Robust Non-Overshoot Time Responses using Cascade Sliding Mode-PID Control

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Abstract. Overshoot is a serious problem in automatic control systems. This paper presents a new method for elimination of the step response overshoot in a conventional PID-controlled system and enhancement of its robustness by cascading a sliding mode controller in the outer loop. The idea is first to use the cascade control principle to model the under-damped system under PID control with a second-order system. Then, by making use of the sliding mode control outer loop, a robust, reduced-order response can be obtained to suppress the control overshoot. The proposed approach can also deal with time delay systems. Its validity is verified through simulation for some dynamic systems subject to highly nonlinear uncertainties, where overshoot remains an issue.

Keywords: sliding mode, cascade control, overshoot, robustness.

1. Introduction

Conventional PID control is quite popular in automatic control systems. The most important issue of a PID controller is that its parameters need to be tuned properly. However, tuning a PID is not easy and in fact, many PID controllers in industry are not well-tuned.

There are some methods for tuning PID parameters. Based on knowledge of characterizing effects of each control parameter, engineers can adjust the P, I, and D gains until a desired response is obtained. However, this manual method is time-consuming and not always yields a desired response because changing one parameter may affect the performance designated by other two parameters. For over half a century the Ziegler and Nichols tuning methods [1] have been widely used in the context of auto-tuning for PID controllers. In the first method, controller parameters are calculated from an open-loop response of the process to a step input (process reaction curve). In the second one, both I and D parameters are set to zero while P parameter is increased gradually until the system oscillates. The period of the oscillation (called ultimate period) and the P gain (called ultimate gain) are used to calculate the desired controller parameters. The Ziegler-Nichols rules can help the tuning process faster than the trial-and-error method. However, they are not practical in many situations when experiments with open-loop or instable closed-loop can damage the process. To avoid this problem, many techniques such as relay feedback [2], approximate system identification [3], and cross-correlation [4] have been developed to estimate the ultimate gain and ultimate period in Ziegler-Nichols rules.

It is well-known that the control performance obtained by the Ziegler-Nichols tuning methods is just acceptable and the controller parameters need to be fine-tuned to provide the desired response [5]. While eliminating the steady error and shortening the settling time, the Ziegler-Nichols rules still result in a reasonable overshoot. This overshoot may be excessive and not acceptable in many processes such as chemical or mechanical systems. In [6] Hang *et al.* proposed a method to reduce the control overshoot to 10% or 20%, depending on applications, by using the set-point weighting. This may still appear inadequate for overshoot-sensitive systems.

When single-loop PID control systems cannot satisfy the control requirement, cascade PID control systems are often used. In [7], both optimization and auto-tuning methods are used for tuning cascade control systems. The results show that a cascade control system gives better responses with shorter settling time and smaller overshoot compared with its single-loop control option.

In this paper, we propose to use the cascade control principle coupled with a sliding mode controller (SMC) at the outer loop to eliminate the overshoot of a step response of the PID-controlled inner loop. It is expected that not only overshoot is suppressed but such SMC prominent property as robustness to external disturbance, uncertainties and nonlinearities can also be achieved [8]. Using this method, the PID controller just needs to be tuned to obtain the desired settling time and steady-state error, while overshoot is not considered in the first stage. Based on the resulting closed-loop transfer function modeled by using the cascade control principle, a sliding mode controller (SMC) is then designed to control the input of the inner loop system in order to entirely suppress the control overshoot. Moreover, in the case the system is subject to an input time delay, which is identified as a cause of deterioration of the overall control performance, the approach can also be shown effective if equipped with a suitable output predictor.

The remainder of the paper is organized as follows. After the introduction, Section 2 presents the modeling and sliding mode control design for cases with and without input time-delay. The approach is illustrated in Section 3 for a DC positioning system. Simulation results are given in Section 4 for the braking control system of a skid-steering uninhabited ground vehicle. Finally, a conclusion is drawn in Section 5.

2. Controller design

2.1 PID-controlled Process Modeling

Figure 1 shows a conventional PID controller in a closed-loop feedback system. Output of the controller is a function of the error between the PID loop reference, $r(t)$, and the output, $y(t)$, i.e. of $e_0 = r - y$:

$$V = K_P e_0 + K_I \int e_0 dt + K_D \frac{de_0}{dt}. \quad (1)$$

Characteristics of a response to a unit step using this PID controller depend on the choice of its parameters. The proportional gain (K_P) has the effect of reducing the rise time and it also reduces, but never eliminates, the steady-state error. The integral gain (K_I) has the effect of annulling the steady-state error, but it may make the transient response worse. The derivative gain (K_D) has the effect of increasing the stability of the system, reducing the overshoot, and improving the transient performance. To obtain a desired response, PID parameters need to be tuned properly. By manually tuning or auto-tuning methods, the desired setting time and steady-state error can be obtained. Nevertheless, in some systems, no matter how well the PID parameters are tuned, overshoot of the step response still exists.

By making use of cascade control, whereby the output of the outer-loop controller is used to manipulate the setpoint of the inner-loop controller [7,9], and the symmetrical optimum principle [10], one can always model a PID-controlled process with overshoot into an equivalent second-order function:

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}, \quad (2)$$

where δ is the damping ratio and ω_n is the natural frequency. The system natural frequency and damping ratio can be calculated from the percentage of overshoot and peak time respectively as [5]:

$$\omega_n = \frac{\pi}{t_P\sqrt{1-\delta^2}}, \quad \delta = -\frac{\ln(M_P)}{\sqrt{\pi^2 + \ln^2(M_P)}}, \quad (3)$$

where M_P and t_P are percentage of overshoot and peak time, respectively. If the system is subject to an input time delay (t_d) due to, e.g., hydraulic actuation or sensing data transmission, the model (2) can be rewritten to incorposate t_d as following

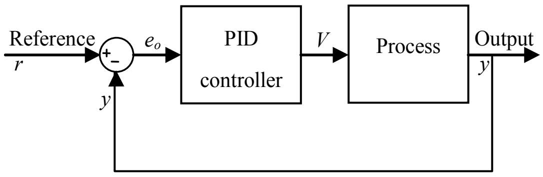

Figure 1. Closed-loop PID controller.

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} e^{-t_d s}. \quad (4)$$

It is noted that the above system may suffer from uncertainties such as nonlinear, modeling error and external disturbance. To deal with these uncertainties as well as overshoot and delay issues, a sliding mode controller (SMC) loop will be proposed in cascade with the PID-controlled system.

2.2 Sliding Mode Controller Design

Figure 2 shows the overall control system, where the input of the PID controller is regulated by the output of the SMC. In this figure, v is an unknown input accounting for external disturbance, modeling error and parametric uncertainties.

Let the control system error be defined as

$$e = y_{ref} - y, \quad (5)$$

where y_{ref} is the system reference or desired output. With the sliding function chosen as $S = \dot{e} + \lambda e$, where λ is a positive scalar to be selected, let a Lyapunov function be $V_L = \frac{1}{2}S^2$. Taking its first time derivative yields $\dot{V}_L = S\dot{S}$, where

$$\dot{S} = \ddot{e} + \lambda\dot{e} = (\ddot{y}_{ref} - \ddot{y}) + \lambda\dot{e}. \quad (6)$$

Equation (2) gives

$$\ddot{y} + 2\delta\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 u. \quad (7)$$

Substitution from (7) into (6) gives

$$\begin{aligned}\dot{S} &= \ddot{y}_{ref} + 2\delta\omega_n\dot{y} + \omega_n^2 y - \omega_n^2 u + \lambda\dot{e} \\ &= \ddot{y}_{ref} + 2\delta\omega_n\dot{y}_{ref} - 2\delta\omega_n(\dot{y}_{ref} - \dot{y}) + \omega_n^2 y_{ref} \\ &\quad - \omega_n^2(y_{ref} - y) - \omega_n^2 u + \lambda\dot{e} \\ &= \ddot{y}_{ref} + 2\delta\omega_n\dot{y}_{ref} + \omega_n^2 y_{ref} - (2\delta\omega_n - \lambda)\dot{e} \\ &\quad - \omega_n^2 e - \omega_n^2 u.\end{aligned}$$

or

$$\dot{S} = \omega_n^2\varphi_{ref} - (2\delta\omega_n - \lambda)\dot{e} - \omega_n^2 e - \omega_n^2 u, \quad (8)$$

where

$$\varphi_{ref} = \frac{(\ddot{y}_{ref} + 2\delta\omega_n\dot{y}_{ref} + \omega_n^2 y_{ref})}{\omega_n^2}.$$

The equivalent control, u_{eq}, is obtained at the nominal regime ($v = 0$) from condition $\dot{S} = 0$:

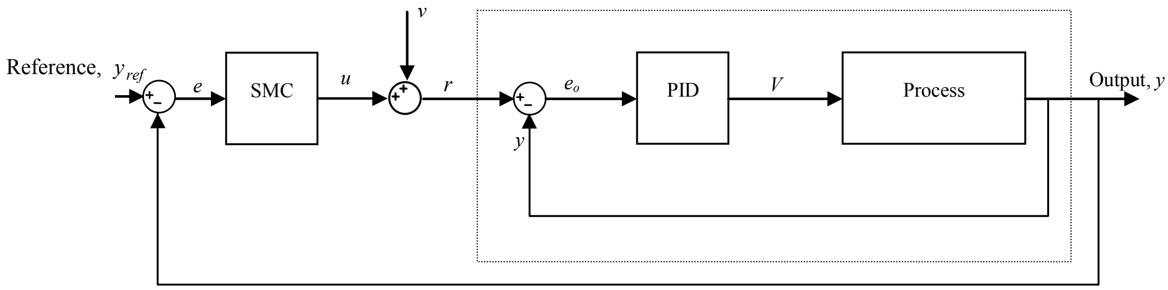

Figure 2. Cascade Sliding Mode - PID control

$$u_{eq} = \varphi_{ref} - \frac{(2\delta\omega_n - \lambda)}{\omega_n^2}\dot{e} - e. \tag{9}$$

Now for $v \neq 0$ the control law for SMC has the form of [11]:

$$u = u_{eq} + u_R. \tag{10}$$

Assuming v is upper-bounded, $|v| \le \rho$, one can easily verify that if the robust control, u_R, is chosen as

$$u_R = \rho sign(S), \tag{11}$$

then the reaching condition $\dot{V}_L \le 0$ is satisfied since

$$\begin{aligned} \dot{V}_L &= S\dot{S} \\ &= S\left[\omega_n^2\varphi_{ref} - (2\delta\omega_n - \lambda)\dot{e} - \omega_n^2 e - \omega_n^2\left(u_{eq} + u_R + v\right)\right] \\ &= -S\left[\omega_n^2(u_R + v)\right]. \end{aligned}$$

The control output of the SMC by (10) is then

$$u = \varphi_{ref} - \frac{(2\delta\omega_n - \lambda)}{\omega_n^2}\dot{e} - e + \rho sign(S). \tag{12}$$

The signum function in (12) usually induces high frequency oscillations in the control output, or so-called chattering. One of the commonly-adopted techniques to reduce this effect is the incorporation of of a saturation function [8].

Remark 1: The proposed method may be applied generally for any overshoot-sensitive systems provided that their PID-controlled inner-loop step responses are known.

Remark 2: With robustness of the SMC, the proposed cascade control may be able to tolerate modeling errors, as well as to deal with such problems as external disturbance, uncertainties and nonlinearities.

2.3 Case with input time delay

Taking into account a time delay t_d as in transfer function (4), system (7) can be rewritten as

$$\ddot{y}(t) + 2\delta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 u(t - t_d), \tag{13}$$

where

$$\ddot{y}(t + t_d) = f(t + t_d) + \omega_n^2 u(t), \tag{14}$$

and

$$f(t + t_d) = -2\delta\omega_n\dot{y}(t + t_d) - \omega_n^2 y(t + t_d). \tag{15}$$

To proceed, let us consider the natural frequency and damping ratio in the following intervals,

$$\omega_{n1} \le \omega_n \le \omega_{n2},\ \delta_1 \le \delta \le \delta_2, \tag{16}$$

whose nominal values can be taken on average as [12]:

$$\hat{\omega}_n = \sqrt{\omega_{n1}\omega_{n2}},\ \hat{\delta} = \sqrt{\delta_1\delta_2}. \tag{17}$$

Then $f(t + t_d)$ in (15) can be approximated with

$$\hat{f}(t + t_d) = -2\hat{\delta}\hat{\omega}_n\dot{y}(t + t_d) - \hat{\omega}_n^2 y(t + t_d), \tag{18}$$

which is assumed to be bounded by

$$\left|\hat{f} - f\right| \le F, \tag{19}$$

where the values in the right hand side of (18) can be estimated by using an output predictor [13] and

$$F = 2\left|\left(\delta_2\omega_{n2} - \hat{\delta}\hat{\omega}_n\right)\dot{y}(t + t_d) + \left(\omega_{n2}^2 - \hat{\omega}_n^2\right)y(t + t_d)\right|.$$

Following the same design procedure described above and chossing the reaching condition $\dot{V}_L \le -\eta|S|$, where η is a positive constant to be determined [8], the equivalent control as in (9) can obtained as

$$u_{eq}(t) = \frac{1}{\hat{\omega}_n^2}\left(-\hat{f}(t + t_d) + \ddot{y}_d(t) - \lambda\dot{e}\right). \tag{20}$$

The robust control of the form (11) can also be derived with the discontinuous gain ρ chosen large enough to counteract the effects of uncertainties [12]:

$$\rho \ge \frac{F + \eta}{\omega_{n1}^2} + \left|\left(1 - \frac{\hat{\omega}_n^2}{\omega_{n1}^2}\right)u_{eq}\right|. \tag{21}$$

3. DC motor positioning system example

The proposed approach is illustrated first with a benchmark DC motor positioning control for linear systems. For this, consider a system modeled by the following transfer function [5]:

$$\frac{X(s)}{V(s)} = \frac{K}{s\left[(J_m s + b_m)(Ls + R) + K^2\right]}, \tag{22}$$

where V and x are applied voltage and position of the motor's shaft. The system parameters are given in Table I below:

TABLE I
DC MOTOR POSITION CONTROL PARAMETERS

Parameter	Value
Motor inertia	J_m=3.2284E-6 kg.m^{2}s^{-2}
Viscous friction coefficient	b_m=3.5077E-6 Nms
Induction	L=2.75E-6 H
Resistance	R=4Ω
Electromotive force constant	K=0.0274 NmA^{-1}
PID parameters	K_P=17, K_I=200, K_D=0

From tuning the PID controller parameters, the system can obtain a fast step input response with zero steady-state error, but overshoot is still rather large. The proposed approach is used to solve the problem.

The results are shown in Fig. 3. When PID is used, a step reference (set-point) at the input (Fig. 3a) creates an oscillated voltage as the motor input (Fig. 3b) and results in a large overshoot at the motor shaft (Fig. 3c). In contrast, the SMC forces the PID input (control output of SMC, u) (Fig. 3a) and the motor input (Fig. 3b) to eliminate completely the overshoot while still keeping the desired settling time for the whole system (Fig. 3c).

4. Hydraulic Braking System for a UGV

We consider next the problem of skid steering of an unmanned ground vehicle (UGV). For this UVG, important variables needed to be controlled at the low level include the engine speed, left and right braking forces, linear velocity and turning rate of the vehicle. The default algorithm for these vehicle controlled variables is the PID. Desirable features such as simple, general-purpose and model-free, make PID controllers suitable for the control of the vehicle. However, there are many components of the vehicle that exhibit nonlinearities and time delays which lead to high overshoots in the responses when using conventional PID controllers. Because most components are correlated, overshoots in one controlled variable can result in adverse responses in the performance of others. For example, overshoot of a braking response (left or right) may cause the turning rate to deviate away from its desired value. On the other hand, a fast rise time is always an essential requirement for the control system for these variables. Therefore, reducing overshoots of PID responses is an important task to enhance the control performance of the UGV [14]. The braking system consists of a voltage to current amplifier to supply for a linear actuator, the actuator comprising a DC servo motor and a ball-screw system, and a hydraulic cylinder driven by the actuator.

The actuator can be described by the following equations,

$$T_m = K_a K_i V, \quad \frac{X}{T_m} = \frac{K_m}{s(J_m s + B_m)}, \tag{23}$$

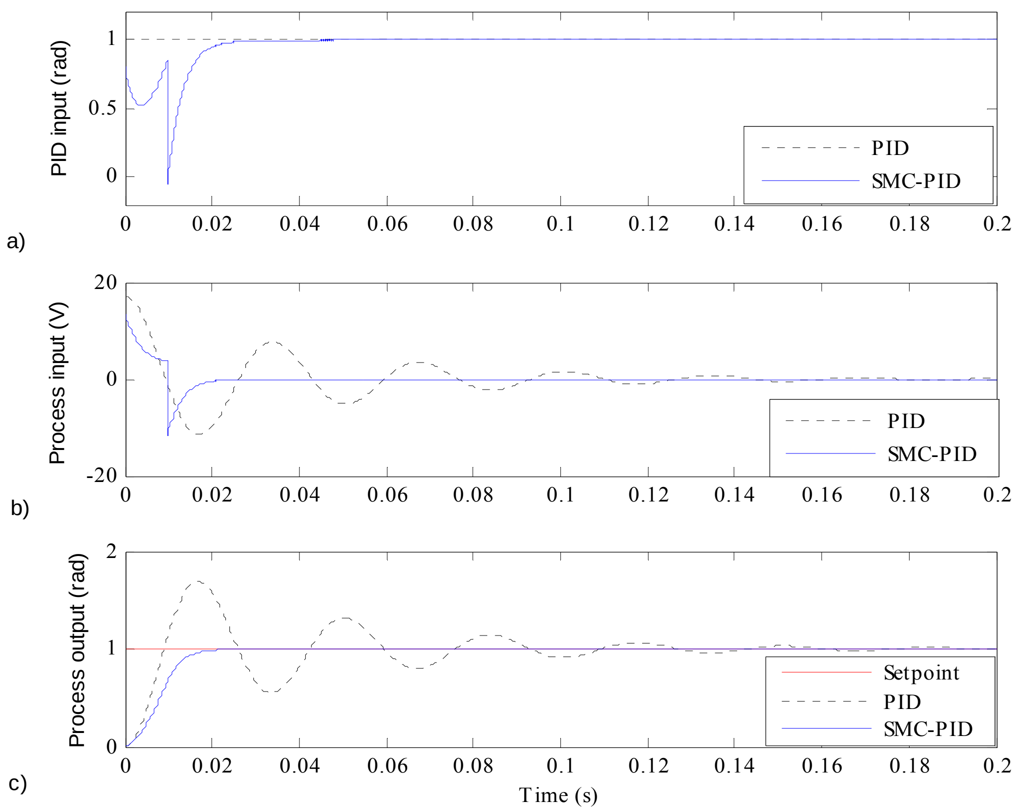

Figure 3. Response of PID controller (---) and SMC-PID (—) for DC motor position

where T_m, x and V are the actuator's torque, position and the applied voltage, K_a, K_i are the voltage-current amplifier coefficient and motor torque coefficient, J_m, B_m are the motor's moment of inertia and viscous damping coefficient, and K_m is the gear ratio inside the actuator. Values of the system parameters are provided in Table II. The complicated relationship between output y and input x of the hydraulic cylinder obtained from experimental data [15] by using the least square identification method, as shown in Fig. 4, is estimated as

$$y' = f(x) = 1.374x^2 - 5.138x + 2.778. \quad (24)$$

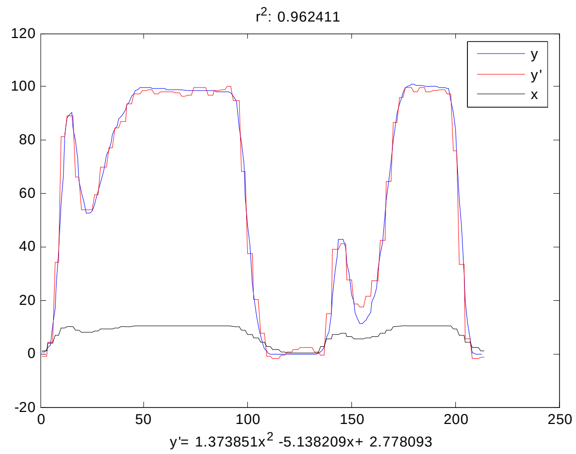

Figure 4. Pressure function approximation

TABLE II
BRAKING SYSTEM PARAMETERS

Parameter	Value
K_a	2 AV^{-1}
K_i	440 NA^{-1}
K_m	0.1815
B_m	33 Nms
J_m	0.3228 $kg.m^2s^{-2}$

For the UGV low-level control, a pressure controller is designed with the assumption that the braking force is proportional to the pressure inside the hydraulic cylinder. A default PID-controller is used in the internal loop for hydraulic pressure control [15]. As overshoot appears to be a problem no matter how the PID controller parameters are tuned, the proposed cascade PID-SMC is used to solve the problem.

The results are shown in Fig. 5 for both the PID and SMC-PID controllers. From step references, it is observed that the PID case alone possesses a large overshoot at the output (pressure) while the cascaded SMC can control the PID input (control output of SMC, *u*) (Fig. 5a) to force the system output to a non-overshoot step response (Fig. 5c).

Fig. 6 shows responses of the controllers with a disturbance representing a load change. The amplitude of disturbance is about 60% of maximum torque provided by the actuator (Fig. 6a). The PID controller cannot regulate the output braking disc to the desired value while the SMC is still able to control the PID input (SMC output, *u*) (Fig. 6b) in a robust way to compensate

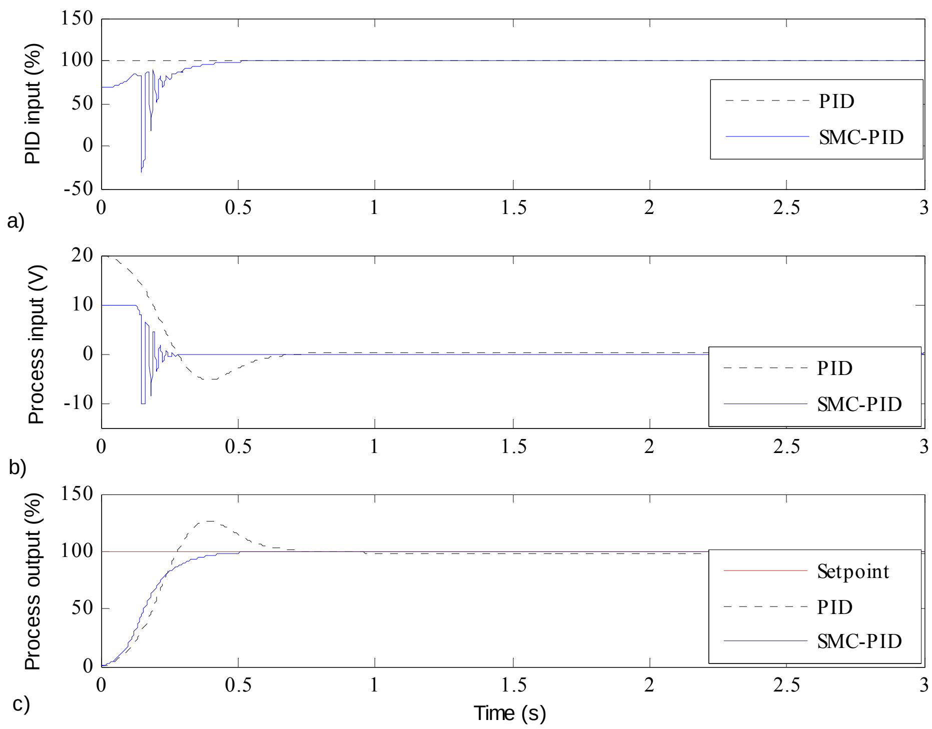

Figure 5. Responses of PID (---) and SMC-PID (—) control for UGV hydraulic braking system

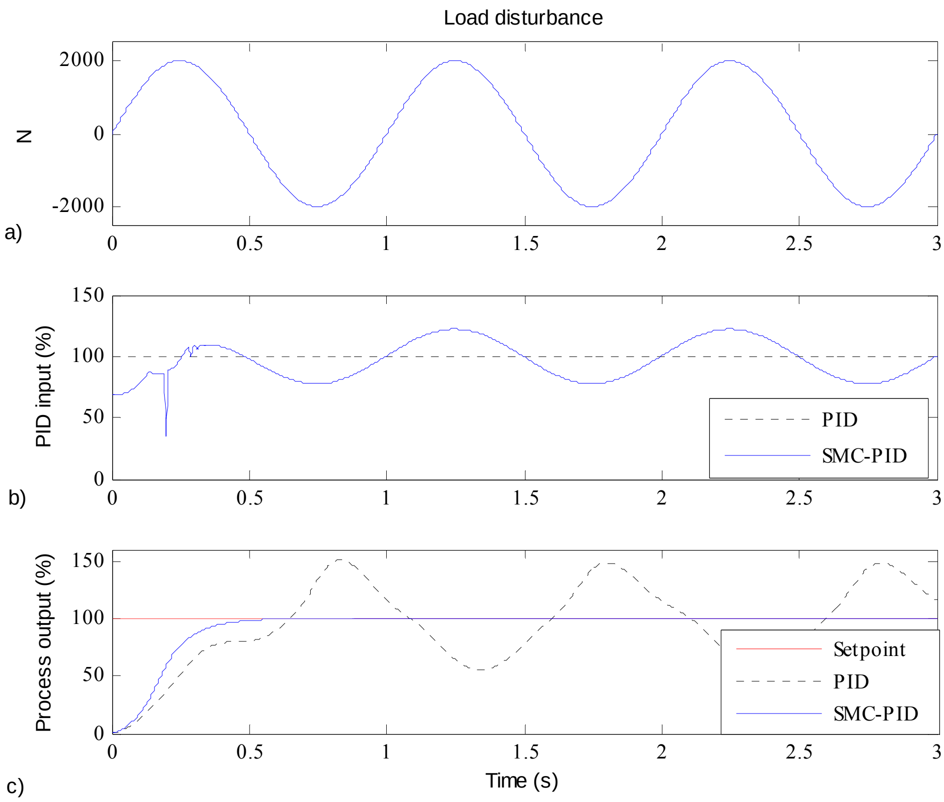

Figure 6. Responses of PID (---) and SMC-PID (—) control with external disturbance

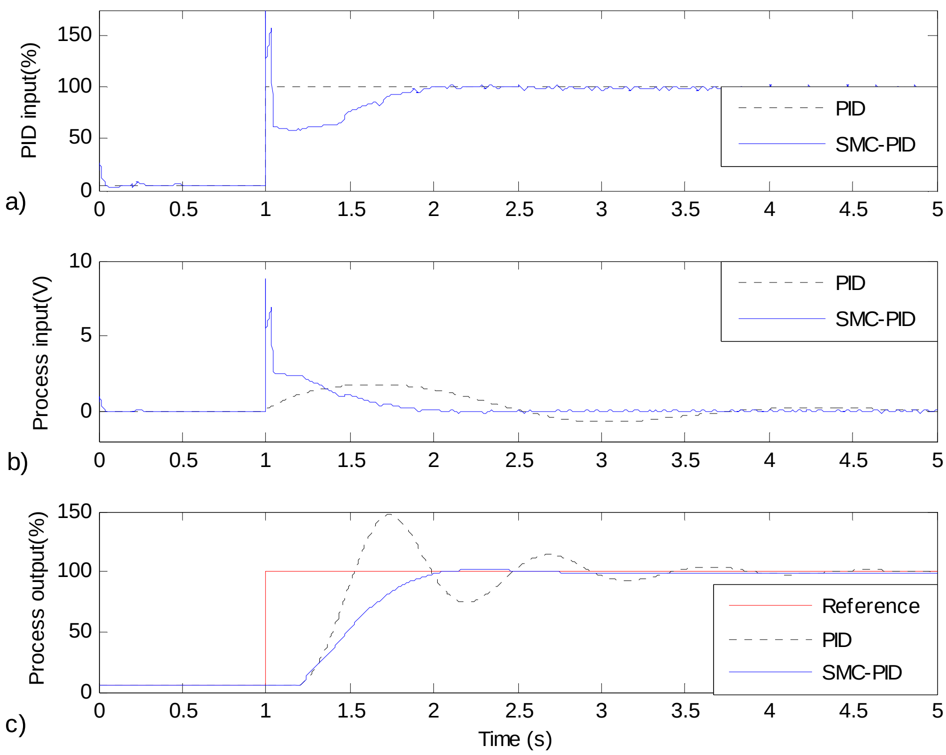

Figure 7. Responses of PID (---) and SMC-PID (—) control for UGV hydraulic braking system, in case of time-delay

for the disturbance. As a result, the error of the SMC-PID is found less than 0.6% compared with 50% of the PID (Fig. 6c). This is explained by the prominent feature of sliding mode control in producing robust, reduced-order time responses, and thus, suppressing successfully the step response control overshoot.

To illustrate the effectiveness of the proposed approach for time-delay systems, the above braking system is considered again taking into account now a time-delay of around 0.2 second between the input and the output [11]. This time-delay leads to around 50% of overshoot if PID controller is used alone (Fig. 7c). With estimation owing to the incorporation of an ouput predictor, the designed SMC-PID controller can suppress the overshoot to a very small value (Fig. 7c). Thus the results show that the proposed method can be effective for systems with an input time-delay.

5. Conclusion

We have presented a cascade sliding mode-PID controller for robust, non-overshoot time responses. The proposed approach can be applied for any PID-controlled system if its step response is known. From an equivalent transfer function of the PID inner-loop system, a sliding mode controller is designed to force the input of the PID so that the control overshoot is fully eliminated. An interesting feature is the approach remains effective for time-delay systems. Its validity is verified through a benchmark DC positioning system and a UGV braking system used for its skid steering. Simulation results for the UGV hydraulic braking system indicate that the proposed method can successfully suppress control overshoots while preserving high quality of other performance criteria such as settling time and steady-state error. By using the proposed approach, the control system can also exhibit strong robustness against uncertainties such as external disturbance, nonlinearities as well as input time delay.

Acknowledgement

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