct from other works in the literature is the incorporation of compress-and-forward relaying. In CF, the relay encodes and stores a compressed (quantized) version of the received signal using, e.g. Wyner-Ziv lossy source coding [29]. The data rate of the source codeword must be bounded by the capacity of the single-input multiple-output (SIMO) channel assuming that the relay and destination are two antennas of the same receiver. Exceeding this data rate will not be reliably decoded by any communication scheme.

$$R_{\mathsf{CF}}[k] < \theta_{\mathsf{CF}}[k] \log\left(1 + \gamma_{\mathsf{SR}}[k] + \gamma_{\mathsf{SD}}[k]\right) \tag{11}$$

Notice that if $R_{\mathsf{CF}}[k] \leq \theta_{\mathsf{CF}}[k] C_{\mathsf{SD}}[k]$, then the destination can decode the source message via direct transmission and the relay does not need to forward anything. For notation, $\gamma_{\mathsf{CF}}[k]$ denotes the SNR required to decode the source's message, which is given by $\gamma_{\mathsf{CF}}[k] = \exp\left(\frac{R_{\mathsf{CF}}[k]}{\theta_{\mathsf{CF}}[k]}\right) - 1$, where the data rate is measured in nats/sec/Hz.

Theorem 1 (Rate of compressed signal at the relay)**.** *Given that* $\gamma'_{\mathsf{SD}}[k] < \gamma_{\mathsf{CF}}[k] < \gamma_{\mathsf{SR}}[k] + \gamma'_{\mathsf{SD}}[k]$, *the data rate of the encoded compressed signal by the relay must satisfy*

$$R^{*}_{\mathsf{CF}}[k] \geq \theta_{\mathsf{CF}}[k] \log \left(1 + \frac{\left(\gamma_{\mathsf{CF}}[k] - \gamma'_{\mathsf{SD}}[k]\right)\left(1 + \gamma'_{\mathsf{SD}}[k] + \gamma_{\mathsf{SR}}[k]\right)}{\left(\gamma'_{\mathsf{SD}}[k] + \gamma_{\mathsf{SR}}[k] - \gamma_{\mathsf{CF}}[k]\right)\left(1 + \gamma'_{\mathsf{SD}}[k]\right)} \right). \tag{12}$$

in order for the destination to be able to reliably decode the source's message.

The proof is provided in Appendix B. Remember that $\gamma'_{\mathsf{SD}}[k] = \gamma_{\mathsf{SD}}[k]$ for channel models (a) and (c), and depends on the operating point of the MAC channel in model (b) (see Fig. 1).

4) Relay Transmission: When the relay transmits, it adjusts its rate based on the channel condition of the RD link. However, it cannot transmit more than the total (whether it is related to decoded or compressed source message) available amount of information bits in its buffers, denoted by $Q[k]$, which is normalized by the size of one channel block TW,

$$R_{\mathsf{RD}}[k] \leq \min\left(\theta_{\mathsf{RD}}[k] C_{\mathsf{RD}}[k], Q[k]\right) \tag{13}$$

Notice that in channel models (b) and (c) in Fig. 1, $\theta_{\mathsf{RD}}[k] = 1$ over all channel blocks.

D. Optimization Problem Formulation

We write the main optimization problem in a generic form that is applied to the three channel models in Fig. 1. We want to maximize the average achievable rate of the relay channel by using an adaptive scheme that combines DT, DF and CF. Therefore, the total rate is the sum of the rates achieved by these three transmission strategies. The relay should transmit sufficient amount of rate to enable the destination to decode the source messages reliably.

$$\max_{\zeta[k]\ \forall k} \quad \bar{R}_{\mathsf{DT}} + \bar{R}_{\mathsf{DF}} + \bar{R}_{\mathsf{CF}} \tag{14a}$$

$$\text{subject to} \quad \bar{R}_{\mathsf{RD}} \geq \bar{R}^{*}_{\mathsf{DF}} + \bar{R}^{*}_{\mathsf{CF}}, \tag{14b}$$

where the long-term rate expressions are given by

$$\bar{X} = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} X[k], \qquad \forall X \in \{R_{\mathsf{DT}}, R_{\mathsf{DF}}, R_{\mathsf{CF}}, R_{\mathsf{RD}}, R^*_{\mathsf{DF}}, R^*_{\mathsf{CF}}\}, \tag{15}$$

and $\zeta[k]$ is the optimization vector that depends on the specific channel model,

$$\zeta_{(a)}[k] = \{\theta_{\mathsf{DT}}[k], R_{\mathsf{DT}}[k], \theta_{\mathsf{DF}}[k], R_{\mathsf{DF}}[k], R^*_{\mathsf{DF}}[k], \theta_{\mathsf{CF}}[k], R_{\mathsf{CF}}[k], R^*_{\mathsf{CF}}[k], \theta_{\mathsf{RD}}[k], R_{\mathsf{RD}}[k]\} \tag{16a}$$

$$\zeta_{(b)}[k] = \{\theta_{\mathsf{DT}}[k], R_{\mathsf{DT}}[k], \theta_{\mathsf{DF}}[k], R_{\mathsf{DF}}[k], R^*_{\mathsf{DF}}[k], \theta_{\mathsf{CF}}[k], R_{\mathsf{CF}}[k], R^*_{\mathsf{CF}}[k], R_{\mathsf{RD}}[k], \omega[k]\} \tag{16b}$$

$$\zeta_{(c)}[k] = \{\theta_{\mathsf{DT}}[k], R_{\mathsf{DT}}[k], \theta_{\mathsf{DF}}[k], R_{\mathsf{DF}}[k], R^*_{\mathsf{DF}}[k], \theta_{\mathsf{CF}}[k], R_{\mathsf{CF}}[k], R^*_{\mathsf{CF}}[k], R_{\mathsf{RD}}[k]\} \tag{16c}$$

Notice that the optimization problem in (14) involves all constraints on achievable rates, i.e. (8), (9), (10), (11), (12), (13), and channel access-ratios, i.e. (6) (for channel models (b) and (c)) or (7) (for channel model (a)).

IV. Optimal Solution

A. Solution Steps of The Optimization Problem (14)

We go through the main steps to be able to obtain the solution of (14).

1) Preliminaries:

Lemma 1 (Adjust rate at capacity bounds)**.** *To achieve the optimal solution of* (14)*,* $R_{\mathsf{DT}}[k]$ *and* $R_{\mathsf{RD}}[k]$ *should be adjusted to be at the maximum bounds (i.e. capacity), and* $R^*_{\mathsf{DF}}[k]$ *and* $R^*_{\mathsf{CF}}[k]$ *should be adjusted to be on the minimum bounds. Thus,* (8)*,* (13)*,* (10) *and* (12) *should be satisfied at equality.*

The proof is straightforward and intuitive since achieving (8) and (13) with strict inequality will be a waste of the channel resources with no prospected benefits. Similarly, achieving (10) and (12) with strict inequality will result in inefficient use of the relay resources by letting the relay forward more than what is actually needed by the destination to be able to decode the source messages reliably. Therefore, $R_{\mathsf{DT}}[k]$, $R_{\mathsf{RD}}[k]$, $R^*_{\mathsf{DF}}[k]$ and $R^*_{\mathsf{CF}}[k]$ can be removed from the set of optimization variables (for all three channel models) in (16) since they can be allocated directly once the other optimization variables (such as the access ratios) are obtained.

Lemma 2 (Use all channel resources)**.** *To achieve the optimal solution of* (14)*, all channel resources should be used. Thus, the sum of channel access-ratios constraint, i.e.* (6) *(for channel models (b) and (c)) or* (7) *(for channel model (a)), should be satisfied at equality.*

The proof is straightforward and intuitive. Let's assume that the optimal solution involves that the sum of channel access ratios is strictly less than one in a given channel block k, then we can increase the value of $\theta_{\mathsf{DT}}[k]$ such that the constraint is achieved at equality. This will increase the value of $R_{\mathsf{DT}}[k]$ without changing the rates of DF and CF. Thus, we increase the total rate, and this contradicts the assumption that the optimal solution is at strict inequality.

Lemma 3 (Queue at edge of non-absorption)**.** *A necessary condition for the optimal solution of* (14) *is that the the queue in the buffer of the relay is at the edge of non-absorption. Consequently, for $K \to \infty$, the impact of the event $Q[k] < \theta_{\mathsf{RD}}[k]C_{\mathsf{RD}}[k]$, $k = 1, \cdots, K$ is negligible. Therefore, the optimal solution will have*

$$\bar{R}_{\mathsf{RD}} = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \theta_{\mathsf{RD}}[k]C_{\mathsf{RD}}[k] \tag{17}$$

and the constraint (14b) *will be satisfied at equality.*

The proof follows the same steps that are known in the literature, e.g. [12, Theorem 1 and Theorem 2].

2) Lagrangian Dual Problem: The Lagrangian dual problem (e.g. [30]) of (14) is given as

$$\min_{\lambda} \quad \mathcal{L}(\lambda), \qquad \text{where } \lambda \geq 0, \text{ and} \tag{18a}$$

$$\mathcal{L}(\lambda) = \max_{\zeta[k]\ \forall k} \quad \bar{R}_{\mathsf{DT}} + \bar{R}_{\mathsf{DF}} + \bar{R}_{\mathsf{CF}} - \lambda\left(\bar{R}^*_{\mathsf{DF}} + \bar{R}^*_{\mathsf{CF}} - \bar{R}_{\mathsf{RD}}\right) \tag{18b}$$

Notice that the optimization variables ($\zeta[k]$, $\forall k$) are obtained by solving the Lagrangian maximization problem (18b) for a given value of λ. The latter should be adjusted globally according to (18a). If we have strong duality between (14) and (18), then the optimal λ will satisfy the constraint (14b) at equality. Therefore, the optimal value of λ depends on the channel statistics of the three links SD, SR and RD, and it is independent of the instantaneous channel gains in a given channel block k.

A direct consequence of Lemma 3, in particular (17), is that the achievable rates in a given channel block k are only dependent on their respective optimization variables $\zeta[k]$, and independent of $\zeta[l]$, where $l \neq k$. Therefore, (18b) can be transformed into a number K of independent optimization problems that are solved independently.

$$\max_{\zeta[k]} \quad R_{\mathsf{DT}}[k] + R_{\mathsf{DF}}[k] + R_{\mathsf{CF}}[k] - \lambda \left(R^*_{\mathsf{DF}}[k] + R^*_{\mathsf{CF}}[k] - R_{\mathsf{RD}}[k] \right) \tag{19}$$

In the next step, we make a change of variables step for the optimization vector ($\zeta[k]$). Notice that $R_{\mathsf{DF}}[k]$ and $R_{\mathsf{CF}}[k]$ are dependent on other optimization variables, which are $\theta_{\mathsf{DF}}[k]$ and $\theta_{\mathsf{CF}}[k]$, respectively. Therefore, we make the optimization variables independent by using

$$C_x[k] = \frac{R_x[k]}{\theta_x[k]}, \quad C^*_x[k] = \frac{R^*_x[k]}{\theta_x[k]}, \quad \forall x \in \{\mathsf{DF}, \mathsf{CF}\} \tag{20}$$

Therefore, based on Lemma 1 and the change of variables defined in (20), we can replace $\zeta[k]$ by a different optimization vector $\mu[k]$ that is given by

$$\mu_{(a)}[k] = \{\theta_{\mathsf{DT}}[k], \theta_{\mathsf{DF}}[k], C_{\mathsf{DF}}[k], \theta_{\mathsf{CF}}[k], C_{\mathsf{CF}}[k], \theta_{\mathsf{RD}}[k]\} \tag{21a}$$

$$\mu_{(b)}[k] = \{\theta_{\mathsf{DT}}[k], \theta_{\mathsf{DF}}[k], C_{\mathsf{DF}}[k], \theta_{\mathsf{CF}}[k], C_{\mathsf{CF}}[k], \omega[k]\} \tag{21b}$$

$$\mu_{(c)}[k] = \{\theta_{\mathsf{DT}}[k], \theta_{\mathsf{DF}}[k], C_{\mathsf{DF}}[k], \theta_{\mathsf{CF}}[k], C_{\mathsf{CF}}[k]\} \tag{21c}$$

For notation, $\psi[k]$ denotes a subset of $\mu[k]$ that includes all elements except $C_{\mathsf{DF}}[k]$ and $C_{\mathsf{CF}}[k]$.

$$\psi[k] = \mu[k] \backslash \{C_{\mathsf{DF}}[k], C_{\mathsf{CF}}[k]\} \tag{22}$$

Based on the new defined notations, we can show that (19) can be written as

$$\max_{\psi[k]} \quad \theta_{\mathsf{DT}}[k]\phi_{\mathsf{DT}}[k] + \theta_{\mathsf{DF}}[k]\phi_{\mathsf{DF}}[k] + \theta_{\mathsf{CF}}[k]\phi_{\mathsf{CF}}[k] + \theta_{\mathsf{RD}}[k]\phi_{\mathsf{RD}}[k] \tag{23}$$

where

$$\phi_{\mathsf{DT}}[k] = C_{\mathsf{SD}}[k], \tag{24a}$$

$$\phi_{\mathsf{DF}}[k] = \max_{C_{\mathsf{DF}}[k]} \quad \left(C_{\mathsf{DF}}[k] - \lambda C^*_{\mathsf{DF}}[k]\right), \tag{24b}$$

$$\phi_{\mathsf{CF}}[k] = \max_{C_{\mathsf{CF}}[k]} \quad \left(C_{\mathsf{CF}}[k] - \lambda C^*_{\mathsf{CF}}[k]\right), \tag{24c}$$

$$\phi_{\mathsf{RD}}[k] = \lambda C_{\mathsf{RD}}[k] \tag{24d}$$

Consequently, the optimization of $C_{\mathsf{DF}}[k]$ and $C_{\mathsf{CF}}[k]$ are modular problems that can be solved independently regardless of the optimal solution of $\psi[k]$. They depend on the value of λ, which is a global variable that is not a function of the instantaneous channel capacities in a given channel block k. This is valid for all three channel models under consideration.

3) Decode-and-Forward: The optimal value of $C_{\mathsf{DF}}[k]$ can be obtained by solving (24b) given (9) and (10), where (10) is satisfied at equality as shown in Lemma 1. The solution yields three possibilities depending on the values of $C_{\mathsf{SR}}[k]$, $C_{\mathsf{SD}}[k]$ and λ; (i) $C_{\mathsf{DF}}[k] = C_{\mathsf{SR}}[k]$ if $\lambda < 1$ or $C_{\mathsf{SR}}[k] < C_{\mathsf{SD}}[k]$, (ii) $C_{\mathsf{DF}}[k] = C_{\mathsf{SD}}[k]$ if $\lambda > 1$ and $C_{\mathsf{SR}}[k] \geq C_{\mathsf{SD}}[k]$, (iii) the optimal solution is not unique, $C_{\mathsf{DF}}[k] \in \left[C_{\mathsf{SD}}[k], C_{\mathsf{SR}}[k]\right]$, if $\lambda = 1$ and $C_{\mathsf{SR}}[k] \geq C_{\mathsf{SD}}[k]$.

Lemma 4 (When is DF useless)**.** *In all channel blocks that have* $C_{\mathsf{SR}}[k] < C_{\mathsf{SD}}[k]$*, using* DF *is useless (for our objective of maximizing expected achievable rate), and it is optimal to make* $\theta_{\mathsf{DF}}[k] = 0$ *in this case.*

The proof is straightforward since DT can achieve higher rates in this case.

Lemma 5 (Optimal C_{DF} allocation)**.** *Given that* $\lambda < 1$*, then* $C_{\mathsf{DF}}[k]$ *should be adjusted at the capacity of the source-relay channel. This means that* (9) *should be satisfied at equality.*

$$C_{\mathsf{DF}}[k] = C_{\mathsf{SR}}[k] \tag{25}$$

A direct consequence of Lemma 5 is that in all channel blocks k that have $C_{\mathsf{SR}}[k] > C_{\mathsf{SD}}[k]$, we will have

$$\phi_{\mathsf{DF}}[k] = (1-\lambda)C_{\mathsf{SR}}[k] + \lambda C_{\mathsf{SD}}[k] \tag{26}$$

4) Compress-and-Forward: The optimal value of $C_{\mathsf{CF}}[k]$ can be obtained by solving (24c) given (11) and (12), where (12) is satisfied at equality as shown in Lemma 1.

Theorem 2 (Optimal C_{CF} allocation)**.** *Given that* $\lambda < 1$, $C_{\mathsf{CF}}[k]$ *should be adjusted according to*

$$C_{\mathsf{CF}}[k] = \max\Big(\log(1-\lambda) + \log\left(1+\gamma'_{\mathsf{SD}}[k] + \gamma_{\mathsf{SR}}[k]\right), C_{\mathsf{SD}}[k]\Big) \tag{27}$$

A sketch of the steps to obtain (27) is shown in Appendix C. Notice that, when we have $C_{\mathsf{CF}}[k] = C_{\mathsf{SD}}[k]$ in (27), then CF is useless and it is optimal to make $\theta_{\mathsf{CF}}[k] = 0$ in this case. This will always be the case when $\lambda \geq 1$, and it depends on the channel conditions when $\lambda < 1$. Furthermore, unlike $C_{\mathsf{DF}}[k]$ in Lemma 5, the optimal allocation of $C_{\mathsf{CF}}[k]$ is a function of λ. Thus, it is dependent on both the channel statistics (which affects the optimal value of λ) and the instantaneous channel conditions.

Based on (27), we can equivalently write

$$\gamma_{\mathsf{CF}}[k] = \max\Big((1-\lambda)(1+\gamma'_{\mathsf{SD}}[k] + \gamma_{\mathsf{SR}}[k]) - 1, \gamma'_{\mathsf{SD}}[k]\Big) \tag{28}$$

A direct consequence of Theorem 2 is that in all channel blocks k that have $C_{\mathsf{CF}}[k] > C_{\mathsf{SD}}[k]$, where $C_{\mathsf{CF}}[k]$ is obtained using (27), we will have

$$\phi_{\mathsf{CF}}[k] = \log\left(1+\gamma_{\mathsf{SR}}[k] + \gamma'_{\mathsf{SD}}[k]\right) + \lambda\log\left(\frac{1+\gamma'_{\mathsf{SD}}[k]}{\gamma_{\mathsf{SR}}[k]}\right) + \lambda\log(\lambda) + (1-\lambda)\log(1-\lambda) \tag{29}$$

5) Operating Point on MAC Channel of Model (b): After characterizing the optimal allocation of $C_{\mathsf{DF}}[k]$ and $C_{\mathsf{CF}}[k]$, we go back to (23) to find the optimal $\psi[k]$. The solution depends on the specific channel model. We start by considering $\omega[k]$, which is specific to channel model (b).

Theorem 3 (Relay message decoded first)**.** *In channel model (b), where the source and relay transmit non-orthogonally to the destination, it is optimal to let the destination decode the relay's* *message first treating the source's message as noise, and then to process the source's message after removing the decoded relay's message. Thus, it is optimal to make* $\omega[k] = 0$ *regardless of the channel conditions or the value of* λ*.*

The proof is provided in Appendix D.

Based on Theorem 3, we obtain that in channel model (b), we always have $\gamma'_{\mathsf{SD}}[k] = \gamma_{\mathsf{SD}}[k]$, and $\gamma'_{\mathsf{RD}}[k] = \frac{\gamma_{\mathsf{RD}}[k]}{1+\gamma_{\mathsf{SD}}[k]}$.

6) Selection of Transmission Strategy: The next step is to find the optimal access ratios for each transmission strategy in a given channel block.

Theorem 4 (Selecting transmission strategy)**.** *For fading channels with continuous probability distribution, and given that* $\lambda < 1$*, the optimal solution of* (18b) *has only one transmission strategy (*DF*,* CF *or* DT*) selected per channel block* k*. Additionally, in channel model (a), either the source or the relay transmits and not both of them. The transmission strategy is selected according to*

$$\xi[k] = \arg\max_{x} \quad \phi_x[k] \tag{30}$$

where $x \in \{\mathsf{DT}, \mathsf{DF}, \mathsf{CF}, \mathsf{RD}\}$ *(for channel model (a)), or* $x \in \{\mathsf{DT}, \mathsf{DF}, \mathsf{CF}\}$ *(for channel models (b) and (c)). Thus, we get* $\theta_x[k] = 1$ *if* $\xi[k] = x$*, and* $\theta_x[k] = 0$ *if* $\xi[k] \neq x$*.*

The proof is straightforward by solving (23). Notice that we assume that the channel gains are random variables with continuous probability distribution. Therefore, ϕ of each transmission strategy (we can call ϕ as the merit function of the corresponding transmission strategy) will also be random, and the probability that two different strategies maximize (30) in a given channel block is zero. Consequently, the solution of (23) is always unique when $\lambda < 1$.

7) Optimal λ*:* The next step is to find the optimal λ by solving (18a).

Lemma 6 (Bound of λ)**.** *The optimal solution of* (18) *must have* $\lambda \leq 1$*.*

This is because if $\lambda > 1$, both DF and CF will be useless and they cannot achieve higher rates than DT regardless of the channel conditions. Thus, the relay resources are not utilized at all in this case, which is intuitively non-optimal.

Lemma 7 (Strong Duality)**.** *A strong duality exists between the primal problem* (14) *and the dual problem* (18)*. Therefore, the optimal solution of* (18) *is also the optimal solution of* (14)*, and it satisfies the constraint* (14b) *at equality.*

The proof is straightforward since the time-sharing condition (refer to [31]) is satisfied in our problem.

The optimal λ can be obtained numerically using different approaches. For example, if the channel PDFs of the SD, SR and RD channels are known, the expected achievable rates can be computed numerically and used in a bisection search over λ to find the value that satisfies (14b) at equality. We can characterize the long-term average achievable rate using

$$\bar{X}(\lambda) = \int_0^\infty \int_0^\infty \int_0^\infty f_\gamma(\gamma_{\mathsf{SD}}, \gamma_{\mathsf{SR}}, \gamma_{\mathsf{RD}}) X(\lambda, \gamma_{\mathsf{SD}}, \gamma_{\mathsf{SR}}, \gamma_{\mathsf{RD}}) d\gamma_{\mathsf{SD}} d\gamma_{\mathsf{SR}} d\gamma_{\mathsf{RD}} \tag{31}$$

where $X \in \{R_{\mathsf{DT}}, R_{\mathsf{DF}}, R_{\mathsf{CF}}, R_{\mathsf{RD}}, R^*_{\mathsf{DF}}, R^*_{\mathsf{CF}}\}$, $\bar{X}(\lambda)$ is the expected achievable rate for a given value of λ, and $X(\lambda, \gamma_{\mathsf{SD}}, \gamma_{\mathsf{SR}}, \gamma_{\mathsf{RD}})$ is the achievable rate given that the optimal resource allocation (i.e. optimal $\zeta[k]$, $\forall k$) is applied for the given channel SNR values and λ. Furthermore, $f_\gamma(\gamma_{\mathsf{SD}}, \gamma_{\mathsf{SR}}, \gamma_{\mathsf{RD}})$ is the probability density function (PDF) of the channel SNR over the three links of the relay channel. In general, there are no simple closed-form analytical representations of $X(\lambda, \gamma_{\mathsf{SD}}, \gamma_{\mathsf{SR}}, \gamma_{\mathsf{RD}})$ based on the optimal resource allocation given by Theorem 4, especially for channel model (a). Therefore, the integration in (31) should be evaluated using numerical methods. With the aid of (31), we can apply a bisection search over λ to find the unique value that makes $\bar{R}_{\mathsf{RD}}(\lambda) - \bar{R}^*_{\mathsf{DF}}(\lambda) - \bar{R}^*_{\mathsf{CF}}(\lambda) = 0$.

8) Practical Methods to Adapt λ in Real-Time Implementations: In a practical deployment scenario, the PDF of the channels may not be perfectly known, or we may have non-ergodic channels. Therefore, off-line calculation of λ might not be feasible in some practical scenarios. Also, in practice, there would be a certain constraint on the size of the relay's buffers. Therefore, we propose for this case to adapt λ in real-time based on the actual queue size $Q[k]$ and a targeted average queue size, $\bar{Q}$, which is related to the buffer size constraint, or the average delay requirement (if it exists). However, the larger $\bar{Q}$, the better in terms of the expected achievable rates.

Assuming that a good initial value of λ is used, it can then be adapted in real-time using

$$\lambda[k] = \lambda[k-1] - \beta\left(\bar{Q} - Q[k]\right), \tag{32}$$

where $\beta \geq 0$ should be adjusted based on how fast the channel statistics varies. However, in general, the smaller β, the better in order to make the variations in λ smaller.

9) Special Case When $\lambda = 1$: At the special case when the average SNR of the SR link is very high relative the average SNR of the RD and SD links, it may happen that the optimal solution of (18) is at $\lambda = 1$. In this particular case, the solution of (18b) will not be unique since $\phi_{\mathsf{DT}}[k] = \phi_{\mathsf{DF}}[k]$ for all values of k at which $\gamma_{\mathsf{SR}}[k] \geq \gamma_{\mathsf{SD}}[k]$, and there are infinite possible solutions to achieve the constraint (14b) at equality. For example, we can always select DF whenever $\gamma_{\mathsf{SR}}[k] \geq \gamma_{\mathsf{SD}}[k]$, but make $C_{\mathsf{DF}}[k] < C_{\mathsf{SR}}[k]$ such that the constraint (14b) is satisfied at equality. Alternatively, we can keep $C_{\mathsf{DF}}[k] = C_{\mathsf{SR}}[k]$ and make $\theta_{\mathsf{DT}}[k] = \rho$ and $\theta_{\mathsf{DF}}[k] = 1-\rho$ whenever $\gamma_{\mathsf{SR}}[k] \geq \gamma_{\mathsf{SD}}[k]$. Then, we find the value of ρ that makes the constraint satisfied at equality. We use the latter approach in our numerical results. Furthermore, as demonstrated in the numerical results, the optimal achievable rate matches the capacity upper-bound when $\lambda = 1$.

B. Important Characteristics of the Optimal Solution

Corollary 1 (When DF is better than DT)**.** *For all channel models in Fig 1, using the relay to decode the source message is better than direct trasnmission whenever $\gamma_{\mathsf{SR}}[k] > \gamma_{\mathsf{SD}}[k]$.*

The proof is straightforward by checking the case at which $\phi_{\mathsf{DF}}[k] > \phi_{\mathsf{DT}}[k]$.

Corollary 2 (Never compress if you can decode)**.** *For all channel models in Fig 1, the relay should not compress a source message if it can decode it reliably.*

The proof is shown in Appendix E. As a consequence of Corollary 2, we can say that it is a necessary condition to have $C_{\mathsf{CF}}[k] > C_{\mathsf{SR}}[k]$ in order for CF to be better than DF. However,

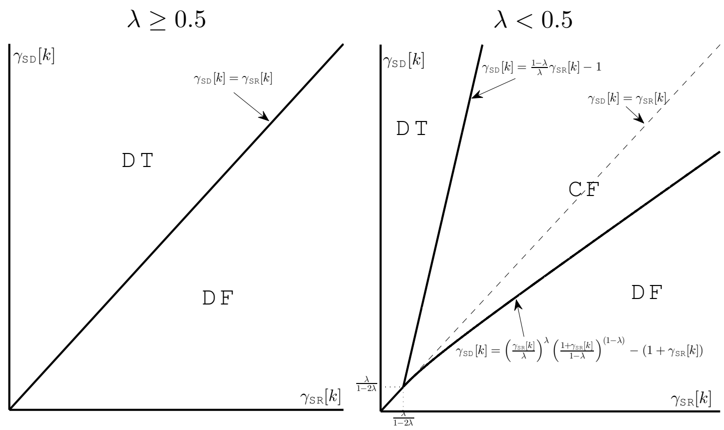

Fig. 2: The regions of selecting DT, DF or CF based on the channel conditions of the SR and SD links.

this condition is not sufficient, and we can actually have cases in which $\phi_{\mathsf{DF}}[k] > \phi_{\mathsf{CF}}[k]$ despite having $C_{\mathsf{CF}}[k] > C_{\mathsf{SR}}[k]$.

Corollary 3 (When CF is never selected)**.** *Compress-and-forward is never selected when the optimal solution of* (18) *is achieved at* $\lambda \geq \frac{1}{2}$.

The proof is shown in Appendix F.

In Fig. 2, we show the regions in the two-dimensional space of $\gamma_{\mathsf{SR}}[k]$ and $\gamma_{\mathsf{SD}}[k]$ in which DT, DF or CF are selected based on the optimal solution of (18).

V. Upper Bounds

In channel models (b) and (c), the ergodic capacity upper bounds are based on the max-flow min-cut bound. This yields for model (b):

$$C_{\text{up}} = \min\left(\lim_{K\to\infty}\frac{1}{K}\sum_k \log\left(1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]\right), \lim_{K\to\infty}\frac{1}{K}\sum_k \log\left(1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{RD}}[k]\right)\right) \tag{33}$$

and for channel model (c):

$$C_{\text{up}} = \min\left(\lim_{K\to\infty}\frac{1}{K}\sum_k \log\left(1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]\right), \lim_{K\to\infty}\frac{1}{K}\sum_k \log\left((1+\gamma_{\mathsf{SD}}[k])(1+\gamma_{\mathsf{RD}}[k])\right)\right) \tag{34}$$

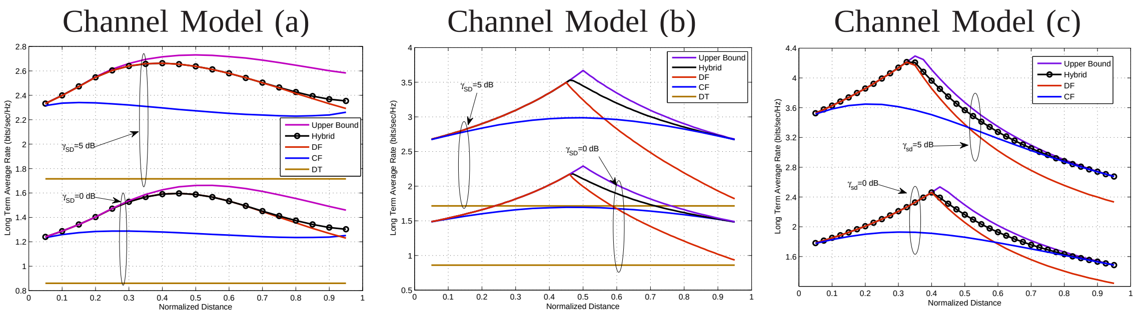

Fig. 3: Acheivable rates results

In channel model (a), the upper bound is obtained by assuming a genie-aided transmission in which the relay can know what the destination receives, but the destination cannot know what the relay receives. In this case, the relay can always decode at a rate of $\log(1 + \gamma_{\mathsf{SR}} + \gamma_{\mathsf{SD}})$. Thus, we do not have CF, and we solve the optimization problem assuming either the source transmits using DF at this giene-aided rate or the relay transmits to the destination according to its channel gain. The selection between these two modes is done such that the long-term average rate is maximized.

VI. Numerical Results

We make our numerical results assuming that the distance between the source and the destination is d_{SD}, and the relay is located on the straight line between the source and the destination such that the distance between the source and the relay is d_{SR}, and the distance between the relay and the destination is $d_{\mathsf{RD}} = d_{\mathsf{SD}} - d_{\mathsf{SR}}$. The channels between the nodes are Rayleigh block-faded, and the average channel qualities are given by $\bar{\gamma}_x = \epsilon \left(\frac{d_x}{d_{\mathsf{SD}}}\right)^{-\alpha}$, where $x \in \{\mathsf{SR}, \mathsf{RD}, \mathsf{SD}\}$, $\alpha = 3$ is the path loss exponent, and ϵ is a constant that is related to the transmission power, antenna gains and total distance. We use two cases in the simulation, $\epsilon = 10^{0.5} \approx 3.1623$, which gives $\bar{\gamma}_{\mathsf{SD}} = 5$ dB, and $\epsilon = 1$, which gives $\bar{\gamma}_{\mathsf{SD}} = 0$ dB.

In Fig. 3, we plot the expected achievable rates versus the normalized distance of the relay to the source $\frac{d_{\mathsf{SR}}}{d_{\mathsf{SD}}}$. Also, we compare the optimal hybrid scheme to the upper-bounds and to three sub-optimal schemes that use DF and DT without CF, or use CF and DT without DF, or use DT

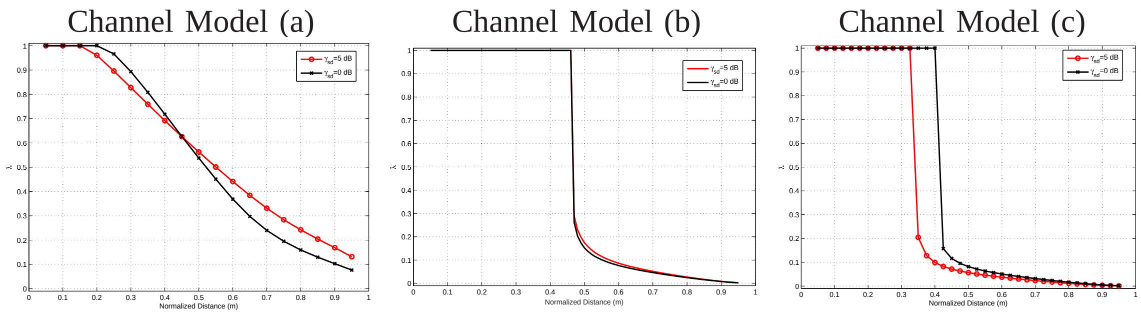

Fig. 4: Optimal λ.

only without any role of the relay. These schemes are optimized using the same approach that is used to optimize the hybrid scheme. We obtain from the achievable rate results that the gains of the hybrid scheme over a sub-optimal scheme that does not use CF are more significant in channel models (b) and (c) that have full-duplex relays. This is valid when the relay is closer to the destination than to the source. Furthermore, in all three channel models, the best achievable scheme matches the capacity upper bounds only when the relay is close to the source. Also, we can see a considerable gain in the achievable rates in models (b) and (c) with respect to (a) since the relay and source transmit together all the time. Furthermore, the gain of model (c) with respect to (b), which is due to having twice the bandwidth, is large when the relay is close to the source, and it is negligible when the relay is close to the destination.

In Fig. 4, we plot the optimal λ for the hybrid scheme versus the normalized distance of the relay to the source $\frac{d_{\mathsf{SR}}}{d_{\mathsf{SD}}}$. The results show that λ is a non-increasing function with respect to the distance of the relay from the source. Furthermore, $\lambda = 1$ when the relay is close to source. By comparing the optimal λ results with the achievable rates results, we can see that the capacity is achievable when $\lambda = 1$. Furthermore, for channel models (b) and (c), the capacity is achievable over a wider range of the source-relay distance in comparison with channel model (a).

In Fig. 5, we plot the average access (i.e. selection) ratios of the the different transmission strategies of the optimal hybrid scheme versus the normalized distance of the relay to the source $\frac{d_{\mathsf{SR}}}{d_{\mathsf{SD}}}$. The results demonstrate that CF becomes more important when the relay is closer to the

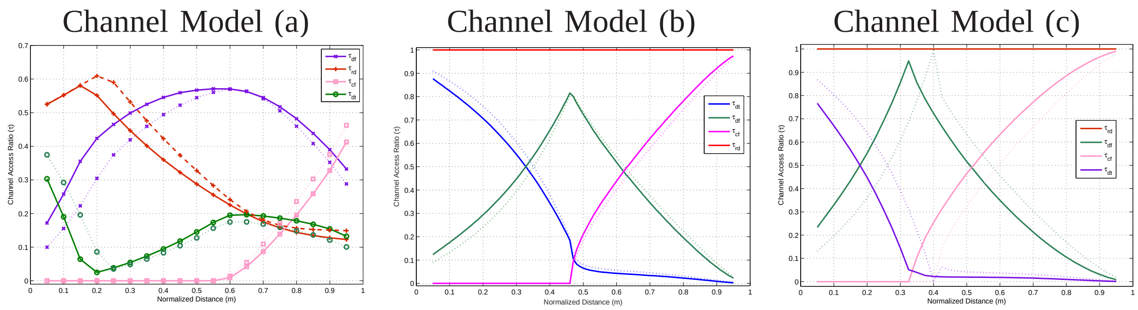

Fig. 5: Average access ratios of DT, DF, CF and RD. The solid lines are for the case when $\bar{\gamma}_{\mathsf{SD}}$ = 5dB, and the dotted lines are for the case when $\bar{\gamma}_{\mathsf{SD}}$ = 0dB.

destination. Furthermore, the use of CF in channel models (b) and (c) is more significant than in channel model (a). By comparing the results with the optimal λ results, we can see that CF is never selected when the relay is closer to the source, where $\lambda \geq \frac{1}{2}$, and this confirms Corollary 3.

VII. Conclusions

We have showed in this paper how to integrate compress-and-forward with decode-and-forward and direct transmission in buffer-aided relaying systems, and we have applied that to three different models of the block-fading relay channel. For optimality, only one transmission strategy is selected in a given channel block based on the channel conditions. The optimization of the data rate for compress-and-forward is obtained using a simple closed-form formula. The numerical results have demonstrated the gains of the proposed scheme. Furthermore, the proposed hybrid scheme can be applied in practice, even if the channel statistics (needed to choose λ) are not known beforehand, by using simple algorithms to adapt λ in real-time.

Appendix A
Definition of $\omega[k]$ in Channel Model (b)

An alternative way to write (5) is

$$C_{\mathsf{SD}}[k] = \omega[k]C_{\mathsf{SD-SFi}}[k]+(1-\omega[k])C_{\mathsf{SD-RFi}}[k], \quad C_{\mathsf{RD}}[k] = \omega[k]C_{\mathsf{RD-SFi}}[k]+(1-\omega[k])C_{\mathsf{RD-RFi}}[k], \tag{35}$$

: $\omega[k] \in [0,1]$, where $C_{\mathsf{SD-SFi}}[k]$ and $C_{\mathsf{RD-SFi}}[k]$ are the instantaneous capacities assuming that the destination decodes the source signal first, removes it and then decodes the relay signal, and $C_{\mathsf{SD-RFi}}[k]$ and $C_{\mathsf{RD-RFi}}[k]$ are the instantaneous capacities assuming that the destination decodes the relay signal first, removes it and then decodes the source signal. They are given by $C_{\mathsf{SD-SFi}}[k] = \log\left(1+\frac{\gamma_{\mathsf{SD}}[k]}{1+\gamma_{\mathsf{RD}}[k]}\right)$, $C_{\mathsf{RD-SFi}}[k] = \log\left(1+\gamma_{\mathsf{RD}}[k]\right)$, $C_{\mathsf{RD-RFi}}[k] = \log\left(1+\frac{\gamma_{\mathsf{RD}}[k]}{1+\gamma_{\mathsf{SD}}[k]}\right)$, $C_{\mathsf{SD-RFi}}[k] = \log\left(1+\gamma_{\mathsf{SD}}[k]\right)$. In (35), $\omega[k]$ represents the time sharing between the two possibilities of successive interference cancellation order at the destination. Therefore, $\omega[k]$ specifies the operating point on the boundary of the MAC channel.

Appendix B

Proof of Theorem 1

The achievable rate by compress-and-forward over constant Gaussian channels was characterized in the literature in terms of the “compression noise”, denoted by σ_w^2 [5, Proposition 3]. The two formulas characterizing the achievable rate, expressed in terms of the notations that are used in this paper, are

$$R_{\mathsf{CF}} = \theta_{\mathsf{CF}} \log\left(1+\gamma'_{\mathsf{SD}}+\frac{\gamma_{\mathsf{SR}}}{1+\sigma_w^2}\right), \tag{36}$$

where

$$\sigma_w^2 \geq \frac{1+\gamma'_{\mathsf{SD}}+\gamma_{\mathsf{SR}}}{\left((1+\gamma'_{\mathsf{RD}})^{\theta_{\mathsf{RD}}/\theta_{\mathsf{CF}}}-1\right)\left(1+\gamma_{\mathsf{SD}}\right)}, \tag{37}$$

where θ_{CF} and θ_{RD} are respectively the bandwidth ratios that are allocated to the source (to send its signal) and to the relay (to send a compressed version of the received signal from the source). The achievable rate of compress-and-froward over constant Gaussian channels is a function of the channel conditions (γ_{SR}, γ'_{SD}, γ'_{RD}) as well as the channel allocation among the source and relay channel (θ_{CF}, θ_{RD}). However, in our case, we have a block-fading channel and the relay does not have to forward the compressed signal in the same channel block. Therefore, we propose an alternative way to present the achievable rate of CF in terms of data rate of the compressed signal R^*_{CF} instead of θ_{RD}. Knowing that $R^*_{\mathsf{CF}} = \theta_{\mathsf{RD}}\log\left(1+\gamma'_{\mathsf{RD}}\right)$, we can write

(37) as, where R^*_{CF} is in nats/sec/Hz,

$$\sigma_w^2 \geq \frac{1+\gamma'_{\mathsf{SD}}+\gamma_{\mathsf{SR}}}{\left(\exp\left(\frac{R^*_{\mathsf{CF}}}{\theta_{\mathsf{CF}}}\right)-1\right)(1+\gamma'_{\mathsf{SD}})} \tag{38}$$

With simple manipulations, we can write (38) as

$$R^*_{\mathsf{CF}} \geq \theta_{\mathsf{CF}} \log\left(1+\frac{1}{\sigma_w^2}\left(\frac{1+\gamma'_{\mathsf{SD}}+\gamma_{\mathsf{SR}}}{1+\gamma'_{\mathsf{SD}}}\right)\right) \tag{39}$$

By using the term γ_{CF}, and given that $\gamma'_{\mathsf{SD}} < \gamma_{\mathsf{CF}} < \gamma'_{\mathsf{SD}}+\gamma_{\mathsf{SR}}$, we can write (36) as $\gamma'_{\mathsf{SD}}+\frac{\gamma_{\mathsf{SR}}}{1+\sigma_w^2} = \gamma_{\mathsf{CF}}$. By simple manipulations, we can write it as

$$\sigma_w^2 = \frac{\gamma'_{\mathsf{SD}}+\gamma_{\mathsf{SR}}-\gamma_{\mathsf{CF}}}{\gamma_{\mathsf{CF}}-\gamma'_{\mathsf{SD}}} \tag{40}$$

By substituting (40) in (39), we obtain (12), where the index of the channel block k is added since we have block-fading channels in our problem.

Appendix C

Solution Steps to Obtain (27)

With straightforward steps, we can write (12) equivalently as (where we have strict equality as shown in Lemma 1)

$$C^*_{\mathsf{CF}}[k] = \log\left(\frac{1+\gamma_{\mathsf{CF}}[k]}{1+\gamma'_{\mathsf{SD}}[k]}\cdot\frac{\gamma_{\mathsf{SR}}[k]}{\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k]-\gamma_{\mathsf{CF}}[k]}\right) \tag{41a}$$

$$= C_{\mathsf{CF}}[k]-C_{\mathsf{SD}}[k]+\log(\gamma_{\mathsf{SR}}[k])-\log\left(\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k]-\gamma_{\mathsf{CF}}[k]\right) \tag{41b}$$

Thus, we can write $\phi_{\mathsf{CF}}[k]$ as

$$\phi_{\mathsf{CF}}[k] = \max_{C_{\mathsf{CF}}[k]} \quad (1-\lambda)C_{\mathsf{CF}}[k]+\lambda C_{\mathsf{SD}}[k]-\lambda\log(\gamma_{\mathsf{SR}}[k])+\lambda\log\left(\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k]-\gamma_{\mathsf{CF}}[k]\right) \tag{42}$$

Notice that $C_{\mathsf{SD}}[k]$ and $\log(\gamma_{\mathsf{SR}}[k])$ are independent of $C_{\mathsf{CF}}[k]$. Thus, the optimal value of $C_{\mathsf{CF}}[k]$ is obtained by solving

$$\max_{C_{\mathsf{CF}}[k]} \quad (1-\lambda)C_{\mathsf{CF}}[k]+\lambda\log\left(1+\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k]-\exp\left(C_{\mathsf{CF}}[k]\right)\right) \tag{43}$$

There is a unique value at which the gradient of the optimization function equals zero, which yields

$$1-\lambda-\frac{\lambda\exp(C_{\mathsf{CF}}[k])}{1+\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k]-\exp\left(C_{\mathsf{CF}}[k]\right)}=0 \tag{44}$$

By simple manipulations, we obtain $C_{\mathsf{CF}}[k]=\log\left(1-\lambda\right)+\log\left(1+\gamma'_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]\right)$. If the value of $C_{\mathsf{CF}}[k]$ at which the gradient equals zero is less than $C_{\mathsf{SD}}[k]$, which is the minimum boundary of the domain of $C_{\mathsf{CF}}[k]$, then the optimal solution is at this boundary.

Appendix D
Proof of Theorem 3

Based on the definition of $\omega[k]$ that is given in Appendix A, we can show that $\frac{\partial C_{\mathsf{SD}}[k]}{\partial\omega[k]}<0$ and $\frac{\partial C_{\mathsf{RD}}[k]}{\partial\omega[k]}>0$.

By substituting (24a), (26), (29) and (24d) in (23), we can write $\theta_{\mathsf{DT}}[k]\phi_{\mathsf{DT}}[k]+\theta_{\mathsf{DF}}[k]\phi_{\mathsf{DF}}[k]+\theta_{\mathsf{CF}}[k]\phi_{\mathsf{CF}}[k]+\phi_{\mathsf{RD}}[k]=G_{\omega}[k]+U[k]$, where $G_{\omega}[k]$ is the sum of the terms that are functions of $\omega[k]$, and $U[k]$ is the sum of the terms that are independent of $\omega[k]$. They are given by $G_{\omega}[k]=(\theta_{\mathsf{DT}}[k]+\theta_{\mathsf{DF}}[k]+\theta_{\mathsf{CF}}[k])\lambda C_{\mathsf{SD}}[k]+\theta_{\mathsf{DT}}[k](1-\lambda)C_{\mathsf{SD}}[k]+\theta_{\mathsf{CF}}[k]\log(1+\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k])+\lambda C_{\mathsf{RD}}[k]$, and $U[k]=\theta_{\mathsf{DF}}[k](1-\lambda)C_{\mathsf{SR}}[k]+\theta_{\mathsf{CF}}[k]\left(-\lambda\log(\gamma_{\mathsf{SR}}[k])+\lambda\log(\lambda)+(1-\lambda)\log(1-\lambda)\right)$.

From Lemma 2, we know that $\theta_{\mathsf{DT}}[k]+\theta_{\mathsf{DF}}[k]+\theta_{\mathsf{CF}}[k]=1$. Also, from (5a), we know that $C_{\mathsf{SD}}[k]+C_{\mathsf{RD}}[k]=C_{\mathsf{MAC}}[k]$, which is a constant regardless of the value of $\omega[k]$. Therefore, we can write $G_{\omega}[k]=\lambda C_{\mathsf{MAC}}[k]+\theta_{\mathsf{DT}}[k](1-\lambda)\log(1+\gamma'_{\mathsf{SD}}[k])+\theta_{\mathsf{CF}}[k]\log(1+\gamma_{\mathsf{SR}}[k]+\gamma'_{\mathsf{SD}}[k])$.

Therefore, given that $\lambda\leq 1$, it is straightforward to show that $\frac{\partial G_{\omega}[k]}{\partial\gamma'_{\mathsf{SD}}[k]}\geq 0$. Consequently, we prove that

$$\frac{\partial}{\partial\omega[k]}\left(\theta_{\mathsf{DT}}[k]\phi_{\mathsf{DT}}[k]+\theta_{\mathsf{DF}}[k]\phi_{\mathsf{DF}}[k]+\theta_{\mathsf{CF}}[k]\phi_{\mathsf{CF}}[k]+\phi_{\mathsf{RD}}[k]\right)\leq 0 \tag{45}$$

Thus, it is optimal to make $\omega[k]=0$ regardless of the optimal solution of $\theta_{\mathsf{DT}}[k]$, $\theta_{\mathsf{DF}}[k]$ and $\theta_{\mathsf{CF}}[k]$. Notice that if $\theta_{\mathsf{DF}}[k]\neq 1$, then $\omega[k]=0$ is the only optimal solution. However, if $\theta_{\mathsf{DF}}[k]=1$, then $\frac{\partial G_{\omega}[k]}{\partial\gamma'_{\mathsf{SD}}[k]}=0$, and all values of $\omega[k]\in[0,1]$ are optimal.

Appendix E
Proof of Corollary 2

Let's assume that

$$\phi_{\mathsf{CF}}[k] > \phi_{\mathsf{DF}}[k], \quad \text{and} \tag{46a}$$

$$C_{\mathsf{SD}}[k] < C_{\mathsf{CF}}[k] < C_{\mathsf{SR}}[k] \tag{46b}$$

This assumption means that CF would be selected according to Theorem 4 although the rate of the source message is below the capacity of the SR link, and hence it can be decoded reliably by the relay. By using (42) and (26) for $\phi_{\mathsf{CF}}[k]$ and $\phi_{\mathsf{DF}}[k]$, respectively, we can write (46a) as $(1-\lambda)C_{\mathsf{CF}}[k] + \lambda C_{\mathsf{SD}}[k] + \lambda \log\left(1 + \frac{\gamma_{\mathsf{SD}}[k]-\gamma_{\mathsf{CF}}[k]}{\gamma_{\mathsf{SR}}[k]}\right) > (1-\lambda)C_{\mathsf{SR}}[k] + \lambda C_{\mathsf{SD}}[k]$. This inequality can be also written as $(1-\lambda)(C_{\mathsf{CF}}[k] - C_{\mathsf{SR}}[k]) + \lambda \log\left(1 + \frac{\gamma_{\mathsf{SD}}[k]-\gamma_{\mathsf{CF}}[k]}{\gamma_{\mathsf{SR}}[k]}\right) > 0$. However, this inequality is invalid since $C_{\mathsf{CF}}[k] < C_{\mathsf{SR}}[k]$ by assumption, $\lambda \leq 1$ as shown in Lemma 6, and $\gamma_{\mathsf{SD}}[k] < \gamma_{\mathsf{CF}}[k]$ as indicated in (46b) (otherwise DT will be used rather than CF). Therefore, the assumptions in (46) can never be valid, and this proves the statement of Corollary 2.

Appendix F
Proof of Corollary 3

A necessary condition for CF to be selected is to have either $\phi_{\mathsf{CF}}[k] > \phi_{\mathsf{DF}}[k] > \phi_{\mathsf{DT}}[k]$ or $\phi_{\mathsf{CF}}[k] > \phi_{\mathsf{DT}}[k] > \phi_{\mathsf{DF}}[k]$. Equivalently, we can say that a necessary condition for CF to be selected is to have either

$$\phi_{\mathsf{CF}}[k] > \phi_{\mathsf{DF}}[k], \quad \text{given that} \quad \gamma_{\mathsf{SR}}[k] > \gamma_{\mathsf{SD}}[k], \tag{47a}$$

$$\text{or} \quad \phi_{\mathsf{CF}}[k] > \phi_{\mathsf{DT}}[k], \quad \text{given that} \quad \gamma_{\mathsf{SR}}[k] < \gamma_{\mathsf{SD}}[k] \tag{47b}$$

As shown in Corollary 2, a necessary condition for (47a) to be valid is to have $\gamma_{\mathsf{CF}}[k] > \gamma_{\mathsf{SR}}[k] > \gamma_{\mathsf{SD}}[k]$. By substituting using (28), we can write $(1-\lambda)(1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]) > 1+\gamma_{\mathsf{SR}}[k]$, which yields $\lambda < \frac{\gamma_{\mathsf{SD}}[k]}{1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]} < \frac{1}{2}$, where the right inequality is justified by the assumption $\gamma_{\mathsf{SR}}[k] > \gamma_{\mathsf{SD}}[k]$. Similarly, a necessary condition for (47b) to be valid is to have $\gamma_{\mathsf{CF}}[k] >$

$\gamma_{\mathsf{SD}}[k] > \gamma_{\mathsf{SR}}[k]$. Therefore, we can write $(1-\lambda)(1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]) > 1+\gamma_{\mathsf{SD}}[k]$, which yields $\lambda < \frac{\gamma_{\mathsf{SR}}[k]}{1+\gamma_{\mathsf{SD}}[k]+\gamma_{\mathsf{SR}}[k]} < \frac{1}{2}$, where the right inequality is justified by the assumption $\gamma_{\mathsf{SD}}[k] > \gamma_{\mathsf{SR}}[k]$. Therefore, in both cases of (47), a necessary condition for the selection of CF is to have $\lambda < \frac{1}{2}$, and this proves the statement of Corollary 3.

References

[1] E. C. V. D. Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, pp. 120–154, 1971.

[2] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity – Part I: System description," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.

[4] J. N. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[5] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2020–2040, Jun. 2005.

[6] Y. Liang and V. Veeravalli, "Gaussian orthogonal relay channels: Optimal resource allocation and capacity," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3284–3289, Sep. 2005.

[7] A. E. Gamal, M. Mohseni, and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1545–1561, Apr. 2006.

[8] M. Cardone, D. Tuninetti, R. Knopp, and U. Salim, "On the Gaussian half-duplex relay channel," *IEEE Transactions on Information Theory*, vol. 57, no. 5, pp. 2542–2562, May 2014.

[9] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[10] D. Gunduz and E. Erkip, "Opportunistic cooperation by dynamic resource allocation," *IEEE Transactions on Wireless Communications*, vol. 6, no. 4, pp. 1446–1454, Apr. 2007.

[11] Y. Liang, V. Veeravalli, and H. V. Poor, "Resource allocation for wireless fading relay channels: Max-min solution," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3432–3453, Oct. 2007.

[12] N. Zlatanov, R. Schober, and P. Popovski, "Buffer-aided relaying with adaptive link selection," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 8, pp. 1530–1542, Aug. 2013.

[13] N. Zlatanov and R. Schober, "Buffer-aided relaying with adaptive link selection – fixed and mixed rate transmission," *IEEE Transactions on Information Theory*, vol. 59, no. 5, pp. 2816–2840, May 2013.

[14] N. Zlatanov, R. Schober, and L. Lampe, "Buffer-aided relaying in a three node network," in *Proceedings IEEE International Symposium on Information Theory (ISIT)*, Cambridge, MA, Jul. 2012, pp. 781–785.

[15] X. Wu and L. Xie, "On the optimal compressions in the compress-and-forward relay schemes," *IEEE Transactions on Information Theory*, vol. 59, no. 5, pp. 2613–2628, May 2013.

[16] A. Zafar, M. Shaqfeh, M.-S. Alouini, and H. Alnuweiri, "Exploiting multi-user diversity and multi-hop diversity in dual-hop broadcast channels," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3314–3325, Jul. 2013.

[17] M. Shaqfeh, A. Zafar, H. Alnuweiri, and M. S. Alouini, "Joint opportunistic scheduling and network coding for bidirectional relay channel," in *Proceedings IEEE International Symposium on Information Theory (ISIT)*, Istanbul, Turkey, Jul. 2013, pp. 1327–1331.

[18] H. Liu, P. Popovski, E. Carvalho, and Y. Zhao, "Sum-rate optimization in a two-way relay network with buffering," *IEEE Communications Letters*, vol. 17, no. 1, pp. 95–98, Jan. 2013.

[19] A. Zafar, M. Shaqfeh, M.-S. Alouini, and H. Alnuweiri, "Resource allocation for two source-destination pairs sharing a single relay with a buffer," *IEEE Transactions on Communications*, vol. 62, no. 5, pp. 1444–1457, May 2014.

[20] M. Shaqfeh, A. Zafar, H. Alnuweiri, and M.-S. Alouini, "Overlay cognitive radios with channel-aware adaptive link selection and buffer-aided relaying," *IEEE Transactions on Communications*, vol. 63, no. 8, pp. 2810–2822, Aug. 2015.

[21] I. Krikidis, T. Charalambous, and J. Thompson, "Buffer-aided relay selection for cooperative diversity systems without delay constraints," *IEEE Transactions on Wireless Communications*, vol. 11, no. 5, pp. 1957–1967, May 2012.

[22] A. Ikhlef, D. Michalopoulos, and R. Schober, "Max-max relay selection for relays with buffers," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1124–1135, Mar. 2012.

[23] A. Ikhlef, J. Kim, and R. Schober, "Mimicking full-duplex relaying using half-duplex relays with buffers," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 7, pp. 3025–3037, Jul. 2012.

[24] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Transactions on Information Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.

[25] D. Tse and S. Hanly, "Multiaccess fading channels – Part 1: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Transactions on Information Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.

[26] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels – Part 1: Ergodic capacity," *IEEE Transactions on Information Theory*, vol. 47, no. 3, pp. 1083–1102, Mar. 2001.

[27] M. Shaqfeh, H. Alnuweiri, and M.-S. Alouini, "Multiuser switched diversity scheduling schemes," *IEEE Transactions on Communications*, vol. 60, no. 9, pp. 2499–2510, Sep. 2012.

[28] D. Slepian and J. Wolf, "Noiseless coding of correlated information sources," *IEEE Transactions on Information Theory*, vol. 19, no. 4, pp. 471–480, Jul. 1973.

[29] A. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Transactions on Information Theory*, vol. 22, no. 1, pp. 1–10, Jan. 1976.

[30] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, March 2004.

[31] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Transactions on Communications*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.