# Structural Parameter Design on Multilayer Conductors of HTS AC Transmission Cable by Means of Particle Swarm Optimization

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For a High Temperature Superconducting (HTS) cable, a non-uniform AC current distribution among the multi-layer conductors gives rise to increased AC losses. To acquire uniform current distribution among the multilayer conductors, a constrained optimization model is constructed with continuous and discrete variables, such as the winding angle, radius and the winding direction of each layer. Under the constraints of the mechanical properties and critical current of the tape, the Particle Swarm Optimization (PSO) algorithm is employed to address the structural parameter optimization. A uniform current distribution among layers is realized by optimizing the structure parameters. PSO is proved to be a more powerful tool than Genetic Algorithm (GA) for structural parameter optimization.

*Key Words:* Current distribution, HTS cable, Particle swarm optimization.

## 1. Introduction

A High Temperature Superconducting (HTS) cable is an interesting part of HTS power application. For the large current transmission, a HTS cable has a multi-layer structure which consists of parallel connected tapes, twisted in each layer. Due to different inductance among layers, the currents flowing in the layers are not the same, as demonstrated by a number of experiments performed on cable conductors [1]. Hence, control of the current distribution among layers is an important issue for design and optimization of a HTS cable conductor because it is related to the current transmission capacity and power loss.

Usually, for an AC HTS cable, the inductive impedance is much greater than the joint resistance. Thus, the current distribution among layers is substantially determined by the inductive impedances of layers. The distribution of inductive impedances is dependent on the structural parameters of a cable conductor, such as the radius, winding pitch and winding direction of a layer, so the main method to obtain a uniform current distribution is to alternate the inductive impedances of layers by adjusting the structural parameters of the cable conductor. The validity of this method has been verified by many experimental results of current distribution.

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By using the Particle Swarm Optimization (PSO) algorithm, a uniform current distribution is realized and demonstrated in this paper through adjusting the structure parameters of the conductor layers. Furthermore, comparison between PSO and Genetic Algorithm (GA) is conducted.

## 2. Calculation of Current Distribution in a Multilayer Conductor HTS Cable

### *2.1 Circuit model of a multilayer conductor*

In long superconducting cables the self and mutual inductances between the layers determine the distribution of AC current [1]. The geometry of a 4-layer HTS cable conductor is illustrated in Fig. 1, and its equivalent circuit is shown in Fig. 2.

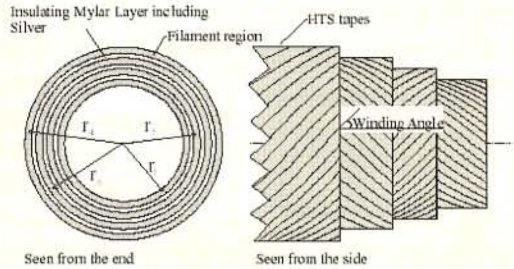



Fig. 1. Geometry of a 4-layer HTS cable conductor

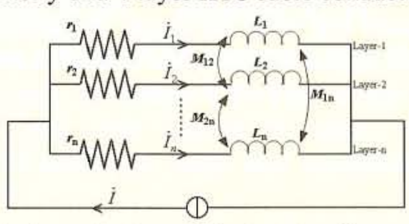



Fig. 2. Equivalent circuit model of HTS cable conductor

In Fig. 2, $r_i$ and $L_i$ ($i$=1,2,...,$n$) are the resistance and self-inductance of the $i$-th layer, respectively, $M_{ij}$ the mutual inductance between the $i$-th and $j$-th layers, and $n$ the total number of layers.

*2.2 Inductances of layers*

According to [1], the self and mutual inductances per unit length can be obtained by

$$L_i = \frac{\mu_0}{4\pi}(\tan^2\beta_i + 2\ln\frac{D}{R_i}) \tag{1}$$

$$M_{ij} = \frac{\mu_0}{2\pi}(\frac{a_i a_j}{2}\frac{R_i}{R_j}\tan(\beta_i)\tan(\beta_j) + \ln\frac{D}{R_j}) \qquad i \neq j \tag{2}$$

where $R_i$ and $R_j$ ($R_i < R_j$) are the radii of the $i$-th and $j$-th layers, respectively, $\beta_i$ and $\beta_j$ are the winding angles of the $i$-th and $j$-th layers, $a_i$ and $a_j$ are constants (-1 or +1) taking into account the relative winding directions of the $i$-th and $j$-th layers, and $D$ the distance between the cable and the return lead.

*2.3 Resistance of layers*

The resistance of layers consists of two components, namely, the joint resistance $r_{j,i}$ and the equivalent non-linear resistance $r_{e,i}$. $r_{j,i}$ is constant while $r_{e,i}$ is determined by the V-I characteristics of the conductor layer.

*2.4 Circuit matrix equation*

For the low-current region, the effect of $r_{e,i}$ can be neglected and the circuit matrix equation can be simplified as a frequency-domain matrix equation.

$$j\omega\begin{bmatrix} L_1+\frac{r_{j,1}}{j\omega} & M_{12} & \cdots & M_{1n} & \frac{-1}{j\omega} \\ M_{21} & L_2+\frac{r_{j,2}}{j\omega} & \cdots & M_{2n} & \frac{-1}{j\omega} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ M_{n1} & M_{n2} & \cdots & L_n+\frac{r_{j,n}}{j\omega} & \frac{-1}{j\omega} \\ \frac{-1}{j\omega} & \frac{-1}{j\omega} & \cdots & \frac{-1}{j\omega} & 0 \end{bmatrix}\cdot\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \\ \dot{U} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -\dot{I} \end{bmatrix} \tag{3}$$

where $\dot{U}$ is the voltage drop across each layer, and $\omega$ angular frequency.

When the current is increased to the vicinity of the critical current of the superconducting tape, considering the nonlinear resistance $r_{e,i}$, a time-domain nonlinear circuit matrix equation should be deduced and applied.

## 3. Optimization Model

*3.1 Objective function*

According to (1) and (2), the self- and mutual-inductance of a conductor layer are correlated with the winding angle $\beta$, the winding direction $\alpha$ and the radius $R$ of the layer. These parameters are chosen as the design variables, which can be expressed as a vector:

$$\boldsymbol{X} = [\beta_1, a_1, R_1, \beta_2, a_2, R_2, \cdots, \beta_n, a_n, R_n]$$

Then the objective function of optimization is derived to find a solution for uniform current distribution in the HTS cable conductor layers.

$$\min f(\boldsymbol{X}) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}|I_{ix}(\boldsymbol{X}) - I_{jx}(\boldsymbol{X})| + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}|I_{iy}(\boldsymbol{X}) - I_{jy}(\boldsymbol{X})| \tag{4}$$

where $I_{ix}(X)$ and $I_{iy}(X)$ are the real and imaginary components of current $\dot{I}_i(X)$ in the $i$-th layer, which is obtained from (3). The current distribution among layers should become more uniform when $f(X)$ approaches the minimum.

*3.2 Constraints*

*A. Constraints of the mechanical properties*

Considering the mechanical properties of a tape, the optimum program should satisfy the following two conditions: the tensile strain of tape must be less than the critical tensile strain; and the bending strain must be less than the critical bending strain [2].

$$\begin{cases} \beta_i - \arcsin(\frac{2R_i\varepsilon_{cb}}{t})^{1/2} \le 0 \\ \beta_i - \arcsin(\frac{\varepsilon_{ct}+\varepsilon_p-\varepsilon_{fc}}{\varepsilon_p-\varepsilon_r})^{1/2} \ge 0 \end{cases} \qquad i=1,2,\cdots,n \tag{5}$$

where $\varepsilon_{cb}$ and $\varepsilon_{ct}$ are the critical bending and tensile strains of a tape at 77 K, $\varepsilon_p$ is the winding pitch thermal shrinkage, $\varepsilon_{fc}$ the thermal shrinkage of tape, $\varepsilon_r$ the radial thermal shrinkage of the former, and $t$ the thickness of tape.

*B. Constraints of radii*

$$\begin{cases} R_1 - \frac{D_{\min}}{2} + (t_f + \frac{t}{2}) \ge 0 & \\ R_{i+1} - R_i - (t_f + t) \ge 0 & i=1,2,\cdots,n-1 \\ \frac{D_{\max}}{2} - R_i - (n-i)(t_f+t) - \frac{t}{2} \ge 0 & i=1,2,\cdots n \end{cases} \tag{6}$$

where $D_{min}$ and $D_{max}$ are used to limit the inner and outer diameters of the cable conductors, and $t_f$ is the thickness of the dielectric between layers.

*C. Constraints of critical current*

These constraints are used to restrict the currents in layers below their critical currents.

$$I_i < N_i I_c k_1 k_2 k_3 k_4 \qquad i=1,2,\cdots,n \tag{7}$$

where $N_i$ is the number of tapes wound on the $i$-th layer, $I_c$ the mean of critical currents of HTS tapes in the cable, $k_1$ $k_2$, and $k_3$ the deterioration factors of the critical current considering the magnetic field and temperature, manufacturing, and thermal cycles, respectively, and $k_4$ the safety margin of design. $k_1$ can be derived from the interpolation surface of critical current - applied magnetic field - applied temperature.

## 4. Particle Swarm Optimization

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Kennedy and Eberhart in 1995 [3]. PSO was inspired by the social behavior of birds flocking and fish schooling. Like other evolutionary computation (EC) techniques, PSO is also initialized with a population of random solution and searches for the optima by updating generations. However, PSO does not seem to have an analogous mechanism (*e.g.* crossover and mutation operator) in traditional EC algorithm. In PSO, each potential solution called a 'particle', flies in the problem search hyperspace looking for the optimal position. As time passes, a particle adjusts its position according to its own 'experience', as well as the experience of neighboring particles.

Suppose that the search space is D-dimensional, and then each particle is treated as a point in a D-dimensional space. The *i*-th particle of the swarm is represented as a D-dimensional vector, $X_i(t)=(X_{i,1}(t),X_{i,2}(t),\cdots,X_{i,D}(t))$. The velocity (position change) of this particle can be represented by another D-dimensional vector, $V_i(t)=(V_{i,1}(t),V_{i,2}(t),\cdots,V_{i,D}(t))$. The *i*-th particle also maintains a memory of its previous best position, $pbest_i=(pbest_{i,1},pbest_{i,2},\cdots,pbest_{i,D})$. At each iterative step, the *gbest* is designated as the index of the best particle in the swarm. Subsequently, the swarm is manipulated according to the following two equations [4]:

$$V_{i,d}(t)=wV_{i,d}(t-1)+c_1r_1\times(pbest_{i,d}-X_{i,d}(t-1))/\Delta t \\ +c_2r_2\times(gbest_d-X_{i,d}(t-1))/\Delta t \quad (8)$$

$$X_{i,d}(t)=X_{i,d}(t-1)+V_{i,d}(t)\times\Delta t \quad (9)$$

where $d$=1,2,…,$D$, $i$=1,2,…,$N$, $N$ is the size of the swarm, $c_1$ and $c_2$ are two positive constants, namely social and cognitive parameters, $r_1$ and $r_2$ are two random numbers distributed within the range [0, 1], $t$ is the iteration number, $\Delta t$=1, and $w$ is the inertia weight.

Equation (8) is used to update the particle's new velocity according to its previous velocity and to the distances of its current position from both its own best historical position and its neighbors' best position at every iterative step. Then the particle flies toward a new position according to (9).

## 5. Structural Parameters Optimization Using PSO

The structural optimum design of a HTS cable is that a series of advanced parameters can be obtained within kinds of constraints, and thus the current distribution becomes as uniform as possible with these parameters. Here, a 4-layer superconducting cable conductor is optimized by PSO. The parameters are given as: $c_1$=2, $c_2$=2. The adaptive inertia weight $w$ is defined as

$$w=(w_{start}-w_{end})C^{gen}+w_{end}$$

where the constant $C$ is chosen as 0.999, and $g_{en}$ is the current number of evolution iteration. The start value of inertia $w_{start}$=1.43, and the end value $w_{end}$=0.1. The inertia weight decreases from $w_{start}$ to $w_{end}$. It can facilitate an initial global exploration and dynamically investigate the refined solution.

Table 1 shows the structural parameters before optimization and after optimization.

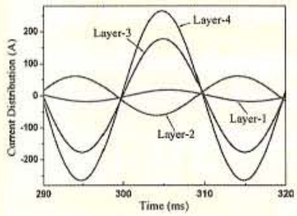
Table 1. Structural parameters of HTS cable conductor

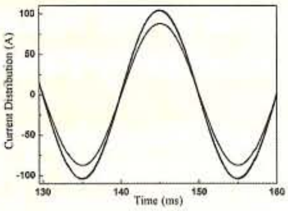
| Layer Index | $a_i^{\Delta}$ | $\beta_i^{\Delta}$ | $R_i^{\Delta}$ (mm) | $a_i^{*}$ | $\beta_i^{*}$ | $R_i^{*}$ (mm) |
|---|---|---|---|---|---|---|
| 1 | +1 | 27.0° | 20.50 | +1 | 43.2° | 20.00 |
| 2 | -1 | 27.0° | 21.00 | +1 | 24.6° | 20.35 |
| 3 | +1 | 27.0° | 21.50 | -1 | 26.8° | 21.34 |
| 4 | -1 | 27.0° | 22.00 | -1 | 43.0° | 21.87 |

Δ represents before optimization, * represents after optimization.

Fig. 3 and Fig. 4 are obtained based on the data in Table 1. From these figures, it can be seen that the currents before optimization differ greatly in the amplitude and phase angle, but the optimized currents become uniform. Especially, as the peak of total cable current is pushed up to 1000 A, the equivalent resistances of layer increase as the outer layer current increases to the vicinity of the critical current, the waveforms of currents before optimization are distorted. However, the waveforms of the optimized currents remain sinusoidal.

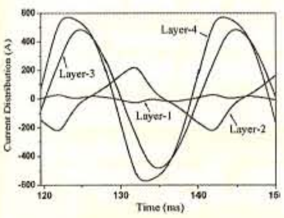
Calculating the AC losses with the method described in [5], the AC losses before optimization are greater than the losses after optimization. Fig. 5 illustrates the comparison of AC losses.

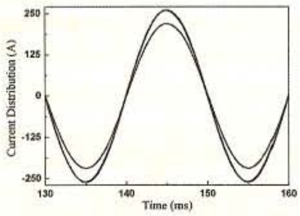



(a) before optimization (b) after optimization

Fig. 3. Current distribution (peak of total current is 400A)




(a) before optimization (b) after optimization

Fig. 4. Current distribution (peak of total current is 1000A)

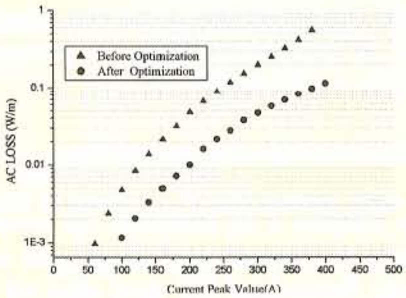



Fig. 5. AC loss Comparison

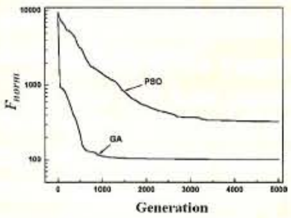
## 6. Comparison between PSO and GA

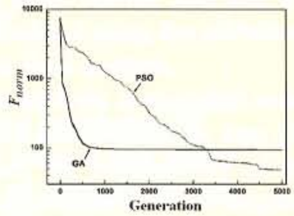
The following conditions are assumed to compare the performance of PSO and GA. In one evolution operating, the maximum number of generation is set as 5000. And a total of 50 evolutions are conducted for populations of 10, 20 and 40 particles respectively. The fitness function, $F_{norm}$ is defined as the average of the best fitness function value of 50 evolutions in the same generation. The performance results are shown in Fig. 6, plotting $F_{norm}$ against the generation. With reference to the results from Fig. 6, GA performs better than PSO when the population is 10. As the population size reaches 20 and 40, GA might lead a premature convergence to a sub-optimal point due to selection and crossover of GA. PSO may converge to a global optimal result.

Table 2 shows the elapsed times of PSO and GA under the same conditions as above. The determination of each particle is very simple than that of an individual of GA. Hence, the PSO computational time is shorter than that of GA.

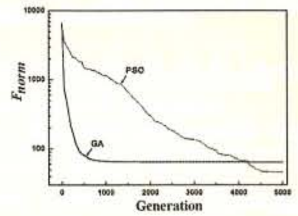
Table 2. Summary of Elapsed Time

| Population Size | PSO | GA |
|---|---|---|
| 10 | 28 s | 73 s |
| 20 | 44 s | 123 s |
| 40 | 76 s | 235 s |




(a) 10 particles (b) 20 particles




(c) 40 particles

Fig. 6. Performance of PSO and GA with different populations

## 7. Conclusion

In this paper, a new heuristic algorithm known as PSO is introduced to handle the structural parameter optimization of a HTS cable conductor for homogeneously distributed layer currents. The optimized results reveal that the current distribution is improved and the AC losses are reduced significantly. The performance of the PSO algorithm is evaluated in comparison with the GA for a number of random optimization tests. It is proved that the PSO algorithm is an attractive alternative for the structural parameter optimization problems.

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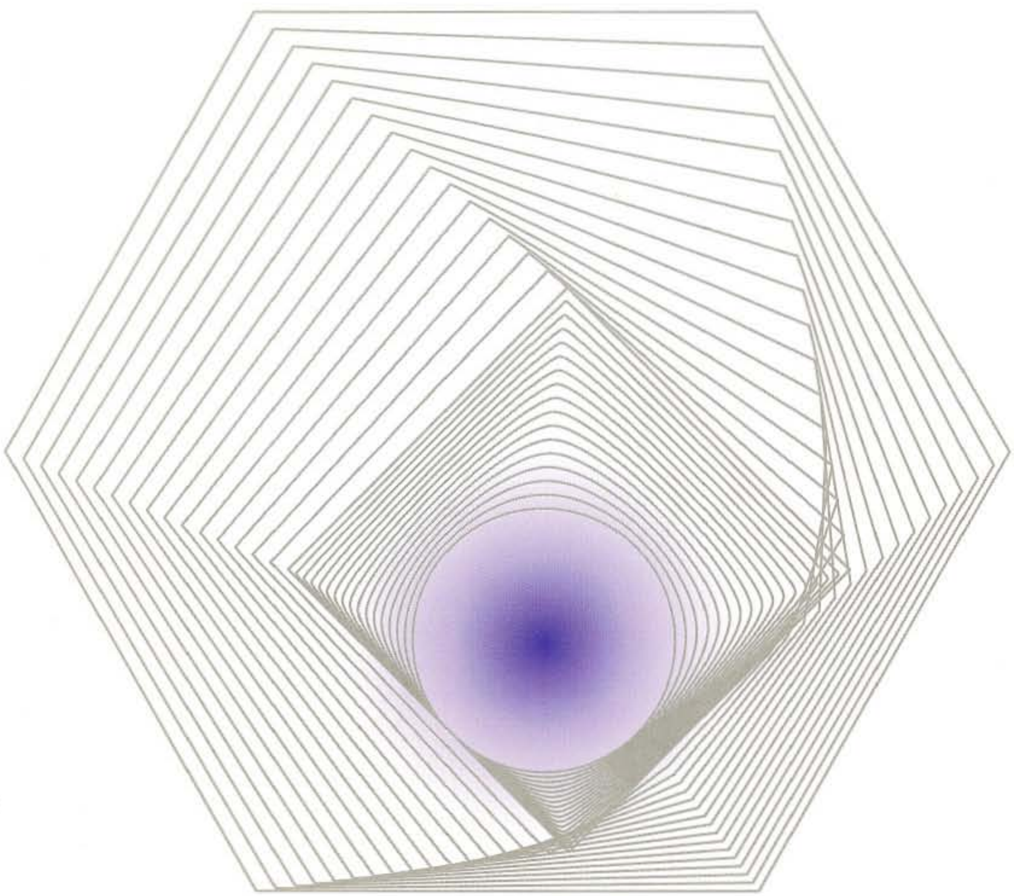

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