International Transactions on Systems Science and Applications
Volume 1 Number 2 (2006) 181-189

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Robust Formation of Multiple Robots Using Reactive Variable Structure Systems

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Abstract: This paper addresses robust control of multiple mobile robots moving in desired formations. A rigorous control technique for such an agent-based robotic system may encounter problems of singularity, parameter sensitivity and inter-robot collision. Our proposed approach focuses on the enhancement of robustness as well as collision-free establishment of formations even in the case of singularity and uncertainties in sensing information of the reference coordinates by using variable structure controllers incorporated with a reactive control scheme. Advantages of the approach are verified in simulation of three robots moving in such formations as a line and a column.
Keywords: reactive control, robotic formation, robustness, variable structure systems.

1. Introduction

The issue of control and coordination for multiple mobile robots has revolved around two major tasks. First, the robot platoon must maintain some desired shapes such as a line, a column or a ring formation. The motivation is that multiple robots are capable of performing many applications that single robots cannot. Second, the robots have to simultaneously avoid collisions between themselves and with obstacles in the environment. Examples of these applications include box pushing [15], load transportation [11] and capturing/enclosing an invader [23]. There exist essentially three approaches in controlling multiple mobile robot rmation, namely: leader-following, behavior-based and .irtual-structure (see, e.g., [7] and references therein).

In leader-following, one robot is designated as the leading reference, with the rest as followers. The basic idea is that the followers track the position and orientation of the leader with some prescribed (possibly time-varying) offset. The leader-following approach to multiple robots has been analyzed in the framework of interconnected systems, focusing on formation stability [22]. Several leader-following control techniques have been proposed. In [4], the problem of leader following is addressed for the case of a multi-robot team with heterogeneous sensing capabilities. Spacecraft control using the leader following concept is reported in [5] with adaptive control laws being proposed for keeping satellite formation in earth orbit. Feedback linearization techniques are used in [6] to derive tracking control laws for non-holonomic robots in a formation that is described as a directed graph.

The behavioral approach is based on the idea of assigning to an agent several desired responses to possible excitations to the system, and for each response or behavior, making the control action a weighted average of all possible actions. The behavioral approach has been used to control a group of robots in line and circle formations [24]. A behavior-based architecture is exploited in [1] for multi-robot teams, where each local platform is controlled appropriately with respect to its neighbors by averaging several competing behaviors. To construct robotic formations, behavioral dynamics of heading direction and path velocity has been proposed recently [16], based on the concept of coupling dynamics to behaviors.

In the virtual structure approach, the entire formation is treated globally as a single structure or so-called virtual structure. If the desired dynamics of the virtual structure can be translated into the desired motion of each robot then one can design local controllers to achieve global performance. The concept of virtual structure in the framework of cooperative robotics is introduced in [15]. The virtual structure is applied to multiple spacecraft flying in [2], where, to achieve global coordination, knowledge of the virtual structure states is shared between each robot through dynamic coordination variables. These variables are similar to the action reference notion introduced in [13] or the platoon-level functions given in [21].

While the virtual structure approach utilizes centralized controllers, the leader-following and behavioral approaches often apply decentralized controllers using local information. To deal with collision avoidance, some researchers used optimal motion planning [14] [20], which may be computationally expensive, while other investigators used feedback control with reactive schemes [3] [6] [19]. These feedback controllers come with formal proofs of satisfactory system performance and formation establishment. One advantage of these schemes is that they can be applied to small, heterogeneous robots with limited communication range.

In robotic formation control, the problem of collision between robots in the transient phase remains important, although has not been explicitly addressed. For the leader-following approach, tracking control of multiple mobile robots using virtual robots (VR) [12] combined with *l-l* control [6] has been proposed in an obstacle-free environment. However, it is observed that the VR control method does not necessarily guarantee acceptable collision avoidance between robots in some cases, particularly in the transient process. Also, there are limitations on the type of the desired formations.

Motivated by Ögren and Leonard [19], who dealt with the

ISSN 1751-1461 print / ISSN 1751-147X cd-rom
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parametric sensitivity problem in robotic formations indirectly by defining uncertainty region around each robot, this paper seeks a control approach that can preserve the formation motion regardless of uncertainties. For this, the variable structure systems (VSS) approach (see, e.g., [8]), known for their prominent feature of robustness or the ability of the system to operate satisfactorily in the presence of parameter perturbations and external disturbances, are employed in this paper to generate appropriate controls for individual robots. Our contribution is on the development of robust controllers that can demonstrate high performance of the whole group of mobile robots when entering any desired formation and simultaneously avoiding inter-robot collision. In the case of potential inter-robot collision, reactive control schemes are proposed for selecting different control parameters to lead the robots to safe positions while still ensuring formation establishment.

The paper is organized as follows. After the introduction given in Section 1, Section 2 presents an overview on multiple robot tracking control. The variable structure systems approach to robot tracking control is developed in Section 3. Reactive control schemes are described in Section 4. Simulation results are provided in Section 5. Finally, Section 6 concludes the paper.

2. Tracking Control Overview

We consider a group of n three-wheeled mobile robots, whose kinematic model is given by

$$\dot{q}_i = B_i u_i = \begin{pmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{pmatrix} u_i, i = 1,2\ldots, n; \qquad (1)$$

where $q_i = (x_i, y_i, \theta_i)^T$ is the state vector, (x_i, y_i) is the position in a global frame and θ_i is the orientation; and $u_i = (v_i, \omega_i)^T$ is the control input, in which vi is the translational velocity and ω_i is the angular velocity, of robot i. It is assumed that

(i) Robots are of the same model and satisfy the constraints of
non-slipping $\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0$, (2)
and pure rolling : $\dot{x}_i \cos\theta_i - \dot{y}_i \sin\theta_i = v_i$. (3)

(ii) The workspace is flat and contains no obstacle.

(iii) The leader maintains positive velocity on a smooth trajectory. Followers are indexed by a distinctive priority number and also aware of others' indices.

(iv) Each robot can extract necessary information via its communication channels.

Giving the initial position and orientation of the reference robot and its motion trajectory, the objective is to design for follower i such that, as $t \to \infty$, a desired formation is established without collision among robot i and any robot j, and the overall motion remains insensitive to sensing perturbations while satisfying the limitation of communication range.

2.1 Virtual Robot Tracking

A virtual robot (VR), proposed in [12], is used to denote a hypothetical one, placed in the workplace such that it has r-l clearances from the follower and has the same orientation with the leader, as shown in Fig. 1.

The geometric relationship between the VR and the follower robot is as follows,

$$\begin{aligned} x_{vi} &= x_i - r\sin\theta_i + l\cos\theta_i \\ y_{vi} &= y_i + r\cos\theta_i + l\sin\theta_i \\ \theta_{vi} &= \theta_i, \end{aligned} \qquad (4)$$

where $q_r = [x_r, y_r, \theta_r]^T$ is the reference state vector, and $q_i = [x_{vi}, y_{vi}, \theta_{vi}]^T$ is the state vector of the VR of follower i, $i = 1, 2, \ldots, n$.

The kinematic model of the VR is then,

$$\dot{q}_{vi} = \begin{pmatrix} \cos\theta_i & -r\cos\theta_i - l\sin\theta_i \\ \sin\theta_i & -r\sin\theta_i - l\cos\theta_i \\ 0 & 1 \end{pmatrix} u_i = \begin{pmatrix} B_{vi} \\ \hline 0 \quad 1 \end{pmatrix} u_i. \qquad (5)$$

The idea is to use the VR to track the reference robot so that the follower will reach the desired position in the formation as its VR approaches the reference robot. In Fig. 1, it is shown that the VR trajectory converges to that of the reference robot in an internal shape defined by x_e and y_e. Details about the controller design can be found in [12]. Note that B_{vi} must be non-singular or l different from zero, which means a line formation cannot, generally, be achieved.

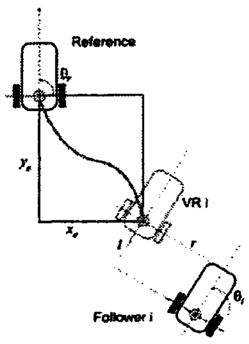

Fig. 1. VR tracking.

2.2 *l-l* Control

The aim of this control [6] is to maintain the desired distances, l_{13}^d and l_{23}^d of a robot from its two nearest ones, which can be represented respectively by robot3 with respect to robots2 and 1 as shown in Fig. 2.

The kinematic equations for robot 3 are given as follows,

$$\begin{aligned} \dot{l}_{13} &= v_3\cos\gamma_1 - v_1\cos\psi_{13} + Dw_3\sin\gamma_1 \\ \dot{l}_{13} &= v_3\cos\gamma_2 - v_2\cos\psi_{23} + Dw_3\sin\gamma_{21} \\ \dot{\theta} &= \omega_3 \end{aligned} \qquad (6)$$

where $\gamma_i = \theta_i + \psi_{i3} - \theta_3, (i = 1, 2)$.

Details about the l-l control can be found in [6], where a singularity case may occur at $\sin(\gamma_1 - \gamma_2)=0$, when the control law is undefined.

2.3 Collision Avoidance

In Fig. 3, the required clearance d between robots is represented by a circle in broken line, having radius $(D + d)$, while the circle in solid line covers the whole region corresponding to the robot itself, centering at the control point, having radius D. Let (x_i, y_i) and (x_j, y_j) denote the control points of robot i and j, then the distance between robot i and j is:

$$\rho_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (7)$$

One has therefore,

$$\rho_{ij} > 2(D+d) \rightarrow \text{safe} \quad (8)$$

$$\rho_{ij} \leq 2(D+d) \rightarrow \text{unsafe} \quad (9)$$

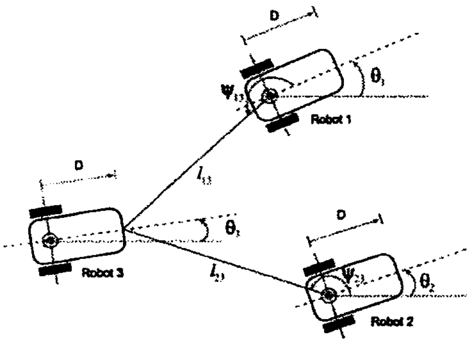

Fig. 2. Notation for *l-l* control.

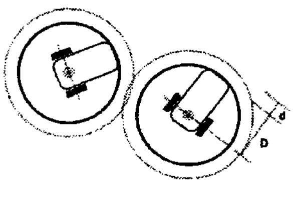

Fig. 3. Collision detection.

In VR tracking [12], when collisions are detected, the follower robots with a lower priority should switch to *l-l* control to avoid collision with those robots having higher priorities, while the latter do not have to change their control laws. To choose the reference distances for *l-l* control, two situations are considered, referring to cases when the target is inside an accessible area of the low priority robot (i.e., the shaded areas in Fig. 4); and when it is outside that area.

Denoting TG_1 and TG_2 respectively the targets of follower d 2, and P_x the point where follower 2 will be led to by .g *l-l* control, then depending on each situation, the design of l_{13}^d and l_{23}^d, which is equivalent to the design of P_x, shall be accomplished accordingly [12].

3. VSS Approach to Robot Tracking Control

The design of VSS with sliding modes involves the design of a sliding function S, and of the control inputs in the common form of:

$$u_j = \begin{cases} u_j^+(x) \text{ when } S_j(x) > 0 \\ u_j^-(x) \text{ when } S_j(x) < 0 \end{cases} \quad (10)$$

where switching between the control structures takes place across the sliding surface, defined by $S_j(x) = 0$, to drive the system state toward this surface. A general condition for the sliding mode to exist is:

$$S^T\dot{S} < 0 \quad (11)$$

To satisfy this, the reaching law [8]:

$$\dot{S} = -Q\text{sgn}(S) - KS \quad (12)$$

where $K = \text{diag}(K_i)$, $Q = \text{diag}(Q_i)$, $K_iS_i > 0$, $Q_i > 0$, can be used. For reduction of the so-called chattering effect, this reaching law can be modified to:

$$\dot{S} = -Qsat(\frac{S}{\varepsilon}) - KS \quad (13)$$

where sat(x) takes the value of x (if $|x| < 1$)or of sgn(x) (elsewhere), and ε defines the radius of a region around the sliding surface.

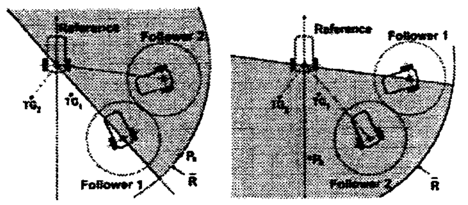

Fig. 4. Target is outside (left) and inside (right) the accessible area.

3.1 Sliding Mode Control for VR Tracking

The error model for follower robot i, which incorporates orientation error, is designed as follows,

$$e = \begin{pmatrix} x_{vi} \\ y_{vi} \\ \theta_{vi} \end{pmatrix} - \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix} = \begin{pmatrix} e_{xi} \\ e_{yi} \\ e_{\theta i} \end{pmatrix}, i = 1,2,\ldots,n. \quad (14)$$

The time-derivative of the error model is:

$$\dot{e} = \begin{pmatrix} \dot{x}_{vi} \\ \dot{y}_{vi} \\ \dot{\theta}_{vi} \end{pmatrix} - \begin{pmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{pmatrix} = \begin{pmatrix} \cos\theta_i & -r\cos\theta_i - l\sin\theta_i \\ \sin\theta_i & -r\sin\theta_i - l\cos\theta_i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_i \\ \omega_i \end{pmatrix} - \begin{pmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_r \\ \omega_r \end{pmatrix} = B_i u_i - B_r u_r \quad (15)$$

The sliding function usedhere is chosen as $S = Ce$, where

$$C = \begin{pmatrix} 1 & 0 & -|r|\sin\theta_i \\ 0 & 1 & |r|\cos\theta_i \end{pmatrix}, \quad (16)$$

or

$$S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} e_{xi} - e_{\theta i}|r|\sin\theta_i \\ e_{xi} + e_{\theta i}|r|\cos\theta_i \end{pmatrix} \quad (17)$$

Taking time-derivative of S:

$$\dot{S} = C\dot{e} + \dot{C}e = (CB_i u_i - CB_r u_r - \dot{C}e) \quad (18)$$

then from condition $\dot{S} = 0$ the equivalent control can be derived as,

$$u_{eq} = (CB)^{-1}CB_r u_r - (CB)^{-1}\dot{C}e \quad (19)$$

or

$$v_{eq} = v_r(\cos e_\theta - \frac{|r|e_\theta}{|r|+l}\sin e_\theta) + \omega_r \frac{r^2 e_\theta}{|r|+l}$$
$$\omega_{eq} = \frac{-v_r}{|r|+l}\sin e_\theta + \frac{|r|}{|r|+l}\omega_r \quad (20)$$

The overall control is then [10]:

$$u_i = u_{eq} + u_R, \tag{21}$$

where the second term, the robust control, is derived to meet the reaching condition. According to the equivalent control chosen above, one has

$$\begin{aligned}\dot{S} &= (CB_i)(u_{eq} + u_R) - CB_r u_r - \dot{C}e \\ &= (CB_i)u_{eq} - CB_r - \dot{C}e + (CB_i)u_R \\ &= (CB_i)u_R\end{aligned} \tag{22}$$

and with the reaching law (13):

$$u_R = -(CB_i)^{-1}\left[KS + Q\mathrm{sat}\left(\frac{S}{\varepsilon_1}\right)\right] \tag{23}$$

Notably, since $(CB_i)^{-1}$ can be obtained as

$$\begin{pmatrix} |r|\cos\theta_i & |r|\sin\theta_i \\ -|r|\sin\theta_i + l\cos\theta_i & +|r|\cos\theta_i + l\cos\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}\frac{1}{|r|+l} \tag{24}$$

there is no reservation in establishing a line formation ($l = 0$) with our proposed controller in comparison with the VR tracking control [12], as commented in [17].

3.2 Sliding Mode Control for Orientation Tracking

To achieve both position and orientation tracking when the initial orientation difference between a follower and the reference robot is large, it is intuitive for orientation tracking to take place prior to VR tracking. The orientation error is defined as:

$$e_{\theta_i} = \theta_i - \theta_r. \tag{25}$$

With the sliding function

$$S = e_{\theta_i} + \lambda \int e_{\theta_i}\, dt \tag{26}$$

and the reaching law chosen according to (13), the resulting control is:

$$\omega_i = \omega_r - \lambda(\theta_i - \theta_r) - \eta_1 \mathrm{sat}\left(\frac{S}{\varepsilon_2}\right) - \eta_2 S, \tag{27}$$

where λ, η_1, η_2 and α are some positive constants.

3.3 Sliding Mode l-l Control

The kinematic equations for the system of three robots are given as follows [6],

$$\begin{aligned}\dot{l}_{13} &= v_3\cos\gamma_1 - v_1\cos\psi_{13} + D\omega_3\sin\gamma_1 \\ \dot{l}_{23} &= v_3\cos\gamma_2 - v_2\cos\psi_{23} + D\omega_3\sin\gamma_2 \\ \dot{\theta} &= \omega_3\end{aligned} \tag{28}$$

Denoting

$$\xi = \begin{pmatrix} l_{13} \\ l_{23}\end{pmatrix} \text{and } \xi^d = \begin{pmatrix} l_{13}^d \\ l_{23}^d\end{pmatrix} \tag{29}$$

one can obtain the time-derivative of ξ

$$\dot{\xi} = \begin{pmatrix} \cos\gamma_1 & D\sin\gamma_1 \\ \cos\gamma_2 & D\sin\gamma_2 \end{pmatrix}\begin{pmatrix} v_3 \\ \omega_3\end{pmatrix} - \begin{pmatrix} v_1\cos\psi_{13} \\ v_2\cos\psi_{23}\end{pmatrix} = Bu_3 - b_r \tag{30}$$

and $\dot{\xi}^d = 0$.

For the following error model,

$$e = \xi - \xi^d, \tag{31}$$

with the sliding function chosen as $S = e$, it is obtained

$$\dot{S} = \dot{e} = Bu_3 - b_r - \dot{\xi}^d = Bu_3 - b_r. \tag{32}$$

By using same method described in Section 3.1, one can derive the following control,

$$u_{l-l} = B^{-1}\left(b_r - H\mathrm{sat}\left(\frac{S}{\varepsilon}\right) - \Lambda S\right), \tag{33}$$

where H contains constant reaching rates and Λ contains proportional reaching rates.

As it is noted that

$$B^{-1} = \frac{1}{D\sin(\gamma_2 - \gamma_1)}\begin{pmatrix} D\sin\gamma_2 & -D\sin\gamma_1 \\ -\cos\gamma_2 & \cos\gamma_1\end{pmatrix} \tag{34}$$

the control is undefined when $\sin(\gamma_2 - \gamma_1) = 0$ or when the third robot is found on the line connecting its two leaders.

4. Reactive Control Scheme

As noted in [18], applying VR control does not necessarily guarantee that VR will track the correct orientation of the reference robot. In fact, in the steady state, the follower robots may have either the same orientation as the reference robot or an opposite orientation, i.e. 180° away. Two scenarios of potential collision are described in the following.

4.1 Observations

It is observed that during the tracking process, the VR rotates and if the follower goes inside the rectangular region defined by x_e and y_e, or initially the follower robot is inside that region, as depicted in Fig. 5, then it may collide with the reference robot.

Another problem arising with the design of P_x is when collision occurs. Since P_x can only be within the accessible area and TG_2 may be outside of that area, it is not guaranteed that when switching back to VR control, follower 2 can track TG_2 without colliding with follower 1 and this may happen repeatedly. Based on the observations, reactive control is proposed as a treatment for these cases.

4.2 Reactive Control

Firstly, if $e_{\theta_i} \geq 90°$ then applying VR tracking may result in increasing position tracking error in the follower robots. Therefore, the VSS control will be applied in this case for orientation tracking until orientation of the controlled follower robot is aligned with the reference robot.

Secondly, should collision occur between any follower robot i and the reference robot according to the collision detection criteria given in [12], *l-l* control will be used to drive the follower robot away from possible collision and also heading to the target position so that collision will most likely not occur after switching back to VR tracking. Here, a VR for the reference robot (VRR) [9] is defined as a VR with r and $-l$ clearances from the reference robot, where r-l are correspondingly the desired clearances of follower i. This virtual robot will be placed at the desired position of follower i in the formation. If a collision occurs between any follower robot and the reference robot, the follower robot will switch to *l-l* control in order to go to P_x, as illustrated in Fig. 6, by using two leaders: the reference robot and the VRR, with l_{13}^d

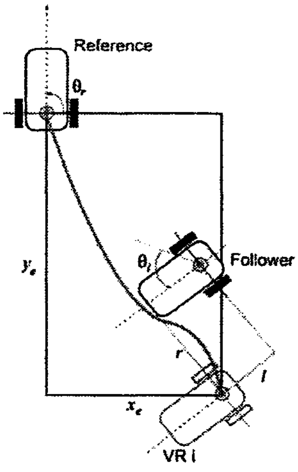

Fig. 5. Potential collision with reference robot.

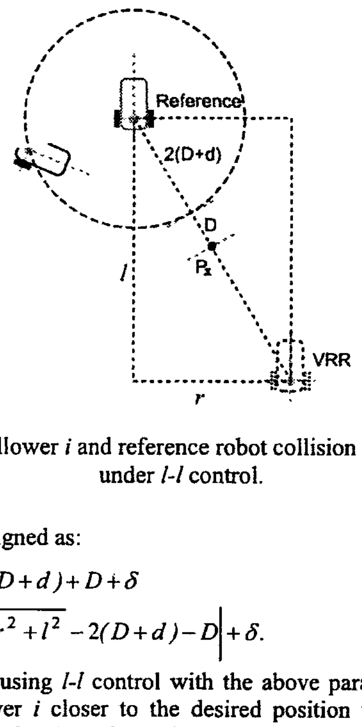

Fig. 6. Follower *i* and reference robot collision avoidance under *l-l* control.

nd l_{23}^d designed as:

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ &= \left|\sqrt{r^2+l^2}-2(D+d)-D\right|+\delta. \end{aligned} \tag{35}$$

Notably, using *l-l* control with the above parameters will lrive follower *i* closer to the desired position while going ıround the safety boundary of the reference robot, which is a :ircle whose diameter is 2(*D* + *d*) centering at the reference obot's control point. Thus, follower *i* is driven not directly to ts desired position as driving it to go around the leader's afety boundary will make follower *i* less likely to collide vith the others.

The reason P_x is 2(*D* + *d*) + *D* away from the reference obot rather than 2(*D* + *d*) is that here, the distance from the ront castor of the third robot (or follower *i*) to the control ıoint of its leader (or the reference robot) is considered nstead of the distance between their control points, or ρ_{ri}. 'hus in order to ensure ρ_{ri} to be sufficiently large for ollision avoidance, l_{13}^d is increased by the distance from the hird robot's control point to its front castor, which is *D*. In ıractice, one cannot drive follower *i* to P_x because P_x lies on he line connecting the reference robot and its VR, where the ontrol is undefined due to singularity. Another reason is that the distance between follower i and the reference robot needs to be strictly greater than 2(*D* + *d*) + *D* to enhance a safe margin for collision avoidance. For these reasons, a small distance δ is deliberately augmented to both l_{13}^d and l_{23}^d.

Thirdly, when there is a possible collision between the two followers, follower 2 (lower priority) shall apply *l-l* control where leader 1 plays the role of the follower 1 and leader 2 of the VRR. In that case, l_{13}^d is the same as in (35) while l_{23}^d is similar to the one in (35) except that $\sqrt{r^2+l^2}$ is replaced by the distance between VRR and follower 1, or ρ_{rr1}. Therefore,

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ l_{23}^d &= \left|\rho_{rr1}-2(D+d)-D\right|+\delta. \end{aligned} \tag{36}$$

However, if target TG_2 is in the opposite half plane divided by the line connecting follower 1 and the reference robot, and the distance between the reference robot and follower 1 is less than or equal to 4(*D* + *d*), then by using (36) it is likely that follower 2 may collide with both the reference and follower 1 in attempting to go to the desired position, as depicted in Fig. 7. In such situation, *l-l* control will be used to drive follower 2 to "go behind" follower 1. The specific strategy is also illustrated in Fig. 7, where follower 2 is driven to P_x and the line connecting P_x and follower 1 is always perpendicular to the line connecting follower 1 and follower 2. This is accomplished by setting, in the *l-l* control framework, leader 1 to play the role of follower 1, and leader 2 as a VR placed at P_x, with l_{13}^d and l_{23}^d defined as follows,

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ l_{23}^d &= \delta. \end{aligned} \tag{37}$$

Apparently, follower 2 will revolve via the rear route around the safety boundary of follower 1 until it becomes collision-free with follower 1, and on the same side with TG_2 with respect to the line connecting follower 1 and the reference robot. This also means follower 2 has been driven to go outside of the accessible area in Fig. 4. Then VR tracking can be applied again to drive follower 2 to its target TG_2. This ensures that follower 2 can always go to the desired position regardless of the positions of the reference and follower 1.

Lastly, in the case of possible collision of follower 2 either with the reference robot or follower 1, follower 2 should apply *l-l* control to go to a safe position. That position should be at least 2(*D* + *d*) + *D* away from other robots. For the same reason as previously explained, δ is augmented to l_{13}^d and l_{23}^d.

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ l_{23}^d &= 2(D+d)+D+\delta. \end{aligned} \tag{38}$$

4.3 Discussion

Using the proposed control framework, collision-free movement for a group of three robots is completely achieved. Firstly, the new VR tracking control guarantees follower robots will track correctly to desired positions regardless of initial positions of the platoon. The combination of VR tracking control and *l-l* control can be used to avoid collision between robots, at the same time driving robots to their desired positions with minimum control effort. When follower 2 has to take the rear route around follower 1, it attempts to maintain the minimum distance to follower 1, which also means that it has maintained minimal communication range with the reference robot while avoiding collision with follower 1. This should sufficiently satisfy the limited communication range restriction. Another advantage comes from the formation control robustness, i.e., insensitivity to parameter perturbations and external disturbances, which is in fact the salient virtue of VSS.

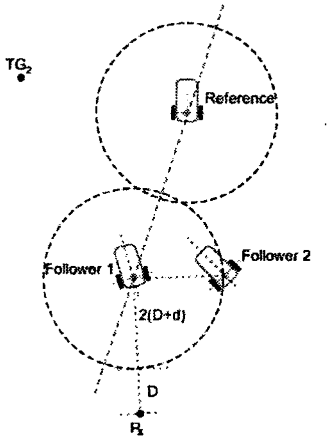

Fig. 7. Driving follower 2 around follower 1 via rear route.

The approach can be extended generally to the multiple robot case, where the platoon of robots can be divided into multiple three-robot groups. Then each of the three-robot groups can be treated as an individual unit, with control points being placed at the reference robots, in order to avoid collision between the robot groups in an obstacle-free environment. To incorporate obstacle avoidance, a strategy similar to that proposed in [6] for *l-l* control may be used, where one of the distances is the distance to the obstacle.

5. Simulation Results

5.1 Collision Avoidance

The aim of this simulation is to validate if collision is detected and avoided properly using the proposed approach. Assuming that there is no parameter variations or external disturbances. Initial parameters are set as follows.

- Common : $D = 3$, $d = 1$, $\delta = 0.3$
- Reference : $q_r(0) = [100, 0, 0]^T$, $u_r = [2.5, 0]^T$
- Follower 1: $\mathbf{Q}$ = diag(0.01, 0.01), $\mathbf{K}$ = diag(0.5, 0.5), ε =0.01, λ =0.01, η_1 =0.005, η_2 =0.05, $\mathbf{H}$ = diag(0.1, 0.1), Λ = diag(0.6, 0.6).
  Follower 2: $\mathbf{Q}$ = diag(0.01, 0.01), $\mathbf{K}$ = diag(0.5, 0.5), ε =0.01, λ =0.01, η_1 =0.005, η_2 =0.05, $\mathbf{H}$ = diag(0.1, 0.1), Λ = diag(0.6, 0.6).

It is assumed that follower robot 2 has the lowest priority and the reference robot the highest one. The first simulation demonstrates the establishment of a column formation, shown in Fig. 8. The initial positions of the followers and formation parameters are given as follows.

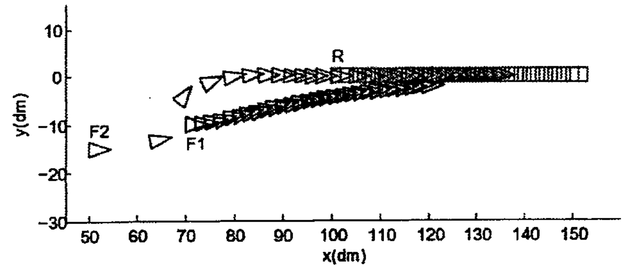

Fig. 8. Collision avoidance when establishing column formation.

- Follower 1: $q_1(0) = [70, -10, 0]^T$, $(r, l) = (0, 30)$,
- Follower 2: $q_2(0) = [50, -15, 0]^T$, $(r, l) = (0, 15)$.

In this figure, R stands for the reference robot, F1 stands for follower robot 1 and F2 stands for follower robot 2. First, follower 2 had apparently an incident collision with follower 1 near $x = 70$ and $y = -10$, when attempting to go to the desired position. It then applied the method described in (36) to avoid collision with follower 1. After avoiding potential collision with follower 1, follower 2 switched the control back to VR control and went to the desired position for a column formation.

The second simulation, shown in Fig. 9, demonstrates the use of the method described in Fig. 7, in the establishment of a line formation. The initial positions of the followers and formation parameters are given as follows,

- Follower 1: $q_1(0) = [80, 20, 0]^T$, $(r, l) = (10, 0)$,
- Follower 2: $q_2(0) = [90, -30, 0]^T$, $(r, l) = (-10, 0)$.

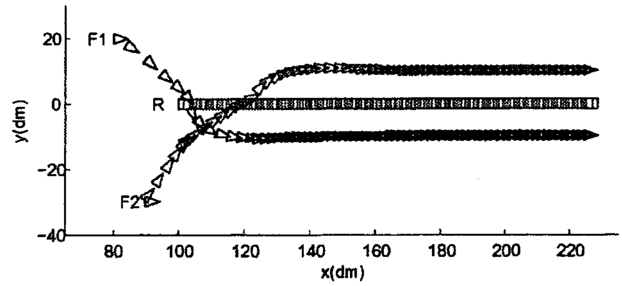

Fig. 9. Collision avoidance when establishing line formation.

Follower 2 detected an incident collision with follower 1 near $x = 100$ and $y = -10$. It then decided to take the rear route because follower 1 and the reference were too close, by applying the strategy described in Fig. 7. After a while, follower 2 went to the same side as its desired position with respect to the line connecting follower 1 and the reference robot. It then applied VR tracking control again, but potentially resulting in another collision possibility with follower 1 near $x = 120$ and $y = 0$. This incident collision was resolved by applying the method described in Section 4.2 (see Fig. 6). Finally, follower 2 approached the desired position to establish safely a line formation.

5.2 Robustness

The issue of uncertainties in robots' coordinates is also taken into account in this simulation. It is assumed that parameter variations come from communication channel with the reference robot, or the state vector of the reference robot contains some sensing error. In other words,

$$q_r = q_r^* + \Delta \tag{39}$$

where q_r^* is the nominal parameter.

This uncertainty is applied to a two-robot group using respectively the VR tracking approach and the VSS-based approach. Simulation results are compared in terms of the tracking error of the follower robot with respect to the leader. The initial parameters are set as follows.

- -Reference: $q_r(0) = [100, 0, 0]^T, u_r = [2.5, 0]^T$,
- -Follower 1: $q_1(0) = [80, -20, 0]^T$, $(r, l) = (10, 15)$.

All other parameters are set as previously. For the sake of simulation, the uncertain perturbations in q_r are chosen as:

$$\begin{cases} x_r = x_r^* + 0.5sin(3t) \\ y_r = y_r^* + 0.5sin(3t) \\ \theta_r = \theta_r^* + 0.2sin(3t). \end{cases} \tag{40}$$

The simulation results are shown in Fig. 10, 11 and 12. Fig. 10 and 11 depict the distances between the reference robot and the follower robot along x and y axes, respectively. Fig. 12 shows the difference in orientation between them. The upper curve in each figure shows the result using the I/O feedback approach and the lower curve shows the result using the proposed approach. The results are shown from the 100[th] second to the 200[th] second to focus on the steady-state responses.

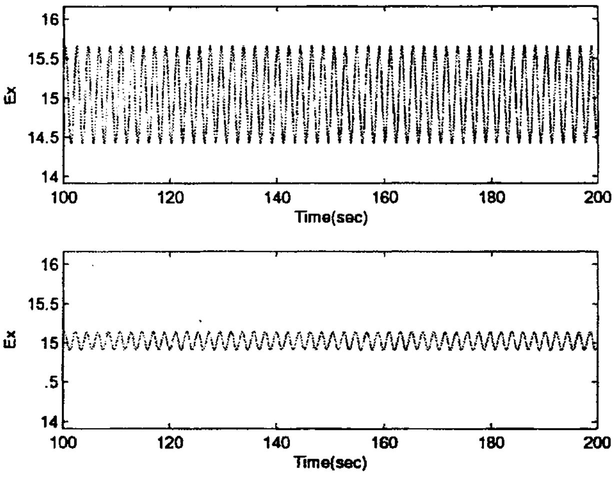

Fig. 10. Distances from follower robot to the reference robot along X axis using VR tracking (top) and VSS approach (bottom).

When reaching the steady state, fluctuations occur due to parameter variations. In Figs. 10, 11 and 12, the reference values are ideally 15, 10 and 0, respectively. It is shown that the magnitude of fluctuations due to parameter variations are smaller with the proposed approach than with the feedback linearization VR tracking approach [12] and also the centres of fluctuation with the VSS-based approach are closer to the ideal values. This indicates that the proposed approach is more robust to parameter perturbations in the state variables of the reference robot. However, robustness of the formation motion will not be well-preserved if perturbations exist with a higher magnitude in the state variables of the follower robots.

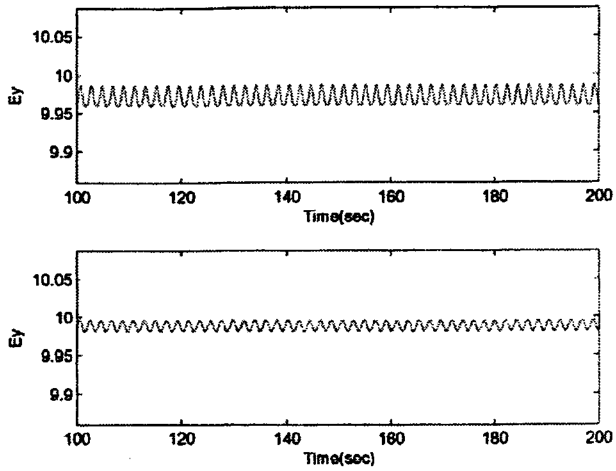

Fig. 11. Distances from follower robot to the reference robot along Y axis using VR tracking (top) and VSS approach (bottom)

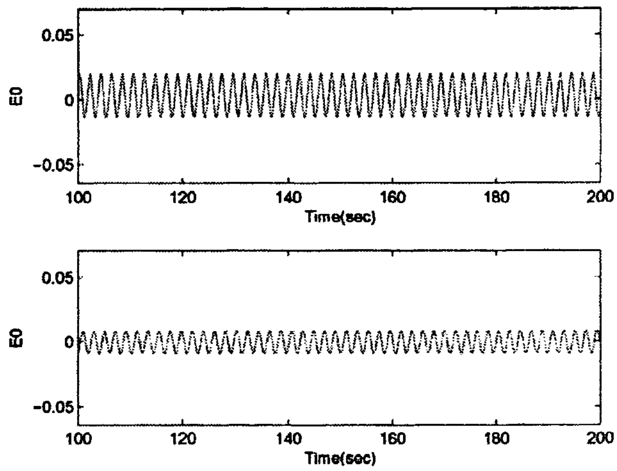

Fig. 12. Orientation difference between follower robot and the reference robot using VR tracking (top) and VSS approach (bottom).

This is one limitation of our approach.

6. Conclusion

This paper has presented a new approach for controlling multiple mobile robots (three-robot case) in formation using the leader-following strategy while ensuring collision-free movement. Sliding mode controllers are developed for virtual robot tracking, robot orientation, and *l-l* control. These controllers, coupled with a reactive control scheme, are used to achieve accurate robot tracking, safe establishment and successful maintenance of robotic formations in the presence of uncertainties in sensing information. The proposed controllers are proved to overcome such disadvantages with virtual robot tracking as incorrect convergence of a robotic formation and potential inter-robot collision. Validity of the reactive control scheme is illustrated through the establishment of column and line formations of a group of three robots. Robustness of the tracking performance is observed in the cases when parameter perturbations exist in the communication channel associated with the reference robot. For extension to the multiple robot case, the platoon of robots may be considered in multiple three-robot groups, where each of the three-robot groups is treated as an individual unit, with control points being placed at the reference robots.

Acknowledgement

This work is supported by the ARC Centre of Excellence programme, funded by the Australian Research Council (ARC) and the New South Wales State Government.

References

[1] T Balch and R C Arkin, Behavior-based formation control for multi-robot teams. IEEE Transactions on Robotics and Automation, Vol. 14, 1998, pp. 926-939.
[2] R W Beard, J Lawton and F Y Hadaegh, A coordination architecture for spacecraft formation control. IEEE Transactions on Control Systems Technology, Vol. 9, 2001, pp. 777-790.
[3] E Bicho and S Monteiro, Formation control of multiple mobile robots: A nonlinear attractor dynamics approach. Proceedings IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'03), 2003, pp. 2016-2022.
[4] S Carpin and L E Parker, Cooperative Leader Following in a Distributed Multi-Robot System. Proceedings IEEE International Conference on Robotics and Automation (ICRA'02), 2002, pp. 2994-3001.
[5] M S De Queiroz, V Kapila and Q Yan, Adaptive nonlinear control of multiple spacecraft formation flying. Journal of Guidance, Control, and Dynamics, Vol. 23, 2000, pp. 385-390.
[6] J P Desai, J Ostrowski and V Kurma, Controlling formations of multiple mobile robots. Proceedings IEEE International Conference on Robotics and Automation (ICRA'98), 1998, pp. 2864-2869.
[7] B Erkin, S Onur and S Erol, A review: Pattern formation and adaptation in multi-robot systems. Robotics Institute -Carnegie Mellon University, Pittsburgh CMU-RI-TR03-43, 2003.
[8] W Gao and J C Hung, Variable structure control of nonlinear systems: A new approach. IEEE Transactions on Industrial Electronics, Vol. 40, 1993, pp. 45-55.
[9] H M Ha, A D Nguyen and Q P Ha, Controlling formations of multiple mobile robots with inter-robot collision avoidance. Proceedings Australiasian Conference on Robotics and Automation (ACRA'05), 2005, pp. 1-7.
[10]Q P Ha, D C Rye and H F Durrant-Whyte, Robust sliding mode control with application. International Journal of Control, Vol. 72, 1999, pp. 1087-1096.
[11]P Johnson and J Bay, Distributed control of simulated autonomous mobile robot collectives in payload transportation. Autonomous Robots, Vol. 2, 1995, pp. 43-64.
[12]J Jongusuk and T Mita, Tracking control of multiple mobile robots: A case study of inter-robot collision-free problem. Proceedings IEEE International Conference on Robotics and Automation (ICRA'01), 2001, pp. 2885-2890.
[13]W Kang, N Xi and A Sparks, Formation control of autonomous agents in 3D workspace. Proceedings IEEE International Conference on Robotics and Automation (ICRA'00), 2000, pp. 1755-1760.
[14]L Kavraki, P Svestka, J C Latombe and M H Overmars, Probabilitic roadmaps for path planning in high dimensional configuration space. IEEE Transactions on Robotics and Automation, Vol. 12, 1996, pp. 566-580.
[15]M A Lewis and K Tan, High precision formation control of mobile robots using virtual structures. Autonomous Robots, Vol. 4, 1998, pp. 387-403.
[16]S Monteiro, M Vaz and E Bicho, Attractor dynamics generates robot formations: from theory to implementation. Proceedings IEEE International Conference on Robotics and Automation (ICRA'04), 2004, pp. 25822587.
[17]AD Nguyen, Q P Ha and H T Nguyen, Virtual-head robot tracking and three-point l-l control for multiple mobile robots. Proceedings IEEE 2006 Workshop on Distributed Intelligent Systems (IEEE-DIS'06), 2006, pp. 73-78.
[18]A D Nguyen, N M Kwok, V T Ngo and Q P Ha, Collision-Free Formations with Reactively-Controlled Virtual Head Robot Tracking. Proceedings IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'06), 2006 (To appear).
[19]P Ögren and N E Leonard, Obstacle avoidance in formation. Proceedings IEEE International Conference on Robotics and Automation (ICRA'03), 2003, pp. 2492-2497.
[20]JA Reeds and L A Shepp, Optimal paths for a car that goes both forwards and backwards. Pacific Journal of Mathematics, Vol. 145, 1990, pp. 367393.
[21]D J Stilwell, Decentralized control synthesis for a platoon of autonomous vehicles. Proceedings IEEE International Conference on Robotics and Automation (ICRA'02), 2002, pp. 744-749.
[22]H G Tanner, G J Pappasand V Kumar, Leader-to-formation stability. IEEE Transactions on Robotics and Automation, Vol. 20, 2004, pp. 443-455.
[23]H Yamaguchi, A cooperative hunting behavior by mobile-robot troops. International Journal of Robotics Research, Vol. 18, 1999, pp. 931940.
[24]X Yun, G Alptekin and O Albayrak, Line and circle formation of distributed physical mobile robots. Journal of Robotic Systems, Vol. 14, 1997, pp. 63-760.

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