Magnetorheological Damper Semiactive Control for Civil Structures with Symmetric Quantised Sliding Mode Controllers

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A symmetric quantised sliding mode controller (SMC) is presented in this paper for semiactive control of civil structures using magnetorheological (MR) dampers. The application of high performance control is needed to suppress the induced vibrations. The SMC is used for its robustness against system uncertainties and external disturbances while the MR damper is technologically-efficient for its vibration absorption capability and fail-safe operations as an ideal semi-active device. A state space for the MR damper embedded building structure is proposed, allowing for direct control of the magnetisation current. The SMC output is quantised correspondingly to the hysteretic force-velocity relationship at given values of currents. Simulation results are included to demonstrate the effectiveness of the proposed controller in a building model under quake-like excitations.

Key Words: Structural control, MR dampers, Symmetric quantised SMC.

1. Introduction

Earthquake is one of the several disasters which frequently occurs and gives rise to a lot of damages to civil structures. In order to protect these structures including buildings and their occupants, many engineers and researchers have been attracted to the investigation and development for effective approaches in structural control.

A common strategy to mitigate structural damages is to reduce their vibration magnitudes during an earthquake. In this regard, the use of active mass dampers and semi-active dampers have been proposed and effectively operated in civil structures in more than a decade [1],[2]. Furthermore, MR dampers, as semi-active devices with the advantage of requiring little energy to operate [3], are becoming a promising candidate in structural control with the incorporation of a suitable controller.

There have been many controller design methods applicable for structural control, such as switching control, pole assignment and linear-quadratic-regulator (LQR) designs [4], [5]. In recent years, sliding mode control has been introduced to this problem domain for its robustness against structural uncertainties, disturbances, actuator non-linearities and hysteresis [6]-[8].

For the reduction of vibrations in civil structures under earthquake excitations, quantised control such as uniform quantised control or symmetric quantised control [9], [10] have been investigated. In this work, a SMC with symmetrically quantised output is developed in order to enhance the system performance.

The paper is organized as follows. In Section 2, the physical characteristics of an MR damper are briefly described. In Section 3, a model is proposed for direct control of the building structure integrated with MR dampers. In Sections 4 and 5, the SMC design and the quantisation process are presented. In Section 6, numerical simulation for the control of a civil structure with MR dampers are provided to illustrate the effectiveness of the proposed technique. Finally, Section 7 concludes the paper.

2. Magnetorheological Damper

An MR damper contains nanoscale magnetizable particles suspended in a carrier ferro-fluid. Under the application of a magnetic field, the particles are aligned in chain-like structures [11], [12], thus, producing controllable damping forces. A schematic is shown in Fig. 1. There are many types of MR damper models available such as the Bingham viscous-plastic model, the Bouc-Wen model, the modified Bouc-Wen model [13]-[16], and recently a current-dependent model [17]. The latter is adopted here for simpler system dynamics which are suitable for the control design.

The MR damper force is given as

$$f = c\dot{x} + kx + \alpha z + f_0 \qquad (1a)$$

$$z = \tanh(\beta\dot{x} + \delta sign(x)) \qquad (1b)$$

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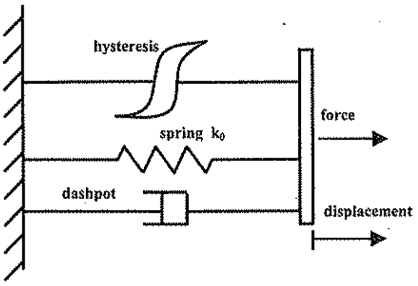

Fig. 1. Schematic of the MR

where x is the damper diaphragm displacement, f is the output force, z is the hysteresis function, f_0 is the damper force offset, β is a constant against the supplied current values, α is the scaling parameter and c, k are the viscous and stiffness coefficients. These parameters can be expressed explicitly as functions of the fluid magnetisation current, i_M [17].

3. Control of Civil Structures with MR Dampers

Consider an n-storey building embedded with r MR dampers, subject to earthquake excitation $\ddot{x}_g$, where f_i is the damper force, x_m, $\dot{x}_m$, $\ddot{x}_m$ are displacement, velocity and acceleration, m_m, c_m, k_m, $(m=1,2,...,n)$ are mass, damping and stiffness of each floor.

The equation of motion of the structure is

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{\Gamma}\mathbf{f}(t)+\mathbf{M}\mathbf{\Lambda}\ddot{x}_g(t) \quad (2)$$

where $\mathbf{x}(t)\in R^n$ is an n-vector of the displacements, $\mathbf{f}=[f_1,f_2,...,f_r]^T$, $\mathbf{f}(t)\in R^r$ is a vector consisting of the control forces, $\ddot{x}_g(t)$ is the earthquake excitation acceleration, and matrices $\mathbf{M}\in R^{n\times n}$, $\mathbf{C}\in R^{n\times n}$, $\mathbf{K}\in R^{n\times n}$ are respectively the mass, damping and stiffness. Matrix $\mathbf{\Gamma}\in R^{n\times r}$ denotes the location of r dampers, and $\mathbf{\Lambda}\in R^n$ is a vector indicating the directional influence of the earthquake excitation.

Equation (2) is rewritten in the state-space form

$$\dot{\mathbf{z}}=\mathbf{A}\mathbf{z}(t)+\mathbf{B}_0\mathbf{f}(t)+\mathbf{E}_0(t) \quad (3)$$

where $\mathbf{z}(t)\in R^{2n}$ is the state vector, $\mathbf{A}\in R^{2n\times 2n}$ is the system matrix, $\mathbf{B}_0\in R^{2n\times r}$ is a constant gain matrix and $\mathbf{E}_0(t)\in R^{2n}$ is a disturbance vector, respectively. They are given by

$$\mathbf{z}(t)=\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{bmatrix},\ \mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix} \quad (4a)$$

$$\mathbf{B}_0=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{\Gamma}\end{bmatrix},\ \mathbf{E}_0(t)=\begin{bmatrix}\mathbf{0}\\ \mathbf{\Lambda}\end{bmatrix}\ddot{x}_g(t) \quad (4b)$$

From (1a) and (1b), the force equation $\mathbf{f}\in R^r$ in (3), whereby each damper current consists of a quiescent component and an alternating component, $i_M=I_Q+i$, can be further cast as

$$\mathbf{f}=\mathbf{B}^*\mathbf{i}+\mathbf{D}^*(\mathbf{i}) \quad (5)$$

where $\mathbf{i}\in R^r$ is the vector of alternating components used for the direct control purpose, $\mathbf{D}^*\in R^r$ is a disturbance vector and $\mathbf{B}^*\in R^{r\times r}$ is a constant diagonal matrix.

Substitution of (5) into (3), the state space equation can be written as

$$\dot{\mathbf{z}}=\mathbf{A}\mathbf{z}+\mathbf{B}\mathbf{i}+\mathbf{E} \quad (6)$$

where $\mathbf{B}\in R^{2n\times r}$ is a gain matrix, $\mathbf{E}\in R^{2n}$ is the disturbance vector.

4. Sliding Mode Control

The main advantage of the SMC is its robustness against variations in system parameters or external disturbances. The selection of the control gain is related to the magnitude of uncertainty in order to keep the state trajectory on the sliding surface. For simplicity, let $\sigma\in R^r$ be an r-dimensional sliding function consisting of a linear combination of the state variables, i.e. $\sigma=\mathbf{S}\mathbf{z}$, where $\mathbf{S}\in R^{r\times 2n}$ is a matrix to be determined such that the sliding motion $\sigma=0$ possesses desired dynamics.

Assuming the availability of the state vector $\mathbf{z}(t)$, and the controllability of the system $(\mathbf{A},\mathbf{B})$, by defining a cost function

$$\mathbf{J}=\int\mathbf{z}^T\mathbf{Q}\mathbf{z}dt \quad (7)$$

then upon the choice of a positive definite matrix $\mathbf{Q}$, one can obtain the LQR gain $\mathbf{F}$ [5] and $\mathbf{S}$ is designed such that the poles of the SMC system coincide with the feedback using $\mathbf{Fz}$ [18].

Indeed, by neglecting the disturbance $\mathbf{E}$, and substitution of the equivalent control $\mathbf{i}=\mathbf{i}_e$ into the time derivative of the sliding function and from condition $\dot{\sigma}=0$, one has

$$\mathbf{i}_e=-(\mathbf{SB})^{-1}\mathbf{SAz} \quad (8)$$

Now, in order to design the switching control, let us first assume the following matching conditions:

$$\mathbf{E}=\mathbf{B}\boldsymbol{\varepsilon} \text{ and } |\varepsilon_i|<\rho_i,\ \rho_i>0 \quad (9)$$

Consider the Lyapunov function $V=0.5\sigma^T\sigma$. Substitution of (5) and (6) into $\dot{V}$, one obtains

$$\dot{V} = \sigma^T \mathbf{S}(\mathbf{Az} + \mathbf{Bi}_e) + \sigma^T \mathbf{S}(\mathbf{Bi}_s + \mathbf{E}) \qquad (10)$$

where $\mathbf{i} = \mathbf{i}_e + \mathbf{i}_s$ and $\sigma^T \mathbf{S}(\mathbf{Az} + \mathbf{Bi}_e) = 0$, one has

$$\dot{V} = \sigma^T \mathbf{SB}(\mathbf{i}_s + \boldsymbol{\varepsilon}) \qquad (11)$$

To satisfy the sliding condition $\dot{V} = \sigma^T \dot{\sigma} = \sum_{i=1}^{r} \dot{V}_i < 0$, the switching control is proposed as

$$\mathbf{i}_s = -diag(\eta_i)\mathbf{sgn}(\mathbf{B}^T \mathbf{S}^T \sigma) \qquad (12)$$

where $\eta_i > \rho_i$, and the i-th entry of vector, $\mathbf{sgn}(\mathbf{B}^T \mathbf{S}^T \sigma) \in R^r$ is $sign((\mathbf{B}^T \mathbf{S}^T \sigma)_i)$, $i = 1,2,...,r$.

5. Symmetric Quantised Sliding Mode Control

The symmetric quantised SMC, shown in Fig. 2, has a convenience in implementation in that it allows for the various magnetisation levels and if the MR damping force exceeds the demand required to suppress the disturbance influence, saturation will occur but the controller still performs well.

The i-th alternating component of the current output, i_{qi}, quantised generally into l levels, is expressed as

$$i_{qi} = k_{i(j-1)} \mu_{i(j-1)} sign(i_i) \text{ for } \mu_{i(j-1)} < |i_i| < \mu_{ij}$$
$$j = 1,2,...,l, \quad l \geq 2 \qquad (13)$$

where $k_{i(j-1)}$ are the slopes, and $\mu_{i(j-1)}$, μ_{ij} are the values of the i-th current correspond to its j-th quantisation level. Fig. 3 shows typically the output current i_{q1} with l=2.

The output of the quantiser, a current vector $\mathbf{i}_q = [i_{q1}, i_{q2}, ..., i_{qr}]^T$, is augmented with the quiescent current vector, $\mathbf{I}_Q$, to formulate the excitation current $\mathbf{i}_M$ for the MR dampers, and $k_{i(j-1)} > 0$, $\mu_{i(j-1)} > 0$, ($\mu_{i0} = 0$), $j = 1,2,...,l$ ($l \geq 2$) are quantisation parameters that should be suitable to the field magnetisation for a particular damper.

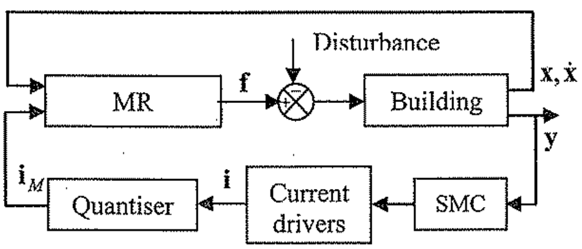

Fig. 2. Block diagram of the quantised SMC

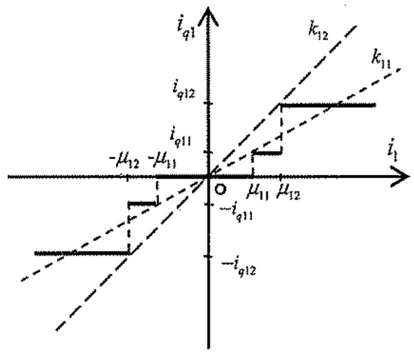

Fig. 3. Quantisation of i_1 into l=2 levels

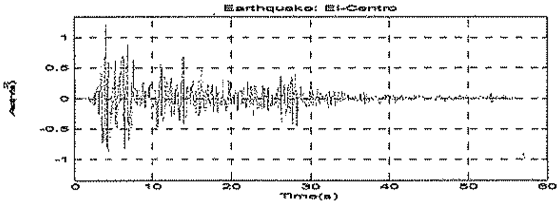

Fig. 4. Trace of El-Centro earthquake

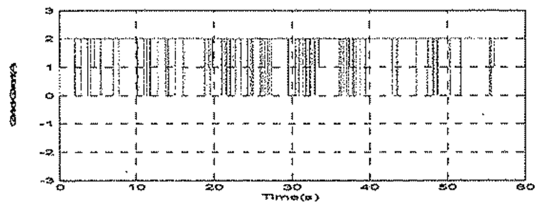

Fig. 5. The quantised current applied to MR damper

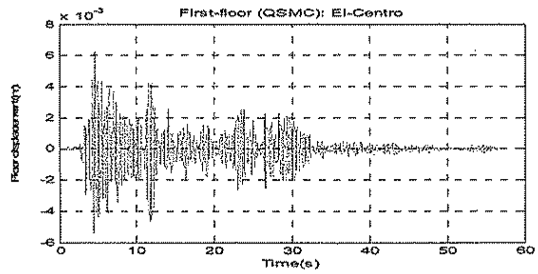

Fig. 6. The uncontrolled floor displacement

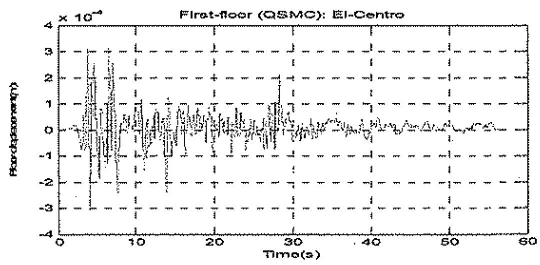

Fig. 7. The controlled floor displacement

6. Simulation Results

Consider the structure of a five-storey building model, available at UTS Structure Lab [16], which has one MR damper installed at the first floor. $\mathbf{x}=[x_1,x_2,x_3,x_4,x_5]^T$ is the displacement vector, and parameters m_m, k_m, c_m, $(m=1,2,...,5)$ are mass, damping and stiffness coefficients.

Acceleration $\ddot{x}_1$ corresponding to the MR damper installed at the building is

$$\begin{aligned}\ddot{x}_1 &= A_{01}+B_1 f_1+E_{01}\\ &= A_{01}+B_1(h_1 i_1+D_1)+E_{01}\\ &= A_{01}+B_1 h_1 i_1+E_1\end{aligned} \tag{14}$$

where $A_{01}=-m_1^{-1}((k_1+k_2)x_1-k_2x_2+(c_1+c_2)\dot{x}_1-c_2\dot{x}_2)$, $E_1=B_1D_1+E_{01}$, and E_1 is the first-floor disturbance.

Simulations were conducted to demonstrate the effectiveness of the building structure under the proposed SMC control scheme using a simple quantiser with l=2. Fig. 4 plots the earthquake excitation acceleration. Fig. 5 shows the output current after quantisation. The uncontrolled floor displacement is depicted in Fig. 6 and the controlled first floor displacement is shown in Fig. 7.

Table 1 shows the maximum (Max) and root-mean-square (RMS) values of the floor displacements resulting from the proposed controller with El-Centro earthquake excitation. The results exhibit a reduction of over 50 % of the peak magnitude and 25 % of the RMS value for the 1st floor vibration, which are comparable with other reported controllers [4], [8] while retaining the implementation convenience.

Table 1. Floor displacements from different controls

Floor No.	El-Centro Earthquake					
	Uncontrolled		SMC		Quantized SMC	
	Max (mm)	RMS (mm)	Max (mm)	RMS (mm)	Max (mm)	RMS (mm)
1	6.3	1.2	3.1	0.7	3.1	0.35
2	9.0	1.7	4.0	1.1	4.0	0.40
3	12.0	2.0	4.2	1.2	4.1	0.22
4	13.5	2.2	4.2	1.2	4.1	0.22
5	13.5	2.3	4.2	1.2	4.1	0.22

7. Conclusion

The control of civil structures using a SMC augmented by a quantiser for the current to be supplied to a MR damper has been presented. Simulation results on the response of a building model under earthquake excitations have illustrated the effectiveness of the method.

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