Performance of Soft Frequency Reuse in random cellular networks in Rayleigh-Lognormal Fading Channels

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***Abstract*—Soft Frequency Reuse (Soft FR) is an effective resource allocation technique that can improve the instantaneous received Signal-to-Interference-plus-Noise ratio (SINR) at a typical user and the spectrum efficiency. In this paper, the performance of Soft FR in Random Cellular network, where the locations of Base Stations (BSs) are random variables of Spatial Point Poisson Process (PPP), is investigated. While most of current works considered the network model with either single RB or frequency reuse with factor of 1, this work assume that the Soft FR with factor of Δ is deployed and there are M users and N ($\Delta > 1, M > 1, N > 1$). The analytical and simulation results show that a network system with high frequency reuse factor create more InterCell Interference than that with low frequency reuse factor. Furthermore, in order to design the parameters to optimize Soft FR, the performance of the Cell-Edge and Cell-Center user should be considered together.**

Keywords: Rayleigh-Lognormal, Poisson Point Process network, frequency reuse, Round Robin Scheduling.

I. Introduction

In Orthogonal Frequency Division Multiple Access (OFDMA) multi-cell networks, the main factor, that directly impacts on the system performance, is intercell interference which is caused by the use of the same frequency band in adjacent cells. Soft FR algorithm [1] is considered as an effective resource allocation technique that improve the performance of users, especially for user experiencing poor serving signal. In this algorithm, the allocated Resource Blocks (RBs) and users are divided into non-overlapping groups, call Cell-Edge and Cell-Center RB group, Cell-Edge and Cell-Center user group.

The performance of Soft FR algorithm has been studied for hexagonal network models such as in [2], [3]. Recently, the Point Poisson Process (PPP) network model has been deployed to analyse network performance using frequency reuse algorithm. In most of works, the authors studied Soft RF with reuse factor of 1 [4], [5], that lead to the fact that all BSs transmit at the same power level. Hence, the concepts of Cell-Center, Cell-Edge users and the transmit power levels on Cell-Edge and Cell-Center RBs have not been discussed.

In [6], [7], the performance of Fractional Frequency Reuse algorithms with reuse factor $\Delta > 1$ were evaluated. In these papers, the effective InterCell Interference was introduced to represent the total InterCell Interference in the network. In fact, in Soft FR network system with factor $\Delta > 1$, the InterCell Interference at a typical user is caused by BSs in two separated groups in which the first group contains the BSs transmitting on Cell-Center RBs and the second group contains the BSs transmitting on the Cell-Edge RBs. Generally, these groups can be distinguished by the differences in the transmit power levels and the densities of BSs. When the location of BSs and the channel power gain are random variables, the powers of interference at the typical user caused by the BSs in each interfering groups are random variables. Hence, the total interference should be the sum of two separated groups of random variables. Consequently, the concept of effective InterCell Interference may be not suitable in this case. The unreasonableness of effective InterCell Interference will be explained with more details in Section II-B.

Furthermore, in the work discussed above, it was assumed that all BSs always cause InterCell Interference to a typical user. This assumption is reasonable when all RBs are used at all adjacent cells, i.e. the number of users is equal or greater than that of allocated RBs.

In this paper, the performance of Soft FR ($\Delta > 1$) network system with Round Robin scheduling is evaluated. The given outcomes of this paper significantly differ from the published results since in this work, instead of using the effective InterCell Interference concept, the interfering BSs are separated into two groups which are distinguished by different transmit power levels and different densities of BSs. In order to analyse the effects of the number of RBs and users when Round Robin scheduling is deployed, the indicator function representing the probability where the BS creates InterCell Interference to a typical user defined.

The average capacities of a typical Cell-Center and Cell-Edge user are presented in this paper. The opposite trend between the average capacity of these users emphasises that the optimization problem of Soft FR should consider the performance of Cell-Edge and Cell-Center together.

II. SYSTEM MODEL

A statistical single-tier cellular network model which the locations of BSs are distributed as the Spatial Poisson Process with density λ is considered. The transmit power of a typical BS is denoted by P. The signals from BSs to the typical user experience propagation path loss with exponent α, and Rayleigh-Lognormal fading with mean $\mu_z\ dB$ and $\sigma_z\ dB$.

Denote r is a random variable which represent the distance from the typical user and the BS. The received signal at the user from the BS is $P\overline{g}r^{-\alpha}$ in which $\overline{g}$ is average power of fading channel. In real network, the typical user try to connect to the strongest BS which provide the highest $P\overline{g}r^{-\alpha}$. Since, in single-tier network, it is assumed that all BSs transmit at the same power level, the average power gain as well as path loss exponent are assumed to be constants. Hence in this case, the strongest BS of the typical user is its nearest BS.

The PDF of the distance r between a typical user and its serving BS is defined by following equation [5].

$$f_R(r) = 2\pi\lambda r \exp\left(-\pi\lambda r^2\right) \tag{1}$$

A. Rayleigh-Lognormal fading channel model

The realistic fading channel is the coherence of fast fading which is caused by scattering from local obstacles such as buildings and slow fading which is caused by the variance of transmission environment. In this work, the fast fading is modelled as Rayleigh fading and the slow fading is modelled as Lognormal fading. The PDF of the Rayleigh-Lognormal channel power gain g is given by.

$$f_{R-Ln}(g) = \int_0^{\infty} \frac{1}{x} e^{-g/x} \frac{1}{x\sigma_z\sqrt{2\pi}} e^{-(10\log_{10}x-\mu_z)^2/2/\sigma_z^2} dx \tag{2}$$

in which μ_z and σ_z are mean and variance of Rayleigh-Lognormal random variable.
Employing the substitution, $t = \frac{10\log_{10}x-\mu_z}{\sqrt{2}\sigma_z}$, then

$$f_{R-Ln}(g) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \frac{1}{\gamma(t)} \exp\left(-\frac{g}{\gamma(t)}\right) \exp(-t^2) dt \tag{3}$$

The integral in Equation 3 has the suitable form for Gauss-Hermite expansion approximation [8]. Thus, the PDF can be approximated by:

$$f_{R-Ln}(g) = \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \frac{1}{\gamma(a_n)} \exp\left(-\frac{g}{\gamma(a_n)}\right) \tag{4}$$

in which

- w_n and a_n are the weights and the abscissas of the Gauss-Hermite polynomial. To achieve high accurate approximation, $Np = 12$ is used.
- $\gamma(a_n) = 10^{(\sqrt{2}\sigma_z a_n + \mu_z)/10}$.

Hence, the CDF of Rayleigh-Lognormal RV $F_{R-Ln}(g)$ is obtained by the integral of PDF from 0 to g:

$$F_{R-Ln}(g) = \int_0^g f(x)dx = 1 - \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \exp\left(-\frac{g}{\gamma(a_n)}\right)$$

Since g is the channel power gain, g is a positive real number $(g > 0)$. The MGF of g can be found as:

$$\begin{aligned} M_{R-Ln}(s) &= \int_0^{\infty} f_{R-Ln}(x) e^{-xs} dx \\ &= \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \frac{1}{1 + s\gamma(a_n)} \end{aligned} \tag{5}$$

The average of the power gain of Rayleigh-Lognormal channel is $\overline{g}_{R-Ln} = 10^{(\mu_z + \frac{1}{2}\sigma_z^2)/10}$. In this paper, it is assumed that the power gain of the channel is normalised, i.e. $\overline{g}_{R-Ln} = 1$.

B. Frequency Reuse Algorithm

It is assumed that all cells are allocated the same N RBs to serve M users. Soft FR with frequency reuse factor Δ is deployed in as shown in Figure 1. As shown in Figure 1, both users and RBs are classified into two types including M_C cell-center users and M_E cell-edge users, N_C cell-center RBs and N_E cell-edge RBs. Since, the cell-edge users are served with higher transmit power level, denote ϕ as the ratio between the serving transmit power of cell-edge and cell-center users.

The optimisation factors $\epsilon^{(e)}$ and $\epsilon^{(c)}$ is defined as the ratios between number of users and the number of RBs at Cell-Edge and Cell-Center areas as Equation 6:
For Cell-Edge area:

$$\frac{M_e}{N_e} = \epsilon^{(e)} \tag{6a}$$

For Cell-Center area:

$$\frac{M_c}{N_c} = \epsilon^{(c)} \tag{6b}$$

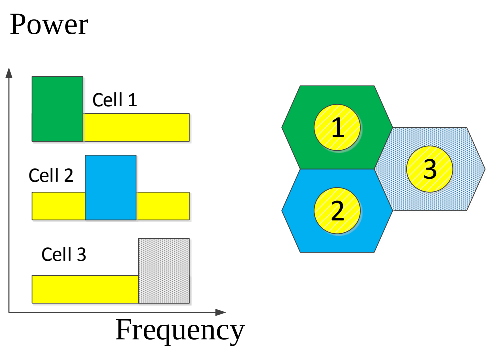

Fig. 1. An example of Soft FR with $\Delta = 3$

It is assumed that the typical user is served on RB b. When FR factor Δ is used, the densities of BSs that transmit on RB b at Cell-Center and Cell-Edge power levels, are $\lambda_C = \frac{\Delta-1}{\Delta}\lambda$ and $\lambda_E = \frac{1}{\Delta}\lambda$, respectively [3], [9].

The intercell interference on the typical user u is given by

$$I_u = \sum_{z_c \in \theta_C} \tau(z_c = b) P g_{z_c} r_{z_c}^{-\alpha} + \sum_{z_e \in \theta_E} \tau(z_e = b) \phi P g_{z_e} r_{z_e}^{-\alpha} \quad (7)$$

in which θ_C and θ_E are the set of BSs transmitting with a cell-center and cell-edge power level ; g_z and r_z are the channel power gain and distance between the user and a BS in cell z where $z = z_c$ corresponds to cell-center area, $z = z_e$ corresponds to cell-edge area; the indicator function $\tau(z = b)$ is defined as below

$$\tau(z = b) = \begin{cases} 1 & \text{RB } b \text{ is used in } z \text{ area} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

the indicator function which represent the event which that take value 1 if the RB b is occupied in area z of a particular cell

When the Round Robin scheduling is assumed to be deployed, the expected values of $\tau(z_c = b)$ and $\tau(z_e = b)$ are given by:

$$\mathbb{E}[\tau(z_c = b)] = \frac{M_C}{N_C} = \epsilon^{(e)} \quad (9a)$$

$$\mathbb{E}[\tau(z_e = b)] = \frac{M_E}{N_E} = \epsilon^{(c)} \quad (9b)$$

If the number of users in a given area such as Cell-Edge area is greater than the number of RBs, all RBs at this area are used at the same time . Hence, there is a BS in this case causes InterCell interference to the typical user, i.e. $\epsilon^{(e)}$.

The main difference between this work and the published work in [6], [7] is that in in [6], [7] it was assumed that the adjacent BSs always create interference on the typical user, i.e. $\tau(z_c = b) = \tau(z_e = b) = 1$. This assumption is valid for the PPP network in which the instantaneous number of users is greater than the number of RBs. Furthermore, in previous work, it is assumed that $g_{z_c} r_{z_c}^{-\alpha} = g_{z_e} r_{z_e}^{-\alpha}$, then the effective InterCell Interference is defined as $I_{eff} = \sum_{z \in \theta} (\lambda_C + \phi \lambda_E) g_z r_z^{-\alpha}$ in which θ is the set of neighboring BSs. This assumption is not reasonable as $g_{z_c} r_{z_c}^{-\alpha}$ and $g_{z_e} r_{z_e}^{-\alpha}$ are random variables with the same distribution but the distribution of the total interference I_u given by equation 4 is different. Hence, the concept of effective InterCell Interference is not feasible in this case.

III. User coverage probability

The coverage probability P_c of the typical user u for the given threshold T is defined as the probability of event in which the instantaneous received SINR of the user is greater than the defined threshold.

$$P_c(T) = \mathbb{P}(SINR(r) > T) \quad (10)$$

in which $SINR(r)$ is the instantaneous SINR of the user u at a distance r from its serving BS and can be obtained by:

$$SINR(r) = \frac{P g r^{-\alpha}}{I_u + \sigma^2} \quad (11)$$

in which I_u is defined in Equation 7; g is the channel power gain from the user u to its serving BS; σ^2 is Gaussian noise.

Since, the expected values of a positive variable x is defined as $E[x] = \int_0^\infty x f_X(x) dx$,

$$P_c(T) = \int_0^\infty \mathbb{P}(T|r) f_R(r) dr = 2\pi\lambda \int_0^\infty r \mathbb{P}(T|r) \exp\left(-\pi\lambda r^2\right) dr \quad (12)$$

where $f_R(r)$ is defined in Equation 1; $\mathbb{P}(T|r) = \mathbb{P}(SINR > T|r)$ is the coverage probability of a user at the distance r from its serving BS.

Lemma 3.1: The coverage probability of the typical user at the distance r from its serving BS is given by

$$\mathbb{P}(T|r) = \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \exp\left[-\frac{T r^\alpha}{\gamma(a_n)} \frac{1}{SNR}\right] \exp\left\{-\pi r^2 \left[\epsilon^{(c)} \lambda_C f_I(T, n, 1) + \epsilon^{(e)} \lambda_E f_I(T, n, \phi)\right]\right\} \quad (13)$$

where

$$f_I(T, n, \phi) = \sum_{n_1=1}^{N_p} \frac{\omega_{n_1}}{\sqrt{\pi}} \left(\frac{2}{\alpha} f(T, n, \phi)^{\frac{2}{\alpha}} \frac{\pi}{\sin\left(\frac{\pi(\alpha-2)}{\alpha}\right)} - \sum_{n_{GL}=1}^{N_{GL}} \frac{c_{n_{GL}}}{2} \frac{f(T, n, \phi)}{f(T, n, \phi) + \left(\frac{x_{n_{GL}}+1}{2}\right)^{\alpha/2}} \right)$$

and $f(T, n, \phi) = \phi T \frac{\gamma(a_{n_1})}{\gamma(a_n)}$; $\gamma(a_n) = 10^{(\sqrt{2}\sigma_z a_n + \mu_z)/10}$; ω_n and a_n, c and x are are weights and nodes of Gauss-Hermite, Gauss-Legendre rule, respectively with order N_{GL}.

Proof: See Appendix A

It is observed from Lemma 3.1 that the coverage probability of a typical user is inversely proportional to exponential function of $1/SNR$ and r for cellular network with $\sigma^2 > 0$.

Here is the coverage probability of a typical user that is served on cell-center RB. If it is served on a cell-edge RB, the coverage probability is also given by Equation 13, but in this case the SNR should be replaced by ϕSNR.

Proposition 3.2: The average coverage probability of the typical user in the PPP network is

$$P_c(T) = \sum_{n_{GL}=1}^{N_{GL}} 4\pi\lambda \frac{c_{n_{GL}}(x_{n_{GL}}+1)}{(1-x_{n_{GL}})^3} e^{-\pi\lambda\left(\frac{x_{n_{GL}}+1}{1-x_{n_{GL}}}\right)^2} \mathbb{P}\left(T|r = \frac{x_{n_{GL}}+1}{1-x_{n_{GL}}}\right) \quad (14)$$

Proof Employing the changes in variable $r = \frac{t}{1-t}$, the Equation 12 equals

$$P_c(T) = 4\phi\lambda \int_0^1 \frac{t}{(1-t)^3}\mathbb{P}(T|r = \frac{t}{1-t})e^{-\pi\lambda(t/(1-t))^2}dt$$

Using Gauss-Legendre quadrature, the Proposition 3.2 is proved.

Proposition 3.3: In special case of interference-limited network, the average coverage probability is expressed as the following equation

$$P_c(T) = \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \frac{1}{1 + \epsilon^{(c)}\frac{\Delta-1}{\Delta}f_I(T,n,1) + \epsilon^{(e)}\frac{1}{\Delta}f_I(T,n,\phi)} \tag{15}$$

where $f_I(T, n, \phi)$ is given in Lemma 3.1.

Proof When $\sigma^2 = 0$, the desired result can be obtained by evaluating the integrand in Equation 12.

When the FR factor $\Delta = 1$ is deployed or the transmit power ratio equals 1, i.e. $\phi = 1$, this expression is comparable to the corresponding results in [5].

IV. AVERAGE USER RATE

The average rate of a typical randomly user is defined as

$$\begin{aligned} R &= \mathbb{E}_R\left[\ln(SINR(r) + 1)\right] \\ &= 2\pi\lambda \int_0^\infty r\mathbb{R}(r)\exp\left(-\pi\lambda r^2\right)dr \end{aligned} \tag{16}$$

where $SINR$ is the received SINR at the user u given in Equation 11; $\mathbb{R}(r)$ is the average rate of the typical user at the distance r from its serving BS (see Appendix B)

$$\mathbb{R}(r) = \int_0^\infty \mathbb{P}_c(T = e^t - 1|r)dt$$

where $\mathbb{P}_c(T|r)$ is given in Lemma 3.1 Hence, the average is obtained by

$$\begin{aligned} R &= 2\pi\lambda \int_0^\infty r\mathbb{P}(T = e^t - 1|r)\exp(-\pi\lambda r^2)dr \\ &= \int_0^\infty P_c(T = e^t - 1)dt \end{aligned} \tag{17}$$

In the special case of the interference-limited network and reuse factor $\Delta = 1$, then the average rate can be simply given by:

$$R = \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \int_0^\infty \frac{1}{1 + f_I(T,n,1)}dt \tag{18}$$

in which $f_I(t, n, 1)$ is given in Lemma 3.1. This is not the closed-form expression of average rate, but it can be evaluated by simple numerical techniques or approximation quadratures.

V. SIMULATION AND DISCUSSION

In this section, numerical method and Monte Carlo simulations are used to validate the theoretical analysis and to visualize the impact of the parameters such as number of RBs and users, the transmission SNR, and FR factor Δ on the network performance.

In simulation, it was assumed that the network model covers service area with a radius of $R(km)$ and s area of $\pi R^2(km^2)$. Hence, the number of BSs is $\pi\lambda R^2$ in which $\frac{\pi\lambda(\Delta-1}{\Delta}R^2$ BSs are transmitting at a lower power level, i.e. P, and $\frac{\pi\lambda}{\Delta}R^2$ BSs are transmitting at a higher power level, i.e. ϕP. It is interesting to note that when R is large enough, for example in this work $R > 30km$, the simulation results are consistent with the changes of R.

It was assumed that the network is allocated 30 RBs of which 10 RBs are allocated to the cell-edge area and 20 RBs are allocated to the cell-center area. From Equation 6, the number of cell-center and cell-edge users are 10ϵ and 20ϵ, respectively. The analytical and simulation parameters that are used are summarised in the Table I.

TABLE I
ANALYTICAL AND SIMULATION PARAMETERS

Parameter	Value
Density of BSs	$\lambda = 0.25$
Power ratio	$\phi = 10$
Number of RBs	$N = 30$
- Number of cell-center RBs	$N_c = 20$
- Number of cell-edge RBs	$N_e = 10$
Path loss exponent	$\alpha = 3.5$

To generate the simulation results shown in subsequent figures, 10^4 network scenarios are generated in which the number of BSs and their locations follow a Poisson distribution with a density λ. In each scenario, the received instantaneous SINR at the user is calculated and compared with the coverage threshold. If the SINR is greater than the coverage threshold, the user will be selected to be under coverage of the network and the coverage event will be counted. Finally, the coverage probability is calculated as a ratio of the number of occurrences of coverage events and number of scenarios.

In the simulation result figures given below, the solid lines represent the results of theoretical analysis which match quite well with the dotted lines that represent the simulation results. These results confirm the accuracy of theoretical analysis.

Figure 2 indicates that the strong effect of the SINR threshold which represents the sensitivity of user devices on the coverage probability. It is observed from this figure that if the sensitivity of user equipment increased by around a factor of 2.5 , (for example 0dB to -4dB), the coverage probability increased by 40% when SNR at the transmitter is $SNR = 0$ dB or 10 dB.

A. Frequency Reuse factor

In the worst case scenario, the typical user is affected by all neighbouring BSs. However, in the case of $\Delta = 1$, all interfering BSs transmit at a low power level, i.e. a cell-center

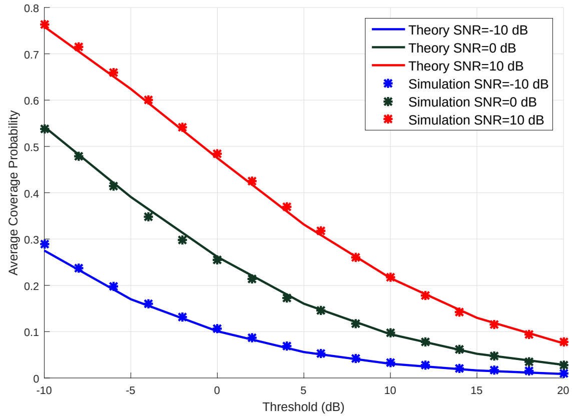

Fig. 2. Coverage probability with different values of SNR and the threshold T

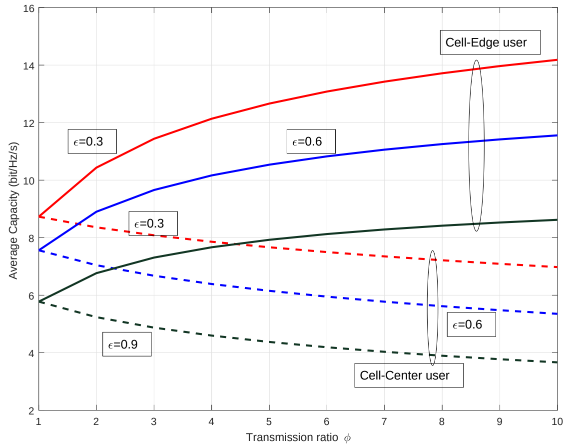

Fig. 4. Average capacity with different values of the power ratio ϕ

power level while in the case of $\Delta > 1$, some of them transmit at a high power level, i.e. cell-edge power level. Hence, the network system with a FR factor $\Delta = 1$ provides a better coverage probability compared to that with FR factor $\Delta > 1$ as shown in Figure 3. This is consistent with the fact that the Soft FR with $\Delta > 1$ can create more intercell interference on both a cell-edge and cell-center user when compared to Strict FR or Soft FR with $\Delta = 1$.

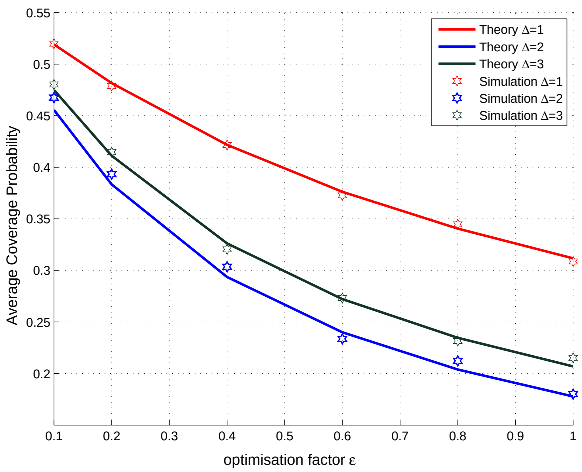

Fig. 3. Average coverage probability with different values of frequency reuse factor Δ

B. Transmission ratio

In Figure 4, the relationship between the average capacity of the typical user and the ratio between transmit power on a Cell-Edge and Cell-Center RB is presented. It is observed from the figure that there is the opposite trend between the capacity of the Cell-Center and Cell-Edge user. When the transmit power ratio $\phi = 1$ that means all users is served with the same power, the capacities of the Cell-Center and Cell-Edge user are the same and equal 8.73 (bit/Hz/s) if $\epsilon = 0.3$ and 7.651 if $\epsilon = 0.6$. When $\epsilon = 0.3$ and the transmit power rato increase by 5 times from 1 to 5, the capacity of Cell-Center user increase significantly by 31.04% to 12.66 (bit/Hz/s) while the capacity of Cell-Center user reduces by 13% to 7.666 (bit/Hz/s). Hence, in order to design the transmit power ratio for a network, there should be a balance between the performance of the Cell-Edge and Cell-Center user.

C. Power of Lognormal fading and path loss exponent

It is noticed from Figure 3 that the coverage probability significantly reduces when the ratio between number of users and RBs increases. For example, when this ratio doubles from 0.2 to 0.4, the coverage probability dropped by 20% in the case $\Delta = 3$ and around 28% in the case $\Delta = 1$.

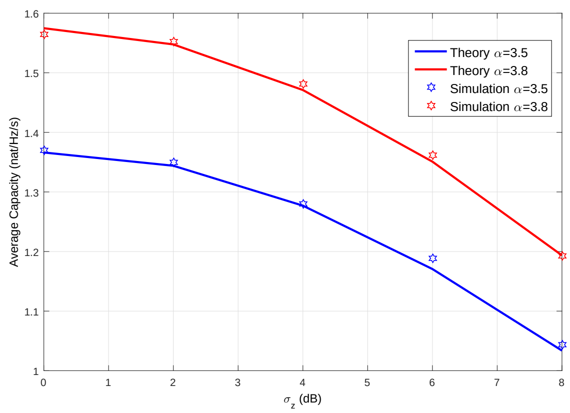

Fig. 5. Capacity with different values of path loss exponent α and Rayleigh-Lognormal variance σ_z

With higher α, total power of interfering signals sees a faster decrease rate over distance than desired signal since the user receives only one useful beam from its serving BS and usually suffers more than one interfering beams. The coverage probability is, hence, inversely proportional to path loss exponential coefficient as shown in Figure 5.

VI. Conclusion

In this paper, the impact of FR factor Δ and the number of users as well as RBs on the network performance in Rayleigh-Lognormal fading channel are presented. The results achieved are comparable with the corresponding results in published works that are only for a reuse factor $\Delta = 1$ and under Rayleigh fading. The analytical result indicates that the coverage is proportional to the FR factor Δ when $\Delta > 1$ and inversely proportional to the ratio of the users to RBs. Furthermore, when $\Delta > 1$, Soft FR created more intercell interference to the users than that with $\Delta = 1$.

VII. Appendix A

The coverage probability in Equation 10 is evaluated by following steps:

$$\begin{aligned}
&\mathbb{P}_c(T|r)\\
&= \mathbb{P}(SINR > T)\\
&= \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \mathbb{E}\left[\exp\left(-\frac{Tr^{\alpha}(I_u+\sigma^2)}{P\gamma(a_n)}\right)\right]\\
&= \sum_{n=1}^{N_p} \frac{\omega_n}{\sqrt{\pi}} \left[\exp\left(-\frac{Tr^{\alpha}}{\gamma(a_n)}\frac{1}{SNR}\right)\mathbb{E}\left\{\exp\left(-\frac{Tr^{\alpha}I_u}{P\gamma(a_n)}\right)\right\}\right]
\end{aligned} \tag{19}$$

in which $SNR = \frac{P}{\sigma^2}$.

Considering the expectation and substituting Equation 7, we obtain

$$\begin{aligned}
&\mathbb{E}\left\{\exp\left(-\frac{Tr^{\alpha}}{P\gamma(a_n)}I_u\right)\right\}\\
=&\mathbb{E}\left\{\prod_{z_c\in\theta_C} 1(z_c=b)\exp\left(-f(T,n,1)g_{z_c}\frac{r_{z_e}{}^{-\alpha}}{r^{-\alpha}}\right)\right\}\\
&\mathbb{E}\left\{\prod_{z_e\in\theta_E} 1(z_e=b)\exp\left(-f(T,n,\phi)g_{z_e}\frac{r_{z_e}{}^{-\alpha}}{r^{-\alpha}}\right)\right\}\\
=&\mathbb{E}_C \text{ x } \mathbb{E}_E
\end{aligned}$$

in which $f(T,n,\phi) = \phi T\frac{\gamma(a_{n_1})}{\gamma(a_n)}$. Evaluating the fist group product $\mathbb{E}_C$, we have

$$\mathbb{E}_C = \mathbb{E}\left\{\prod_{z_c\in\theta_C}\epsilon^{(c)}\mathbb{E}_{g_z}\left[\exp\left(-f(T,n,1)g_z\left(\frac{r_{z_e}}{r}\right)^{-\alpha}\right)\right]\right\}$$

Since g_z is Rayleigh-Lognormal fading channel then

$$=\mathbb{E}\left\{\prod_{z_e\in\theta_C}\epsilon^{(c)}\sum_{n_1=1}^{N_p}\frac{\omega_{n_1}}{\sqrt{\pi}}\frac{1}{1+f(T,n,1)\left(\frac{r_{z_e}}{r}\right)^{-\alpha}}\right\}$$

Using the properties of PPP generating function. Hence, the expectation equals:

$$=\exp\left(-2\pi\lambda^{(c)}\epsilon^{(c)}\int_r^{\infty}1-\frac{1}{1+f(T,n,1)\left(\frac{r_{z_e}}{r}\right)^{-\alpha}}\right)dr$$

The integral can be evaluated by using the properties of Gamma function and Gauss-Legendre rule as in [10], then

$$E_C = \exp\left(-\pi\lambda_C r^2\epsilon^{(c)}f_I(T,n,1)\right) \tag{20}$$

in which

$$\begin{aligned}
f_I(T,n,1) = \sum_{n_1=1}^{N_p}\frac{\omega_{n_1}}{\sqrt{\pi}}\Bigg(&\frac{2}{\alpha}f(T,n,1)^{\frac{2}{\alpha}}\frac{\pi}{\sin\left(\frac{\pi(\alpha-2)}{\alpha}\right)}\\
&-\sum_{n_{GL}=1}^{N_{GL}}\frac{c_{n_{GL}}}{2}\frac{f(T,n,1)}{f(T,n,1)+\left(\frac{x_{n_{GL}}+1}{2}\right)^{\alpha/2}}\Bigg)
\end{aligned} \tag{21}$$

Similarly, $\mathbb{E}_E$ is achieved by

$$\mathbb{E}_E = \exp\left(-\pi\lambda_E r^2\epsilon^{(e)}f_I(T,n,\phi)\right) \tag{22}$$

Substituting Equation 20 and 22 into Equation 19, the Theorem is proved.

VIII. Appendix B

The average rate of the typical user in this case is

$$\begin{aligned}
\mathbb{E}\left[\ln(1+SINR(r))\right] &= \int_0^{\infty}\mathbb{P}\left[\ln(1+SINR(r))>t\right]dt\\
&= \int_0^{\infty}\mathbb{P}\left[SINR(r)>e^t-1\right]dt\\
&= \int_0^{\infty}\mathbb{P}_c(T=e^t-1|r)dt
\end{aligned}$$

The Lemma is proved.

References

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