NEW ZEALAND JOURNAL OF MATHEMATICS
Volume 32 Supplementary Issue (2003), 135–144

WHAT ON EARTH IS SUSTAINABILITY IN MATHEMATICS?

PETER PETOCZ AND ANNA REID

Abstract. We examine the problems of integrating issues of sustainability within the mainstream curriculum of university mathematics, taking our cue from the discussion at the Johannesburg Earth Summit. Our approach to the problem is to synthesise various strands of our research programme, combining information about lecturers' conceptions of sustainability in the curriculum, children's notions about environment, and students' ideas about the nature of mathematics. We suggest ways in which university mathematics teachers can provide a quality learning experience for their students that includes notions of sustainability within the mathematical context, without compromising the mathematical content of their courses.

1. Introduction

Between the Warthog Delta conference in 2001 and the Remarkable Delta conference in 2003, an important event has taken place. At the Johannesburg Earth Summit in September 2002, recognising the vital importance of education for sustainable development, a plea was made to the academic community to integrate issues of sustainability within mainstream curriculum in order to prepare students for the needs of the Earth in the twenty-first century. Some of the relevant recommendations published in the report [**18**] are:

> 1. We, the representatives of the peoples of the world, assembled at the World Summit on Sustainable Development in Johannesburg, South Africa, from 2 to 4 September 2002, reaffirm our commitment to sustainable development.
> 5. Accordingly, we assume collective a responsibility to advance and strengthen the interdependent and mutually reinforcing pillars of sustainable development—economic development, social development and environmental protection—at the local, national, regional and global levels.
> 11. We recognise that poverty eradication, changing consumption and production patterns and protecting and managing the natural resource base for economic and social development are overarching objectives of and essential requirements for sustainable development.
> 116. Education is critical for promoting sustainable development.
> 121. Integrate sustainable development into educational systems at all levels of education in order to promote education as a key agent for change.
> 124. Support the use of education to promote sustainable development, including ... (d) recommend to the United Nations General Assembly that it consider adopting a decade of education for sustainable development, starting in 2005.

Key words and phrases: Sustainability, mathematics education, curriculum, conceptions.

Most universities in the Western World are signatories to agreements that express their commitment to the notion of sustainable development: for example, the University of Technology, Sydney (UTS) signed the Talloires Declaration in 1998 [**15**], and on the basis of that, developed a "Sustainability Policy" [**19**]:

> UTS notes with concern the growing body of evidence indicating that human activities are threatening the life support systems on Earth. The composition of the atmosphere is being altered, soils are being lost, species are being threatened or extinguished at an accelerated rate, and air and water are being polluted. These changes, together with massive upheavals in world economies, are challenging ecological integrity, social stability, economic security, cultural diversity and human rights. The University recognises a special responsibility to apply its intellectual resources to ameliorating, even reversing, these trends by identifying, verifying and promoting policies, mechanisms and procedures which lead to sustainability in all aspects of human endeavour. This means that the University will ensure that in its curricula, teaching practices, research and consulting, community service activities, institutional practices, emphasis will be given to ensuring that UTS demonstrates and promotes the achievement of sustainable futures embracing ecological, economic and social aspects of human existence.

Such commitments have often led to peripheral improvements such as the greening of university campuses, or to more substantial improvements such as the setting up of institutes or departments that focus on the problems of sustainability (UTS has the *Institute for Sustainable Futures* and Macquarie University, Sydney has a *Graduate School of the Environment*). However, it seems harder to actually integrate issues of sustainability into all aspects of the institutional practices of universities, and particularly the curriculum.

In an area such as mathematics, it seems difficult to conceive of how this could be done, a view that is supported by the paucity of available discussion and materials. Even a search of the internet reveals the dimensions of the problem. Looking for "mathematics" and "sustainability" (using Google) results in almost 60,000 documents, mostly syllabuses from educational institutions in which courses in sustainability and courses in mathematics are listed. There is only a very small number of documents talking about the incorporation of sustainability into the mathematics curriculum. As an example, here is the (complete) section on mathematics from a guide for "integrating environmental responsibility into curricula" developed at another Australian university [**6**]:

> *Mathematics*: Understanding, estimating and calculating using contemporary environmental data and environmental trends and developments. Developing mathematical skills using case studies from the local environment. Surveying a range of mathematical models used by industry and government in the estimation of environmental impacts. *Example*: According to [one lecturer at] UNSW, "I already teach the fundamental sustainability issue whenever I teach first-year mathematics. Basically, the issue is that exponential growth is not sustainable in any finite system. Really, that is the fundamental point. The point is well illustrated and expanded on by considering various models for population growth." (p. 29)

Another example is a recent survey of education for sustainability [**17**], whose only substantive reference to mathematics is to a statement from the (US) National Commission on Mathematics and Science Teaching for the 21^{st} Century:

> The future well-being of the United States and it people depends not just on how well its children are educated generally, but on how well they are educated in mathematics and science specifically. From mathematics and the sciences, the commission argued, will come the products, services standard of living, and economic and military security that will sustain the country at home and around the world – reflecting the worldview that all problems are amenable to technical solutions and that one country must maintain an economic and military superiority over other countries. (p. 190)

2. What is Sustainability?

The survey in the previous section prompts us to consider the question "What is sustainability or sustainable development?" and in particular— from our point of view—"How is it viewed by academics and their students?" The official and institutional definitions are contained in the quotations above: paragraph 5 of the Earth Summit report, and the first paragraph of the UTS Sustainability Policy point to the environmental, economic and social aspects of the notion of sustainability.

As part of a recent study funded by *Environment Australia*, we have found that academics in a wide range of subject areas (including mathematics) understand the ideas of sustainability in the curriculum in a variety of different ways [**16**]. The context of the study is a series of interviews with university academics involved in teaching at the postgraduate level, asking them a series of questions about sustainability and their teaching. The transcripts of the interviews were analysed using a phenomenographic approach [**9**], focusing on the critically *different* aspects of people's views in a particular situation. It is these differences that make one way of seeing a phenomenon such as sustainability qualitatively different from another.

We isolated two important dimensions. Teaching (in the context of sustainability) describes the actions and aspects that the academics have control over. There are three qualitatively different conceptions, listed here from the narrowest to the broadest:

- Disparate—teaching and sustainability are seen as two completely unrelated ideas; teaching focuses on content of subject and "covering" a syllabus, sustainability is seen as keeping something going, or the "green" approach.
- Overlapping—the notion of sustainability overlaps to an extent with the activity of teaching; teaching is seen as ensuring that students understand the substantive content of the course; teachers see that specific ideas such as environmental or cultural sustainability can be incorporated in their teaching (as examples) but only to the extent that the situation allows.
- Integrated—sustainability in all its guises is an essential component of teaching; teaching is seen as encouraging students to make a personal commitment to the area represented by course content, incorporating the notion of sustainability.

Sustainability (in the context of teaching) describes the ideas or thinking (rather than actions) that academics could have. Again, there are three qualitatively different conceptions, listed from narrowest to broadest:

- Distance—sustainability is approached via a definition, maybe a dictionary definition of "keeping something going", but essentially to keep the concept at a distance and avoid engagement with it.
- Resources—sustainability is approached by focusing on various resources, either material resources (minerals, water, soil), or biological (fish, crops), or human (minority languages, populations, economies).
- Justice—sustainability is approached by focusing on the notion of "fairness" from one generation to the following one, or even within one generation; the idea is that sustainability can essentially only happen under these conditions.

In each dimension, the conceptions are hierarchical; lecturers whose characteristic approach to sustainability was via the broader conceptions were also aware of the features of the narrower, more atomistic conceptions, and were able to use them when necessary in their discussions. However, this was not the case in the other direction; those lecturers who thought of sustainability using the narrower conceptions gave no indication of being able to use the broader, more holistic conceptions. It is in this sense that the narrowest conceptions are described as "limiting" and the broadest conceptions as "expansive". Such information is vital for addressing problems of sustainability in an educational setting. Institutional policy is implemented by individual academics, who work to incorporate ideas of sustainability and sustainable development into their teaching and help students develop an appreciation of the issues. The approaches identified in this study are presently being used to inform an action research project investigating ways in which academics can do this.

Further studies are needed to examine how students view idea of sustainability, and what effects these views have on their learning. In one component of sustainability—the notion of environment—we have described an important qualitative difference between conceptions that focus on the environment as an object, a place possibly including living things and even people, and conceptions that focus on the relation between people and the environment, whether environment sustaining people, or people caring for the environment, or a combination of both ideas [**7**]. The conceptions were isolated in the context of a large survey b of school children's ideas about the environment. Statistical analysis revealed that only a minority of children (about 1 in 8) held a "relation" conception, and that primary school children were between five and six times as likely as high school children to hold a "relation" rather than an "object" conception of environment [**8**]. These findings present a disturbing picture of the effects of current efforts in environmental education, quite at odds with the hopes espoused at the Earth Summit, and imply that other aspects of education in sustainability are also likely to need attention.

3. What is Mathematics?

Our discussion now moves to mathematics. In order to plan more effective curriculum in mathematics that incorporates issues of sustainability, it seems important to examine students' and lecturers' ideas of the nature of their subject. We are presently carrying out a research project investigating mathematics students' ideas about mathematics and working as professionals in the mathematical sciences, and the impact of these ideas on their learning of mathematics. An initial report on the findings is presented in another paper in this journal [**14**]. A phenomenographic analysis of the transcripts of interviews with 22 undergraduate students majoring in the mathematical sciences revealed three qualitatively different conceptions of mathematics, which we have arranged from the narrowest and most limiting to the broadest and most holistic view:

- Components—here, students see mathematics as made up of individual components; they focus their attention on disparate mathematical activities or aspects of mathematics, including the notion of calculation, interpreted in the widest sense; nevertheless, these components are viewed as part of a mathematical investigation, for instance, the statistical components of a census of population.
- Models—here, students see mathematics as being about building and using models, translating some aspect of reality into mathematical form; in some cases, such models are representations of specific situations, such as a production line; in other cases, the models are universal principles, such as the law of gravity.
- Life—here, students view mathematics as an approach to life and a way of thinking. They believe that reality can be represented in mathematical terms and their way of thinking about reality is mediated by mathematics. They make a strong personal connection between mathematics and their own lives.

We carried out an earlier investigation into students' conceptions of *statistics*, a component of our mathematics degree, and a possible area of specialisation for them [**13**]. In this area, we found a conception of statistics (and hence of an area of mathematics) that we would place before the first conception (Components) in our list.

- Techniques—here, students focus on mathematics or statistics as consisting of isolated mathematical or statistical techniques, without any connection between the techniques and their meaningful use in mathematical or statistical problems.

This is supported by the results of an earlier study of first-year students who were for the most part not planning to major in mathematics [**2**], although the authors described their narrowest conception as "numbers, rules and formulae". Again, the conceptions are hierarchical: the narrowest conceptions are limiting precisely because they result in a student approaching mathematics by focusing only on its techniques or components. The broadest conceptions are expansive, in that they allow students to look at mathematics using the full range of approaches, including techniques and components, the notion of modelling, and the idea that mathematics is an holistic approach to life in general.

These investigations report on students' ideas about mathematics, essential knowledge for developing mathematics curriculum that incorporates notions of sustainability. However, it is important to note that professional mathematicians, statisticians and academics in these fields express views that are broadly consistent with those of students [**5**, **13**], and this seems true of other subject areas [**12**].

4. What is Sustainability in Mathematics?

Having described the various conceptions of sustainability and of mathematics, in their nature limited or expansive, we can now address the problem of integrating issues of sustainability within mainstream curriculum in mathematics. Essentially, our approach is to look at the possibilities afforded by the various levels of conceptions of sustainability and mathematics. At this stage, we move into speculation, however, this speculation is supported by a small number of interviews addressing issues of sustainability with mathematics lecturers, and a large amount of experience with our own and our colleagues' teaching of mathematics. It becomes obvious that the narrowest conceptions of both areas limit the opportunities for incorporating sustainability in mathematics classes, while the broadest conceptions allow scope for a whole range of opportunities.

If a student of mathematics holds a narrow conception of mathematics as "techniques" or "components", they will focus their study of mathematics on acquiring, practicing and perfecting such techniques or components. If a teacher or lecturer views mathematics in this way, they will spend their mathematics classes on "drill and practice" of basic techniques, an approach that can sound more familiar from discussions of school mathematics or "servicing" mathematics classes. However, it is also the guiding principle behind concern over "covering" a syllabus and making sure that students are able to reproduce the material (in an examination, for instance). The corresponding approach to sustainability is likely to be "distance", avoidance of engagement with ideas that seem to be "disparate", unconnected with the mathematics. Lecturers may acknowledge the importance of sustainability but maintain that it should be 'taught' somewhere else, and not in their own lectures and tutorials where it will take valuable time away from the real mathematics. Students will agree, and encourage their lecturers to focus on material that is "assessable" or "on the examination". This is a familiar cluster of views about mathematics and its relationships with other areas of learning and with life in general.

If a student views mathematics as "models", the second conception that we have described, they will necessarily engage with various aspects of the reality around them, building and using mathematical models of this reality. Then there is scope for a lecturer to use an "overlapping" approach to teaching sustainability in a mathematical context, focusing the notion of sustainability on "resources", which provides a large source of examples and models that can be examined mathematically. Thus, at this second level, students can examine issues of sustainability of (say) biological populations by looking at mathematical population models ranging from a model of exponential growth to complex models of interaction between predators and their prey. Students will be interested to see applications of the mathematics that they are studying, and will be encouraged to discuss ideas of sustainability in the context of the mathematics they are learning.

One example is a group assignment (1–3 students) set by one of our colleagues (Leigh Wood) in a course on differential equations:

> Assignment: Investigate and summarise models using first-order systems for the interaction between two species. It is expected that you will find at least eight models. Each model could contain: historical note, applications, equations and explanation of terms, phase portrait, judgement of the value of the model. Prepare a poster to present your findings to the rest of the class.

The assignment stimulated substantial interest in the topic, including the ecological as well as the mathematical aspects, and when the resulting posters were displayed around the department they were read by other students and staff. The explicit marking criteria included aspects of the mathematics, presentation of the poster, originality and discussion of the broader context of each example.

A second example is another group assignment set by one of the authors in his course on regression analysis. Students were asked to examine a set of economic data published by *The Economist* on 45 world cities, giving the cost (in minutes of labour) of a *Big Mac* and various other economic indicators (including a primary teacher's salary and tax rate). The final part of the question asked students to write a short summary of their findings for the university magazine. One group of students wrote this as part of their response:

> One student who assisted with the assignment commented: "A person would have to work just over two hours in Lagos to buy a Big Mac. If the average wage in Australia was $10 then relatively a Big Mac would cost over $20. That's incredible. You might as well eat a Big Mac with a knife and fork at that price. What would have to be the state of the country for a person's income to be so low? Poverty is widespread and an ever increasing concern for all of us." The assignments done by the students enabled them to see outside their own country and analyse the situations of other countries in depth.

A third example concerns a series of projects in mathematics used as learning materials in preparatory courses [**10**]. The projects focus on various topics in mathematics, and allow groups of students to investigate areas of interest to them. Here is what one student wrote in her "reflections" on her study of the Greenhouse effect:

> The reason I selected this project initially was simply due to my genuine interest and concern in our changing environment. I was further drawn to this option as I was most curious to know how mathematics came into the Greenhouse equation. I must say that I got a lot more than I bargained for. As interesting as it was to learn more about greenhouse gases, the most satisfaction came from learning so much more about the importance and value of maths. ... But obviously, the bottom line is moderation, and this does rely quite a lot on the human race. Educating people seems the best solution. Perhaps showing society these mathematical figures achieved in the project may open their eyes: they've certainly opened mine.

This example (and the previous ones) shows that it is quite possible to incorporate notions of sustainability into mathematics courses without compromising the mathematical content. Indeed, it seems that such examples can enhance our students' understanding of and connection with the mathematics that they are studying.

If a student views mathematics as an approach to life and a way of thinking, the broadest and most holistic conception that we found in our interviews, then they will already be making a strong personal connection between mathematics and aspects of their own lives. They will need little encouragement from their teachers to include ideas of sustainability in this connection between their academic study and their view of the world. Lecturers can present a holistic approach to mathematics, coupled with an "integrated" approach to sustainability, including notions of "justice" as well as "resources" (using the hierarchical nature of the conceptions). In the first instance, this can be done by extending the use of mathematical models to a wider range of situations, and highlighting aspects of sustainability in as many of these situations as possible. The cumulative effect of such an approach in a particular subject, or even a whole mathematics course, can be dramatic.

Sometimes, though, it seems necessary to move beyond this and acknowledge that the mathematics itself takes second place to important aspects of our and our students' lives. Galpin [**4**] discusses the problem of teaching statistics in a context where a substantial proportion of her students are likely to be infected with HIV/AIDS, and those that aren't are quite likely to become infected unless they take precautionary measures. In such a situation, it seems much more important to discuss topics in statistics related to AIDS, and investigate official government statistics (and their reliability) than to continue through the usual sequence of statistical topics. Other examples of such an approach in the context of ethnomathematics [**3**] and peace education [**1**] show that a coherent approach to mathematics can be presented from a social justice perspective.

5. Conclusion

We have argued that the broader and more holistic conceptions of mathematics allow room for integration with the broader conceptions of sustainability, while the narrower and more atomistic conceptions seem to preclude any serious engagement with the ideas. Happily, there is evidence that students' conceptions of a subject can be broadened by appropriate pedagogy [**11, 13**]. It is obviously important to develop learning environments that encourage students to use the broader conceptions, and to look for meaning in the mathematics and relate this meaning to their own personal situations. It is also true, however, that a limiting approach to pedagogy can also encourage students with broader, more holistic conceptions to engage with their course of study using a more limiting approach. This can happen, for example, in mathematics subjects where the assessment (which can include an examination) concentrates on the memorisation and reproduction of theorems and proofs, without any attempt at assessing students' deeper understanding.

To conclude, here is a quote from a student who does seem to hold the broadest conception of mathematics, as a way of thinking and an approach to life. His view of mathematics comes across clearly, despite the obvious fact that he is (at least) bilingual:

> *Joseph:* Some students are very hate the mathematics, you know, some students love the mathematics, because you have to use your brain all the time, you know, to solve the problem, Anyway if you want to solve any problem,

> anyway you have to use your brain, if you didn't use your brain for a long time, so its difficult to solve any problems. So that's why I enjoy then using my brain all the time, that's why I love mathematics, because if you do mathematics, you have to use your brain all the time, all the time. ... Yeah, so if you have like problem in your family, or in your friends or something like that, you have to use your brain how to solve this problem. I have a lot of experience in solving the problem, because I love mathematics very, very much, that's why I always enjoy to do mathematics questions, it's like chess playing, I want to use my brain all the time, and then if I have a problem with my family or with my friend or with whatever, I can use my brain how to solve that problem, systematically. So that benefit come from, I think that benefit come from mathematics, you know, because if you use your brain all the time, it's very good.

If Joseph can view mathematics as a way of solving problems with his family or friends, systematically, then it seems possible to extend his approach to solving a broader range of problems, even those involving the planet, and to incorporate the broadest ideas of sustainability in his study of mathematics.

References

1. U. D'Ambrosio, *Mathematics and peace: Our responsibilities*, Zentralblatt fur Didaktik der Mathematik, **30**(3) (1998), 67–73.
2. K. Crawford, S. Gordon, J. Nicholas, and M. Prosser, *Conceptions of mathematics and how it is learned: The perspectives of students entering university*, Learning and Instruction, **4** (1994), 331–345.
3. M. Frankenstein and A. B. Powell (Eds.), *Ethnomathematics: Challenging Eurocentrism in mathematics education*, State University of New York Press, 1997.
4. J. Galpin, *Does statistical education include social issues?*, In B. Phillips, (Ed.), Proceedings of the Sixth International Conference on Teaching Statistics, ICOTS6, Capetown, International Association for Statistics Education, 2002.
5. K. Houston, *Teaching modelling as a way of life*, Quaestiones Mathematicae, Supplement **1** (2001), 105–113.
6. Institute for Environmental Studies, *Education for Sustainability: Integrating environmental responsibility into curricula*, University of New South Wales, 1999, online `www.ies.unsw.edu.au/Documents/EducationforSustainability.pdf`.
7. T. Loughland, A. Reid, and P. Petocz, *Young people's conceptions of environment: A phenomenographic analysis*, Environmental Education Research, **8**(2) (2002), 187–197.
8. T. Loughland, A. Reid, K. Walker, and P. Petocz, *Factors influencing young people's conceptions of environment*. Environmental Education Research, **9**(1) (2003), 3–19.
9. F. Marton and S. Booth, *Learning and awareness*, Lawrence Erlbaum, NJ, 1997.
10. P. Petocz, D. Petocz, and L. N. Wood, *Introductory mathematics*, Thomas Nelson, Melbourne, 1992.

11. A. Reid, *The hierarchical nature of meaning in music and the understanding of teaching and learning*, Advancing International Perspectives, **20** (1997), 626–31.
12. A. Reid, *Musicians' experience of the musical world: Relations with teaching and learning*, In C. Rust (Ed.), Improving student learning through the disciplines, Oxford Brookes, Oxford, 2000.
13. A. Reid and P. Petocz, *Students' conceptions of statistics: A phenomenographic study*, Journal of Statistics Education, **10**(2), 2002.
14. A. Reid, P. Petocz, G. H. Smith, L. N. Wood, and E. Dortins, *Maths students' conceptions of mathematics*, Proceedings of Delta'03 Symposium on Undergraduate Mathematics, Queenstown, New Zealand, 2003.
15. Talloires Declaration, University Presidents for a Sustainable Future, Oct. 1988, online `iisd1.iisd.ca/educate/declarat/talloire.htm`.
16. D. Tilbury and A. Reid *Action research for university staff: Change in curricula and graduate skills towards sustainability. Stage 1 Report.* Environment Australia, 2003.
17. D. Tilbury, R. B. Stevenson, J. Fein, and D. Schrender (Eds.), *Education and sustainability: Responding to the global challenge*, IUCN, Gland, Switzerland, 2002.
18. United Nations, *Report of the world summit on sustainable development*, New York, 2002, online `www.johannesburgsummit.org/html/documents/documents.html`
19. University of Technology, Sydney, *Sustainability policy*, online accessed 19/6/03, `www.sustainability.uts.edu.au/commitment/sustpolicy.html`.

Peter Petocz
University of Technology, Sydney
Australia
Peter.Petocz@uts.edu.au

Anna Reid
Macquarie University, Sydney
Australia
Anna.Reid@mq.edu.au

New Zealand Journal of Mathematics

Proceedings of Remarkable Delta:03
Fourth Southern Hemisphere Symposium on
Undergraduate Mathematics and Statistics
Teaching and Learning

Guest Editors: Michael O. J. Thomas and Greg Oates

Volume 32, Supplementary Issue November 2003

Published by the
New Zealand Mathematical Society
and
Department of Mathematics
The University of Auckland

ISSN 1171-6096

New Zealand Journal of Mathematics

Publisher

A Joint Committee of the New Zealand Mathematical Society and the Department of Mathematics, The University of Auckland.

Contact address: New Zealand Journal of Mathematics, Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland, NEW ZEALAND.

Electronic mail address: nzjmath@math.auckland.ac.nz

WWW location: http://www.math.auckland.ac.nz/NZJM/

Subscriptions Rates

Institutions:	$NZ 90.00	$US 50.00	$A 80.00
Individuals:	$NZ 40.00	$US 20.00	$A 36.00

Members of the New Zealand Mathematical Society receive the Journal at a discounted rate. Back issues for both New Zealand Journal of Mathematics and the Mathematical Chronicle are available and inquiries should be sent to the Treasurer at the address above.

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Publication dates of recent issues

Volume 26	Number 1	April 1997
Volume 26	Number 2	October 1997
Volume 27	Number 1	April 1998
Volume 27	Number 2	October 1998
Volume 28	Number 1	April 1999
Volume 28	Number 2	October 1999
Volume 29	Number 1	April 2000
Volume 29	Number 2	October 2000
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Volume 30	Number 2	October 2001
Volume 31	Number 1	April 2002
Volume 31	Number 2	October 2002
Volume 32	Number 1	April 2003
Volume 32	Number 2	October 2003

Editorial and Production Staff

The Editor of the New Zealand Journal of Mathematics is Professor G.J. Martin, and he is supported by the Executive Editor Dr J.L. Schiff and Associates Dr J. An, Ms B. Fong and Miss Min–Young Lee.

Acknowledgement

This Journal is produced with LaTeX (LaTeX 2_ε) and $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX, the TeX macro package of the American Mathematical Society. The assistance of Blue Sky Research, makers of *Textures*, is gratefully acknowledged.

Preface

The Remarkable Delta03 conference is co-hosted by the Department of Mathematics and Statistics of the University of Otago and the Department of Mathematics and the Department of Statistics of the University of Auckland. Delta03 is backed by an International Steering Committee made up of representatives from the Southern Hemisphere countries of Argentina, Australia, New Zealand, South Africa, and Uruguay. This steering committee currently consists of: Derek Holton (New Zealand, Chair), Nestor Aguilera (Argentina), Pat Cretchley (Australia), Johann Engelbrecht (South Africa), Victor Martinez Luaces (Uruguay), Jan Persens (South Africa), Ivan Reilly (New Zealand) and Christina Varavsky (Australia).

This Conference is the fourth in a series of conferences on the undergraduate teaching and learning of Mathematics and Statistics, as part of a collaboration between Southern Hemisphere countries. The first conference, Delta97 (delta implying change) was held in Brisbane, Australia, in November 1997. The second conference, Delta99, took place in November 1999 at Laguna Quays, Queensland, Australia. The third conference, Warthog Delta01, was held in Kruger National Park, South Africa, in July 2001. The Delta conferences take place biennially, and the next one, Delta05, will probably be in Australia.

The theme for Delta97 was What Can We Do to Improve Learning?, for Delta99 it was The Challenge of Diversity, and for Delta01 it was Gearing for Flexibility. For Delta03 the theme is From all Angles.

The 120 delegates, attending from 12 different countries worldwide, are responsible for about 80 contributed presentations, panel discussions and round table discussions. The conference has two publications: the Proceedings, consisting of peer reviewed research papers, and the Communications (subject to editorial scrutiny) largely comprising reports on teaching experiences and research in progress.

We believe that the deliberations at this meeting will influence the course of future tertiary mathematics and statistics education worldwide. It is broadly accepted that skills and training in the quantitative sciences will be crucial to success in the future. Thus our belief is that this meeting will be one of the most important of 2003.

Our thanks to the group who worked on editing, compiling and producing this issue of the Proceedings, especially to, Mike Thomas, Greg Oates, Olita Moala, Betty Fong and Philip Sharp.

All papers in these Proceedings have been subject to the normal process of journal peer review, including the papers from the two plenary speakers who chose to submit a paper. In all 57 papers were submitted to the conference for consideration as full papers, and only the 22 that were accepted as full papers appear in this volume.

All opinions expressed herein are those of the authors themselves, and not necessarily of the Delta03 organising committee.

Derek Holton and Ivan Reilly
Co-Convenors
Remarkable Delta:03

Acknowledgements

It is a pleasure to acknowledge the work of the team which has put together these Proceedings.

Editorial work has been handled by Mike Thomas and Greg Oates, of the Mathematics Education Unit, Department of Mathematics, The University of Auckland.

Editorial assistance has been provided by Olita Moala of the Department of Mathematics, The University of Auckland and Betty Fong, of the New Zealand Journal of Mathematics.

Special thanks to the sponsors of this conference:

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New Zealand Journal of Mathematics

VOLUME 32, Supplementary Issue November 2003