Improved Phase Variable Model and Field-Circuit Coupling Method for Performance Analysis of High Speed PM Brushless DC Motor

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This paper presents an improved phase variable model and field-circuit coupling method to evaluate the comprehensive performance of brushless DC (BLDC) motors in both steady and dynamic conditions. In the proposed model, major motor parameters such as inductances, back electromotive force and cogging torque are obtained based on time-stepping nonlinear finite element analyses. The phase variable model is built and implemented in the MATLAB/Simulink through look-up tables to decide the rotor position dependence of the parameters. Furthermore, a mathematical method is proposed for determining the central point voltage of the Y-connected three phase windings, so that the model can obtain the input voltages of both energized and non-energized phase windings, and can be directly applied to BLDC motors. By using the developed model, the comprehensive performance of a high-speed surface mounted permanent magnet BLDC motor prototype is investigated.

Key Words: Improved phase variable model, Field-circuit coupling, BLDC motor, Performance evaluation.

1. Introduction

Thanks to the advantages such as high efficiency, high power density and high drive performance, permanent magnet (PM) brushless DC (BLDC) motors have been widely applied in industrial and domestic appliances [1]. As a crucial part in the electrical driving system design, a fast and accurate model for predicting, assessing and optimizing the performance of BLDC motors would be always useful.

For performance evaluation, compared with an equivalent electrical circuit model, the time-stepping nonlinear magnetic field finite element analysis (FEA) can provide accurate results but is more time consuming. A phase variable model of BLDC motor based on FEA and coupled with external circuits, which behaves much faster with the same level of accuracy, has been introduced and verified in [2], [3]. In the model, the inductances, back electromotive force *(emf)* and cogging torque were obtained by nonlinear FEA. However, the equation-based model cannot be applied to BLDC directly and a model composed of several circuit components has to be employed. To solve this problem, a pure mathematic method is proposed in this paper. By using the method, the central point potential (voltage) of the Y-type three phase windings can be worked out, so that the port voltages of three phase windings can be obtained and the model can be directly applied to BLDC motors. The theoretical procedure is given in detail.

The improved phase variable model has been implemented in the Simulink environment and used to analyze the performance of a high-speed surface mounted PM BLDC motor for driving embroidery machines [4]. In the model, key motor parameters such as winding flux, back *emf*, inductance and cogging torque are accurately determined based on magnetic field FEAs, which can take into account the details of motor structure and dimensions and the nonlinear properties of ferromagnetic cores. The simulations agree with the experimental results on the motor prototype operated with the BLDC control scheme.

2. Equation-based Phase Variable Model

The equation-based phase variable model of BLDC motor is given as

$$V_{abc} = r_{abc} i_{abc} + \frac{d\psi_{abc}}{dt} + e_{abc} \tag{1}$$

$$\psi_{abc} = L_{abc} i_{abc} \tag{2}$$

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$$T_m = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_r} + T_{cog} \tag{3}$$

$$J\frac{d\omega_r}{dt} = T_m - B\omega_r - T_L \tag{4}$$

$$\begin{bmatrix} \frac{d\psi_{sa}}{dt} \\ \frac{d\psi_{sb}}{dt} \\ \frac{d\psi_{sc}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\partial\psi_{sa}}{\partial i_a} & \frac{\partial\psi_{sa}}{\partial i_b} & \frac{\partial\psi_{sa}}{\partial i_c} \\ \frac{\partial\psi_{sb}}{\partial i_b} & \frac{\partial\psi_{sb}}{\partial i_b} & \frac{\partial\psi_{sb}}{\partial i_a} \\ \frac{\partial\psi_{sc}}{\partial i_a} & \frac{\partial\psi_{sc}}{\partial i_b} & \frac{\partial\psi_{sc}}{\partial i_c} \end{bmatrix} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} + \begin{bmatrix} \frac{\partial\psi_{sa}}{\partial\theta} \\ \frac{\partial\psi_{sb}}{\partial\theta} \\ \frac{\partial\psi_{sc}}{\partial\theta} \end{bmatrix} \frac{d\theta}{dt}$$

$$= \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} + \begin{bmatrix} \frac{dL_{aa}}{d\theta} & \frac{dL_{ab}}{d\theta} & \frac{dL_{ac}}{d\theta} \\ \frac{dL_{ba}}{d\theta} & \frac{dL_{bb}}{d\theta} & \frac{dL_{bc}}{d\theta} \\ \frac{dL_{ca}}{d\theta} & \frac{dL_{cb}}{d\theta} & \frac{dL_{cc}}{d\theta} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} p\omega_r \tag{5}$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} r_a & 0 & 0 \\ 0 & r_b & 0 \\ 0 & 0 & r_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{dL_{aa}}{d\theta} & \frac{dL_{ab}}{d\theta} & \frac{dL_{ac}}{d\theta} \\ \frac{dL_{ba}}{d\theta} & \frac{dL_{bb}}{d\theta} & \frac{dL_{bc}}{d\theta} \\ \frac{dL_{ca}}{d\theta} & \frac{dL_{cb}}{d\theta} & \frac{dL_{cc}}{d\theta} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} p\omega_r + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{6}$$

$$L_{ab} = L_{ba},\ L_{bc} = L_{cb},\ L_{ca} = L_{ac} \tag{7}$$

$$r_a = r_b = r_c \tag{8}$$

$$i_a + i_b + i_c = 0 \tag{9}$$

where L_{abc} is the inductance matrix and the difference between apparent and differential inductances is ignored here, ψ_{sj} ($j=a,b,c$) is the flux linkage of phase winding j, and p is the number of pole-pairs. The rest of parameters are used as their conventional meanings. The profiles of L_{abc}, e_{abc} and T_{cog} are obtained from the nonlinear transient FE solutions, in which the rotor position dependence and the saturation effect are considered. The stator windings are three-phase symmetrical.

3. Calculation of the Central Point Voltage

Suppose the electrical potentials (voltages) of terminals a, b, c and N (the central point) are U_a, U_b, U_c and U_N, respectively, one can obtain

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} U_a - U_N \\ U_b - U_N \\ U_c - U_N \end{bmatrix} \tag{10}$$

Then

$$U_N = \frac{\sum_{j=a}^{c}(U_j - v_j)}{3} \tag{11}$$

Substituting (6) into (11) and considering (9), the central point voltage is expressed as

$$U_N = \frac{[U_a - (L_{aa}+L_{ba}+L_{ca})\frac{di_a}{dt} - \frac{d(L_{aa}+L_{ba}+L_{ca})}{d\theta} i_a\omega]}{3}$$
$$+ \frac{[U_b - (L_{ab}+L_{bb}+L_{cb})\frac{di_b}{dt} - \frac{d(L_{ab}+L_{bb}+L_{cb})}{d\theta} i_b\omega]}{3}$$
$$+ \frac{[U_c - (L_{ac}+L_{bc}+L_{cc})\frac{di_c}{dt} - \frac{d(L_{ac}+L_{bc}+L_{cc})}{d\theta} i_c\omega]}{3}$$
$$- \frac{[e_a(\theta)+e_b(\theta)+e_c(\theta)]}{3} \tag{12}$$

The values of U_a, U_b and U_c are determined by the switching state of inverter with three phases, the state of PWM and the phase currents. When one phase current, e.g. i_a of phase a, is zero, and the associated circuit is open-circuited (i.e. the winding of phase a is in a non-energized condition), under the consideration of (7)-(9), U_N and U_a can be obtained by

$$U_N = \frac{[U_b - L_{bb}\frac{di_b}{dt} - \frac{dL_{bb}}{d\theta} i_b\omega]}{2}$$
$$+ \frac{[U_c - L_{cc}\frac{di_c}{dt} - \frac{dL_{cc}}{d\theta} i_c\omega]}{2} - \frac{[e_b(\theta)+e_c(\theta)]}{2} \tag{13}$$

$$U_a = U_N + (L_{aa}+L_{ba}+L_{ca})\frac{di_a}{dt} + e_a(\theta)$$
$$+ (\frac{dL_{ab}}{d\theta} i_b + \frac{dL_{ac}}{d\theta} i_c)\omega + (L_{ab}\frac{di_b}{dt} + L_{ac}\frac{di_c}{dt}) \tag{14}$$

When the winding current is not equal to zero and PWM is under the state of duty-off, the voltage of input port of phase a can be decided by

$$\text{if } i_a>0, \text{ then } U_a=U_{bus} \tag{15}$$

$$\text{if } i_a<0, \text{ then } U_a=0 \tag{16}$$

where U_{bus} is the voltage of input power line. According to (13)-(16), one can work out the input port voltages of three phases and their central point, and hence the three phase voltages v_a, v_b and v_c.

Referring to (6), the voltage equation of phase a is

$$v_a=(r_a i_a+L_{aa}\frac{di_a}{dt})+(L_{ab}\frac{di_b}{dt}+L_{ac}\frac{di_c}{dt}) +(\frac{dL_{aa}}{d\theta}i_a+\frac{dL_{ab}}{d\theta}i_b+\frac{dL_{ac}}{d\theta}i_c)\omega+e_a \tag{17}$$

By defining that

$$v_{am}=(L_{ab}\frac{di_b}{dt}+L_{ac}\frac{di_c}{dt}) +(\frac{dL_{aa}}{d\theta}i_a+\frac{dL_{ab}}{d\theta}i_b+\frac{dL_{ac}}{d\theta}i_c)\omega \tag{18}$$

we have

$$v_a=(r_a i_a+L_{aa}\frac{di_a}{dt})+v_{am}+e_a \tag{19}$$

$$v'_a=v_a-v_{am}=(r_a i_a+L_{aa}\frac{di_a}{dt})+e_a \tag{20}$$

Similarly,

$$v_{bm}=(L_{ba}\frac{di_a}{dt}+L_{bc}\frac{di_c}{dt}) +(\frac{dL_{ba}}{d\theta}i_a+\frac{dL_{bb}}{d\theta}i_b+\frac{dL_{bc}}{d\theta}i_c)\omega \tag{21}$$

$$v_b=(r_b i_b+L_{bb}\frac{di_b}{dt})+v_{bm}+e_b \tag{22}$$

$$v'_b=v_b-v_{bm}=(r_b i_b+L_{bb}\frac{di_b}{dt})+e_b \tag{23}$$

$$v_{cm}=(L_{ca}\frac{di_a}{dt}+L_{cb}\frac{di_b}{dt}) +(\frac{dL_{ca}}{d\theta}i_a+\frac{dL_{cb}}{d\theta}i_b+\frac{dL_{cc}}{d\theta}i_c)\omega_r \tag{24}$$

$$v_c=(r_c i_c+L_{cc}\frac{di_c}{dt})+v_{cm}+e_c \tag{25}$$

$$v'_c=v_c-v_{cm}=(r_c i_c+L_{cc}\frac{di_c}{dt})+e_c \tag{26}$$

4. Performance Simulation of BLDC Motor

4.1 Simulink-based phase variable model

According to (1)-(16), a complete Matlab/Simulink-based phase variable model is built as shown in Fig. 1, where v_{am}, v_{bm} and v_{cm}, v_a, v_b and v_c, v'_a, v'_b and v'_c can be obtained from Matlab functions based on (17)-(26). The rest of work is similar to the modeling of a conventional DC motor, so the proposed model can be easily realized in the Simulink environment.

4.2 Performance Evaluation

As an example, the presented improved phase variable model has been used to analyze a surface mounted PM BLDC motor, which has been developed for driving high-speed embroidery machines [4]. Major parameters and dimensions of the motor include 4 poles, 12 stator slots, 3 phase single-layer windings, and the stator core dimensions are 38 mm for the inner diameter, 76 mm for the outer diameter, and 38 mm for the axial length, respectively. The motor is designed to deliver an output torque of 1.0 Nm at a speed of not less than 5000 rpm. The calculations of winding flux, back *emf*, inductances and cogging torque are reported in [4].

By using the improved phase variable model, comprehensive performances of the BLDC motor can be simulated, such as the curves of speed, current and torque during the start-up or transients when the load or power supply changes. For example, Fig. 2 illustrates the speed curve during the start-up with the full load of 1.0 Nm and the rated inverter voltage of 310 VDC. It can be seen that the motor speed can smoothly increase to the rated speed of 5000 rpm. Fig. 3 shows the bus current waveform.

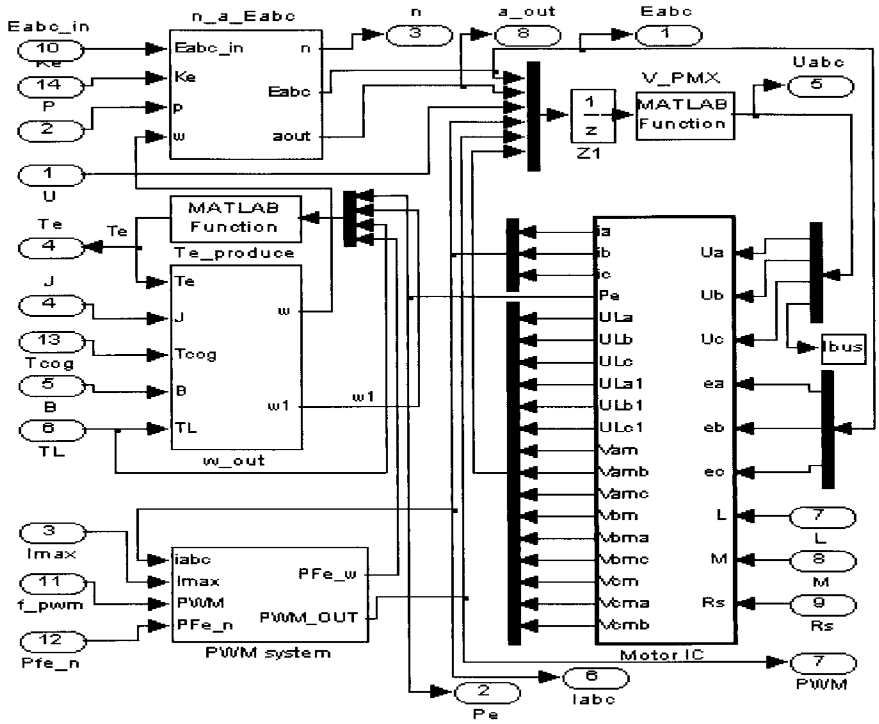

Fig. 1. Simulink-based improved phase variable model of BLDC motors

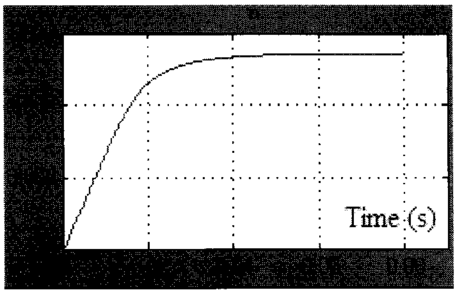

Fig. 2. Speed curve during start-up

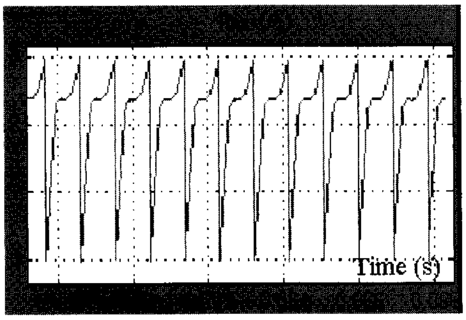

Fig. 3. Bus current waveform

5. Conclusion

This paper presents an improved phase variable model to evaluate the comprehensive performance of a PM brushless DC motor in both dynamic and steady conditions. A pure mathematical method is proposed to achieve the central point voltage of the Y-connected three phase windings so that the phase voltages can be obtained and the model can be directly applied to analyze the BLDC motor. The simulations on a high speed BLDC motor prototype show the validity of the model.

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