Calculation of Differential Inductances of a Tubular Linear PM Actuator

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The inductances of a multi-phase electrical machine are key parameters for the performance analysis of the machine. Commonly, apparent inductances are employed. This, however, is incorrect when the non-linear characteristic of the magnetic core of the machine is considered in dynamic analysis. Instead, the differential inductances should be employed in non-linear analysis. A numerical method for calculating the differential inductances is presented in the paper and it is used for the prediction of the inductances of a tubular linear permanent magnet (PM) actuator. The measurement of the inductances of the prototype is also carried out for the verification of the method.

Key Words: Differential inductance, Numerical method, Permanent magnet, Linear actuator.

1. Introduction

The phase inductances are those of the most important parameters of an electric machine. The machine modeling and performance analysis greatly depend on the valid inductances obtained. Usually, the inductance profiles in an electric machine are complicated functions of machine geometry, winding currents, saturation effect of the magnetic core, and the displacement between the stationary and moving windings or magnets. Accurate dynamic analysis of an electric machine requires the profiles of machine phase windings' self and mutual inductances with respect to the rotor positions so as to capture their variations due to machine movement and load conditions.

Nowadays, numerical solutions are widely used in electrical machine analysis and the machine inductances can be readily obtained by those methods. In general, the apparent inductances are usually employed in most applications. This, however, is incorrect when the non-linear characteristic of the magnetic core is considered in dynamic analysis. For the non-linear analysis, the differential inductances should be used.

This paper presents a numerical method for the calculation of the differential inductances and the method was applied for the prediction of the inductances of a newly developed tubular linear permanent magnet (PM) actuator as a function of the position of the moving armature. The results show the effect of the magnetic saliencies due to both the machine structure and magnetic saturation, as well as the end effect due to the limited length of the actuator. The measurement of inductances of a prototype provides a verification of the proposed method.

2. Apparent and Differential Inductances

An electrical machine can be considered as a system with a set of N-discrete, coupled windings. The flux linkage of each winding is a complicated function of the winding currents, machine geometry, material properties, and the angle, $\theta$, between the stationary and moving windings [1]. For the $j$-th winding, the flux linkage can be represented as follows,

$$\lambda_j = \lambda_j(i_1, i_2, \ldots, i_N, \theta)$$  \hspace{1cm} (1)

And the apparent inductance between windings $j$ and $k$, $L_{j}^{\text{app}}$, is defined as,

$$L_{j}^{\text{app}} = \lambda_j / i_j$$  \hspace{1cm} (2)

where $\lambda_{jk}$ is the $k$-th component of $\lambda_j$ that is produced by the current in the $k$-th winding $i_k$. When $k=j$, $L_{j}^{\text{app}}$ is the apparent self-inductance of the winding $j$.

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3. Inductances in Dynamic Model

The following equation is commonly used for analyzing the dynamic performance of a multi-phase electromagnetic device.

\[ v_j = R_j i_j + d\lambda_j / dt \quad , \quad j = 1, 2, ..., N \]  

where \( R_j \) is the resistance of phase \( j \), and \( v_j \) the phase voltage. According to (1), the derivative of the flux linkage in (5) can be further expressed as,

\[ d\lambda_j / dt = \sum_{i=1}^{N} L_{jk}^{\mu} di_k + \omega \frac{d\lambda_j}{d\theta} \]  

where \( \omega = d\theta/dt \).

Therefore, the differential inductances should be used in the dynamic model of the electromagnetic device.

4. Computation of Differential Inductances

4.1 Energy and current perturbation

One of the methods for computing the differential inductances is to use the finite element (FE) magnetic field solutions in conjunction with energy and current (E/C) perturbation technique.

For a conservative magnetic system with \( N \) windings, as shown in Fig. 1, the sum of the magnetic energy, \( W \), and co-energy, \( W_c \), is given by,

\[ W + W_c = \lambda_1 i_1 + \lambda_2 i_2 + \cdots + \lambda_N i_N \]  

The magnetic energy in a non-linear magnetic system is illustrated in Fig. 2 for a single coil. The change in the stored magnetic co-energy due to an infinitesimal change in the winding current at a fixed set of flux linkages can be obtained as,

\[ dW_c = \lambda_1 di_1 + \lambda_2 di_2 + \cdots + \lambda_N di_N \]  

This differential can also be expressed as,

\[ dW_c = \frac{\partial W}{\partial i_1} di_1 + \frac{\partial W}{\partial i_2} di_2 + \cdots + \frac{\partial W}{\partial i_N} di_N \]  

Comparing the terms in (8) and (9) reveals that,

\[ \lambda_j = \frac{\partial W}{\partial i_j} , \quad j = 1, 2, ..., N \]  

By differentiating both sides of (10) with respect to the \( k \)-th winding current, the differential inductance defined in (3) can be acquired by,

\[ L_{jk}^{\mu} = \frac{\partial W}{\partial i_j} / \frac{\partial \lambda_k}{\partial \theta} , \quad j, k = 1, 2, ..., N \]  

In case of self-inductance, (11) can be rewritten as,

\[ L_{jj}^{\mu} = \frac{\partial W}{\partial i_j} / \frac{\partial \lambda_j}{\partial \theta} , \quad j = 1, 2, ..., N \]  

4.2 Numerical solution of E/C perturbation

In order to make the E/C technique to be used in numerical solutions, discretization of the above method is required. By applying the central divided difference to (11) and (12), the following can be yielded,

\[ L_{jk}^{\mu} \triangleq \left[ W_i (i_j + \Delta i_j, i_k + \Delta i_k) - W_i (i_j, i_k) \right] / 4 \Delta i_j \Delta i_k \]  

\[ L_{jj}^{\mu} \triangleq \left[ W_i (i_j, i_k) - 2 W_i (i_j + \Delta i_j) + W_i (i_j + \Delta i_j, i_k) \right] / (\Delta i_j)^2 \]  

By using (13) and (14), the differential inductances can be readily obtained by computing the co-energy of the machine under the conditions described above. Nonetheless, precision problem might arise when dealing with the machine containing the permanent magnets, and this will lead to the method not to be directly applicable in numerical solutions.
5. Inductances of a Tubular Linear PM Actuator

The above numerical E/C method is applied to predict the inductances of a newly developed tubular linear permanent magnet actuator. Fig. 4 shows the schematic structure of the proposed tubular linear actuator [2]. The outer diameter of the actuator is 32 mm and the length of the stator is 40 mm. There are twelve annular windings mounted inside the stator core. Each phase contains four windings connected in series.
An FE model of the motor considering the non-linear properties of magnetic core is established for the analysis of the phase inductances. Factors related to the construction of the prototype are considered during modeling, such as the lamination factor of the magnetic core, the tolerances required for the machining and installation, etc., so that the model could reproduce the reality as much as possible.

5.1 Computed inductances

When computing the inductances, the parts of the actuator that are made up of non-linear magnetic material, such as the stator cylinder and pole pieces on the armature, are discretized by the elements generated by the FE method. Via non-linear solution, the element-based saturation characteristics of these non-linear magnetic parts at the operational point are obtained. Then the model is linearized and the phase inductances including self and mutual inductances are computed via linear solution.

The computed results are shown in Fig. 5. It can be seen that the inductances vary with respect to the position of the armature. The inductances increase when the armature approaches the corresponding phase windings. The magnetic saliency, however, is not remarkable because the magnetic core is not saturated so much.

5.2 Measured inductances

The inductances of the prototype are measured at different armature positions. For the sake of comparison, the results are plotted by solid symbols in the same figure with the predicted values. It can be seen that the predicted inductances are close to the measured values. The variation of the self-inductance caused by the armature position can be clearly seen in both predicted and measured results. The variation of mutual inductances, however, is not as much as predicted.

6. Conclusion

A numerical solution for the differential inductance computation is discussed. The method is successfully implemented based on the FE analysis and is applied to the prediction of inductances of a newly developed tubular linear PM actuator. Comparison between the predicted and the measured value shows the validity of the proposed solution.

References


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