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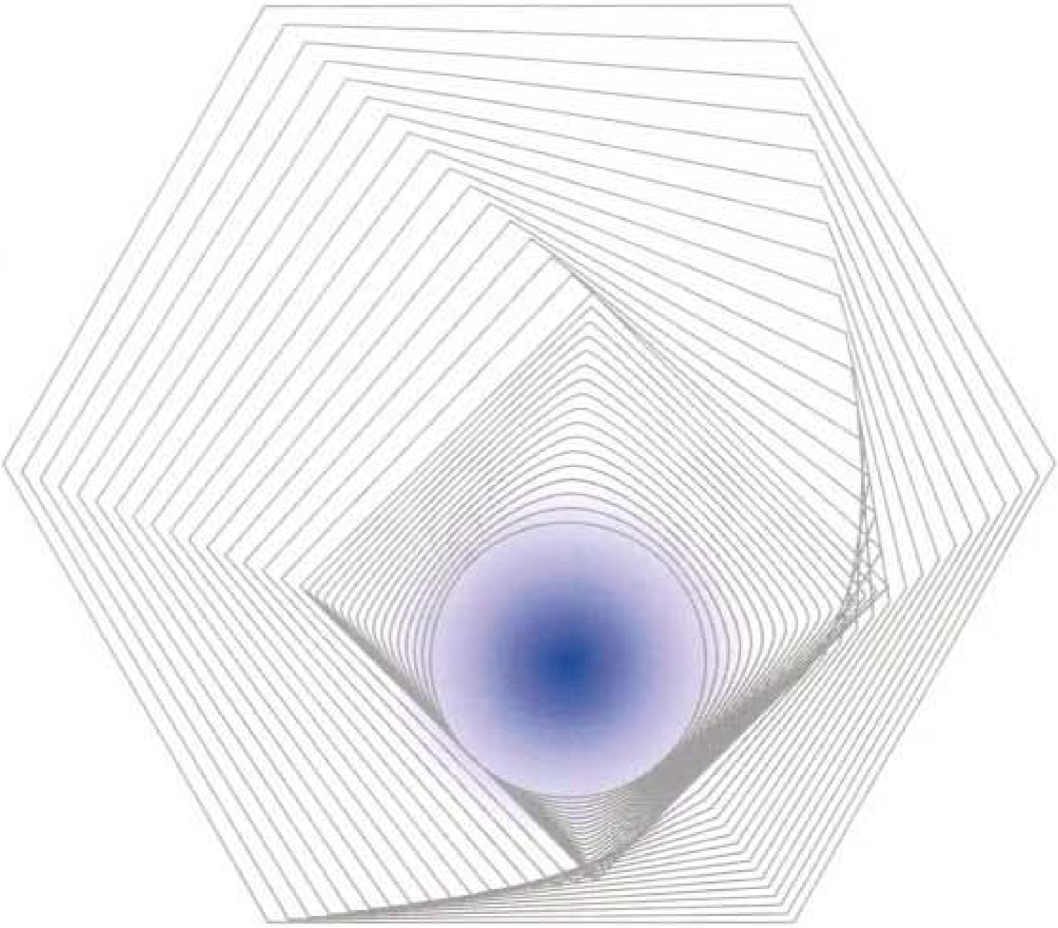

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Super-High Q Resonate Circuit: Theory and Device

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A high Q R-L-C series resonant circuit has been analyzed with the consideration of using high T_c superconductor (HTS) technology. With HTS techniques, a very high Q circuit can be achieved; consequently special aspects such as high voltage generation can be practically realized. Theoretical study has been carried out, as well as a prototype device has been made for the experimental verification. This paper will describe this high voltage generation method with theoretical analysis of the prototype device.

Key Words: High temperature superconductor; Inductor; Resonant circuit; Quality factor.

1. Introduction

A resistor R - capacitor C - inductor L series resonant circuit has been explored with regard to its voltage aspects of using a high T_c superconductor (HTS) [1]-[2]. The relation of the circuit quality factor Q and its voltage aspects have been studied, and formed a method of high voltage generation. Then a method to generate high voltage from a low voltage source has been explored. This high voltage generator using the resonant circuit mainly consists of an inductor, a capacitor, a DC battery source, and an electronic switch. As a fundamental principle, the resonant circuit generates the voltage which is proportional to the circuit Q value. As a basic principle of operational approach, a low voltage DC power source can be used and its polarity is reversed at a certain frequency and this control is achieved with an electronic switch. Resistance in the circuit will limit the Q value and therefore voltages that can be achieved in practice; however HTS technology can dramatically reduce the resistance and present a very high Q value. Both theoretical analysis and practical device operation principle will be presented in this paper with the advantages of using a high Q circuit made with HTS Bi-2223/Ag multifilament wires.

2. Operational Theory

2.1 Q value and voltage feature of a resonant circuit

The build-up voltage in a resistive R-C-L series resonant circuit with a DC power supply as shown in Fig. 1, is related to the circuit quality factor Q. The quality factor of a resonant circuit is $Q = \omega_o L/R$, sometimes called the magnification factor, and $\omega_o = [(1/LC)-(R^2/4L^2)]^{1/2}$ is the resonant frequency in rad s^{-1}. The potential difference across the capacitor at resonance is Q times as great as the applied emf V_p (rms). For a sinusoidal power supply, the voltage across the capacitor V_{Cmax} at resonance is given by

$$V_{Cmax} = Q\, V_p \qquad (1)$$

For using a DC power supply with an electronic switch to reverse the polarity, the maximum voltage V_{Cmax} can be expressed as

$$V_{Cmax} = Q\, V_B' \qquad (2)$$

where V_B' is the effective voltage of the low voltage power source. If the switching controller used has rectangular switching wave form with low voltage source polarity switching frequency f, the build-up voltage wave form F(t) can be expressed by Fourier series as

$$F(t) = (4V_B/\pi)[\sin\omega t + (1/3)\sin3\omega t + (1/5)\sin5\omega t + ...] \qquad (3)$$

The generator only resonates on the first harmonic with amplitude of $4V_B/\pi$. Therefore

$$V_B' = 4\, V_B/\pi \qquad (4)$$

and the maximum build-up voltage is related with the R-C-L resonant circuit Q value by

$$V_{Cmax} = Q\,(4V_B/\pi) \qquad (5)$$

Reducing the R-C-L resonant circuit resistance is achieved by introducing the superconducting

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[...]ctor; the circuit Q value then will be [...]matically increased, therefore leading to a very [...]h voltage across the capacitor.

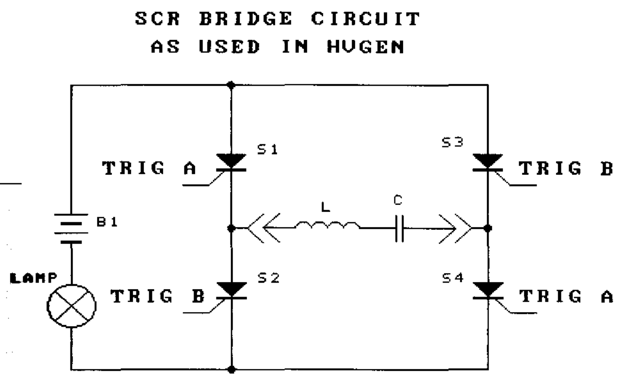

Fig. 1. The resonant circuit with an electronic switch

2.2 Resistance-less circuit

In a R-C-L resonant circuit as shown in Fig. 1, when switches s1 and s4 is turn on, the instantaneous current i(t) and capacitor voltage $V_C(t)$ solutions respectively are

$$i(t) = e^{\frac{-Rt}{2L}}\left[\frac{(V_{CO}+V_B)\sin\omega t}{\omega L}\right] \quad (6)$$

$$V_C(t) = (V_{CO}+V_B)\left[1-e^{\frac{-Rt}{2L}}\left(\cos\omega t+\frac{R}{2L\omega}\sin\omega t\right)\right]-V_{CO} \quad (7)$$

where $R < 2(L/C)^{1/2}$, V_{CO} is the initial capacitor voltage. Both equations describe decaying sinusoids, with $V_C(t)$ approaching a steady state value of V_B, and i(t) approaching a steady state value of zero.

Now in a circuit using a superconducting inductor and no separate resistor, then R will become very small. If R = 0 Ω, then (6) and (7) can be simplified to

$$i(t) = \frac{(V_{CO}+V_B)\sin\omega t}{\omega L} \quad (8)$$

$$V_C(t) = -(V_{CO}+V_B)\cos\omega t + V_B \quad (9)$$

where $\omega = (LC)^{-1/2}$ rad s^{-1}. These two equations describe constant magnitude sinusoids, with the average values of i(t) and $V_C(t)$ being zero and V_B respectively.

When switch S in Fig. 1 is closed, $V_C(t) = V_C(0) = -V_{CO}$. One half a resonant cycle later, this voltage will have increased to

$$V_C(t) = V_C(\pi/\omega) = -(V_{CO}+V_B)(-1) + V_B$$

$$= V_{CO} + 2V_B \quad (10)$$

If at this point of time the battery is disconnected, and then reconnected in the opposite polarity for the next half cycle, then the initial capacitor voltage V_{CO} is changed to $V_{CO\text{-}new}$, and is given by

$$V_{CO\text{-}new} = -V_C(t) = -(V_{CO}+2V_B) \quad (11)$$

Half a cycle later, $V_C(t) = V_C(2\pi/\omega)$ becomes

$$V_C(t) = -[-(V_{CO}+2V_B)+(-V_B)](-1)+(-V_B)$$
$$= -(V_{CO}+4V_B) \quad (12)$$

If the battery polarity is reversed every half cycle thereafter then

$$V_C(t) = (V_{CO}+6V_B);\ -(V_{CO}+8V_B);\ (V_{CO}+10V_B);\ \ldots\text{etc} \quad (13)$$

This is the build-up voltage for an ideal non-resistive circuit. Therefore the positive and negative peak voltages can be described by (14). Consequently in a resistance-less circuit, the voltage across the capacitor C after n cycles will be

$$V_C(n) = (-1)^{n+1}(V_{CO}+2nV_B) \quad (14)$$

where n is the iteration number, and V_{CO} is the initial capacitor voltage.

2.3 Practical resistive circuit

From (6), when $t = \pi/\omega$, i = 0, if the electronic bridge in Fig. 1 changes DC source polarity, the capacitor voltage is given by

$$V_{C1} = (V_{CO}+V_B)(1+e^{-R\pi/2L\omega})-V_{CO} \quad (15)$$

After the polarity is changed n times, the capacitor voltage becomes

$$V_{Cn} = (V_{Cn-1}+V_B)(1+e^{-R\pi/2L\omega})-V_{Cn-1} \quad (16)$$

If $V_{CO} = 0$, then

$$V_{C1}=V_B(1+e^{-R\pi/2L\omega})$$
$$V_{C2}=(V_{C1}+V_B)(1+e^{-R\pi/2L\omega})-V_{C1}$$
$$=V_B(1+2e^{-R\pi/2L\omega}+e^{-2R\pi/2L\omega})$$
$$\ldots$$
$$V_{Cn}=(V_{Cn-1}+V_B)(1+e^{-R\pi/2L\omega})-V_{Cn-1}$$
$$=V_B(1+2e^{-R\pi/2L\omega}+2e^{-2R\pi/2L\omega}\ldots+2e^{-(n-1)R\pi/2L\omega}+e^{-nR\pi/2L\omega})$$
$$=V_B(1+e^{-nR\pi/2L\omega})+2V_B\sum_{i=1}^{n-1}e^{-iR\pi/2L\omega} \quad (17)$$

Assuming that the electronic bridge changes the power supply polarity at $t = n\pi/\omega$, after the polarity is changed n times, the build-up voltage V_{Cn} is

$$V_{Cn} = V_B(1+e^{-nR\pi/2L\omega}) + 2V_B \frac{e^{-R\pi/2L\omega} - e^{-(n-1)R\pi/2L\omega}}{1-e^{-R\pi/2L\omega}} \quad (18)$$

When $n \to \infty, V_{Cn} \to V_{Cmax}$, therefore

$$V_{Cmax} = \lim_{n\to\infty} V_{Cn} = V_B(1+\frac{2}{e^{R\pi/2L\omega}-1}) \quad (19)$$

From (19), when $R \to 0$, $V_{Cmax} \to \infty$. When $R \to \infty$, $V_{Cmax} \to V_B$. Since $Q = \omega L/R$, therefore (19) can be expressed as

$$V_{Cmax} = V_B + \frac{2V_B}{e^{\pi/2Q}-1} \quad (20)$$

3. Practical Operation Theory

A low voltage DC source is readily used as the power supply, and a practical resonant device central circuit comprises an electronic bridge which applies and periodically reverses the DC source polarity to the circuit, *e.g.* a bridge of four silicon controlled rectifiers (SCRs) connected as shown in Fig. 1. This is accomplished by a control circuit triggering the alternate pairs of SCRs at a selected rate.

If the initial state of the bridge circuit in Fig. 1 is: SCRs off; C discharged; no current flowing, then if S1 and S4 are both triggered, battery voltage V_B is applied to the series resonant circuit comprising L and C. If the trigger pulse is maintained until the SCR latching current is reached and ignoring any losses, a current will rise sinusoidally to a maximum and down to zero whereupon the SCRs will cease conducting due to load commutation. The length of this charge pulse is one half of the natural resonant period of the C-L circuit. The voltage left on C will be twice the battery voltage V_B. Now if at a later time S2 and S3 are triggered the battery voltage will be placed in series with the voltage left on C and this cycle will again add twice the battery voltage to C but with opposite sign, thus: $2V_B$, $-4V_B$, $6V_B$, $-8V_B$, $10V_B$, etc., will be the sequence of voltages produced across the capacitor. Consequently repeated cycles will raise the absolute voltage of C until losses in the resonant circuit cause a voltage plateau to be reached in a practical resistive circuit. Now if the period between SCR switching is enlarged (lower cycle repetition rate) the slow voltage loss in C between charge sinusoids will cause the voltage left on C to be stabilized at lower levels. This gives some means of voltage control, although if the Q is changed while the system is running, the voltages and currents will change as a result. Fig. 2 shows a graph illustrating the voltage build up and the principle of operation; and Fig. 3 shows the practically designed device circuit frame.

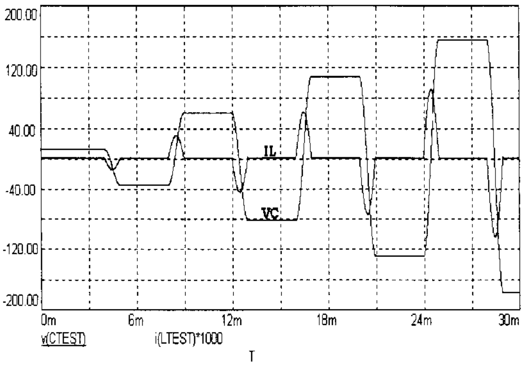

Fig. 2. Build-up of the voltage V_c(V) and the circuit current i_L(A)

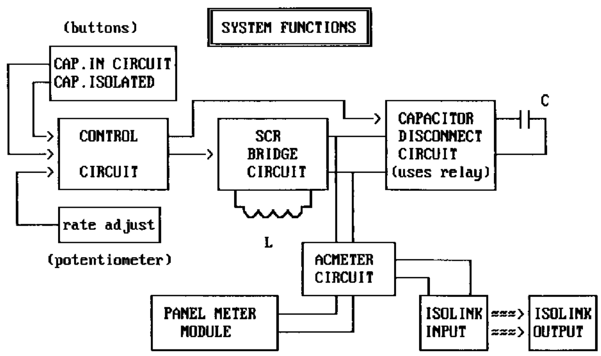

Fig. 3. The practical device principle circuit

4. Discussion

As above analysis, with a HTS the resonant circuit can be developed to be a method of generating high voltage from a low voltage source. This method can achieve high voltages by using a high Q circuit and with the appropriate choice of circuit components, as an example shown in Fig. 4. By limiting the number of iterations or by employing voltage sensing, it is possible to generate a pre-determined set voltage.

Practically, any circuit resistance causes energy dissipation, and for each reversal of the DC source polarity the corresponding increase in voltage is less than $2V_B$, and the magnitude of the voltage increase gets smaller with each iteration. Any resistance in the circuit limits the final achievable voltage. The circuit resistance is mainly caused by the inductor as well as the DC source, electronic switches, wires and connections. In practice the final voltage across the capacitor would reach a finite maximum value. This occurs as a result of the voltage increase per cycle being balanced by the same magnitude voltage loss per cycle due to leakage effects.

In a series resonant circuit with a power supply $V_p = V_m \sin\omega t$, the resonant frequency is given by $\omega_o = [(LC)^{-1} - (R/2L)^2]^{1/2}$. To reduce the circuit resistance

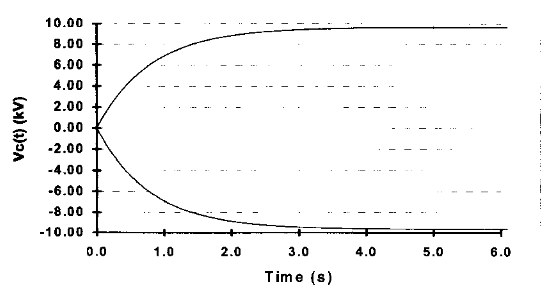

Fig. 4. A $V_C(t)$ peak curve for a sample circuit with $R = 0.05\ \Omega$, $C = 20\ \mu F$ and $L = 20$ mH

to near zero leads to an infinite circuit quality factor Q (= $\omega_o L/R = X/R$) at resonance. The rms current I_{max} in the circuit at resonance is

$$I_{max} = V_m/(R\sqrt{2}) \quad (21)$$

and the voltage across the capacitor V_{Cmax} (rms) is

$$V_{Cmax} = X_C\, I_{max} = (\omega_o C)^{-1}\, [V_m/(R\sqrt{2})] \quad (22)$$

By assuming that the circuit resistance R is zero, the circuit then has infinite Q at resonant frequency $\omega_o = 1/\sqrt{(LC)}$ rad s^{-1}. This leads to an infinite value of V_C generated and a very large potential current. This device therefore is able to provide controls of both high voltages and high currents.

HTS wire can be used to achieve the very high Q inductor to make this method viable; on the other hand the conventional inductor technique can not make this method applicable. The high Q inductor is able to be realized by using $(Bi,Pb)_2Sr_2Ca_2Cu_3O_{10+x}$ Ag-clad HTS wires, which have potential capability to make the inductor winding. This HTS wire has high engineering critical current density $J_e > 10^4$ A/cm^2 for 77 K operation, high magnetic field tolerance when lower the operational temperature, mechanical flexibility, and long length; and can be employed for design of the HTS inductor [3]-[5]. The HTS inductor virtually has no resistance for a DC current operation; however loss will be generated even at a relative low value for low frequency AC application, which however does not affect this application significantly [6]. The circuit current i, which can be calculated from V_C, is required for the design of a HTS inductor, and the circuit di/dt is also required for the HTS inductor design and the design of the electronic switch.

This voltage generation method can provide high voltages at a low load current for universal high voltage applications. When the resistance load across the capacitor is low, the maximum build up voltage is considerably reduced. The possible applications in electrical engineering include partial discharge testing and pressure testing of electrical insulation systems, etc, which do not require low impedance loads to be driven.

5. Conclusions

A resonant circuit with a power electronic controller has been verified with theoretical and practical operation analysis, which enables voltages to be increased rapidly to a value many times greater than the input low voltage source. A potential very high voltage can be achieved by using a very high Q inductor, which can be made by using HTS technology. The method can be developed to build a HTS device to generate very high voltages for any high impedance load applications.

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