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Initialization for Multi-Robot Formations with Virtual-Head Robot Tracking and Three-Point *l-l* Control

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Abstract: Robotic formation is a group of mobile robots coordinated to get into and maintain a certain geometric shape. This paper presents an effective methodology for initialization of mobile robots to establish desired formation shapes. To enter a formation pattern from arbitrary positions, virtual head robot tracking is proposed for two robots and three-point *l-l* control for three robots. These controllers are incorporated with reactive control schemes to achieve inter-robot collision avoidance. A generic procedure is suggested to deploy multiple robots into a desired shape. Advantages of our approach include the establishment and maintenance of any formation type from arbitrary initial conditions with collision avoidance for a large group of mobile robots. Extensive simulation results are provided to illustrate the capability of handling singularities and avoiding inter-robot collision in a group of *N* robots.

Keywords: collision avoidance, formation, three-point *l-l* control, virtual head robot.

1. Introduction

Significant achievements of distributed control of multi-agent systems have been recognized with the use of advances in communication and computation. This trend results in the next generation of automated highway systems [22], co-ordination of multiple aircraft in future air traffic management systems [21], as well as formations of aircraft, satellites, and mobile robots [1] [2] [6]. The deployment of multi-robot systems would be made simpler when the agents' mission can be executed by means of a formation, defined as a motion pattern of these robots. Maintaining a formation shape remains an issue in multiple aircraft used for investigation of aerodynamic effects [3] or in robotic exploration of large areas with restricted sensor capabilities [5].

The robotic formation problem has attracted intense research effort over recent years. The use of a group of robots to operate in a group has several advantages over that of single robots, including overall system enhanced performance (increased instrument resolution, reduced cost), and the capability of executing tasks that single robots cannot accomplish. Potential applications can range from industrial coordination in agriculture, construction, mining and to diverse missions such as surveillance, wide-area search and rescue, environmental mapping, defense, and healthcare. Examples that have been described in the literature include box pushing [17], load transportation [12] and invader capturing/enclosing [23].

The motion patterns of multiple robots or formation types can be a column, a line, a wedge, a ring or a chain [10]. Solutions for the coordination problem are currently applied in search and rescue operations, landmine removal, remote terrain and space exploration, and also the control of satellites and unmanned aerial vehicles. This problem of control and coordination for multiple mobile robots has revolved around two major tasks. Firstly, the robot platoon must maintain desired shapes. Secondly, the robots have to simultaneously avoid collisions between themselves and with obstacles in the environment.

Fundamentally, approaches to formation control for multiple robots can be categorized into three broad groups: behavior-based, virtual structure and leader-follower. In behavior-based approaches [1] [4] [24], some simple and intuitive behaviors or motion primitives for individual agent should be firstly assigned. Then, by using a weighted sum of these simple primitives, more complex motion patterns are generated through the interaction of several agents. Although it would be difficult to analyze rigorously the characteristics of a behavior-based approach, the system stability and convergence can be proved for some simple schemes [11]. In virtual structure approaches [2] [17], the entire formation is treated globally as a single entity or the so-called virtual structure. If desired dynamics of the virtual structure can be translated into the desired motion of each agent then one can design local controllers to achieve global performance.

In leader-follower approaches [16] [19], one or more robots are designated as leaders and responsible for guiding the formation shape. The rest of the robots are required to follow the leader with a predefined offset. Considering a group of mobile robots in the framework of interconnected systems, the leader-to-formation stability problem is addressed in [20], based on input-to-state stability in the control theory. Practically, there are two main problems in leader-follower approaches: leader tracking and collision avoidance. For tracking, feedback controllers called l-ψ and l-l have been proposed in [9] for maintaining formations of multiple mobile robots. In another approach three types of controllers, namely basic leader-following, leader-obstacle, and three-robot shape, are used to maintain a formation under appropriate assumptions on the motion of the leader robot

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[7]. Path following and mobile object tracking is tackled in [14] using respectively "vertical" and "horizontal" tracking control schemes. Formation robustness is considered in [15] by using the variable structure systems methodology for control design.

The above-mentioned approaches are not concerned directly with the problem of deploying a large group of robots from arbitrary initial conditions to enter into a desired formation shape of robots. In [13], the virtual robot tracking control and modified *l-l* control approach could be used to initialize a group of three mobile robots. However, this approach faces a difficulty in forming the group into a formation line and at the same time avoiding inter-robot collisions. In this paper, virtual-head robot tracking and three-point *l-l* control is proposed along with a general procedure to initialize and maintain a desired formation shape for a large group of mobile robots while ensuring collision avoidance and singularity alleviation.

The paper is organized as follows. After the introduction, Section 2 presents the model and problem formulation. The control design is detailed in Section 3. Collision avoidance with reactive control schemes and the initialization procedure are given in Section 4. Section 5 presents the simulation results and a conclusion is drawn in Section 6.

2. Preliminaries and Problem Statement

This section describes the robot model, provides some definitions and formulates the problem.

2.1 Robot Model

A mobile robot can be described by a conventional kinematic model as:

$$\dot{x} = v\cos\theta,\ \dot{y} = v\sin\theta,\ \dot{\theta} = \omega, \tag{1}$$

where (x, y) is the centre point on the wheel axis, $\theta \in R$ is the orientation angle, inputs v and ω are respectively the translational and angular velocities. From the non-holonomic velocity constraints, it is required the robots satisfy strictly the pure rolling and non-slipping conditions, i.e. $\dot{x}\cos\theta + \dot{y}\sin\theta = v$ and $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$.

2.2 Definitions

Definition 1. Virtual robot (VR): is a hypothetical robot whose orientation is identical to that of its host robot, but position is displaced apart from the predefined *R-L* clearances. The symbols L and R denote respectively the longitudinal clearance and clearance along rear wheel axis.

The relation between VR and its host in terms of positions and orientation can be written as:

$$\begin{cases} x_{vi} = x_i + R\sin\theta_i - L\cos\theta_i \\ y_{vi} = y_i - R\cos\theta_i - L\sin\theta_i \\ \theta_{vi} = \theta_i, \end{cases} \tag{2}$$

where (x_i, y_i, θ_i) and $(x_{vi}, y_{vi}, \theta_{vi})$ are respectively coordinates of the host (robot i itself) and its virtual robot. Clearances R and L here are defined to be strictly positive for the case VR is located in the right-bottom corner of its host, as shown in Fig. 1.

Definition 2. Head robot (HR): is a hypothetical robot whose orientation is identical to that of its host, but position is placed at distance $d > 0$ ahead from its host.

The relation between HR and its host can be written as:

$$\begin{cases} x_{hj} = x_j + d\cos\theta_j \\ y_{hj} = y_j + d\sin\theta_j \\ \theta_{hj} = \theta_j, \end{cases} \tag{3}$$

where (x_j, y_j, θ_j) and $(x_{hj}, y_{hj}, \theta_{hj})$ denote coordinates of the host (robot j) and its head robot, respectively. Note that HR is designated to serve as a virtual robot of the follower with d to be chosen adequately small as a tracking margin. HR is identical with its host when $d = 0$. Position errors between VR of the leader i and HR of the follower j are:

$$\begin{cases} e_{xjl} = x_{hj} - x_{vi} = (x_j + d\cos\theta_j) - (x_i + R\sin\theta_i - L\cos\theta_i) \\ e_{yjl} = y_{hj} - y_{vi} = (y_j + d\sin\theta_j) - (y_i - R\cos\theta_i - L\sin\theta_i). \end{cases} \tag{4}$$

Definition 3. Collision measure. Possible collision between any two robots may be detected by associating each robot with a circle whose centre located at the control point on the robot's wheel axis, and radius r determined by the robot's dimensions with an additional distance as a safety margin. Denoting ρ_{ij} the distance between control points of two robots i and j, a measure describing the possibility of collision between robots is defined as

$$f_{ij} = \rho_{ij} - 2r, \tag{5}$$

where $f_{ij} > 0 \ \rightarrow safe,\ f_{ij} \le 0 \ \rightarrow unsafe.$

2.3 Problem Formulation

Before stating the problem, the following assumptions are made:

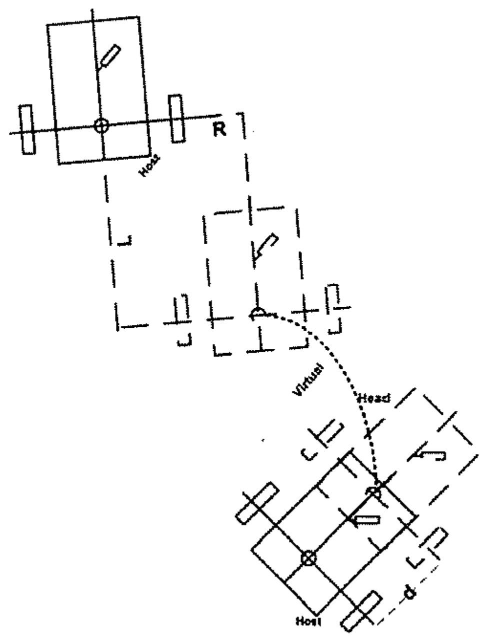

Fig. 1. Virtual Robot and Head Robot.

Assumption 1. Each robot is indexed by a unique number indicating the priority according to its role in the group: the lower the index, the higher the priority.
Assumption 2. Each robot can get from its communication channel any necessary information of its position and orientation and of its leader in a global coordinates.
Assumption 3. The leader follows a smooth trajectory and the workspace is flat and obstacle-free.

Our objective here is to design controllers for a large group of robots to achieve
1) any desired formation shape,
2) no inter-robot collision, and
3) the desired group motion that satisfies the limitation of the communication range.

3. Control Development

The control design in this paper is based on the virtual-head robot tracking (VHRT) and three-point *l-l* control (3PLL) coupled with reactive schemes.

3.1 Virtual-Head Robot Tracking (VHRT)

Virtual robot (VR) tracking control has been proposed in [13] to form a desired configuration of a pair of leader-follower robots, ensuring the convergence of the position of the virtual robot to that of the reference. However, because the configuration parameter l must be non-zero, some desired shape (e.g. a line) may not be formed directly. In addition, in some cases of initial conditions the orientation of the VR (and its host) may not converge to that of the reference, implying that the desired formation shape may not be established and potential collisions may happen in those situations. The Virtual-Head Robot Tracking model, motivated by VR and the idea of virtual reference point used in [14], is proposed here as a remedy for these circumstances. The objective is to design a control law for follower robot j such that its head robot can track the virtual robot of leader i with tracking errors decreasing monotonically versus time. The controller should ensure a desired *R-L* configuration of leader i and follower j with position errors smaller than or equal to the chosen margin d as $t \to \infty$.

From Eq. (1) and (4) an error model can be derived as

$$\dot{e}_{ji} = B_j u_j - b_i u_i, \qquad (6)$$

where

$$e_{ji} = \begin{bmatrix} e_{xji} \\ e_{yji} \end{bmatrix},\ B_j = \begin{bmatrix} \cos\theta_j & -d\sin\theta_j \\ \sin\theta_j & d\cos\theta_j \end{bmatrix},\ u_j = \begin{bmatrix} v_j \\ \omega_j \end{bmatrix},$$

$$b_i = \begin{bmatrix} \cos\theta_i & R\cos\theta_i + L\sin\theta_i \\ \sin\theta_i & R\sin\theta_i - L\cos\theta_i \end{bmatrix},\ u_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}.$$

A standard I/O linearization technique is used to generate the control law:

$$u_j = B_j^{-1}\left[b_i u_i - \Lambda_j e_{ji}\right], \qquad (7)$$

where $\Lambda_j = \begin{bmatrix} \lambda_{j1} & 0 \\ 0 & \lambda_{j2} \end{bmatrix}$ is a positive-definite diagonal matrix and $B_j^{-1} = \frac{1}{d}\begin{bmatrix} d\cos\theta_j & d\sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{bmatrix}$.

Time responses of the controlled system errors are then

$$\begin{cases} e_{xji}(t) = e_{xji}(0).e^{-\lambda_{j1}t} \\ e_{yji}(t) = e_{yji}(0).e^{-\lambda_{j2}t}, \end{cases}$$

which demonstrate exponential convergence to zero. The control u_j can always be defined if $d > 0$ is chosen such that matrix B_j is non-singular. By applying Eq. (7), the time derivative for θ_j can be obtained as

$$\dot{\theta}_j = d^{-1}[(\lambda_1 e_{xji} - v_i'\cos\theta_i')\sin\theta_j + (-\lambda_2 e_{yji} + v_i'\sin\theta_i')\cos\theta_j], \qquad (8)$$

where

$$v_i' = \sqrt{v_i^2 + (R\omega_i)^2 + (L\omega_i)^2 + 2Rv_i\omega_i},$$
$$\theta_i' = \text{atan2}(X, Y)$$
$$X = v_i\sin\theta_i + (R\sin\theta_i - L\cos\theta_i)\omega_i$$
$$Y = v_i\cos\theta_i + (R\cos\theta_i + L\sin\theta_i)\omega_i.$$

Eq. (8) can be then rewritten as:

$$\dot{\theta}_j(t) = A_j\cos(\theta_j + \beta_j), \qquad (9)$$

where $A_j = d^{-1}\sqrt{(\lambda_1 e_{xji} - v_i'\cos\theta_i')^2 + (-\lambda_2 e_{yji} + v_i'\sin\theta_i')^2}$,

and

$$\beta_j = \text{atan2}(-\lambda_1 e_{xji} + v_i'\cos\theta_i',\ -\lambda_2 e_{yji} + v_i'\sin\theta_i').$$

By assigning the right hand side of Eq. (9) to a feasible reference angular velocity ω_{ri} and linearizing Eq. (9) along the solution $\theta_j^e = -\beta_j + \arccos(\omega_{ri}/A_j)$, one can obtain

$$\delta\dot{\theta}_j = -A_j\sin(\theta_j + \beta_j)|_{\theta_j=\theta_j^e}\delta\theta_j = -\sqrt{A_j^2 - \omega_{ri}^2}\,\delta\theta_j, \qquad (10)$$

which implies the stability of zero dynamics.

The control law ensures collision avoidance between two robots i and j if at the beginning the centre of leader i is not found in the "critical area", determined by initial positions of the two robots as illustrated in Fig. 2, where the safe distance, referred to the centre of any robot in the group for avoidance of collision with others, is defined as

$$D_{safe} = d + d_{safe}, \qquad (11)$$

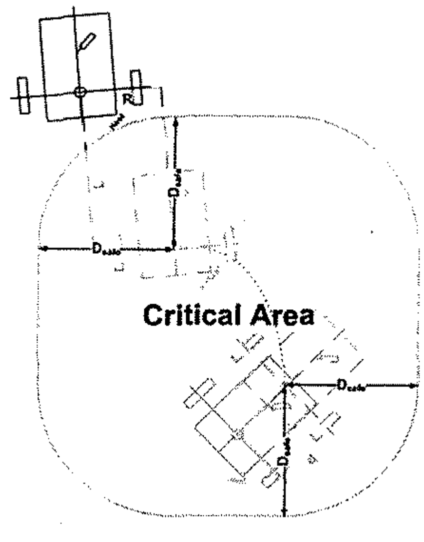

Fig. 2. Critical area of potential collision.

with $d_{safe} = 2r$. It is clear from the expression of B_j^{-1} that the margin d cannot be chosen too small as this will involve very high angular velocity of robot j. Note that should potential collision with a third robot happen then one may apply *l-l* control law to achieve desired distances from the leader and follower with this third robot, as detailed in the next section.

3.2 Three-Point *l-l* (3PLL) Control

The *l-l* control approach, proposed in [8] for collision avoidance, is subject to a singularity when three considering robots lie on the same line connecting them. Therefore, three *l-l* controllers will be used here, whereby distances to robot 1 and robot 2 are taken from three virtual points. These points are located around the centre of robot 3 to form a certain triangle. An appropriate *l-l* controller shall be chosen in the singularity case. First, velocities of a virtual robot are defined in the following proposition.

Proposition 1. Virtual Robot velocities. A virtual robot of robot i, having predefined configuration values of $R = R^*, L = 0$, can be considered apparently as an "independent" robot with velocities $v_i' = v_i + R^*\omega_i$ and $\omega_i' = \omega_i$, where v_i and ω_i are velocities of robot i.

Indeed, by considering a virtual robot of robot i with pre-defined clearances $R = R^*, L = 0$, one has from Eq. (3):

$$\begin{cases} x_{vi} = x_i + R^* \sin\theta_i \\ y_{vi} = y_i - R^* \cos\theta_i \\ \theta_{vi} = \theta_i, \end{cases} \tag{12}$$

and hence,

$$\begin{cases} \dot{x}_{vi} = \left(v_i + R^*\omega_i\right)\cos\theta_i \\ \dot{y}_{vi} = \left(v_i + R^*\omega_i\right)\sin\theta_i \\ \dot{\theta}_{vi} = \dot{\theta}_i = \omega_i. \end{cases} \tag{13}$$

The virtual velocities of robot i given in Proposition 1 can then be obtained by comparing Eq. (13) to model (1).

Consider now the case when distances to robot 1 and robot 2 are from point K, determined by r_K, l_K, which is different from the centre of robot 3, as shown in Fig. 3. Here, K is considered as a head point located, along the longitudinal axis, at distance l_K from the centre of a virtual robot R_{v3} of robot 3, where R_{v3} is defined with $R = r_K, L = 0$.

According to Proposition 1, the velocities of virtual robot R_{v3}:

$$\begin{cases} v_{v3} = v_3 + r_K\omega_3 \\ \omega_{v3} = \omega_3. \end{cases} \tag{14}$$

The kinematic model for R_{v3} under *l-l* control is:

$$\begin{bmatrix} \dot{l}_{13K} \\ \dot{l}_{23K} \end{bmatrix} = \begin{bmatrix} \cos\gamma_{1K} & l_K \sin\gamma_{1K} \\ \cos\gamma_{2K} & l_K \sin\gamma_{2K} \end{bmatrix} \begin{bmatrix} v_{v3} \\ \omega_{v3} \end{bmatrix} - \begin{bmatrix} v_1 \cos\psi_{13K} \\ v_2 \cos\psi_{23K} \end{bmatrix} = B_{uK} u_{v3} - v_{uK}, \tag{15}$$

$$\dot{\theta}_{v3} = \omega_{v3}. \tag{16}$$

Therefore, similarly to the control design proposed in [13], one can apply the following control law for virtual robot R_{v3}:

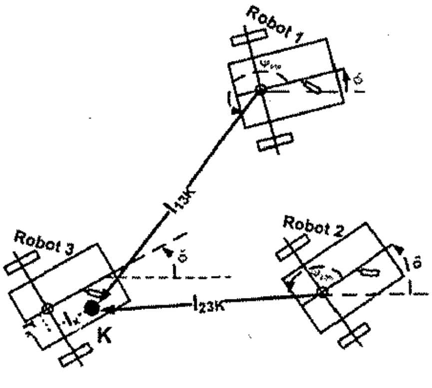

Fig. 3. *l-l* control with respect to virtual point K.

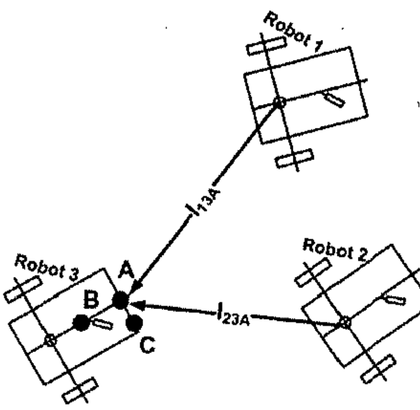

Fig. 4. Switching among three *l-l* controllers.

$$u_{v3} = \begin{bmatrix} v_{v3} \\ \omega_{v3} \end{bmatrix} = B_{uK}^{-1}(v_{uK} + \alpha_K . l_{eK}),\ 0 \le t \le T_r; \tag{17}$$

where
$$\begin{cases} \gamma_{iK} = \theta_i + \psi_{i3K} - \theta_3, (i = 1,2) \\ l_{eK} = \begin{bmatrix} \left(l_{13K}^d - l_{13K}\right)^{\frac{2}{3}} \\ \left(l_{23K}^d - l_{23K}\right)^{\frac{2}{3}} \end{bmatrix}, \alpha_K = \begin{bmatrix} \alpha_{1K} & 0 \\ 0 & \alpha_{2K} \end{bmatrix}, \\ \alpha_{1K} = sign(l_{13K}^d - l_{13K}(0))\left|l_{13K}^d - l_{13K}(0)\right|^{\frac{1}{3}} / \left(\frac{T_r}{3}\right) \\ \alpha_{2K} = sign(l_{23K}^d - l_{23K}(0))\left|l_{23K}^d - l_{23K}(0)\right|^{\frac{1}{3}} / \left(\frac{T_r}{3}\right). \end{cases}$$

From Eq. (14), the control law in terms of velocities for robot 3 can be calculated as:

$$u_3 = \begin{bmatrix} v_3 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} v_{v3} - r_K\omega_{v3} \\ \omega_{v3} \end{bmatrix}. \tag{18}$$

As stated, the three *l-l* controllers are to be selected correspondingly, for example, to three virtual head points of robot 3, namely A, B, and C, forming a triangle, as shown in Fig. 4. By that way, the singularity problem associated with *l-l* control, i.e. when the head point of robot 3 is aligned with the line connecting two other robots, can be completely overcome. Triggering the switching for A, B, or C in practice depends on the level of sensitivity of the robot actuator control signal with respect to singularities.

4. Collision-Free Initialization Procedure

The previous section is concerned with the formation control of two and three robots. The approach can be extended for the establishment and maintenance of a large group of robots. For this, a generic procedure is proposed in this section for initialization of the robots in the group, using control laws Eq. (7) and (17). An essential requirement for the formation problem is inter-robot collision avoidance. It is noted that in the tracking phase if the safe distance given Eq. (11) is not preserved, the proposed controllers may not ensure the collision avoidance. Therefore, reactive control schemes are proposed to deal with this problem.

4.1 Reactive Control Schemes

The idea of reactive control here is that should collision occur among robots according to the collision detection criteria described in Eq. (2), 3PLL control will be used to drive the lowest priority robot (i.e. robot with the highest index) among the ones in potential collision to diverge from them but still heading to the target position [18]. Generally, cases necessitating reactive control can be dealt with by using following schemes.

Scheme 1. Potential collision between two robots. This scheme is used for a potential collision between any two robots, as illustrated in Fig. 5. The lower priority robot, robot 3, will switch to 3PLL control with respect to two leaders: the higher priority robot as the first leader (robot 1) and a VR of this robot as the second leader (robot 2) with predefined clearances $R = 2r + D_{\max}$ or $R = -(2r + D_{\max})$ and $L = 0$. Here, $D_{\max}$ is the largest distance from one of three head points of robot 3 to its centre, and the sign of clearance R is decided such that the second leader should be close to the target position of robot 3. The control parameters l_{13}^d and l_{23}^d can be designed as follows

$$\begin{aligned} l_{13}^d &= 2r + D_{\max} + \delta_1 \\ l_{23}^d &= \delta_2. \end{aligned} \tag{19}$$

Using 3PLL control with the above parameters will drive robot 3 closer to the VR (robot 2) while going around the safe boundary of robot 1, which is a circle with centre of robot 1 and radius $2r$.

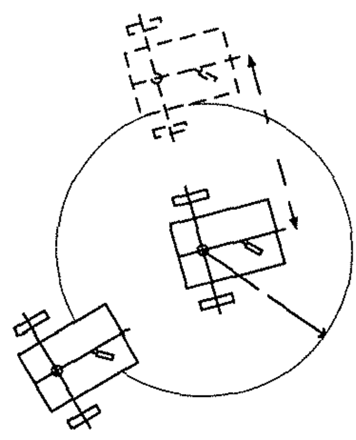

Fig. 5. Case $R = 2r + D_{\max}$ with 3PLL control.

The reason that l_{13}^d is $2r + D_{\max}$ rather than $2r$ is that in 3PLL control, the distances from one of three head points of robot 3 to the centres of its leaders is referred to instead of the distances among their centres. Thus in order to ensure collision avoidance, l_{13}^d has to be increased by the largest distance from one of three head points of robot 3 to its centre. Margins δ_1 and δ_2 are deliberately augmented to l_{13}^d and l_{23}^d to ensure the distance between the centre of robot 1 and robot 3 to be strictly greater than $2r$.

Scheme 2. Potential collision between three robots. This scheme is used when potential collision is between a robot and two other robots. The lowest priority robot will then apply 3PLL control with respect to two leaders, which are the two other robots in consideration. The control parameters l_{13}^d and l_{23}^d are proposed as,

$$\begin{aligned} l_{13}^d &= 2r + D_{\max} + \delta_1 \\ l_{23}^d &= 2r + D_{\max} + \delta_2, \end{aligned} \tag{20}$$

where again, $D_{\max}$, δ_1 and δ_2 are augmented to l_{13}^d and l_{23}^d for the same reason as explained above.

4.2 Initialization Procedure

A step-by-step procedure is proposed in this section for initialization of a group of N mobile robots to enter a desired formation shape. At each step, robots in the group are classified as *active* or *inactive*. *Active* robots will participate in the process while *inactive* robots stay at its initial position. The initialization procedure will then run until all robots in the group become *active* and a desired formation shape is obtained. In addition, in the proposed framework, each robot can play the role of a follower (tracking another robot) or of a leader (guiding another one). The leader of the whole group is indexed by 1. Under the proposed reactively controlled VHRT and 3PLL control, inter-robot collision avoidance can be achieved with some margin in distance between robots, consequently inactive robots are designated not to obstruct any active robot. Note that in any desired motion pattern, distance between any two robots shall preserve a certain lower limit determined in a practical application. In the paper, this limit distance is taken as double of the safe distance, i.e. $4r$.

The proposed initialization process is summarised in the following algorithm.

Step1. Make all robots in the group *inactive*. Choose the leader robot, indexed by i=1, to guide the whole group. Let it become *active*.

Step2. Index (or reindex) all *inactive* robots from j=(i+1) to N, based on their initial position with respect to the motion of the leader robot.

Step3. Let one or two *inactive* robots with smallest indices become *active*. Use VHRT-3PLL to get them into desired positions while avoiding reactively collision with other robots until all i *active* robots have reached their positions in the group. Go to Step 2 if j<N.
If there is no *inactive* robot left (or i=N), the desired formation shape has established.

Step 4. Exit.

By applying the proposed methodology, a large group of mobile robots can be controlled to form and maintain a desired formation shape without inter-robot collision.

5. Simulation Results

Extensive simulation has been conducted using the proposed approach. Some typical illustrations are included in the following.

5.1 Singularity case

To illustrate the capability of avoiding singularities and possibilities of collision among robots the case of three mobile robots moving to form a wedge is first considered. Parameters and conditions used in this simulation were set as:

- Initial conditions of robots:

Robot 1: $x_1(0)=30, y_1(0)=0, \theta_1(0)=0(rad), v_1=5, \omega_1=0$,

Robot 2: $x_2(0)=0, y_2(0)=-50, \theta_2(0)=0(rad)$,

Robot 3: $x_3(0)=20, y_3(0)=-90, \theta_3(0)=1(rad)$;

- Parameters for the desired wedge:

Robot 2: $R_2=40, L_2=30$, Robot 3: $R_3=-40, L_3=30$;

- Tracking margin for head robots: $d=1$;
- Safe distance between any two robots: $d_{safe}=26$;
- Parameters for Virtual-Head tracking control:

Robot 2: $\lambda_{21}=1, \lambda_{22}=2$, Robot 3: $\lambda_{31}=1, \lambda_{32}=2$;

- Parameters for *l-l* control with point A:

$r_A=0,\ l_A=12,\ T_r=3s,\ l^d_{13A}=100,\ l^d_{23A}=50$;

- Parameters for *l-l* control with point B:

$r_B=0,\ l_B=6,\ T_r=3s,\ l^d_{13B}=100,\ l^d_{23B}=50$.

Fig. 6, 7, 8 and 9 show respectively the global trajectories and time responses of position x, y and orientation θ for the three robots. With these initial conditions, potential collision appeared at time points $t=0.11s$; $3.21s$; $6.33s$; $9.46s$; and $12.6s$ when using the *l-l* controller with head point A of robot 3. At $t=9.46s$, due to singularity, the system switched to the *l-l* controller with respect to head point B. The results obtained illustrated that the three robots could successfully get into and maintain a wedge formation without inter-robot collision even in the presence of singularities.

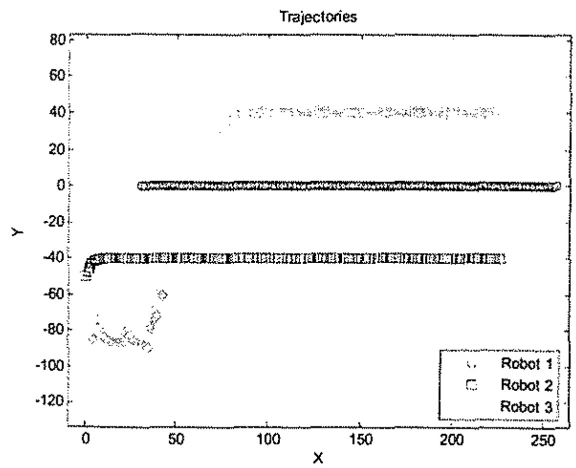

Fig. 6. Wedge trajectories.

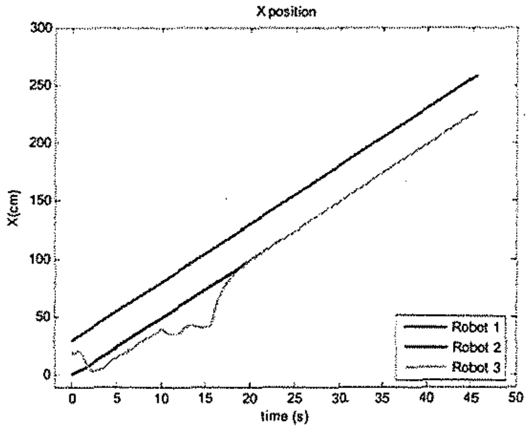

Fig. 7. x position – a wedge.

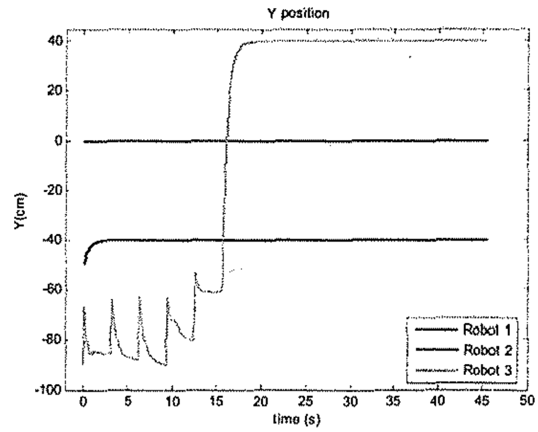

Fig. 8. y position – a wedge.

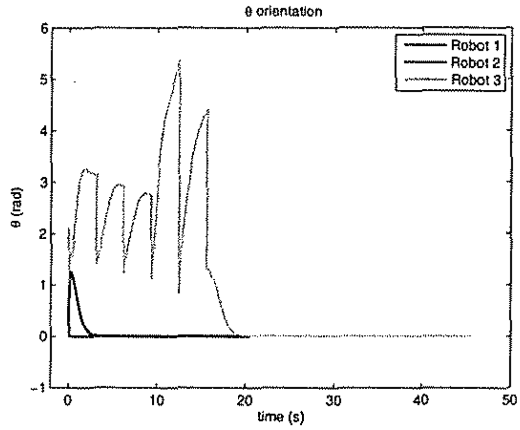

Fig. 9. θ - orientation – a wedge.

5.2 Multi-Robot Initialization

To illustrate the procedure proposed for a group of robots to enter a shape, we choose typically the case of five mobile robots moving to form a diamond-like formation. Parameters and conditions used were set as:

- Initial conditions of robots:

Robot 1: $x_1(0)=30, y_1(0)=0, \theta_1(0)=0(rad), v_1=5, \omega_1=0$,

Robot 2: $x_2(0)=0, y_2(0)=0, \theta_2(0)=0(rad)$,

Robot 3: $x_3(0)=10, y_3(0)=50, \theta_3(0)=0(rad)$,

Robot 4: $x_3(0) = 150, y_3(0) = 180, \theta_3(0) = 2(rad)$,
Robot 5: $x_3(0) = 150, y_3(0) = -150, \theta_3(0) = \pi/2(rad)$;

- Parameters for the desired diamond:

Robot 2: $R_2 = 85, L_2 = 40$, Robot 3: $R_3 = -85, L_3 = 40$,
Robot 4: $R_3 = 0, L_3 = 40$, Robot 5: $R_3 = 0, L_3 = 80$;

- Tracking margin for choosing head robots: $d = 1$;
- Safe distance between any two robots: $d_{safe} = 22$.

Following the proposed procedure, in the first step, three robots 1, 2, 3 formed a part of the desired diamond, which is a wedge. This step took 30 seconds. In second step, robot 4 and robot 5 tracked robot 1 to form the desired shape. Figures 10, 11, 12 and 13 show respectively the global trajectories and time responses of position X, Y and orientation θ of the robots. From the simulation, robot 5 could possibly collide with robot 2 at $t = 30.22s$, and robot 4 with robot 3 at $t = 30.23s$. After avoiding collision by using the proposed reactive control schemes, robot 4 and robot 5 switched back to tracking control to eventually establish the desired diamond formation.

The simulation results have shown the validity of the proposed reactively-controlled VHRT and 3PLL control approach in the initialization and establishment of any desired formation shape for a large group of mobile robots while ensuring inter-robot collision avoidance.

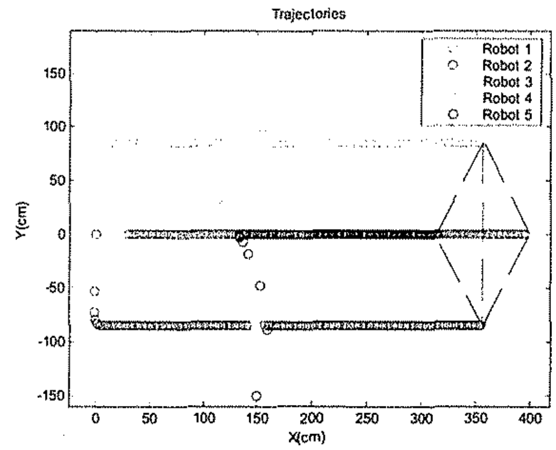

Fig. 10. Diamond-shape trajectories.

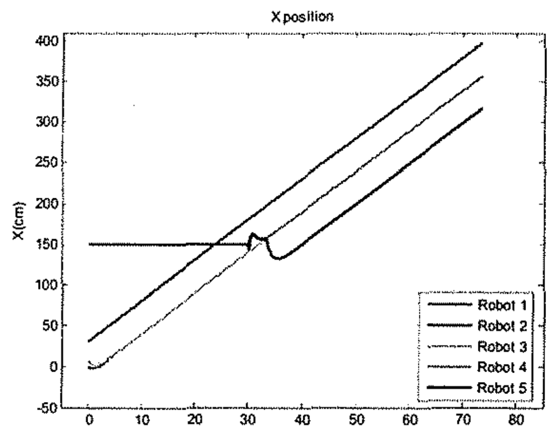

Fig. 11. x position – a diamond.

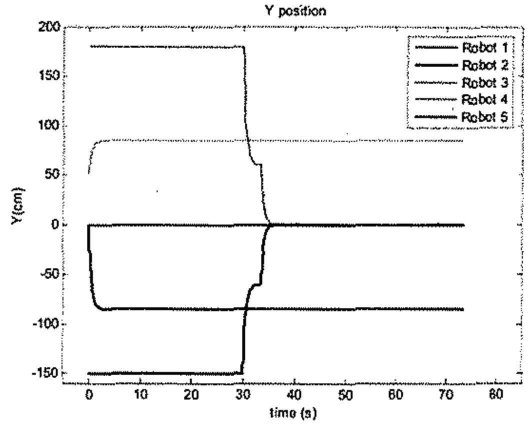

Fig. 12. y position – a diamond.

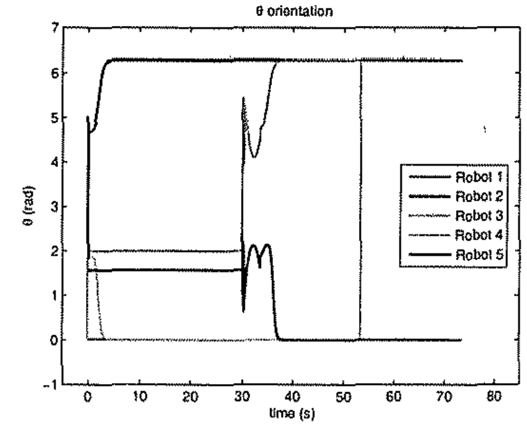

Fig. 13. θ-orientation – a diamond.

6. Conclusion

An effective approach has been presented in this paper for the initialization of multiple mobile robots into desired formation groups by using the leader-following strategy while ensuring collision-free group motion. For the control design, virtual head robot tracking (VHRT) is proposed for a follower to form with its leader any desired shape with a given tracking margin, and three-point *l-l* (3PLL) control for avoiding singularities in establishing a formation shape of a follower with respect to two leaders. Reactive control schemes are suggested to ensure a safe distance between robots. For generally a group of N robots, a step-by-step procedure is formulated for initialization of the multi-robot group. The proposed approach has been tested through extensive simulations to demonstrate its validity.

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