FACULTY OF ENGINEERING AND INFORMATION TECHNOLOGY

Multilevel Decision Making for Supply Chain Management

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CERTIFICATE OF AUTHORSHIP/ORIGINALITY

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ABSTRACT

Multilevel decision-making techniques aim to handle decentralized decision problems that feature multiple decision entities distributed throughout a hierarchical organization. Decision entities at the upper level and the lower level are respectively termed the *leader* and the *follower*. Three challenges have appeared in the current developments in multilevel decision-making: (1) large-scale - multilevel decision problems become large-scale owing to high-dimensional decision variables; (2) uncertainty - uncertain information makes related decision parameters and conditions imprecisely or ambiguously known to decision entities; (3) diversification - multiple decision entities that have a variety of relationships with one another may exist at each decision level. However, existing decision models or solution approaches cannot completely and effectively handle these large-scale, uncertain and diversified multilevel decision problems.

To overcome these three challenges, this thesis addresses theoretical techniques for handling three categories of unsolved multilevel decision problems and applies the proposed techniques to deal with real-world problems in supply chain management (SCM). First, the thesis presents a heuristics-based particle swarm optimization (PSO) algorithm for solving large-scale nonlinear bi-level decision problems and then extends the bi-level PSO algorithm to solve tri-level decision problems. Second, based on a commonly used fuzzy number ranking method, the thesis develops a compromise-based PSO algorithm for solving fuzzy nonlinear bi-level decision problems. Third, to handle tri-level decision problems with multiple followers at the middle and bottom levels, the thesis provides different tri-level multi-follower (TLMF) decision models to describe various relationships between multiple followers and

develops a TLMF *K*th-Best algorithm; moreover, an evaluation method based on fuzzy programming is proposed to assess the satisfaction of decision entities towards the obtained solution. Lastly, these proposed multilevel decision-making techniques are applied to handle decentralized production and inventory operational problems in SCM.

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CHAPTER 1 INTRODUCTION

1.1 BACKGROUND

Multilevel decision-making techniques, motivated by Stackelberg game theory (Stackelberg 1952), have been developed to address compromises between the interactive decision entities that are distributed throughout a hierarchical organization (Zhang, Lu & Gao 2015). In a multilevel decision-making process, decision entities at the upper level and the lower level are respectively termed the *leader* and the *follower* (Bard 1998), and make their individual decisions in sequence, from the leader to the follower, with the aim of optimizing their respective objectives. This decision-making process means that the leader has priority in making its own decision and the follower reacts after and in full knowledge of the leader's decision; however, the leader's decision is implicitly affected by the follower's reaction.

Bi-level decision problems and tri-level decision problems are the typical cases of multi-level decision-making, which have motivated a number of significant efforts in decision models, solution approaches and applications in areas of both mathematics/computer science and business (Bard 1998; Dempe 2002; Zhang, Lu & Gao 2015). To achieve a quick understanding of multilevel decision problems, a tri-level decision-making case in relation to the hierarchical production-inventory planning in a conglomerate enterprise can be taken as an example. The conglomerate is composed of a sales company, a logistics center and a manufacturing factory, which are distributed throughout a three-stage supply chain. To fully satisfy market demand and shorten time-to-market, the sales company and the logistics center have to hold a certain amount of inventory using their respective warehouses but both of them nonetheless seek to minimize their individual inventory holding costs. When making the production-inventory plan within a stable sales cycle, the sales company (the leader) takes the lead in developing an optimal inventory plan which considers the current market demand and implicit reactions of other decision entities. The logistics center (the middle-level follower) then makes an optimal inventory plan according to the decision given by the sales company and considers the implicit production planning of the manufacturing factory (the bottom-level follower). Lastly, the manufacturing factory makes the production plan to minimize its own cost of production in light of the fixed inventory plans. The decision process will not stop until each decision entity is unwilling to change its decision; which implies that a compromised result or the equilibrium between the decision entities is achieved. The example describes a typical tri-level decision problem in which decisions are sequentially and repeatedly executed with all decision entities seeking to optimize their individual objectives until the equilibrium between them is achieved.

Nowadays, multilevel decision problems have increasingly appeared in decentralized management situations in the real world and have become highly complicated, particularly with the development of economic integration and in the current age of big data. For example, business firms usually work in a decentralized manner in a complex supply chain network comprised of suppliers, manufacturers, logistics companies, customers and other specialized service functions. The latest developments of multilevel decision-making manifest three typical features: (1) large-scale - multilevel decision problems become large-scale because of high-dimensional decision variables; (2) uncertainty - related decision parameters and conditions always involve uncertain information that is imprecisely or ambiguously known to decision entities; (3) diversification - there may exist multiple decision entities at each decision level, in which multiple decision entities at the same level have a variety of relationships with one another.

In general, there are two fundamental issues in supporting a multilevel decision-making process: one is how to develop a multilevel decision model to describe such a hierarchical decision-making process, and the other is how to find an optimal solution to the resulting decision model (Lu, Shi & Zhang 2006). However, existing decision models or solution approaches cannot completely and effectively handle large-scale, uncertain and diversified multilevel decision problems, which: (1) are still time-consuming or almost impossible for solving large-scale nonlinear bi-level and tri-level decision problems; (2) are limited to solving linear bi-level decision problems with special uncertainty, e.g. triangular fuzzy numbers; (3) are not applied to deal with tri-level decision problems involving multiple decision entities at each decision level. Moreover, for the sake of handling real-world cases that appear in highly complex decision situations, e.g. where there is uncertainty in data or multiple decision entities are involved at each decision level, it is crucial to investigate much more practical decision models together with solution approaches.

To support large-scale, uncertain and diversified multilevel decision-making, this thesis addresses theoretical techniques for handling three categories of unsolved multilevel decision problems, involving large-scale nonlinear bi-level and tri-level decision problems, fuzzy nonlinear bi-level decision problems, and tri-level decision problems with multiple decision entities at the middle and bottom levels; moreover, the proposed multilevel decision-making techniques are applied to handle decentralized management problems in supply chain management (SCM).

1.2 RESEARCH QUESTIONS AND OBJECTIVES

This research aims to present practical decision models and effective solution approaches for handling large-scale, uncertain and diversified multilevel decision problems. The research questions are summarized as follows:

Question 1. How to solve large-scale nonlinear bi-level decision problems using an effective algorithm and extend the algorithm to solve tri-level decision problems?

Question 2. How to solve uncertain bi-level decision problems with general fuzzy parameters, known as fuzzy bi-level decision problems?

Question 3. How to model and solve tri-level decision problems with multiple decision entities at the middle and bottom levels?

Question 4. How to apply the proposed multilevel decision-making techniques to handle decentralized decision problems in applications?

This research aims to achieve the following objectives, which are expected to answer the above research questions:

Objective 1. To develop an effective particle swarm optimization (PSO) algorithm for solving nonlinear and large-scale bi-level decision problems. Moreover, the PSO algorithm can be extended to solve tri-level decision problems.

This objective corresponds to research Question 1. PSO is a heuristic global optimization algorithm first proposed by Kennedy and Eberhart (1995)，which is inspired by the social behavior of organisms such as fish schooling and bird flocking. As PSO requires only primitive mathematical operators, and is computationally inexpensive in terms of both memory requirements and speed (Eberhart & Kennedy 1995), it has a good convergence performance and has been successfully applied in many fields. Multilevel decision problems have been proved to be NP-hard (Bard 1991; Ben-Aved & Blair 1990). Since traditional exact algorithmic approaches lack universality and efficiency, this study will develop a heuristics-based PSO algorithm for solving nonlinear and large-scale bi-level decision problems and it will extend the bi-level PSO algorithm to solve tri-level decision problems.

Objective 2. To handle general fuzzy parameters and develop a compromise-based PSO algorithm for solving fuzzy bi-level decision problems.

This objective corresponds to research Question 2. Fuzzy parameters involved in a fuzzy bi-level decision problem are always characterized by fuzzy numbers (Zhang, Lu & Gao 2015). A commonly used fuzzy number ranking method will be adopted to handle fuzzy numbers, which can transform the fuzzy problem to a crisp problem for ease of solving. However, the crisp problem keeps features of uncertainty, which are determined by different understanding and identification of the leader and the follower in relation to the uncertain decision situation. For the sake of solving the crisp problem, the leader and the follower between themselves need to achieve a compromised selection of uncertain decision conditions. The bi-level PSO algorithm proposed above can be extended to solve the crisp problem.

Objective 3. To develop tri-level multi-follower (TLMF) decision models with various relationships between multiple followers at the same level.

This objective corresponds to research Question 3. In a tri-level decision problem, multiple decision entities are often involved at the middle and bottom levels; these multiple decision entities are called multiple followers. Moreover, multiple followers at the same level may have a variety of relationships with one another. For example, followers may control their decisions independently without any information exchange, which is called the uncooperative relationship; may make decisions cooperatively with each other in line with the shared information, which is called the cooperative relationship, may consider the actions of their counterparts for reference, known as the reference-based relationship; or may be confronted with a hybrid situation of some of the above relationships. Such diversified situations make the decision problem complex and generate different decision processes. This category of tri-level decision problems is known as the tri-level multi-follower (TLMF) decision problem. This study will carefully analyze the various relationships and describe related decision processes using different TLMF decision models.

Objective 4. To develop an effective TLMF *K*th-Best algorithm for solving TLMF decision problems and present an evaluation method to assess the solution obtained.

This objective corresponds to research Question 3. This study will discuss theoretical properties of TLMF decision problems in relation to the existence and optimality of solutions. Based on related theoretical properties, a TLMF *K*th-Best algorithm will be developed. Moreover, Since the TLMF decision problem involve multiple followers with various relationships, the solution, known as the final decision result, cannot completely reflect the operations of the complex decision-making process in applications. To assess the satisfaction of decision entities towards the solution obtained, a solution evaluation method needs to be proposed.

Objective 5. To apply the proposed multilevel decision-making techniques to handle decentralized production and inventory operational problems in SCM.

This objective corresponds to research Question 4. Driven by a new round of industrial revolution, business firms are always distributed in a hierarchical and networked supply chain, and it becomes very difficult for a company to be competitive without working in close collaboration with external partners (Aguezzoul 2014). When making decisions in relation to SCM, each decision entity has to consider decision reactions of its upstream and downstream decision entities. In this situation, it is reasonable to apply multilevel decision-making techniques to handle such decentralized SCM problems. This research will focus on how to model and solve decentralized production and inventory operational problems in SCM using multilevel decision-making techniques.

1.3 RESEARCH SIGNIFICANCE

The significance of this research work can be summarized from the following aspects.

Significance 1: the research develops a novel PSO algorithm for solving large-scale nonlinear multilevel decision problems.

A big challenge in solving nonlinear and large-scale multilevel decision problems is how to handle high-dimensional decision variables, nonlinear objective functions and complex constraint conditions. In contrast to existing PSO algorithms that are limited to solving linear or small scale bi-level decision problems, the novel PSO algorithm is able to overcome the challenge effectively. The PSO algorithm not only provides a practical way to solve large-scale nonlinear bi-level decision problems, but also can be extended to solve tri-level decision problems.

Significance 2: the research develops a compromise-based PSO algorithm for solving fuzzy nonlinear bi-level decision problems.

Within the compromise-based PSO algorithm, the leader and follower are able to choose acceptable decision conditions based on rules of compromise due to uncertainty, which can provide not only better solutions to benchmarks under the specific decision situation but also different solution options due to various decision environments. Whereas existing solution approaches are limited to handling linear problems with special fuzzy numbers, the compromise-based PSO algorithm aims to solve nonlinear bi-level decision problems with general fuzzy numbers.

Significance 3: the research develops theoretical techniques for handling TLMF decision problems.

There still lack effective theoretical techniques for handling TLMF decision problems. Accordingly, the research first proposes different TLMF decision models that can be used to describe various relationships between multiple followers at the same level. Second, a TLMF *K*th-Best algorithm is developed to solve TLMF decision problems, which can be also used to solve large-scale problems in reasonable computing time. Moreover, a fuzzy programming approach is proposed to evaluate the satisfaction of decision entities towards the solution obtained. The evaluation method of solutions can quantitatively analyze the operation of a decision-making process due to the changing decision environment. The above techniques provide the theoretical foundation for TLMF decision-making research and overcomes the lack of solution algorithms for solving TLMF decision problems.

Significance 4: the proposed multilevel decision-making techniques provide a practical way to handle decentralized decision problems in applications.

Many decentralized decision problems have increasingly appeared in highly complex decision situations in the real world, e.g. where there is uncertainty in data or multiple followers are involved. The proposed multilevel decision-making techniques can provide much more practical decision models and solution approaches for handling these real world cases. Specifically, this research displays how to handle decentralized production and inventory operational problems in SCM using multilevel decision-making techniques.

1.4 RESEARCH METHODOLOGY AND PROCESS

Research methodology is the "collections of problem solving methods governed by a set of principles and a common philosophy for solving targeted problems" (Gallupe 2007). This research belongs to the information system domain. A number of research methodologies have been proposed and applied in the information system domain, such as case study, field study, design research, archival research, field experiment, laboratory experiment, survey and action research (Niu 2009; Shambour 2012; Vaishnavi & Kuechler Jr 2007).

1.4.1 RESEARCH METHODOLOGY

In this study, design research is utilized as the research methodology according to the analysis of the research questions and objectives. Design research focuses on crafting and analyzing artifacts in order to gain insights into research problems. Examples of the artifacts include physical product prototypes, computer-based information systems and human-computer interfaces. The methodology of design research is illustrated in Figure 1.1. Generally speaking, a design research effort includes five basic steps (Vaishnavi & Kuechler Jr 2007).

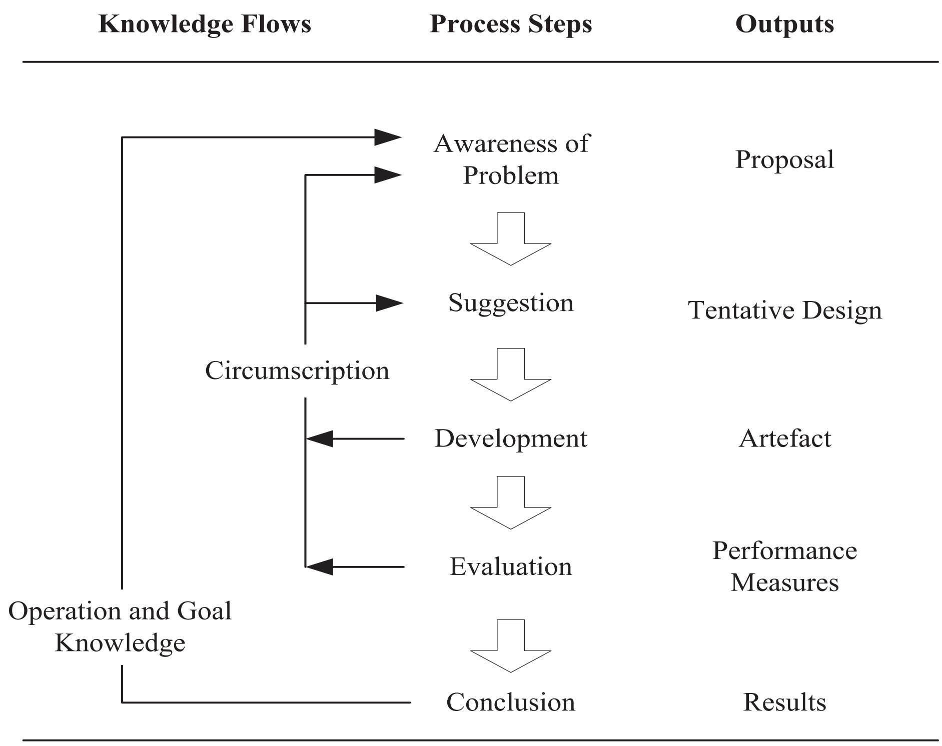


Figure 1.1 Reasoning in the general design cycle (Vaishnavi & Kuechler Jr 2007)

1) Awareness of problem

This is the starting point of a design research, at which limitations of existing research are examined and meaningful research problems are identified. The research problems reflect a gap between existing research and the expected status. The awareness of problems can come from different sources: industry experience, observations on practical applications and literature review. The corresponding output of this step is a research proposal (Vaishnavi & Kuechler Jr 2007).

2) Suggestion

This step follows the identification of research problems, and a tentative design is suggested. The tentative design describes what the prospective artifacts will be and how they can be developed. Suggestion is a creative process during which new concepts, models and approaches of artifacts are demonstrated. The resulting tentative design of this step is usually one part of the research proposal. Thus, the output of the suggestion step is feedback to the first step, so that the research proposal can be revised (Vaishnavi & Kuechler Jr 2007).

3) Development

In this step, artifacts are actually built based on the suggested design. The development of artifacts can testify to the reasonability and feasibility of the original design and improve the original design. As a result, the development of artifacts is often an iterative process in which an initial prototype is first built and this then evolves when the researcher gains a deeper understanding of the research problems. The knowledge obtained in this step is fed back to the previous two steps, which helps researchers revise the design and the proposal (Vaishnavi & Kuechler Jr 2007).

4) Evaluation

This step considers the evaluation of the developed artifacts. The performance of artifacts can be evaluated according to criteria defined in the research proposal and the suggested design. The evaluation results, which might or might not meet the expectations, are fed back to the first two steps. Thus, the proposal and design might be revised and the artifacts might be improved (Vaishnavi & Kuechler Jr 2007).

5) Conclusion

This is the final step of a design research effort. A conclusion or end is reached as a result of satisfaction with the evaluation results of the developed artifacts. There might still be deviations between the suggested proposal and the artifacts that are actually developed. However, a design research effort concludes as long as the developed artifacts are considered as "good enough" (Vaishnavi & Kuechler Jr 2007).

1.4.2 RESEARCH PROCESS

This research was planned according to the methodology of design research. First, a subject of multilevel decision-making was chosen as a very broad research topic of this research. A literature review of previous research in the topic area was conducted, and existing literature was retrieved and critically reviewed. The results of the literature review helped to identify specific research questions to be directly addressed in this research. As the research questions grew clearer and more definite, more literature closely related to the research questions was reviewed. Because the existing work in the literature lacks the ability to deal with large-scale, uncertain and diversified multilevel decision problems, this research proposed theoretical techniques involving decision models and solution approaches for solving such problems. The proposed models and approaches were implemented and evaluated by numerical experiments and/or real-world cases. According to the methodology of design research, this research is an iterative process. As indicated in Figure 1.1, the output of each research step might be fed back to its previous step when deviations between expectations and evaluation results are found. Through the feedback, research outcomes are progressively improved until satisfying results are drawn from evaluations. Finally, writing up the PhD thesis is done at the end of the research.

1.5 THESIS STRUCTURE

This thesis contains seven chapters. Chapter 1 presents the research background, research questions, objectives, significance, research methodology and process, and the thesis structure. Chapter 2 reviews the literature relevant to this study, including bi-level decision-making, tri-level decision-making, fuzzy multilevel decision-making, and the applications of multilevel decision-making techniques. Chapter 3 presents a PSO algorithm for solving large-scale nonlinear bi-level decision problems, which can be also extended to solve tri-level decision problems. Chapters 4 addresses related theoretical properties of fuzzy nonlinear decision problems and develops a compromise-based PSO algorithm. Chapter 5 proposes different TLMF decision models to describe various relationships between multiple followers at the same level; also, an effective solution algorithm is given. Chapter 6 applies these multilevel decision-making techniques to handle decentralized production and inventory operational problems in SCM. Chapter 7 provides conclusions and recommendations for further study. The structure of the thesis is shown in Figure 1.2.

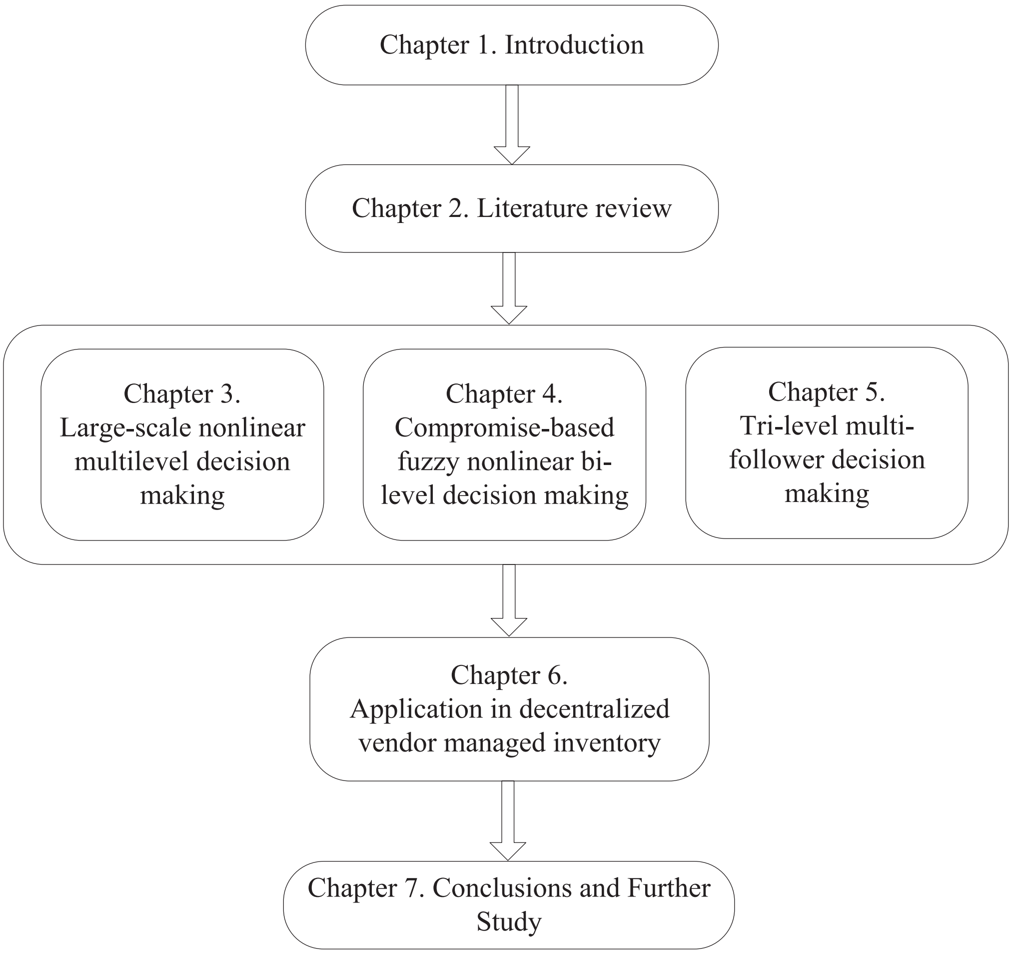


Figure 1.2 Thesis structure

1.6 PUBLICATIONS RELATED TO THIS THESIS

The related papers of this thesis that are under review or published in referred international journals and conferences are listed below.

1) Jialin Han, Jie Lu, Yaoguang Hu, Guangquan Zhang. 2015, 'Tri-level decision-making with multiple followers: Model, algorithm and case study', *Information Sciences*, vol. 311, pp. 182-204.

2) Jialin Han, Guangquan Zhang, Yaoguang Hu, Jie Lu. 2016, 'A solution to bi/tri-level programming problems using particle swarm optimization', *Information Sciences*, vol. 370-371, pp. 519-537.

3) Jialin Han, Guangquan Zhang, Jie Lu. 'Decentralized vendor-managed inventory in a three-echelon supply chain network using tri-level programming', *European Journal of Operational Research*. (Under 2nd-round review)

4) Jialin Han, Jie Lu, Guangquan Zhang. 'Fuzzy bi-level decision model and solution algorithm for integrated production planning and scheduling', IEEE Transactions on Fuzzy Systems. (Under 1st-round review)

5) Jie Lu, Jialin Han, Yaoguang Hu, Guangquan Zhang. 2016, 'Multilevel decision-making: A survey', *Information Sciences*, vol. 346-347, pp. 463-487.

6) Guangquan Zhang, Jialin Han, Jie Lu. 2016, 'Fuzzy bi-level decision-making: A survey', *International Journal of Computational Intelligence Systems*, vol. 9, pp. 25-34.

7) Jialin Han, Guangquan Zhang, Jie Lu, Yaoguang Hu, Shuyuan Ma. 2014, 'Model and algorithm for multi-follower tri-level hierarchical decision-making', *Lecture Notes in Computer Science*, vol. 8836, pp. 398-406.

8) Jialin Han, Jie Lu, Guangquan Zhang, Yaoguang Hu. 2015, 'Solving tri-level programming problems using a particle swarm optimization algorithm'. *The 10th IEEE Conference on Industrial Electronics and Applications*, pp. 569-574.

9) Jialin Han, Jie Lu, Guangquan Zhang, Yaoguang Hu. 2015, 'A compromise-based particle swarm optimization algorithm for solving bi-level programming problems with fuzzy parameters', *The 10th International Conference on Intelligent Systems and Knowledge Engineering*, pp. 214-221.

10) Jialin Han, Jie Lu, Guangquan Zhang, Shuyuan Ma. 2014, 'Multi-follower tri-level decision making with uncooperative followers', *Decision Making and Soft Computing: The 11th International FLINS Conference*, pp. 524-529.

CHAPTER 2 LITERATURE REVIEW

This chapter reviews the research on multilevel decision-making involving theoretical research results and applications, which are clustered into four categories: bi-level decision-making, tri-level decision-making, fuzzy multilevel decision-making, and applications of multilevel decision-making techniques. In Section 2.1, the bi-level decision-making models and solution approaches are reviewed and analyzed. Section 2.2 presents the tri-level decision-making models and solution approaches. Section 2.3 addresses fuzzy multilevel (including bi-level and tri-level) decision-making techniques. Section 2.4 discusses the applications of multilevel decision-making techniques. A summary is given in Section 2.5.

2.1 BI-LEVEL DECISION-MAKING

This section first reviews the development of techniques for solving basic bi-level decision-making problems. It then addresses the developments of bi-level decision-making with multiple optima involving bi-level multi-objective decision-making, bi-level multi-leader decision-making and bi-level multi-follower decision-making.

2.1.1 BASIC BI-LEVEL DECISION-MAKING

Basic bi-level decision-making, as found in a bi-level programming situation, has only one decision entity attempting to optimize a unique objective at each decision level. The general formulation for basic bi-level decision-making is described by a bi-level program as Definition 2.1.

Definition 2.1 (Bard 1998) For $x \in X \subset R^p$, $y \in Y \subset R^q$, a general bi-level decision problem is defined as:

$$\min_{x \in X} F(x, y) \qquad \text{(Leader)} \qquad (2.1a)$$

$$\text{s.t.} \quad G(x, y) \leq 0, \qquad (2.1b)$$

where, for each x given by the leader, y solves problem (2.1c-2.1d)

$$\min_{y \in Y} f(x, y) \qquad \text{(Follower)} \qquad (2.1c)$$

$$\text{s.t.} \quad g(x, y) \leq 0, \qquad (2.1d)$$

where x, y are the decision variables of the leader and the follower respectively; $F, f : R^p \times R^q \to R^1$ are the objective functions of the leader and the follower respectively; $G : R^p \times R^q \to R^m$, $g : R^p \times R^q \to R^n$ are the constraint conditions of the leader and the follower respectively. The sets X and Y place additional restrictions on the decision variables, such as upper and lower bounds or integrality requirements (Bard 1998).

It can be seen from Definition 2.1 that, for each value of x is given by the leader, the follower will choose the value of its decision variable y under the constraint condition (2.1d) with the aim of optimizing the objective function (2.1c); also, the selected value of y will affect the leader's objective function (2.1a). Thus, the leader needs to consider the implicit reaction y of the follower when making its own decisions; that is, the follower's decision problem (2.1c-2.1d) can be considered as the constraint condition of the leader's decision problem. It is clear that the constraint domain associated with a bi-level decision problem is implicitly determined by two optimization problems that must be solved in a predetermined sequence from the leader to the follower. Although the basic bi-level decision problem has been proved to be NP-hard by Ben-Aved and Blair (1990) and Bard (1991), many methods/algorithms have been developed for solving linear, nonlinear and discrete bi-level decision problems. This section reviews the research of these three categories of basic bi-level decision-making techniques.

2.1.1.1 LINEAR BI-LEVEL DECISION-MAKING

Definition 2.2 (Bard 1998) Based on Definition 2.1, for $x \in X \subset R^p$, $y \in Y \subset R^q$, and $F, f: R^p \times R^q \to R^1$, the linear bi-level decision problem can be written as follows:

$$\min_{x \in X} F(x, y) = c_1 x + d_1 y \qquad \text{(Leader)} \qquad (2.2a)$$

$$\text{s.t. } A_1 x + B_1 y \le b_1, \qquad (2.2b)$$

where, for each *x* given by the leader, *y* solves problem (2.2c-2.2d)

$$\min_{y \in Y} f(x, y) = c_2 x + d_2 y \qquad \text{(Follower)} \qquad (2.2c)$$

$$\text{s.t. } A_2 x + B_2 y \le b_2, \qquad (2.2d)$$

where $c_1, c_2 \in R^p$, $d_1, d_2 \in R^q$, $b_1 \in R^m$, $b_2 \in R^n$, $A_1 \in R^{m \times p}$, $B_1 \in R^{m \times q}$, $A_2 \in R^{n \times p}$, $B_2 \in R^{n \times q}$.

In terms of solving linear bi-level decision problems, the traditional algorithms can be classified into three main categories: the vertex enumeration approaches (Bialas & Karwan 1984; Candler & Townsley 1982; Shi, Lu & Zhang 2005; Tuy, Migdalas & Värbrand 1993) based on an important characteristic of bi-level programming whereby an optimal solution occurs at a vertex of the constraint region; the Kuhn-Tucker approaches involving branch and bound algorithms (Bard & Falk 1982; Bard & Moore 1990; Fortuny-Amat & McCarl 1981; Shi et al. 2006) and complementary pivot algorithms (Bialas & Karwan 1984; Júdice & Faustino 1992; Önal 1993), in which the upper-level problem includes the lower-level's optimality conditions as extra constraints; and the penalty function approaches (Anandalingam &

White 1990; White & Anandalingam 1993) which append a penalty term of the lower-level problem to the objective function of the upper-level problem.

In recent years, Audet, Haddad and Savard (2007) proposed a disjunctive cuts method for a linear bi-level decision problem with continuous variables. Audet, Savard and Zghal (2007) considered the equivalences between linear mixed 0-1 integer programming problems and linear bi-level decision problems, and proposed a finite and exact branch-and-cut algorithm for solving such problems. Glackin, Ecker and Kupferschmid (2009) addressed the relationship between linear multi-objective programs and linear bi-level programs and presented an algorithm for solving linear bi-level programs that uses simplex pivots on an expanded tableau. Calvete and Galé (2012) addressed linear bi-level programs in which the coefficients of both objective functions are interval numbers and developed two algorithms based on ranking extreme points to solve such problems. Ren and Wang (2014) proposed a cutting plane method to solve the linear bi-level decision problem with interval coefficients in both objective functions.

A range of heuristic algorithms have been also developed to solve bi-level decision problems. Gendreau, Marcotte and Savard (1996) used an adaptive search method related to the tabu search meta-heuristic to solve the linear bi-level decision problem. Hejazi et al. (2002) proposed a method based on genetic algorithm for solving linear bi-level decision problems. Lan et al. (2007) proposed a hybrid algorithm that combines neural network and tabu search for solving linear bi-level decision problems. Calvete, Galee and Mateo (2008) developed a genetic algorithm for solving a class of linear bi-level decision problems in which both objective functions are linear and the common constraint region is a polyhedron; the authors also presented a method for the test set construction of linear bi-level decision problems especially for generating large-scale problems, which can be employed to assess the efficiency performance of related algorithms. Kuo and Huang (2009) developed a particle swarm optimization (PSO) algorithm with swarm intelligence to solve linear bi-level decision problems. Hu et al. (2010) presented a neural network approach for solving linear bi-level decision problems.

2.1.1.2 NONLINEAR BI-LEVEL DECISION-MAKING

With respect to Definition 2.1, if the objective functions $F(x, y)$, $f(x, y)$ or the constraint conditions $G(x, y) \leq 0$, $g(x, y) \leq 0$ are nonlinear formulations, the bi-level program is known as a nonlinear bi-level decision problem, which is much more difficult to solve than linear versions.

In early research in solving nonlinear bi-level decision problems, Bard (1988) extended the traditional branch and bound algorithm to solve nonlinear convex bi-level decision problems. Edmunds and Bard (1991) used a branch-and-bound algorithm and a cutting-plane algorithm to solve various versions of nonlinear bi-level decision problems when certain convexity conditions hold. Al-Khayyal, Horst and Pardalos (1992) developed a branch and bound algorithm and a piecewise linear approximation method to find the global minimum for a class of nonlinear bi-level decision problems based on an equivalent system of convex and separable quadratic constraints. Vicente and Calamai (1994) introduced two descent methods for a special instance of bi-level programs where the second-level problem is strictly convex quadratic.

In recent years, Tuy, Migdalas and Hoai-Phuong (2007) showed that a nonlinear bi-level decision problem can be transformed into a monotonic optimization problem which can then be solved by a branch-reduce-and-bound method using monotonicity cuts. Mitsos, Lemonidis and Barton (2008) presented a bounding algorithm for the global solution of nonlinear bi-level programs involving nonconvex objective functions in both decision levels. Mersha and Dempe (2011) studied the application of a class of direct search methods and solved bi-level decision problems containing convex lower level problems with strongly stable optimal solutions.

In regard to related heuristic algorithms, Wang, Jiao and Li (2005) transformed a special nonlinear bi-level decision problem into an equivalent single objective nonlinear programming problem that can be solved by an evolutionary algorithm. Wan, Wang and Sun (2013) presented a hybrid intelligent algorithm of PSO and chaos searching technique for solving nonlinear bi-level decision problems. Wan, Mao and Wang (2014) also developed a novel evolutionary algorithm, called the estimation of distribution algorithm, for solving a special class of nonlinear bi-level decision problems in which the lower-level problem is a convex program for each given upper-level decision. Lv et al. (2008), Lv, Chen and Wan (2010) and He et al. (2014) proposed neural network methods for solving nonlinear bi-level decision problems. It is notable that Sinha, Malo and Deb (2014) proposed a procedure for designing the test set of nonlinear bi-level decision problems and presented the corresponding computational results for these test problems using a nested bi-level evolutionary algorithm. Researchers can consider these test problems as the benchmark for examining the effectiveness of their own algorithms.

2.1.1.3 Discrete bi-level decision-making

In many bi-level decision-making problems, a subset of the variables is restricted to take on discrete values (Bard 1998). A problem can be considered to be a general discrete bi-level decision problem when the decision variables in Definition 2.1 are discrete, e.g. integer programming. Clearly, if the decision variables are discrete rather than continuous, the linear bi-level decision problem (2.2) will become a discrete linear bi-level program.

Discrete variables can complicate bi-level decision problems by several orders of magnitude and render all but the smallest instances unsolvable (Bard 1998). Wen and Yang (1990), Moore and Bard (1990), and Bard and Moore (1992) therefore proposed traditional branch and bound algorithms for finding solutions to integer linear bi-level decision-making problems. Edmunds and Bard (1992) developed a branch and bound algorithm to solve a mixed-integer nonlinear bi-level decision problem. Vicente, Savard and Judice (1996) designed penalty function methods for solving discrete linear bi-level decision problems.

Recently, Faísca, Dua, et al. (2007) proposed a global optimization approach to solve quadratic bi-level and mixed integer linear bi-level problems, with or without right-hand-side uncertainty. Mitsos (2010) presented an algorithm based on the research by Mitsos, Lemonidis and Barton (2008) for the global optimization of nonlinear bi-level mixed-integer programs, which relies on a convergent lower bound and an optional upper bound. Köppe, Queyranne and Ryan (2010) proposed a parametric integer programming algorithm for solving a mixed integer linear bi-level decision problem where the follower solves an integer program with a fixed number of variables. Domínguez and Pistikopoulos (2010) addressed two algorithms using multiparametric programming techniques respectively for solving two categories of integer bi-level decision problems: one category consists of pure integer problems where integer variables of the first level appear in the linear or polynomial problem of the second level, and the other consists of mixed-integer problems where integer and continuous variables of the first level appear in the linear or polynomial problem of the second level. Xu and Wang (2014) solved a mixed integer linear bi-level decision problem using an exact algorithm. The algorithm relies on three simplifying assumptions, explicitly considers finite optimal, infeasible and unbounded cases, and is proved to terminate finitely and correctly. Sharma, Dahiya and Verma (2014) discussed an integer bi-level decision problem with bounded variables in which the objective function of the first level is linear fractional, the objective function of the second level is linear and the common constraint region is a polyhedron. They proposed an iterative algorithm to find an optimal solution to the problem.

In relation to heuristic algorithms for solving discrete bi-level decision problems, Wen and Huang (1996) reported a mixed-integer linear bi-level decision-making formulation in which zero-one decision variables are controlled by the first level and real-value decision variables are controlled by the second level. An algorithm based on the short term memory component of tabu search, called simple tabu search, was developed to solve the problem. Nishizaki and Sakawa (2005) presented a method using genetic algorithms for obtaining optimal solutions to integer bi-level decision problems.

2.1.2 Bi-level multi-objective decision-making

When multiple conflicting objectives for each decision entity exist in a bi-level decision problem, this is known as a bi-level multi-objective (BLMO) decision problem.

Definition 2.3 (Deb & Sinha 2010) For $x \in X \subset R^p$, $y \in Y \subset R^q$, a general BLMO decision problem is formulated as:

$$\min_{x \in X} F(x,y) = (F_1(x,y), F_2(x,y), \ldots, F_M(x,y)) \quad \text{(Leader)} \quad (2.3a)$$

$$\text{s.t. } G(x,y) \le 0, \quad (2.3b)$$

where, for each x given by the leader, y solves problem (2.3c-2.3d)

$$\min_{y \in Y} f(x,y) = (f_1(x,y), f_2(x,y), \ldots, f_N(x,y)) \quad \text{(Follower)} \quad (2.3c)$$

$$\text{s.t. } g(x,y) \le 0, \quad (2.3d)$$

where x, y are the decision variables of the leader and the follower respectively; $F_i, f_j : R^p \times R^q \to R^1$, $i = 1,2,\ldots,M$, $j = 1,2,\ldots,N$ are the conflicting objective functions of the leader and the follower respectively; $G : R^p \times R^q \to R^m$, $g : R^p \times R^q \to R^n$ are the constraint conditions of the leader and the follower respectively. The sets X and Y place additional restrictions on the decision variables, such as upper and lower bounds or integrality requirements.

Many algorithms have been developed to solve bi-level multi-objective (BLMO) decision problems in various versions. Ankhili and Mansouri (2009) addressed a class of linear bi-level programs where the upper level is a linear scalar optimization problem and the lower level is a linear multi-objective optimization problem; they approached the problems via an exact penalty method. Calvete and Galé (2010) presented a number of methods of computing efficient solutions to solve linear bi-level decision problems with multiple objectives at the upper level; all the methods result in solving linear bi-level problems with a single objective function at each level based on both weighted sum scalarization and scalarization techniques. Eichfelder (2010) discussed a nonlinear nonconvex BLMO decision problem using an optimistic approach in which the feasible points of the upper-level objective function can be expressed as the set of minimal solutions of a single-level multi-objective optimization problem. The BLMO decision problem is then solved by an iterative process, again using sensitivity theorems. Emam (2013) proposed an interactive approach for solving bi-level integer fractional multi-objective decision problems.

From the aspect of using heuristic algorithms for solving BLMO decision problems, Deb and Sinha (2010) proposed a viable and hybrid evolutionary-cum-local-search based algorithm for solving BLMO decision problems. Note that Deb and Sinha (2009) also presented a method for constructing the test set of BLMO decision problems. Calvete and Galé (2011) developed an exact algorithm and a metaheuristic algorithm to solve linear bi-level decision problems with multiple objectives at the lower level. Zhang et al. (2013) proposed a hybrid PSO algorithm with crossover operator to solve high dimensional bi-level multi-objective decision problems. Alves and Costa (2014) presented an improved PSO algorithm to solve linear bi-level decision problems with multiple objectives at the upper level.

2.1.3 Bi-level multi-leader and/or multi-follower decision-making

In a bi-level decision problem, multiple decision entities may exist at each level, and this is known as a bi-level multi-leader and/or multi-follower decision problem. A general bi-level multi-leader (BLML) decision problem can be defined as Definition 2.4.

Definition 2.4 (Zhang, Lu & Gao 2015) For $x_i \in X_i \subset R^{p_i}$, $y \in Y \subset R^q$, $i = 1,2,\ldots,L$, a general BLML decision problem in which L leaders and one follower are involved can be described as:

$$\min_{x_i \in X_i} F_i(x, y) \qquad \text{(Leader } i\text{)} \tag{2.4a}$$

$$\text{s.t. } G_i(x, y) \le 0, \tag{2.4b}$$

where, for each x given by the leaders, y solves problem (2.4c-2.4d)

$$\min_{y \in Y} f(x, y) \qquad \text{(Follower)} \tag{2.4c}$$

$$\text{s.t. } g(x, y) \le 0, \tag{2.4d}$$

where $x = (x_1, x_2, \ldots, x_L)$, x_i and y are the decision variables of the ith leader and the follower respectively; $F_i, f : R^{p_1} \times \ldots \times R^{p_L} \times R^q \to R^1$ are the objective functions of the ith leader and the follower respectively; $G_i : R^{p_1} \times \ldots \times R^{p_L} \times R^q \to R^{m_i}$, $g : R^{p_1} \times \ldots \times R^{p_L} \times R^q \to R^n$ are the constraint conditions of the ith leader and the follower respectively. The sets X and Y place additional restrictions on the decision variables, such as upper and lower bounds or integrality requirements. It is clear in Definition 2.4 that, when leaders make their individual decisions, they need to not only take into account the implicit reaction of the follower but also consider the decision results given by their counterparts at the first level.

In relation to research on bi-level multi-leader decision-making, DeMiguel and Huifu (2009) studied a stochastic BLML decision model and proposed a computational approach to find a Stochastic Multiple-leader Stackelberg-Nash-Cournot (SMS) equilibrium based on the sample average approximation method. Zhang, Lu and Gao (2015) introduced a framework for the bi-level multi-leader (BLML) decision problem, in which they presented different BLML decision models in line with various relationships between multiple leaders.

The authors also proposed a PSO algorithm to find a solution for BLML decision problems based on the related solution concepts.

In contrast to the limited discussion on BLML decision-making, researchers have paid considerably more attention to bi-level multi-follower (BLMF) decision-making. A general BLMF decision problem in which one leader and k followers are involved can be defined as Definition 2.5.

Definition 2.5 (Zhang, Lu & Gao 2015) For $x \in X \subset R^p$, $y_i \in Y_i \subset R^{q_i}$, $i = 1,2,\ldots,k$, a general BLMF decision problem in which one leader and k followers are involved can be written as:

$$\min_{x \in X} F(x, y) \qquad \text{(Leader)} \qquad (2.5a)$$

$$\text{s.t. } G(x, y) \le 0, \qquad (2.5b)$$

where, for each x given by the leader, y_i solves problem (2.5c-2.5d)

$$\min_{y_i \in Y_i} f_i(x, y) \qquad \text{(Follower } i\text{)} \qquad (2.5c)$$

$$\text{s.t. } g_i(x, y) \le 0, \qquad (2.5d)$$

where $y = (y_1, y_2, \ldots, y_k)$, x and y_i are the decision variables of the leader and the ith follower respectively; $F, f_i : R^p \times R^{q_1} \times \ldots \times R^{q_k} \to R^1$ are the objective functions of the leader and the ith follower respectively; $G : R^p \times R^{q_1} \times \ldots \times R^{q_k} \to R^m$, $g_i : R^p \times R^{q_1} \times \ldots \times R^{q_k} \to R^{n_i}$ are the constraint conditions of the leader and the ith follower respectively. The sets X and Y place additional restrictions on the decision variables, such as upper and lower bounds or integrality requirements. It can be seen in Definition 2.5 that followers need to consider the decision results of their counterparts as references when making their individual decisions in view of the decision given by the leader.

Anandalingam and Apprey (1991) first presented a linear BLMF decision model, known as a linear bi-level multi-agent system, and developed a penalty function approach to solve the problem. Liu (1998) designed a genetic algorithm for solving Stackelberg-Nash equilibrium of nonlinear BLMF decision problems in which there might be an information exchange between the followers. Based on previous research, Lu, Shi and Zhang (2006) proposed a general framework of BLMF decision-making that considers three main relationships between multiple followers: the uncooperative relationship, the referential-uncooperative relationship, and the partial-cooperative relationship. The research on BLMF decision-making after Lu, Shi and Zhang (2006) was structured on the general framework. Calvete and Galé (2007) subsequently presented a approach for solving the linear BLMF decision problem with uncooperative followers, which converted the BLMF problem to a bi-level problem with one leader and one follower. Shi, Zhang and Lu (2005) and Shi et al. (2007) extended the *K*th-Best algorithm to solve linear BLMF decision problems with uncooperative and partial-cooperative relationships respectively between followers. Lu et al. (2007) and Lu and Shi (2007) respectively adopted the extended Kuhn-Tucker algorithm and the extended branch and bound algorithm to solve the referential-uncooperative linear BLMF decision problem.

Nie (2011) developed and characterized discrete-time dynamic bi-level multi-leader and multi-follower (BLMLMF) games with leaders in turn, and a dynamic programming algorithm was employed to solve this problem. Gao (2010) developed PSO-based algorithms to solve BLML, BLMF and BLMLMF decision problems. Sinha et al. (2014) used a computationally intensive nested evolutionary algorithm to find an optimal solution for a multi-period BLMLMF decision problem with nonlinear and discrete variables.

2.2 TRI-LEVEL DECISION-MAKING

Decentralized decision-making problems within a hierarchical system are often comprised of more than two levels in many applications, which is known as tri-level and multilevel decision-making.

Definition 2.6 (Faísca, Saraiva, et al. 2007) For $x \in X \subset R^p$, $y \in Y \subset R^q$, $z \in Z \subset R^r$, a general tri-level decision problem is defined as:

$$\min_{x \in X} f_1(x, y, z) \qquad \text{(Leader)} \tag{2.6a}$$

$$\text{s.t.} \quad g_1(x, y, z) \le 0, \tag{2.6b}$$

where, for each x given by the leader, (y, z) solves the problems (2.6c-2.6f) of the middle-level and bottom-level followers:

$$\min_{y \in Y} f_2(x, y, z) \qquad \text{(Middle-level follower)} \tag{2.6c}$$

$$\text{s.t.} \quad g_2(x, y, z) \le 0, \tag{2.6d}$$

where, for each (x, y) given by the leader and the middle-level follower, z solves the problem (2.6e-2.6f) of the bottom-level follower:

$$\min_{z \in Z} f_3(x, y, z) \qquad \text{(Bottom-level follower)} \tag{2.6c}$$

$$\text{s.t.} \quad g_3(x, y, z) \le 0, \tag{2.6d}$$

where x, y, z are the decision variables of the leader, the middle-level follower and the bottom-level follower respectively; $f_1, f_2, f_3 : R^p \times R^q \times R^r \to R^1$ are the objective functions of the three decision entities respectively; $g_i : R^p \times R^q \times R^r \to R^{k_i}, i = 1,2,3$ are the constraint conditions of the three decision entities respectively.

While the majority of studies on multilevel decision-making have focused on bi-level decision-making, research on tri-level decision problems has increasingly attracted investigations into solution approaches since tri-level decision-making can be applied to handle many decentralized decision problems in the real world (Lu et al. 2012). Bard (1984) first presented an investigation into linear tri-level decision-making and designed a cutting plane algorithm to solve such problems, based on which White (1997) proposed a penalty function approach for linear tri-level decision problems. Anandalingam (1988) and Sinha (2001) developed Kuhn-Tucker transformation methods to find local optimal solutions for linear tri-level decision problems. Ruan et al. (2004) discussed the optimality conditions and related geometric properties of a linear tri-level decision problem with dominated objective functions. Faísca, Saraiva, et al. (2007) studied a multi-parametric programming approach to solve tri-level hierarchical and decentralized optimization problems based on parametric global optimization for bi-level decision-making (Faísca, Dua, et al. 2007). Zhang et al. (2010) developed a tri-level *K*th-Best algorithm to solve linear tri-level decision problems.

A category of approaches based on fuzzy programming has been also developed to solve multilevel decision problems involving bi-level and tri-level programs. Lai (1996) first proposed a fuzzy approach to find a satisfactory solution to the linear multilevel decision problem using concepts of membership functions of individual optimality and the satisfactory degree of individual decision power. Shih, Lai and Lee (1996) extended Lai's concepts and adopted tolerance membership functions and multiple objective optimization to develop a fuzzy approach for solving the above problems. Sakawa, Nishizaki and Uemura (1998) presented an interactive fuzzy programming approach for linear multilevel decision problems by updating the satisfactory degrees of decision entities at the upper level with considerations of overall satisfactory balance between all levels. Their interactive fuzzy programming approach overcomes the inconsistency between the fuzzy goals of objectives and decision variables that existed in the research developed by Lai (1996) and Shih, Lai and Lee (1996). Sinha (2003a, 2003b) developed an alternative multilevel decision technique based on fuzzy mathematical programming, which considered a sequential order of the multilevel hierarchy and took into account the preference of the decision entity at each level. Pramanik and Roy (2007) and Arora and Gupta (2009) each proposed a fuzzy goal programming approach to solve linear multilevel decision problems using definitions of tolerance membership functions and satisfactory degree of decision entities.

To solve tri-level decision-making problems with multiple optima, Shih, Lai and Lee (1996) proposed a tri-level decision model with multiple followers and developed a fuzzy approach to solve the model. Lu et al. (2012) presented a framework for tri-level multi-follower (TLMF) decision-making research and discussed various relationships between multiple followers.

2.3 FUZZY MULTILEVEL DECISION-MAKING

A multilevel decision problem in which the parameters are described by fuzzy values, often characterized by fuzzy numbers, is called a fuzzy multilevel decision problem (Zhang & Lu 2007; Zhang, Lu & Gao 2015). For the sake of simplicity, this section presents a general fuzzy linear bi-level decision problem based on Definition 2.2, described as Definition 2.7.

Definition 2.7 (Zhang & Lu 2005; Zhang, Lu & Gao 2015) For $x \in X \subset R^p$, $y \in Y \subset R^q$, and $F, f : R^p \times R^q \to F(R)$, a general fuzzy linear bi-level decision problem can be written as follows:

$$\min_{x \in X} F(x, y) = \tilde{c}_1 x + \tilde{d}_1 y \qquad \text{(Leader)} \tag{2.7a}$$

$$\text{s.t.} \quad \tilde{A}_1 x + \tilde{B}_1 y \le \tilde{b}_1, \tag{2.7b}$$

where, for each x given by the leader, y solves problem (2.7c-2.7d)

$$\min_{y \in Y} f(x, y) = \tilde{c}_2 x + \tilde{d}_2 y \qquad \text{(Follower)} \tag{2.7c}$$

$$\text{s.t.} \quad \tilde{A}_2 x + \tilde{B}_2 y \le \tilde{b}_2, \tag{2.7d}$$

where $\widetilde{c}_1, \widetilde{c}_2 \in F^p(R)$, $\widetilde{d}_1, \widetilde{d}_2 \in F^q(R)$, $\widetilde{b}_1 \in F^m(R)$, $\widetilde{b}_2 \in F^n(R)$, $\widetilde{A}_1 \in F^{m\times p}(R)$, $\widetilde{B}_1 \in F^{m\times q}(R)$, $\widetilde{A}_2 \in F^{n\times p}(R)$, $\widetilde{B}_2 \in F^{n\times q}(R)$, $F(R)$ is the set of all finite fuzzy numbers.

Like multilevel decision-making under certainty, the majority of the research on fuzzy multilevel decision-making has focused on bi-level versions that have motivated numerous solution approaches (Zhang, Lu & Gao 2015). Zhang and Lu (2005) proposed a general fuzzy linear bi-level decision problem and developed an approximation Kuhn-Tucker approach to solve this problem. They also presented an approximation *K*th-Best algorithm to solve the fuzzy linear bi-level decision problem (Zhang & Lu 2007). Gao et al. (2008) proposed a programmable λ-cut approximation algorithm to solve a λ-cut set based fuzzy goal bi-level decision problem. Budnitzki (2013) used the selection function approach and a modified version of the *K*th-Best algorithm to solve a fuzzy linear bi-level decision problem. Sakawa, Nishizaki and Uemura (2000a) proposed an interactive fuzzy programming approach to find a satisfactory solution to a fuzzy linear bi-level decision problem. Pramanik (2012) adopted a fuzzy goal programming approach to solve fuzzy linear bi-level decision problems.

Fuzzy bi-level decision-making with multiple optima has attracted numerous studies. Zhang, Lu and Dillon (2007a) developed an approximation branch-and-bound algorithm to solve a fuzzy linear BLMO decision problem. Gao et al. (2010) proposed a λ-cut and goal-programming-based algorithm to solve fuzzy linear BLMO decision problems. Gao, Zhang and Lu (2009) focused on the fuzzy linear bi-level decision problem with multiple followers who share the common constraints and developed a PSO algorithm to solve the problem. Gao and Liu (2005) integrated fuzzy simulation, neural network and genetic algorithm to produce a hybrid intelligent algorithm for solving a fuzzy nonlinear bi-level decision problem with multiple followers. Zhang, Lu and Dillon (2007b) proposed a set of fuzzy linear bi-level multi-objective multi-follower (BLMOMF) decision models and developed an extended branch and bound algorithm to solve such problems. Zhang and Lu (2010) developed an approximation *K*th-Best algorithm to solve fuzzy linear BLMOMF decision problems with a cooperative relationship among multiple followers. Zhang, Lu and Gao (2008) developed an approximation branch-and-bound algorithm to solve a fuzzy linear BLMOMF decision problem with a partial cooperative relationship among multiple followers.

In terms of the discussion about fuzzy tri-level and multilevel decision-making, Sakawa, Nishizaki and Uemura (2000a) extended their bi-level interactive fuzzy programming approach to solve fuzzy linear multilevel decision problems. They also extended the fuzzy approach to solve fuzzy linear multilevel fractional decision problems (Sakawa, Nishizaki & Uemura 2000b), fuzzy multilevel 0-1 decision problems (Sakawa, Nishizaki & Hitaka 2001) and fuzzy multilevel non-convex decision problems (Sakawa & Nishizaki 2002). Based on interactive fuzzy programming approaches, Osman et al. (2004) studied a fuzzy non-linear tri-level decision problem with multiple objectives.

2.4 APPLICATIONS OF MULTILEVEL DECISION-MAKING TECHNIQUES

Multilevel decision-making techniques have been widely applied to handle decentralized decision problems in the real world, in particular in the last five years. These applications largely fall into the following four areas: (1) supply chain management; (2) traffic and transportation network design; (3) energy management; and (4) safety and accident management.

2.4.1 SUPPLY CHAIN MANAGEMENT

Supply chain management (SCM) requires decentralized decisions to be made at several stages in a complex hierarchical system which includes the location of business firms, the acquisition of raw materials, production planning and operations, inventory control, and the delivery and pricing of commodities. It is increasingly important to develop an efficient and easily-applicable decision-making methodology to handle conflict coordination and the decentralized nature of SCM (Li & Cruz Jr 2009; Sana 2011). In recognition of this, multilevel decision-making techniques have been applied to deal with many of the decentralized decision-making problems found in SCM.

Multilevel decision-making techniques in SCM have largely been applied to deal with the competitive location of facilities, production planning and operations, commodities distribution and pricing. With respect to the competitive location of facilities in SCM, Plastria and Vanhaverbeke (2008) used bi-level programs to adapt the competitive location model based on maximal covering to include the knowledge that a competitor will eventually enter the market with a single new facility. Küçükaydin, Aras and Kuban Altınel (2011) studied a problem in which a firm or franchise enters a market by locating new facilities near existing facilities belonging to a competitor and formulated the problem as a mixed-integer nonlinear bi-level decision model in which the firm entering the market is the leader and the competitor is the follower. Rider et al. (2013) presented a bi-level decision model for determining optimal location and contract pricing of distributed generation in radial distribution systems where the upper-level optimization determines the allocation and contract prices of the distributed generation units, whereas the lower-level optimization models the reaction of the distribution company. Gang et al. (2015) proposed a bi-level multi-objective optimization model for a stone industrial park location problem with a hierarchical structure consisting of a local government and several stone enterprises in a random environment. The problem was solved using a bi-level interactive method based on a satisfactory solution and adaptive chaotic PSO.

For decentralized production planning and operations, Lukač, Šorić and Rosenzweig (2008) designed a mixed 0-1 integer bi-level decision model for a production planning problem with sequence dependent setups, in which the objective of the leader is to assign the products to the machines in order to minimize the total sequence dependent setup time, while the objective of the follower is to minimize the production, storage and setup cost of the machine. They developed a heuristic algorithm based on tabu search to solve the problem. Calvete, Galé and Oliveros (2011) proposed a bi-level program to model a hierarchical production-distribution planning problem in which two decision makers respectively controlling the production process and the distribution process do not cooperate because of different optimization strategies. An ant colony optimization approach was developed to solve the bi-level model. Kasemset and Kachitvichyanukul (2012) presented a bi-level multi-objective mathematical model for a TOC (theory of constraints)-based job-shop scheduling problem and developed a PSO algorithm to solve the problem.

In terms of using multilevel decision-making techniques to handle commodities distribution and pricing problems in SCM, Gao et al. (2011) established two bi-level pricing models for pricing problems between the vendor and the buyer, designated as the leader and the follower respectively, in a two-echelon supply chain. They developed a PSO-based algorithm to solve problems defined by these bi-level pricing models. Kuo and Han (2011) applied linear bi-level programming to model a supply chain distribution problem and developed an efficient method based on a hybrid of the genetic algorithm and PSO algorithm to solve the resulting decision model. Kis and Kovács (2013) studied an extension of the classical uncapacitated lot-sizing problem with backlogs, in which two autonomous and self-interested decision makers constitute a two-echelon supply chain. Qiu and Huang (2013) presented a bi-level decision model and an enumerative algorithm to describe and solve a SCM problem in which a supply hub in an industrial park and manufacturers interact to make their decisions on pricing, replenishment and delivery. Calvete, Galé and Iranzo (2014) addressed a mixed integer bi-level optimization model for the planning of a decentralized distribution network consisting of manufacturing plants, depots and customers, and a metaheuristic approach based on evolutionary algorithms was developed to solve the optimization model. Ma, Wang and Zhu (2014) considered a two-echelon supply chain system with one manufacturer and one retailer, in which the manufacturer first purchases raw materials from the supplier; following production and processing by the manufacturer, the end products are sold to the retailer. By switching the leader and follower roles between the manufacturer and the retailer, the authors established two bi-level decision models for joint pricing and lot-sizing and developed a PSO algorithm to solve the resulting models.

A number of researchers have applied multilevel decision-making techniques to handle product design, raw materials supply and inventory control problems in SCM. Yang et al. (2015) formulated a mixed 0-1 nonlinear bi-level decision model for the joint optimization of product family configuration and scaling design, in which a bi-level decision structure reveals coupled decision making between module configuration and parameter scaling. Based on a conditional value-at-risk (CVaR) measure of risk management, Xu, Meng and Shen (2013) proposed a tri-level decision model for the three-echelon SCM in which the material supplier and the manufacturer maximize their own profit while the retailer maximizes its CVaR of the expected profit. The authors showed that the proposed tri-level decision model can be transformed into a bi-level decision model that can be solved by existing methods.

2.4.2 TRAFFIC AND TRANSPORTATION NETWORK DESIGN

Severe traffic and transportation delays are incurred in most road networks as a result of continuously growing travel demand, increasing traffic congestion, transportation allocation problems between supply and demand nodes, and optimal transportation route problems. The rapid growth of overload in traffic and transportation networks has motivated decision makers to apply multilevel decision-making techniques to cope with the related decision-making and optimization problems in decentralized situations.

Extensive research on the basis of multilevel decision-making has been devoted to road network design problems as a result of insufficient provision of link capacity for travel demand surges. Cao et al. (2007) used a discrete bi-level decision model to describe the relationship of the benefit-cost of the traffic flow guidance system (TFGS) and the equilibrium of users, and presented an arithmetic based on sensitivity analysis. In a system which allows buses of different sizes to be assigned to public transport routes, dell'Olio, Ibeas and Ruisánchez (2012) addressed a discrete bi-level optimization model with constraints on bus capacity to size buses and set frequencies on each route in an attempt to optimize the headways on each route in accordance with observed levels of demand. Ukkusuri, Doan and Aziz (2013) formulated a combined dynamic user equilibrium and traffic signal control problem as a discrete bi-level optimization model and solved the problem using a solution technique based on the iterative optimization and assignment method. Wang, Meng and Yang (2013) addressed a discrete network design problem with multiple capacity levels which determines the optimal number of lanes to add to each candidate link in a road network. They formulated the problem as a bi-level decision model, where the upper level aims to minimize the total travel time by adding new lanes to candidate links and the lower level is a traditional Wardrop user equilibrium (UE) problem. Han et al. (2015) proposed a nonlinear bi-level decision model for traffic network signal control, which was formulated as a dynamic Stackelberg game and solved as a mathematical program with equilibrium constraints. Angulo et al. (2014) proposed a nonlinear bi-level decision model for the expansion of a highway network by adding several highway corridors within a geographical region, in which the upper level problem determines the location of the highway corridors by taking into account budgetary and technological restrictions, while the lower level problem models user behavior in the located transport network (choice of route and transport system). Fontaine and Minner (2014) designed a linear bi-level decision model for the discrete network design problem which adds arcs to an existing road network at the leader stage and anticipates traffic equilibrium for the follower stage. They proposed a new fast solution method for the resulting model with binary leader and continuous follower variables under the assumption of partial cooperation.

In regard to solving transportation planning and scheduling, origin-destination allocation and routing optimization problems, Chiou (2009) proposed a nonlinear bi-level decision model for a logistics network design problem with system-optimized flows and developed a novel solution algorithm to efficiently solve the problem. Ge, Chen and Wang (2013) established a discrete bi-level decision model to analyze an integrated inventory-transportation optimization problem and adopted a layer-iterative algorithm to solve the resulting model. Liu, Zheng and Cai (2013) presented a novel real-time path planning approach for unmanned aerial vehicles, in which the planning problem is described as a nonlinear bi-level decision model. In particular, a discretization solution algorithm embedded with five heuristic optimization strategies was designed to speed up the planning. Konur and Golias (2013) studied the scheduling of inbound trucks at the inbound doors of a cross-dock facility under truck arrival time uncertainty and formulated this problem as a pessimistic and optimistic discrete bi-level decision problem respectively. They developed a genetic algorithm to solve the bi-level formulations of the pessimistic and the optimistic approaches. Hajibabai, Bai and Ouyang (2014) studied an integrated facility location problem that simultaneously considers traffic routing under congestion and pavement rehabilitation under deterioration and formulated this problem as a nonlinear mixed-integer bi-level program with facility location, freight shipment routing and pavement rehabilitation decisions in the upper level and traffic equilibrium in the lower level.

Researchers have also applied multilevel decision-making techniques to handle traffic and transportation problems under uncertainty. For example, Chiou (2015) developed a bi-level decision support system for a normative road network design with uncertain travel demand in order to simultaneously reduce travel delay to road users and mitigate the vulnerability of the road network. Xu and Gang (2013) investigated a transportation scheduling problem in a large-scale construction project under a fuzzy random environment and formulated this problem as a fuzzy and random bi-level multi-objective optimization model which was solved by a PSO algorithm. Shao et al. (2014) proposed a nonlinear bi-level optimization model to estimate the variation in peak hour origin-destination traffic demand from day-to-day hourly traffic counts throughout the whole year. A heuristic iterative estimation-assignment algorithm for solving the bi-level optimization problem was proposed.

2.4.3 ENERGY MANAGEMENT

Growing environmental concerns have motivated worldwide attention to energy management. Multilevel decision-making techniques have been applied to handle many energy management problems, such as energy transmission and marketing, reducing pollution and promoting cleaner production.

In relation to the transmission and marketing of natural gas, Dempe, Kalashnikov and Ríos -Mercado (2005) presented a mathematical framework for the problem of minimizing the cash-out penalties of a natural gas shipper and modeled the problem as a mixed-integer bi-level decision problem having one Boolean variable in the lower level problem, in which the decision making process for the shipper (leader) is to determine how to carry out its daily imbalances to minimize the penalty that will be imposed by the pipeline (follower). For the sake of justifying the daily imbalance swings made by the gas shipper as result of variations in the selling price of gas, Kalashnikov, Pérez and Kalashnykova (2010) extended the model presented by Dempe, Kalashnikov and Ríos -Mercado (2005) to another bi-level optimization model, in which the upper level objective function includes additional terms that account for the gas shipping company's daily actions with the aim of taking advantage of the price variations. Dempe et al. (2011) also adopted a linear bi-level decision model to describe a natural gas cash-out problem between a natural gas shipping company and a pipeline operator and a penalty function method was developed to solve the model.

To handle marketing problems in electricity markets, Zhang et al. (2009) built a nonlinear bi-level optimization model for a strategic bidding problem in competitive day-ahead electricity markets and developed a PSO algorithm for solving the resulting model. Also, Zhang et al. (2011) presented a general nonlinear bi-level multi-leader one-follower decision model for strategic bidding optimization in day-ahead electricity markets. The resulting model allows each generating company to choose its biddings to maximize its individual profit; while a market operator can find its minimized purchase electricity fare, which is determined by the output power of each unit and the uniform marginal prices. The authors then developed a PSO algorithm to solve the problem. Garcés et al . (2009) presented a bi-level multi-follower decision model for electricity transmission expansion planning within a market environment. The upper-level problem represents the decisions to be made by the transmission planner with the target of deciding transmission investments while maximizing average social welfare and minimizing investment cost. The lower-level problems represent a market clearing for each market scenario and consider known investment decisions. Using duality theory, the proposed bi-level model was recast as a mixed-integer linear programming problem, which was solvable by branch-and-cut solvers. Fernandez-Blanco, Arroyo and Alguacil (2014) discussed an alternative day-ahead auction based on consumer payment minimization for pool-based electricity markets and solved this problem by discrete bi-level optimization. In the upper-level optimization, generation is scheduled with the goal of minimizing the total consumer payment while taking into account the fact that locational marginal prices are determined by a multiperiod optimal power flow in the lower level. Hesamzadeh and Yazdani (2014) proposed a mixed-integer linear bi-level multi-follower decision model for transmission planning in an environment where there is imperfect competition in the electricity supply industry, and the problem was solved using Kuhn-Tucker optimality conditions and a binary mapping approach. Street, Moreira and Arroyo (2014) developed a tri-level decision model for energy reserve scheduling in electricity markets with transmission flow limits and found a solution using a Benders decomposition approach. Fernandez-Blanco, Arroyo and Alguacil (2012) presented a nonlinear mixed-integer bi-level decision-making formulation for alternative market-clearing procedures in restructured power systems that are dependent on market-clearing prices rather than on offers. Taha, Hachem and Panchal (2014) presented a nonlinear bi-level optimization formulation for Quasi-Feed-In-Tariff (QFIT) policy which integrates the physical characteristics of the power-grid, in which the upper-level problem corresponds to the policy makers, whereas the lower-level decisions are made by generation companies.

Multilevel decision-making techniques have been also applied to handle water exchange problems in relation to the consumption of water resources and the generation of waste. Aviso et al. (2010) developed a fuzzy bi-level optimization model to explore the effect of charging fees for the purchase of freshwater and the treatment of wastewater in optimizing the water exchange network of plants in an eco-industrial park (EIP). Tan et al. (2011) extended the optimization model developed by Aviso et al. (2010) to a fuzzy bi-level decision model by modifying the role of the EIP authority to include water regeneration and redistribution via a centralized hub and found a reasonable compromise between the EIP authority's desire to minimize fresh water usage, and the participating companies' desire to minimize costs. Skulovich, Perelman and Ostfeld (2014) presented a discrete bi-level optimization approach for the placement and sizing of closed surge tanks in the water distribution system subjected to transient events. Based on the optimization of comprehensive social, economic, agricultural, environment and groundwater preservation benefits, Guo et al. (2014) presented a bi-level multi-objective optimization model that allocates water resources rationally between all sectors and prevents over-exploitation.

2.4.4 Safety and accident management

Safety and accident management has increasingly attracted concern in relation to man-made disasters such as terrorist attacks and hazmat leakage, and natural disasters such as hurricanes and earthquakes. Multilevel decision-making techniques have been widely applied to assist authorities in making decisions associated with safety and accident management, e.g. electric power network defense, hazmat transportation, pollution abatement and emergency evacuation.

From the aspect of the prevention and defense of man-made and natural disaster, Yao et al. (2007) built a tri-level optimization model for resource allocation in electric power network defense which identifies the most critical network components to defend against possible terrorist attacks, and a decomposition approach was proposed to find an optimal solution to the resulting model. Alguacil, Delgadillo and Arroyo (2014) applied a tri-level decision model to describe an electric grid defense planning problem and solved it using a novel two-stage solution approach. Erkut and Gzara (2008) proposed a discrete bi-level decision model for the problem of network design for hazardous material transportation where the government designates a network and the carriers choose the routes on the network. The authors developed a heuristic solution method that always finds a stable solution. Bianco, Caramia and Giordani (2009) proposed a linear bi-level decision model for a hazmat transportation network design problem which was then transformed into a single-level mixed integer linear program by Kuhn-Tucker conditions for finding an optimal solution. Scaparra and Church (2008) developed a mixed-integer bi-level program for critical infrastructure protection planning in which the upper-level problem involves the decisions about which facilities to fortify to minimize the worst-case efficiency reduction due to the loss of unprotected facilities, whereas worst-case scenario losses are modeled in the lower-level interdiction problem. He, Huang and Lu (2011) presented two mixed integer bi-level decision-making models for integrated municipal solid waste management and greenhouse gas emissions control. Shih et al. (2012) applied nonlinear bi-level programming to determine a subsidy rate for Taiwan's domestic glass recycling industry. Hajinassiry, Amjady and Sharifzadeh (2014) presented a new adaptive discrete bi-level optimization approach to solve a short-term hydrothermal coordination problem with AC (alternating current) network constraints.

To achieve emergency evacuation and provide rapid aid after a catastrophic disaster, Lv et al. (2010) proposed a bi-level optimization model to reduce traffic congestion of the transportation network while evacuating people to safe shelters during disasters or special events, in which the upper level aims to minimize the total evacuation time, while the lower level functions on the basis of user equilibrium assignment. A solution method based on discrete PSO and the Frank-Wolfe algorithm was employed to solve the bi-level optimization problem. Camacho-Vallejo et al. (2014) proposed a linear bi-level decision model for humanitarian logistics to optimize decisions related to the distribution of international aid after a catastrophic disaster. Apivatanagul, Davidson and Nozick (2012) introduced nonlinear bi-level optimization for risk-based regional hurricane evacuation planning where the upper level develops an evacuation plan to minimize both risk and travel time while the lower level is a dynamic user equilibrium traffic assignment model. Ren et al. (2013) proposed a bi-level bi-objective decision model based on the concept of robust optimization for determining flows on emergency evacuation routes and traffic signals at intersections in the presence of uncertain background travel demands. A non-dominated sorting genetic algorithm was employed to determine the Pareto solutions of this optimization problem.

2.5 SUMMARY

Although multilevel decision-making techniques have been the subject of great developments in decision models, solution approaches and practical applications, several challenges still require further research.

(1) A number of solution approaches involving exact algorithms and heuristic algorithms have been developed for solving a variety of bi-level decision problems. Nevertheless, these algorithms are still time-consuming for solving nonlinear and large-scale bi-level decision problems. Also, it is difficult and sometimes almost impossible to extend these algorithms to solve tri-level decision problems.

(2) Although research on tri-level decision problems has increasingly attracted investigations into solution approaches, there are three noticeable drawbacks to

adopting these approaches to solve tri-level decision problems. First, the existing approaches are limited to solving tri-level decision problems in linear format or in a special situation where all decision entities from different levels share the same constraint conditions. Second, the fuzzy approaches for solving tri-level decision problems can only be used to find satisfactory solutions rather than optimal solutions, because cooperation is inhibited in classical multilevel decision-making problems, as has been commented on by Dempe (2011). Lastly, there still lack effective decision models and solution approaches for handling TLMF decision problems.

(3) Although solution approaches have been developed to solve a range of fuzzy bi-level decision problems. However, these solution approaches are limited to handling linear problems with special fuzzy numbers, e.g. triangular fuzzy numbers. In particular, these interactive fuzzy approaches can only solve fuzzy bi-level decision problems in which decision entities from different levels share the same constraint conditions and prefer to cooperate with one another; under this special situation, the fuzzy approaches aim to find satisfactory solutions rather than optimal solutions.

(4) Multilevel decision-making techniques have been widely applied to handle real-world problems. However, the majority of these application research uses basic bi-level decision-making techniques. Since many real-world cases appear in highly complex decision situations, e.g. where there is uncertainty in data or multiple followers are involved, it is necessary to handle these problems using much more practical decision models and solution approaches.

To overcome these issues, this study first develops a PSO algorithm for solving large-scale nonlinear bi-level decision problems; moreover, this algorithm is extended to solve tri-level decision problems. Second, based on fuzzy set theory, this study presents a compromise-based PSO algorithm to solve fuzzy nonlinear bi-level decision problems involving general fuzzy numbers. Third, for the sake of handling TLMF decision problems, different TLMF decision models are proposed to describe various relationships between multiple followers at the same level; also, a TLMF

*K*th-Best algorithm is developed to find an optimal solution. Lastly, this study applies these multilevel decision-making techniques to handle decentralized production and inventory operational problems in SCM.

CHAPTER 3 LARGE-SCALE NONLINEAR MULTILEVEL DECISION MAKING

3.1 INTRODUCTION

The multilevel decision problem is strongly NP-hard and traditional exact algorithmic approaches lack efficiency and universality in solving such problems, thus, heuristics-based PSO algorithms have been used to generate an alternative for solving such problems. However, the existing PSO algorithms are limited to solving linear or small-scale bi-level decision problems.

Nowadays, large-scale and nonlinear features have increasingly appeared in multilevel decision problems. For example, business firms usually work in a decentralized manner in a complex supply chain network, thus, high-dimensional decision variables and nonlinear objectives/constraints are often involved when handling related multilevel decision problems. This chapter first aims to develop a novel PSO algorithm to solve bi-level programs involving nonlinear and large-scale problems, called the bi-level PSO algorithm. It then extends the bi-level PSO algorithm to a tri-level PSO algorithm for solving tri-level decision problems. Lastly, for the sake of exploring the algorithms' performance, the proposed bi-level/tri-level PSO algorithms are applied to solve 62 benchmark problems from references and 810 large-scale problems which are randomly constructed.

This chapter is organized as follows. Following the introduction, Section 3.2 presents the bi-level PSO algorithm for solving bi-level decision problems. In Section 3.3, the tri-level PSO algorithm is developed to solve tri-level decision problems. In

Section 3.4, the proposed bi-level/tri-level PSO algorithms are applied to solve 62 benchmark problems and 810 large-scale problems. A summary is given in Section 3.5.

3.2 Bi-level PSO algorithm

This section first proposes a general bi-level decision problem and presents related solution concepts. Second, a bi-level PSO algorithm is developed for solving the bi-level decision problem.

3.2.1 General bi-level decision problem and solution concepts

The general bi-level decision problem presented by Bard (1998) is defined as follows.

Definition 3.1 (Bard 1998) For $x \in X \in R^p$, $y \in Y \in R^q$, a general bi-level decision problem is defined as:

$$\min_{x \in X} F(x, y) \qquad \text{(Leader)} \qquad (3.1a)$$

$$\text{s.t.} \quad G(x, y) \le 0, \qquad (3.1b)$$

where *y*, for each *x* fixed, solves the follower's problem (3.1c-3.1d)

$$\min_{y \in Y} f(x, y) \qquad \text{(Follower)} \qquad (3.1c)$$

$$\text{s.t.} \quad g(x, y) \le 0, \qquad (3.1d)$$

where *x*, *y* are the decision variables of the leader and the follower respectively; $F, f : R^p \times R^q \to R^1$ are the objective functions of the leader and the follower respectively; $G : R^p \times R^q \to R^m, g : R^p \times R^q \to R^n$ are the constraint conditions of the leader and the follower respectively.

To find an optimal solution for the bi-level decision problem (3.1), solution concepts in relation to operations of the decision-making process are presented as follows.

Definition 3.2 (Bard 1998)

(1) The constraint region of the bi-level decision problem (3.1):

$$S = \{(x, y) \in X \times Y : G(x, y) \le 0, g(x, y) \le 0\}.$$

(2) The feasible set of the follower for each fixed *x*:

$$S(x) = \{y \in Y : g(x, y) \le 0\}.$$

(3) The rational reaction set of the follower:

$$P(x) = \{y \in Y : y \in \arg\min[f(x, y) : y \in S(x)]\}.$$

(4) The inducible region (*IR*) of the bi-level decision problem (3.1):

$$IR = \{(x, y) : (x, y) \in S, y \in P(x)\}.$$

(5) The optimal solution set of the bi-level decision problem (3.1):

$$OS = \{(x, y) : (x, y) \in \arg\min[F(x, y) : (x, y) \in IR]\}.$$

It is clear from Definition 3.2 that the constraint domain associated with a bi-level decision problem is implicitly determined by two optimization problems which must be solved in a predetermined sequence from the leader to the follower (Kalashnikov & Ríos-Mercado 2006). To ensure the bi-level decision problem is well posed in respect to the existence of solutions, the following assumptions based on Definition 3.2 are commonly made.

Assumption 3.1 F, f, G, g are continuous functions, while f, g are continuously differentiable.

Assumption 3.2 f is strictly convex in y for $y \in S(x)$ where $S(x)$ is a compact convex set.

Assumption 3.3 F is continuous convex in x and y.

Under Assumptions 3.1 and 3.2, the rational reaction set of the follower $P(x)$ is a point-to-point map and closed; this implies that the *IR* is compact. Thus, under the Assumption 3.3, solving the bi-level decision problem (3.1) is equivalent to optimizing the leader's continuous function F over the compact set *IR*. It is well known that the solution to such a problem is guaranteed to exist. A bi-level PSO algorithm is thereby developed for the purposes of finding a solution for the bi-level decision problem (3.1).

3.2.2 THE BI-LEVEL PSO ALGORITHM DESCRIPTION

PSO is a category of the population-based heuristic algorithm that is motivated by the social behavior of organisms such as fish schooling and bird flocking. The population of PSO is known as a swarm, while each element in the swarm is termed a particle. In a swarm with the size N, the position vector of each particle with index $i(i=1,2,\cdots,N)$ is denoted as $X_i^t=(x_i^t, y_i^t)$ at iteration t, which represents a potential solution to the problem (3.1). For the sake of convenient discussion, let $X_i^t=(x_i^t, y_i^t)=(x_{i1}^t, x_{i2}^t)$. At iteration t, each particle i moves from X_i^t to X_i^{t+1} in the search space at a velocity $V_i^{t+1}=(v_{i1}^{t+1}, v_{i2}^{t+1})$ along each dimension. Each particle keeps track of its coordinates in hyperspace which are associated with the best solution (fitness), called *pbest* solution ($p_i=(p_{i1}, p_{i2})$), it has achieved so far; while the PSO algorithm is divided into two versions, respectively known as the GBEST version and the LBEST version, due to different definitions of the global best solution (Eberhart & Kennedy 1995). In the GBEST version, the particle swarm optimizer keeps track of the overall best value, called *gbest* solution ($p_g=(p_{g1}, p_{g2})$), and its location obtained thus far by any particle in the population, known as the global neighborhood. For the LBEST version, particles only contain their own and their nearest array neighbors' best information within a local topological neighborhood, rather than that of the entire group. However, in either PSO version, the PSO concept , at each iteration, always consists of an aggregated acceleration of each particle towards its *pbest* and *gbest* position. In this study, the GBEST version of PSO is followed, and in this section, detailed procedures for solving the problem (3.1) are developed based on Definition 3.2.

(1) Initial population

In an initial population of particles with the number *N*, each particle $i(i=1,2,\ldots,N)$ can be represented as $X_i^0=(x_i^0,y_i^0)=(x_{i1}^0,x_{i2}^0)$. As an initial population needs to be randomly constructed for the bi-level PSO algorithm, a random method is proposed to construct an initial population with the size *N*.

First, the required number of the leader's decision variables x_i^0 $(i=1,2,\ldots,N)$ is randomly generate. Second, the existing simplex method or interior point method is adopted (in MATLAB) to solve the follower's decision problem $\min_{y\in Y}\{f(x,y):g(x,y)\le 0\}$ under $x=x_i^0$ and obtain the corresponding solution y_i^0. In this way, the construction of the initial population is completed and $X_i^0=(x_i^0,y_i^0)=(x_{i1}^0,x_{i2}^0)$.

Nevertheless, a number of particles of the initial population may occur outside the constraint region *S* particularly in relation to solving large-scale problems with complex constraints. To ensure many more particles of the initial population occur in the constraint region, another construction method is proposed to supplement part particles to the initial population.

First, obtain two solutions $X^{\min}=(x^{\min},y^{\min})$ and $X^{\max}=(x^{\max},y^{\max})$ respectively for solving the problems $\min\{F(x,y):(x,y)\in S\}$ and $\max\{F(x,y):(x,y)\in S\}$.

Second, a formula is defined to construct the initial population:

$(x_i^0,y_i^0)=(x^{\min},y^{\min})+[(x^{\max}-x^{\min})*r_1,(y^{\max}-y^{\min})*r_2]$, where $i=1,2,\ldots,N$, r_1 and r_2 are random numbers uniformly distributed between 0 and 1.

The second method provides more particles occurring in the constraint region. Even though the particles may be not uniformly distributed throughout the constraint region only using the second method, all particles gathered by both methods will be uniformly distributed in the search space. Consequently, when the bi-level PSO algorithm is performed for solving small-scale problems, the first method can be only used to construct the initial population; whereas both methods mentioned are able to be combined to construct the initial population for solving large-scale problems. Moreover, the percentage of the population generated by the second method should goes up with the increase in the problem size. Although some particles of the initial population still occur outside the constraint region *S* using these above construction methods, the particles will be tugged to return towards the constraint region *S* at the following iterations if there exist better solutions in *S* (Eberhart & Kennedy 1995); this is an advantage of the PSO algorithm in constructing the initial population.

(2) The updating rules of particles

In the bi-level PSO algorithm, each particle *i* moves toward $X_i^{t+1}=(x_i^{t+1},y_i^{t+1})=(x_{i1}^{t+1},x_{i2}^{t+1})$ in the search space at a velocity $V_i^{t+1}=(v_{i1}^{t+1},v_{i2}^{t+1})$ at each iteration t. In this study, the velocity and position of each particle *i* are updated as follows for $j=1,2,i=1,2,\ldots,N$ based on related definitions proposed by Shi and Eberhart (1998):

$$v_{ij}^{t+1} = wv_{ij}^{t} + c_1 r_1 (p_{ij}^{t} - x_{ij}^{t}) + c_2 r_2 (p_{gj}^{t} - x_{ij}^{t}), \tag{3.2}$$

$$x_{ij}^{t+1} = x_{ij}^{t} + v_{ij}^{t+1}. \tag{3.3}$$

The selection of parameters involved in the formula (3.2) need to be determined. For the updating velocity, there are usually maximum and minimum velocity levels $v_{\max}$ and $v_{\min}$. If the current velocity $v_{ij}^{t+1} > v_{\max}$, set $v_{ij}^{t+1} = v_{\max}$; while $v_{ij}^{t+1} = v_{\min}$ if $v_{ij}^{t+1} < v_{\min}$. In the beginning, set $v_{ij}^{0} = v_{\max}$.

w is inertia weight, which controls the impact of the previous velocities on the current velocity. The inclusion of the inertia weight involves two definitions proposed by Shi and Eberhart (1998): a fixed constant and a decreasing function with time. Since some particles of the initial population may occur outside the constraint region *S*, large inertia weight is needed to enhance the search ability of the bi-level PSO algorithm at the beginning of iterations for the sake of tugging such particles to return towards the constraint region *S*. In contrast, small inertia weight can improve the convergence ability of the bi-level PSO algorithm later on in the search for finding a convergent solution quickly. Thus, the decreasing inertia weight with time is used in the bi-level PSO algorithm. The inertia weight is represented as:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{Iter\_\max} * t, \tag{3.4}$$

where $w_{\max}$ and $w_{\min}$ are the upper and lower bounds on the inertia weight, which are determined by the practical problem; *Iter_max* is the maximum number of iterations while *t* represents the current iteration number.

c_1 and c_2 are known as learning factors or acceleration coefficients, which control the maximum step size that the particle can do. A recommended choice for constant c_1 and c_2 is integer 2 as proposed by Kennedy and Eberhart (1995).

r_1 and r_2 are uniform random numbers between 0 and 1.

(3) Fitness evaluation

For each particle i at the iteration t $X_i^t = (x_i^t, y_i^t)$, adopt the existing simplex method or interior point method to solve the problem $\min_{y \in Y}\{f(x, y) : g(x, y) \le 0\}$ under $x = x_i^t$ and obtain the solution (x_i^t, y^*) where $y^* \in P(x_i^t)$. If the solution $(x_i^t, y^*) \in S$, update $X_i^t = (x_i^t, y_i^t) = (x_i^t, y^*)$. Note that $y^* \in P(x_i^t)$ and $(x_i^t, y^*) \in S$ mean $(x_i^t, y^*) \in IR$ based on Definition 3.2, that is, (x_i^t, y^*) is a feasible solution for the bi-level decision problem (3.1). The *pbest* solution is $p_i = (p_{i1}, p_{i2}) = (x_i^t, y_i^t)$, if $F(x_i^t, y_i^t) \le F(p_{i1}, p_{i2})$ where $p_i = (p_{i1}, p_{i2}) = (x_i^0, y_i^0)$ and $F(x_i^0, y_i^0) = +\infty$ are set at the beginning. The global best solution *gbest* of the swarm at the iteration t is $p_g = (p_{g1}, p_{g2})$ where $F(p_{g1}, p_{g2}) = \min\{F(p_{i1}, p_{i2}), i = 1,2,\ldots,N\}$.

(4) Termination criterion

The bi-level PSO algorithm will be terminated after a maximum number of iterations *Iter_max* or when it achieves a maximum CPU time.

(5) Computational procedures of the bi-level PSO algorithm

Based on the theoretical basis proposed above, the complete computational procedures of the bi-level PSO algorithm are presented for solving the bi-level decision problem (3.1).

Algorithm 3.1: Bi-level PSO algorithm

[Begin]

Step 1: *Initialization*.

a) Construct the population size N and generate the initial population of particles $X_i^0 = (x_i^0, y_i^0), i = 1,2,\ldots,N$;

b) Initialize the *pbest* solution as $p_i = (p_{i1}, p_{i2}) = (x_i^0, y_i^0)$ and the fitness $F(x_i^0, y_i^0) = +\infty$;

c) Set the maximum and minimum velocity levels $v_{\max}$ and $v_{\min}$, and initialize $v_{ij}^0 = v_{\max}$;

d) Set the upper and lower bounds on the inertia weight $w_{\max}$ and $w_{\min}$, acceleration coefficients c_1 and c_2, and the maximum iteration number *Iter_max*;

e) Set the current iteration number t=0 and go to Step 2.

Step 2: *Compute the fitness value and update the pbest solution for each particle*. Set i=1 and go to Step 2.1.

Step 2.1: Under $x = x_i^t$, solve the problem $\min_{y \in Y}\{f(x,y) : g(x,y) \le 0\}$ and obtain the solution (x_i^t, y^*). Go to Step 2.2.

Step 2.2: If the solution $(x_i^t, y^*) \in S$, update $X_i^t = (x_i^t, y_i^t) = (x_i^t, y^*)$; otherwise, set $F(x_i^t, y_i^t) = +\infty$. Go to Step 2.3.

Step 2.3: If $F(x_i^t, y_i^t) < F(p_{i1}, p_{i2})$ or $f(x_i^t, y_i^t) < f(p_{i1}, p_{i2})$ under $F(x_i^t, y_i^t) = F(p_{i1}, p_{i2})$, update $p_i = (p_{i1}, p_{i2}) = (x_i^t, y_i^t)$. If $i<N$, set $i=i+1$ and go to Step 2.1; otherwise, go to Step 3.

Step 3: *Update the gbest solution*. Set $p_g = (p_{g1}, p_{g2})$ where $F(p_{g1}, p_{g2}) = \min\{F(p_{i1}, p_{i2}), i = 1,2,\ldots,N\}$. Go to Step 4.

Step 4: *Termination criterion*. If $t<Iter\_max$, go to Step 5; otherwise, stop and $p_g = (p_{g1}, p_{g2})$ is a solution for the bi-level decision problem (3.1).

Step 5: *Update the inertia weight, and the velocity and the position of each particle* by the formulas (3.2), (3.3) and (3.4). If the current velocity $v_{ij}^{t+1} > v_{\max}$, set $v_{ij}^{t+1} = v_{\max}$; while $v_{ij}^{t+1} = v_{\min}$ if $v_{ij}^{t+1} < v_{\min}$. Set t=t+1 and go to Step 2.

[End]

3.3 Tri-level PSO algorithm

In this section, the proposed bi-level PSO algorithm is extended to a tri-level PSO algorithm for solving tri-level decision problems.

3.3.1 General tri-level decision problem and related theoretical properties

The general tri-level decision problem presented by Faísca, Saraiva, et al. (2007) is defined as follows.

Definition 3.3 (Faísca, Saraiva, et al. 2007) For $x \in X \subset R^p$, $y \in Y \subset R^q$, $z \in Z \subset R^r$, a general tri-level decision problem is defined as:

$$\min_{x \in X} f_1(x, y, z) \quad \text{(Leader)} \tag{3.5a}$$

$$\text{s.t.} \quad g_1(x, y, z) \le 0, \tag{3.5b}$$

where (y, z), for each x fixed, solves the problems (3.5c-3.5f)

$$\min_{y \in Y} f_2(x, y, z) \quad \text{(Middle-level follower)} \tag{3.5c}$$

$$\text{s.t.} \quad g_2(x, y, z) \le 0, \tag{3.5d}$$

where z, for each (x, y) fixed, solves the problem (3.5e-3.5f)

$$\min_{y \in Y} f_3(x, y, z) \qquad \text{(Bottom-level follower)} \tag{3.5e}$$

$$\text{s.t.} \quad g_3(x, y, z) \le 0, \tag{3.5f}$$

where x, y, z are the decision variables of the leader, the middle-level follower and the bottom-level follower respectively; $f_1, f_2, f_3 : R^p \times R^q \times R^r \to R$ are the objective functions of the three decision entities respectively; $g_i : R^p \times R^q \times R^r \to R^{k_i}, i = 1,2,3$ are the constraint conditions of the three decision entities respectively.

To find an optimal solution for the tri-level decision problem (3.5), relevant solution concepts in relation to operations of the tri-level decision-making process are presented as follows.

Definition 3.4 (Faísca, Saraiva, et al. 2007)

(1) The constraint region of the tri-level decision problem (3.5):

$$S = \{(x, y, z) \in X \times Y \times Z : g_i(x, y, z) \le 0, i = 1,2,3\}.$$

(2) The feasible set of the middle-level follower for each fixed x:

$$S(x) = \{(y, z) \in Y \times Z : g_2(x, y, z) \le 0, g_3(x, y, z) \le 0\}.$$

(3) The feasible set of the bottom-level follower for each fixed (x, y):

$$S(x, y) = \{z \in Z : g_3(x, y, z) \le 0\}.$$

(4) The rational reaction set of the bottom-level follower:

$$P(x, y) = \{z \in Z : z \in \arg\min[f_3(x, y, z) : z \in S(x, y)]\}.$$

(5) The rational reaction set of the middle-level follower:

$$P(x) = \{(y, z) \in Y \times Z : (y, z) \in \arg\min[f_2(x, y, z) : (y, z) \in S(x), z \in P(x, y)]\}.$$

(6) The inducible region of the tri-level decision problem (3.5):

$$IR = \{(x, y, z) : (x, y, z) \in S, (y, z) \in P(x)\}.$$

(7) The optimal solution set of the tri-level decision problem (3.5):

$$OS = \{(x, y, z) : (x, y, z) \in \arg\min[f_1(x, y, z) : (x, y, z) \in IR]\}.$$

For the sake of developing an effective algorithm to solve the tri-level decision problem (3.5), the geometry of the solution space and related theoretical properties need to be explored. To ensure the problem (3.5) is well posed in respect to the existence of solutions, it is common to make the following assumptions based on Definition 3.4.

Assumption 3.4 $f_1, f_2, f_3, g_1, g_2, g_3$ are continuous functions, whereas f_2, f_3, g_2, g_3 are continuously differentiable.

Assumption 3.5 f_3 is strictly convex in z for $z \in S(x, y)$ where $S(x, y)$ is a compact convex set, while f_2 is strictly convex in (y, z) for $(y, z) \in S(x)$ where $S(x)$ is a compact convex set.

Assumption 3.6 f_1 is continuous convex in x, y, and z.

Under the Assumptions 3.4 and 3.5, the rational reaction sets of the bottom-level follower and the middle-level follower $P(x, y)$ and $P(x)$ are point-to-point maps and closed, which implies that IR is compact. Thus, under the Assumption 3.6, solving the tri-level decision problem (3.5) is equivalent to optimizing the leader's continuous function f_1 over the compact set IR. It is well known that the solution to such a problem is guaranteed to exist.

It is noticeable that, if the bottom-level follower's problem is a convex parametric programming problem that satisfies the Manasarian-Fromowitz constraint qualification (MFCQ) for each fixed (x, y) (Bard 1998; Dempe 2002), the bottom-level follower's problem is equivalent to the following Kuhn-Tucker conditions (3.6-3.9):

$$\nabla_z L(x, y, z, u) = \nabla_z f_3(x, y, z) + u\nabla_z g_3(x, y, z), \tag{3.6}$$

$$ug_3(x, y, z) = 0, \tag{3.7}$$

$$g_3(x, y, z) \leq 0, \tag{3.8}$$

$$u \geq 0, \tag{3.9}$$

where $L(x, y, z, u) = f_3(x, y, z) + ug_3(x, y, z)$ is the Lagrangian function of the bottom-level follower, $\nabla_z L(x, y, z, u)$ denotes the gradient of the function $L(x, y, z, u)$ with respect to *z*, and *u* is the vector of Lagrangian multipliers.

Theorem 3.1 (Dempe 2002) A necessary and sufficient condition that $(y, z) \in P(x)$ is that the row vector u exists such that (*x*, *y*, *z*, *u*) satisfies the Kuhn-Tucker conditions (3.6-3.9).

Based on Theorem 3.1, the tri-level decision problem (3.5) can be transformed into the bi-level decision problem (3.10) by replacing the bottom-level follower's problem with the Kuhn-Tucker conditions (3.6-3.9).

$$\min_x f_1(x, y, z) \qquad \text{(Leader)} \tag{3.10a}$$

$$\text{s.t.} \quad g_1(x, y, z) \leq 0, \tag{3.10b}$$

where (*y*, *z*), for the given *x*, solves the follower's problem (3.10c-3.10h)

$$\min_{y,z,u} f_2(x, y, z) \qquad \text{(Follower)} \tag{3.10c}$$

$$\text{s.t.} \quad g_2(x, y, z) \leq 0, \tag{3.10d}$$

$$\nabla_z f_3(x, y, z) + u\nabla_z g_3(x, y, z) = 0, \tag{3.10e}$$

$$ug_3(x, y, z) = 0, \tag{3.10f}$$

$$g_3(x, y, z) \leq 0, \tag{3.10g}$$

$$u \geq 0. \tag{3.10h}$$

Therefore, the following theorem is obtained.

Theorem 3.2 (x, y, z) solves the tri-level decision problem (3.5) if and only if (x, y, z, u) solves the bi-level decision problem (3.10).

In this study, the bi-level PSO algorithm is extended to a tri-level PSO algorithm for finding a solution (x, y, z) for the tri-level decision problem (3.5) based on Theorems 3.1 and 3.2.

3.3.2 THE TRI-LEVEL PSO ALGORITHM DESCRIPTION

In a swarm with the size N, the position vector of each particle with index i $(i = 1,2,\ldots,N)$ is denoted as $X_i^t = (x_i^t, y_i^t, z_i^t)$ at iteration t, which represents a potential solution to the problem (3.5). For the sake of accessibility, let $X_i^t = (x_i^t, y_i^t, z_i^t) = (x_{i1}^t, x_{i2}^t, x_{i3}^t)$. At iteration t, each particle i moves from X_i^t to X_i^{t+1} in the search space at a velocity $V_i^{t+1} = (v_{i1}^{t+1}, v_{i2}^{t+1}, v_{i3}^{t+1})$ along each dimension. Also, set the *pbest* solution $p_i = (p_{i1}, p_{i2}, p_{i3})$ and *gbest* solution $p_g = (p_{g1}, p_{g2}, p_{g3})$. Based on Theorems 3.1 and 3.2, the tri-level PSO algorithm for solving the tri-level decision problem (3.5) is developed in this section.

1) Initial population

The method of constructing the initial population is similar to the bi-level PSO algorithm. First, the required number of the leader's decision variables x_i^0 $(i = 1,2,\ldots,N)$ is randomly generated. Second, solve the following problem (3.11) under $x = x_i^0$ using the branch and bound algorithm (Bard 1998) or interior point method and obtain the corresponding solution (y_i^0, z_i^0, u_i^0). In this way, the construction of the initial population is completed and $X_i^0 = (x_i^0, y_i^0, z_i^0) = (x_{i1}^0, x_{i2}^0, x_{i3}^0), i = 1,2,\ldots,N$.

$$\min_{y,z,u} f_2(x,y,z) \tag{3.11a}$$

$$\text{s.t.} \quad g_2(x,y,z) \le 0, \tag{3.11b}$$

$$\nabla_z f_3(x,y,z) + u\nabla_z g_3(x,y,z) = 0, \tag{3.11c}$$

$$u g_3(x,y,z) = 0, \tag{3.11d}$$

$$g_3(x,y,z) \le 0, \tag{3.11e}$$

$$u \ge 0. \tag{3.11f}$$

2) The updating rules of particles

Within the tri-level PSO algorithm, each particle *i* moves toward $X_i^{t+1} = (x_i^{t+1}, y_i^{t+1}, z_i^{t+1}) = (x_{i1}^{t+1}, x_{i2}^{t+1}, x_{i3}^{t+1})$ in the search space at a velocity $V_i^{t+1} = (v_{i1}^{t+1}, v_{i2}^{t+1}, v_{i3}^{t+1})$ at each iteration *t*. The velocity and position of each particle *i* are updated as well as the bi-level PSO algorithm developed in Section 3.2.2 by the formulas (3.2), (3.3) and (3.4).

3) Fitness evaluation

For each particle *i* at the iteration *t* $X_i^t = (x_i^t, y_i^t, z_i^t)$, solve the problem (3.11) under $x = x_i^t$ using the branch and bound algorithm (Bard 1998) or interior point method and obtain the solution (x_i^t, y^*, z^*, u^*). If the solution $(x_i^t, y^*, z^*) \in S$, update $X_i^t = (x_i^t, y_i^t, z_i^t) = (x_i^t, y^*, z^*)$. The *pbest* solution is $p_i = (p_{i1}, p_{i2}, p_{i3}) = (x_i^t, y_i^t, z_i^t)$, if $f_1(x_i^t, y_i^t, z_i^t) \le f_1(p_{i1}, p_{i2}, p_{i3})$ where $p_i = (p_{i1}, p_{i2}, p_{i3}) = (x_i^0, y_i^0, z_i^0)$ and $f_1(x_i^0, y_i^0, z_i^0) = +\infty$ are set at the beginning. The global best solution *gbest* of the swarm at the iteration t is $p_g = (p_{g1}, p_{g2}, p_{g3})$ where $f_1(p_{g1}, p_{g2}, p_{g3}) = \min\{f_1(p_{i1}, p_{i2}, p_{i3}), i = 1,2,\ldots,N\}$.

4) Termination criterion

The tri-level PSO algorithm will be terminated after a maximum number of iterations *Iter_max* or when it achieves a maximum CPU time.

5) Computational procedures of the tri-level PSO algorithm

Based on the bi-level PSO algorithm and the theoretical basis proposed above, the complete computational procedures of the tri-level PSO algorithm are presented.

Algorithm 3.2: Tri-level PSO algorithm

[Begin]

Step 1: *Initialization*.

a) Construct the population size N and generate the initial population of particles $X_i^0 = (x_i^0, y_i^0, z_i^0), i = 1,2,\ldots,N$ by solving the problem (3.11);

b) Initialize the *pbest* solution as $p_i = (p_{i1}, p_{i2}, p_{i3}) = (x_i^0, y_i^0, z_i^0)$ and the fitness $f_1(x_i^0, y_i^0, z_i^0) = +\infty$;

c) Set the maximum and minimum velocity levels $v_{\max}$ and $v_{\min}$, and initialize $v_{ij}^0 = v_{\max}$;

d) Set the upper and lower bounds on the inertia weight $w_{\max}$ and $w_{\min}$, acceleration coefficients c_1 and c_2, and the maximum iteration number *Iter_max*;

e) Set the current iteration number t=0 and go to Step 2.

Step 2: *Compute the fitness value and update the pbest solution for each particle*. Set i=1 and go to Step 2.1.

Step 2.1: Under $x = x_i^t$, solve the problem (3.11) using the branch and bound algorithm or interior point method and obtain the solution (x_i^t, y^*, z^*, u^*). Go to Step 2.2.

Step 2.2: If the solution $(x_i^t, y^*, z^*) \in S$, update $X_i^t = (x_i^t, y_i^t, z_i^t) = (x_i^t, y^*, z^*)$; otherwise, set $f_1(x_i^t, y_i^t, z_i^t) = +\infty$. Go to Step 2.3.

Step 2.3: If $f_1(x_i^t, y_i^t, z_i^t) \le f_1(p_{i1}, p_{i2}, p_{i3})$, update $p_i = (p_{i1}, p_{i2}, p_{i3}) = (x_i^t, y_i^t, z_i^t)$. If i<N, set i=i+1 and go to Step 2.1; otherwise, go to Step 3.

Step 3: *Update the gbest solution*. Set $p_g = (p_{g1}, p_{g2}, p_{g3})$ where $f_1(p_{g1}, p_{g2}, p_{g3}) = \min\{f_1(p_{i1}, p_{i2}, p_{i3}), i = 1,2,\ldots,N\}$. Go to Step 4.

Step 4: *Termination criterion*. If t<*Iter_max*, go to Step 5; otherwise, stop and $p_g = (p_{g1}, p_{g2}, p_{g3})$ is a solution for the tri-level decision problem (3.5).

Step 5: *Update the inertia weight, and the velocity and the position of each particle* by the formulas (3.2), (3.3) and (3.4) for $j = 1,2,3, i = 1,2,\ldots,N$. If the current velocity $v_{ij}^{t+1} > v_{\max}$, set $v_{ij}^{t+1} = v_{\max}$; while $v_{ij}^{t+1} = v_{\min}$ if $v_{ij}^{t+1} < v_{\min}$. Set t=t+1 and go to Step 2.

[End]

3.4 COMPUTATIONAL STUDY

A completed computational study is conducted to show the performance of the proposed bi-level/tri-level PSO algorithms. First, the bi-level/tri-level PSO algorithms are applied to solve 25 bi-level and 8 tri-level benchmark problems involving linear and nonlinear versions. Second, the bi-level PSO algorithm is applied to solve 29 large-scale nonlinear bi-level benchmark problems. Lastly, for the sake of exploring the algorithm performance in depth, 810 large-scale bi-level decision problems are generated using the random method proposed by Calvete, Galé and Mateo (2008). It is noticeable that the objective value of the upper level is put in a prior position when comparing the computational results with other algorithms. These computational experiments are operated in MATLAB(2014a) programs performed on a 3.47GHz Inter Xeon W3690 CPU with 12G of RAM under a Red Hat Enterprise Linux Workstation. Also, these large-scale problems are randomly generated using the MATLAB(2014a) environment.

3.4.1 SMALL-SCALE BENCHMARK PROBLEMS

In this section, the bi-level/tri-level PSO algorithms are applied to solve 25 bi-level and 8 tri-level benchmark problems involving linear and nonlinear versions. Moreover, the computational results respectively obtained by the bi-level/tri-level PSO algorithms and other algorithms are compared. The benchmark problems and their related sources are listed in Table 3.1.

To solve the benchmark problems 1-33, related parameters involved in the bi-level/tri-level PSO algorithms are chosen in Table 3.2. Under the parameters in Table 3.2, the PSO algorithms are performed in 20 independent runs on each of the above 33 benchmark problems. The computational results for bi-level decision problems 1-25 are shown in Table 3.3. In Table 3.3, the solution and the corresponding objective values obtained by the bi-level PSO algorithm are respectively denoted by (x^*, y^*) and (F^*, f^*), while the values obtained by other algorithms are respectively denoted by $(\bar{x}, \bar{y})$ and $(\bar{F}, \bar{f})$.

Table 3.1 Benchmark problems and their related sources

Problems	Sources
1-14	Problems 1-14 (Wan, Wang & Sun 2013)
15	Ex 1. (Wan, Mao & Wang 2014)
16	Ex 3. (Wan, Mao & Wang 2014)
17	Ex 5. (Wan, Mao & Wang 2014)
18	Ex 7. (Wan, Mao & Wang 2014)
19-20	Problems 1-2 (Wang, Jiao & Li 2005)
21-25	Problems 5-9 (Wang, Jiao & Li 2005)
26	Example 1 (Bard 1984)
27	The tri-level numerical illustration (Anandalingam 1988) Example 1 (Sinha 2001) The tri-level example (Lai 1996) Example 3 (Shih, Lai & Lee 1996)
28	Example 2 (Sinha 2001) Example 1 (Sinha 2003b) Example 1 (Sinha 2003a) Example 1 (Pramanik & Roy 2007)
29	Example 2 (Sinha 2003a)
30	Example 4.1 (Ruan et al. 2004) Illustrative example 1 (Faísca, Saraiva, et al. 2007)
31	Example 4.2 (Ruan et al. 2004)
32	The numerical example (Zhang et al. 2010)
33	The case study (Zhang et al. 2010)

It can be seen from Table 3.3 that for problems 4, 6-8, 15-16, 18-20 and 24, the solutions obtained by the bi-level PSO algorithm are equal or extremely close to those found by the PSO-CST (particle swarm optimization with chaos searching technique) algorithm (Wan, Wang & Sun 2013) and the evolutionary algorithm in (Wang, Jiao & Li 2005). In terms of problems 1, 3, 5, 9-11, 13-14, 17, 23 and 25, the solutions obtained by the bi-level PSO algorithm are better or much better than those found by the compared algorithms in (Wan, Mao & Wang 2014; Wang, Jiao & Li 2005). In particular for problems 9-11, 23 and 25, the objective values of the leader respectively obtained by the bi-level PSO algorithm and the compared algorithm are extremely close to one another under different solutions, which implies that there exist multiple solutions for problems 9-11, 23 and 25. Under this situation, using the bi-level PSO algorithm can achieve better or much better objective values for the follower than the compared PSO-CST algorithm and evolutionary algorithm.

Table 3.2 Parameters employed in the bi-level/tri-level PSO algorithms for solving problems 1-33

Problems	N	v_{max}	v_{min}	w_{max}	w_{min}	c_1	c_2	*Iter_max*
1	30	1.0	-1.0	0.5	0.01	2.0	2.0	100
2	30	1.0	-1.0	0.5	0.01	2.0	2.0	150
3	20	1.0	-1.0	0.5	0.01	2.0	2.0	60
4	50	1.0	-1.0	0.5	0.01	2.0	2.0	100
5	30	1.0	-1.0	0.5	0.01	2.0	2.0	60
6	50	1.0	-1.0	1.0	0.01	2.0	2.0	150
7	30	1.0	-1.0	0.5	0.01	2.0	2.0	60
8	30	1.0	-1.0	0.5	0.01	2.0	2.0	60
9	60	0.5	-0.5	0.5	0.01	2.0	2.0	100
10	60	1.0	-1.0	0.5	0.01	2.0	2.0	80
11	60	1.0	-1.0	0.5	0.01	2.0	2.0	80
12	40	1.0	-1.0	0.5	0.01	2.0	2.0	60
13	40	1.0	-1.0	0.5	0.01	2.0	2.0	60
14	40	1.0	-1.0	0.5	0.01	2.0	2.0	60
15	50	1.0	-1.0	0.5	0.01	2.0	2.0	150
16	80	1.0	-1.0	1.0	0.01	2.0	2.0	100
17	20	1.0	-1.0	0.5	0.01	2.0	2.0	50
18	30	1.0	-1.0	0.5	0.01	2.0	2.0	60
19	20	1.0	-1.0	0.5	0.01	2.0	2.0	60
20	30	1.0	-1.0	0.5	0.01	2.0	2.0	60
21	50	1.0	-1.0	1.0	0.01	2.0	2.0	150
22	60	0.5	-0.5	0.5	0.01	2.0	2.0	100
23	40	1.0	-1.0	0.5	0.01	2.0	2.0	60
24	40	1.0	-1.0	0.5	0.01	2.0	2.0	60

25	40	1.0	-1.0	0.5	0.01	2.0	2.0	60
26	20	1.0	-1.0	1.0	0.01	2.0	2.0	30
27	20	1.0	-1.0	1.0	0.01	2.0	2.0	30
28	30	1.0	-1.0	1.0	0.01	2.0	2.0	40
29	30	1.5	-1.5	1.0	0.01	2.0	2.0	40
30	20	1.0	-1.0	0.5	0.01	2.0	2.0	30
31	20	1.0	-1.0	0.5	0.01	2.0	2.0	20
32	30	2.0	-2.0	1.0	0.01	2.0	2.0	40
33	30	3.0	-3.0	1.0	0.01	2.0	2.0	40

In relation to problems 2, 12 and 21-22, it seems in Table 3.3 that the solutions found by the bi-level PSO algorithm are worse than those obtained by the compared algorithm. With respect to problem 2, the follower will choose $y=(y_1, y_2, y_3)=(0, 0, 0)$ to achieve an optimal objective value $f=0.4832$ (better than $\bar{f}=2.3641$) in view of $x=(x_1,x_2)=(0.1324, 0.1754)$. Clearly, the solution $\bar{y}=(\bar{y}_1,\bar{y}_2,\bar{y}_3)=(0.6935, 0.7327, 0.2273)$ given by the PSO-CST algorithm in (Wan, Wang & Sun 2013) occurs outside the rational reaction set $P(x)$, which implies that $(\bar{x},\bar{y})=(0.1324, 0.1754, 0.6935, 0.7327, 0.2273)$ is not a feasible solution for problem 2 according to Definition 3.2 in Section 3.2. Similarly, the solution $(\bar{x},\bar{y})=(0.8606, 1.4599, 0.3188)$ given by the evolutionary algorithm in (Wang, Jiao & Li 2005) is not a feasible solution for problem 12, because the follower will choose $y=(y_1, y_2)=(1.5382, 0.2166)$ to achieve an optimal objective value $f=2.4980$ (better than $\bar{f}=2.5621$) in view of $x=0.8606$. Clearly, the algorithms in (Wan, Wang & Sun 2013; Wang, Jiao & Li 2005) cannot find an optimal solution for problems 2 and 12. In addition, the objective values of the leader for problems 21 and 22 are $\bar{F}=-8.9087$ and $\bar{F}=-7.5766$ respectively under the solutions $(\bar{x},\bar{y})=(1.03, 3.097, 2.59, 1.79)$ and $(\bar{x},\bar{y})=(0.27, 0.49, 2.34, 1.036)$, thus, the computational results given in (Wang, Jiao & Li 2005) are wrong and the bi-level

PSO solutions are better than the compared algorithm for problems 21 and 22. In general, the bi-level PSO algorithm performance better than the compared algorithms in (Wan, Wang & Sun 2013; Wang, Jiao & Li 2005) in terms of solving problems 2, 12 and 21-22.

Table 3.4 reports the computational results for tri-level decision problems 26-33. The solution and the corresponding objective values obtained by the tri-level PSO algorithm are respectively denoted by (x^*, y^*, z^*) and (f_1^*, f_2^*, f_3^*), while the values obtained by other solution approaches are denoted by $(\bar{x}, \bar{y}, \bar{z})$ and $(\bar{f}_1, \bar{f}_2, \bar{f}_3)$. Table 3.4 clearly shows that the tri-level PSO algorithm can find the same solutions as the compared approaches or much better solutions. Note $(\bar{x}, \bar{y}^*, \bar{z}^*)$ that $(\bar{y}^*, \bar{z}^*)$ denotes the best reactions of the middle-level follower and the bottom-level follower in the light of $\bar{x}$ determined by the leader. According to Definition 3.4, $(\bar{x}, \bar{y}, \bar{z}) = (\bar{x}, \bar{y}^*, \bar{z}^*)$ under $(\bar{x}, \bar{y}, \bar{z}) \in S$ means the solution $(\bar{x}, \bar{y}, \bar{z}) \in IR$, which implies that $(\bar{x}, \bar{y}, \bar{z})$ is a feasible solution for the tri-level decision problem; otherwise, $(\bar{x}, \bar{y}, \bar{z})$ is not a feasible solution. Thus, it can be seen from Table 3.4 that the solutions $(\bar{x}, \bar{y}, \bar{z})$ in (Bard 1984) for problem 26, in (Lai 1996) for problem 27, in (Pramanik & Roy 2007; Sinha 2003a, 2003b) for problem 28, in (Faísca, Saraiva, et al. 2007) for problem 30 and in (Zhang et al. 2010) for problem 33 are not feasible solutions. In addition, although the solutions $(\bar{x}, \bar{y}, \bar{z})$ in (Anandalingam 1988; Shih, Lai & Lee 1996) for problem 27, in (Pramanik & Roy 2007; Sinha 2001) for problem 28 and in (Sinha 2003a) for problem 29 occurs over *IR*, they can be only considered as local optimal solutions since the tri-level PSO algorithm can find much better solutions for such problems. Thus, the tri-level PSO algorithm provides a better way to solve tri-level decision problems.

Table 3.3 The computational results for bi-level decision problems 1-25

Problems	(x^*, y^*)	(F^*, f^*)	$(\bar{x}, \bar{y})$	$(\bar{F}, \bar{f})$
1	(0, 2, 1.875, 0.9063)	(-18.6787, -1.0156)	(0.3844, 1.6124, 1.8690, 0.8041)	(-14.7772, -0.2316)
2	(0, 0.9, 0, 0.6, 0.4)	(-29.2, 3.2)	(0.1324, 0.1754, 0.6935, 0.7327, 0.2273)	(-29.2064, 2.3641)
3	(0, 1, 0)	(1000, 1)	(0.1511, 0.6256, 0.369)	(640.7139, 0.9946)
4	(9.9998, 9.9998)	(99.996, 0)	(10.0020, 9.9961)	(100.0393, 0)
5	(2.0345, 0.8838, 0)	(-1.2312, 7.7818)	(1.8602, 0.9073, 0.005)	(-1.1660, 7.4441)
6	(7.0696, 7.0696, 6.9279, 6.9278)	(1.98, -1.98)	(7.0321, 6.84204, 5.9071, 6.8312)	(1.9816, -1.9816)
7	(20.0282, 14.8381, 0.0282, -5.1619)	(0, 0)	(17.5039, 29.8906, -2.4994, 9.8894)	(0.0527, 0)
8	(17.8377, 20.1712, -2.1623, 0.1712)	(0, 0)	(12.4124, 19.3109, -7.5859, -0.6899)	(0.0004, 0)
9	(20, 5, 10, 5)	(0, 100)	(17.2024, 7.4665, 7.2189, 2.4251)	(0.0075, 125.0854)
10	(10.9317, 9.6004, 10, 9.6004)	(0, 0.868)	(0.1946, 14.9870, 6.1019, 7.9628)	(0, 84.2367)
11	(6.4462, 11.9941, 6.4462, 10)	(0, 3.9763)	(10.6084, 10.0550, 9.4545, 5.1257)	(0.0001, 25.6292)
12	(1.8888, 0.889, 0)	(0, 7.6167)	(0.8606, 1.4599, 0.3138)	(0.0082, 2.5621)
13	(0.6648, 1.5746, 0.0722)	(0, 2.5)	(0.9099, 1.5294, 0.1762)	(0.0374, 2.6969)
14	(0.6648, 1.5746, 0.0722)	(0, 2.5)	(0.9233, 1.5083, 0.1899)	(0.0337, 2.7442)
15	(4, 15, 9.2, 2)	(41.2, -9.2)	(4.000517, 14.999931, 9.199862, 2)	(41.199207, -9.198828)
16	(0, 30, -10, 10)	(0, 100)	(0, 30, -10, 10)	(0, 100)
17	(1, 0)	(1, 0)	(10, 0)	(82, 0)
18	(0, 30, -10, 10)	(0, 100)	(0, 30, -10, 10)	(0, 100)

19	(20, 5, 10, 5)	(225, 100)	(20, 5, 10, 5)	(225, 100)
20	(0, 30, -10, 10)	(0, 100)	(0, 30, -10, 10)	(0, 100)
21	(1.0312, 3.0978, 2.597, 1.7929)	(-8.9172, -6.136)	(1.03, 3.097, 2.59, 1.79)	(-8.92, -6.14)
22	(0.281, 0.4754, 2.3437, 1.0328)	(-7.5774, -0.5777)	(0.27, 0.49, 2.34, 1.036)	(-7.58, -0.574)
23	(38.0907, 60.5204, 2.9985, 2.9985)	(-11.9985, -219.2618)	(12.47, 67.511, 2.999, 2.999)	(-11.999, -163.42)
24	(2, 0, 2, 0)	(-3.6, -2)	(2, -2.84e-8, 2, 0)	(-3.6, -2)
25	(-0.4009, 0.8023, 1.9998, 0)	(-3.9194, -2.0109)	(-0.381, 0.8095, 2, 0)	(-3.92, -2)

Table 3.4 The computational results for tri-level decision problems 26-33

Problems	(x^*, y^*, z^*)	(f_1^*, f_2^*, f_3^*)	$(\bar{x}, \bar{y}, \bar{z})$	$(\bar{f}_1, \bar{f}_2, \bar{f}_3)$	$(\bar{x}, \bar{y}^*, \bar{z}^*)$
26	(6.6667, 8, 0)	(-10.6667, -8, 0)	(4.6667, 1, 0) (Bard 1984)	(-16.6667, -1, 0) (Bard 1984)	(4.6667, 6.5, 4.5)
27	(1.5, 0, 0.5)	(8.5, 0, 0.5)	(0.5, 1, 0.5) (Anandalingam 1988)	(4.5,1,0.5) (Anandalingam 1988)	(0.5, 1, 0.5)
			(1.5, 0, 0.5) (Sinha 2001)	(8.5, 0, 0.5) (Sinha 2001)	(1.5, 0, 0.5)
			(1.66, 1, 0.34) (Lai 1996)	(13.26, 1, 0.34) (Lai 1996)	No solution
			(0.92, 0.58, 0.5) (Shih, Lai & Lee 1996)	(6.18, 0.58, 0.5) (Shih, Lai & Lee 1996)	(0.92, 0.58, 0.5)
28	(2.3329, 0.0006,0.3335, 0)	(14.9979, 1.0012, 5.0)	(0.86, 1.86, 0, 0.71) (Sinha 2001)	(13, 4.7, 4.29) (Sinha 2001)	(0.86, 1.86, 0, 0.71)
			(1.59,1.08,0.62,0.06) (Sinha 2003a, 2003b)	(12.01,3.18,4.94) (Sinha 2003a, 2003b)	(1.59, 1.08, 0.705, 0)
			(1.106, 1.525, 0, 0.631) (Pramanik & Roy 2007)	(13.58, 4.05, 4.37) (Pramanik & Roy 2007)	(1.106, 1.525, 0.581, 0.244)
			(0.857, 1.857, 0, 0.714) (Pramanik & Roy 2007)	(13, 4.71, 4.28) (Pramanik & Roy 2007)	(0.857, 1.857, 0, 0.714)
29	(1, 2, 0, 2, 0)	(14, 2, 8)	(2, 1.99, 1.004, 0, 0.009) (Sinha 2003a)	(12.964,5.001,10.188) (Sinha 2003a)	(2, 1.99, 1.01, 0, 0.02)
30	(0.5, 1, 1)	(4.5, -2, 1)	(0.5, 1, 1) (Ruan et al. 2004)	(4.5, -2, 1) (Ruan et al. 2004)	(0.5, 1, 1)
			(1, 0.5, 1) (Faísca, Saraiva, et al. 2007)	(5, -2, 1) (Faísca, Saraiva, et al. 2007)	(1, 1, 0.5)
31	$(x^* \leq 2{,}0{,}0)$	(0, 0, 0)	$(\bar{x} \leq 2{,}0{,}0)$ (Ruan et al. 2004)	(0, 0, 0) (Ruan et al. 2004)	$(\bar{x} \leq 2{,}0{,}0)$
32	(4, 6, 0)	(-20, 10, -8)	(4, 6, 0) (Zhang et al. 2010)	(-20, 10, -8) (Zhang et al. 2010)	(4, 6, 0)
33	No solution	---	(10, 28.33, 11.66) (Zhang et al. 2010)	(146.6667,176.6,343.3) (Zhang et al. 2010)	Unbounded solution

3.4.2 LARGE-SCALE BENCHMARK PROBLEMS

In this section, the bi-level PSO algorithm is applied to solve the large-scale nonlinear bi-level decision problems 34-62. The sources of the benchmark problems 34-57 are the problems SMD1-SMD12 with five and 10 dimensions constructed by Sinha, Malo and Deb (2014), while the problems 58-62 with 20 dimensions are cited from the problems (Exs.12-16) solved in (Wan, Mao & Wang 2014). When solving the problems 34-57, the population size and iteration number are chosen as N=30, $Iter\_max$=60 and N=50, $Iter\_max$=100 respectively for solving five-dimensional and 10-dimensional problems. The other parameters in the bi-level PSO algorithm are chosen as follows: $v_{\max}$=1.0, $v_{\min}$=-1.0, c_1=c_2=2, $w_{\max}$=0.5, $w_{\min}$=0.01. In response to solving problems 58-62, the related parameters are chosen as $v_{\max}$=0.5, $v_{\min}$=-0.5, c_1=c_2=2, $w_{\max}$=0.5, $w_{\min}$=0.01, N=30, $Iter\_max$=100.

The computational results for the problems 34-45 and problems 46-57 are respectively provided in Tables 3.5 and 3.6. In Tables 3.5 and 3.6, the solution and the corresponding objective values obtained by the bi-level PSO algorithm are respectively denoted by (x^*, y^*) and (F^*, f^*), while the objective values obtained by the nested bi-level evolutionary algorithm developed in (Sinha, Malo & Deb 2014) are denoted by $(\bar{F}, \bar{f})$. Let (F, f) be the objective values under the exact solution. $\Delta(F^*, f^*) = (|F^* - F|, |f^* - f|)$ and $\Delta(\bar{F}, \bar{f}) = (|\bar{F} - F|, |\bar{f} - f|)$ are adopted to reflect the accuracy of the solution respectively obtained by both of the algorithms. The smaller number of $\Delta(F^*, f^*)$ and $\Delta(\bar{F}, \bar{f})$ means the higher accuracy of the solution obtained. It can be seen from Tables 3.5 and 3.6 that the bi-level PSO algorithm is able to find a more accurate solution than the nested bi-level evolutionary algorithm.

Table 3.5 The computational results for five-dimensional test problems 34(SMD1) - 45(SMD12)

Problems	(x^*, y^*)	(F^*, f^*)	$\Delta(F^*, f^*)$	$\Delta(\bar{F}, \bar{f})$
34 (SMD1)	(0, 0, 0, 0, 0)	(0, 0)	(0, 0)	(0.000114, 0.000087)
35 (SMD2)	(-9.1024e-11, 1.3609e-10, -6.4516e-09, 1.0)	(-9.3916e-17, 9.3952e-17)	(9.3916e-17, 9.3952e-17)	(0.000073, 0.000016)
36 (SMD3)	(0, 0, 0, 0, 0)	(0, 0)	(0, 0)	(0.000054, 0.000055)
37 (SMD4)	(-6.3714e-06, 2.7123e-06, -1.2107e-08, 1.3537e-08, 4.1916e-04)	(-1.7330e-07, 1.7339e-07)	(1.7330e-07, 1.7339e-07)	(0.000023, 0.000057)
38 (SMD5)	(-6.0842e-08, -3.3604e-06, 1.0, 1.0, 0.0039)	(-1.2665e-10, 1.3795e-10)	(1.2665e-10, 1.3795e-10)	(0.000002, 0.000009)
39 (SMD6)	(-3.4925e-08, 2.5489e-05, 4.4706e-06, 4.4706e-06, 2.5474e-05)	(6.8968e-10, 1.4570e-15)	(6.8968e-10, 1.4570e-15)	(0.000108, 0.000061)
40 (SMD7)	(3.8445e-09, -1.1977e-11, -6.4516e-09, -6.4516e-09, 1.0)	(-9.3943e-17, 9.3943e-17)	(9.3943e-17, 9.3943e-17)	(0.000016, 0.000177)
41 (SMD8)	(4.2671e-11, 2.4561e-09, 1.0, 1.0, 0.0233)	(8.8549e-12, 2.0450e-10)	(8.8549e-12, 2.0450e-10)	(0.000174, 0.000027)
42 (SMD9)	(3.2473e-05, -2.7497e-07, -1.6235e-04, -1.6235e-04, 5.1978e-04)	(-3.2198e-07, 3.2409e-07)	(3.2198e-07, 3.2409e-07)	(0.000017, 0.000054)
43 (SMD10)	(1.0, 1.0, 1.0, 1.0, 0.7854)	(4.0, 3.0)	(0, 0)	(0.034759, 0.018510)
44 (SMD11)	(8.2462e-06, -6.2919e-04, 1.0379e-07, 1.0379e-07, 2.7166)	(-1.0, 1.0)	(0, 0)	(0.0131643, 0.129893)
45 (SMD12)	(1.0, 1.0, 1.0, 1.0, 0.7849)	(4.9990, 3.0)	(0.001, 0)	(0.032372, 0.000206)

Table 3.6 The computational results for 10-dimensional test problems 46(SMD1) - 57(SMD12)

Problems	(x^*, y^*)	(F^*, f^*)	$\Delta(F^*, f^*)$	$\Delta(\bar{F}, \bar{f})$
46 (SMD1)	(0, 0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0)	(0, 0)	(0.000332, 0.000018)
47 (SMD2)	(1.6899e-08, -3.0972e-08, 8.2156e-08, -2.1181e-07, 2.5143e-07, -6.4507e-09, -6.4507e-09, -6.4507e-09, 1.0, 1.0)	(1.1593e-13, 8.1410e-15)	(1.1593e-13, 8.1410e-15)	(0.000066, 0.000011)
48 (SMD3)	(1.4331e-06, -1.0599e-06, -1.4075e-06, -4.3816e-07, 3.8293e-06, -3.7340e-09, -3.7340e-09, 5.1703e-09, 1.3951e-08, 1.3951e-08)	(2.0014e-11, 5.1590e-12)	(2.0014e-11, 5.1590e-12)	(0.000359, 0.000033)
49 (SMD4)	(2.3924e-07, 4.5629e-08, 3.7434e-07, 1.0472e-06, 1.6518e-07, -1.4097e-08, -1.4097e-08, -1.4097e-08, 3.1005e-04, 3.0963e-04)	(-1.9119e-07, 1.9119e-07)	(1.9119e-07, 1.9119e-07)	(0.000286, 0.000027)
50 (SMD5)	(2.4397e-05, -3.1364e-06, -5.7256e-06, -2.1074e-05, -4.7159e-06, 1.0, 1.0, 1.0, 0.0040, 0.0038)	(9.9401e-10, 7.4795e-10)	(9.9401e-10, 7.4795e-10)	(0.000052, 0.000009)
51 (SMD6)	(-8.7492e-06, -7.0808e-06, 7.6839e-05, 4.9603e-05, 5.0376e-05, 1.6532e-08, 5.3412e-05, 5.3412e-05, 4.9566e-05, 5.0338e-05)	(1.6735e-08, 6.0309e-09)	(1.6735e-08, 6.0309e-09)	(0.001435, 0.000082)
52 (SMD7)	(-1.9409e-09, 1.4642e-08, 1.4642e-08, -7.3262e-09, -7.1216e-09, -6.4688e-09, -6.4688e-09, 1.1635e-04, 1.0, 1.0)	(-2.6032e-08, 1.0577e-16)	(2.6032e-08, 1.0577e-16)	(0.006263, 0.000127)
53 (SMD8)	(2.3591e-07, 4.3256e-05, 1.5413e-06, 1.2043e-07, 2.3549e-06, 1.0, 1.0, 1.0, 0.0320, 0.0324)	(9.9992e-05, 4.5035e-05)	(9.9992e-05, 4.5035e-05)	(0.003122, 0.000157)
54 (SMD9)	(0.0012, 3.6938e-04, 3.2828e-05, -2.9128e-04, -3.1246e-04, -9.4424e-04, -9.0683e-04, -9.3510e-04, 0.0055, -0.0157)	(-2.7495e-04, 2.7829e-04)	(2.7495e-04, 2.7829e-04)	---
55 (SMD10)	(0.50, 0.50, 0.50, 0.50, 0.50, 0.50, 0.50, 0.50, 0.4636, 0.4636)	(12.0, 7.50)	(0, 0)	---
56 (SMD11)	(-1.8430e-06, 4.6479e-08, -6.4905e-07, -3.6628e-07, 1.6103e-06, 6.6159e-08, 6.6159e-08, 6.6159e-08, 2.0281, 2.0281)	(-1.0, 1.0)	(0, 0)	---
57 (SMD12)	---	---	---	---

Table 3.7 The computational results for 20-dimensional test problems 58-62

Problems	(x^*, y^*)	(F^*, f^*)	$(\bar{x}, \bar{y})$	$(\bar{F}, \bar{f})$
58	(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)	(0,1)	(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)	(0,1)
59	(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)	(0,1)	(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)	(0,1)
60	(-0.0859,-0.6008,0.2454,-0.1412,-1.2518, -0.1978,-0.9856,-0.1297,-0.7022,-5, 0,0,0,0,0,0,0,0,0,0)	(1.2388e-6,1)	(1.149034,0.08833383,1.254797,1.182997, 2.130051, 1.742112,0.3082794, 1.591319,1.409942,-0.2195419, 0,0,0,0,0,0,0,0,0,0)	(4.64e-6,1)
61	(0.8882,0.7552,5,4.3309,-0.5017,0.8485, -1.2183,2.2813,-1.5316,0.6639, 0,0,0,0,0,0,0,0,0,0)	(1.7316e-7,1)	(-1.275612,0.4240169,-1.292204,-0.57017, 1.238698,2.83057,1.313386, 0.65589,-2.799304,-1.467915)	(1.12e-5,1)
62	(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)	(0,1)	(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0)	(0,1)

Table 3.7 displays the computational results for problems 58-62 with 20 dimensions. As shown in Table 3.7, the solutions found by the bi-level PSO algorithm are equal to those obtained by the compared algorithm in (Wan, Mao & Wang 2014) for problems 58-59 and 62. With regard to problems 60-61, the bi-level PSO algorithm can achieve little better in terms of objective values. Also, it can be found from Table 3.7 that problems 60-61 have multiple solutions that can achieve objective values extremely close to each other. To conclude, the results indicate that the bi-level PSO algorithm can find the same solutions as the compared algorithm or better solutions for 20-dimensional nonlinear bi-level problems.

3.4.3 ASSESSING THE EFFICIENCY PERFORMANCE OF THE BI-LEVEL PSO ALGORITHM

This section aims to assess the efficiency performance of the proposed bi-level PSO algorithm in relation to solve large-scale problems. In Section 3.4.2, the related parameters employed in the bi-level PSO algorithm are provided for solving large-scale nonlinear benchmark problems. Much less iterations need to be executed by the bi-level PSO algorithm than the evolutionary algorithm (Sinha, Malo & Deb 2014) that needs 330 and 678 iterations at least respectively for solving five-dimensional and 10-dimensional problems. Clearly, the bi-level PSO algorithm has a better convergence and efficiency performance than the evolutionary algorithm in solving large-scale nonlinear bi-level decision problems. However, the increase in the number of decision variables (e.g. more than 20 dimensions) may result in bi-level problems having no solutions apart from some special versions (Sinha, Malo & Deb 2014); thus, there are not sufficient benchmark nonlinear problems in the existing research that can be used to explore the algorithm efficiency. In this study, to explore the algorithm efficiency in solving much larger-scale (e.g. 20 dimensions or much more) problems, the bi-level PSO algorithm is applied to solve sufficient large-scale (40, 60 and 100 dimensions) linear bi-level problems that can be randomly generated.

Sufficient large-scale linear bi-level decision problems are randomly generated using the method proposed by Calvete, Galé and Mateo (2008). The problems are constructed by the following formulation format:

$$\min_{x \geq 0} F(x, y) = c_1 x + d_1 y \qquad \text{(Leader)}$$

$$\min_{y \geq 0} f(x, y) = c_2 x + d_2 y \qquad \text{(Follower)}$$

$$\text{s.t.} \quad Ax + By \leq b\,.$$

Table 3.8 Test problem dimensions

G_1: n=40			G_2: n=60			G_3: n=100		
n_1	n_2	m	n_1	n_2	m	n_1	n_2	m
28	12	12	42	18	18	70	30	30
28	12	20	42	18	30	70	30	50
28	12	32	42	18	48	70	30	80
20	20	12	30	30	18	50	50	30
20	20	20	30	30	30	50	50	50
20	20	32	3	30	48	50	50	80
8	32	12	12	48	18	20	80	30
8	32	20	12	48	30	20	80	50
8	32	32	12	48	48	20	80	80

The objective functions' coefficients (c_1, d_1, c_2, d_2) of both decision entities are randomly generated from the uniform distribution on [-10, 10]. For the sake of ensuring the problem is well posed, the coefficients of one constraint condition are chosen from uniform random numbers between 0 and 10, whereas the remainder elements of the coefficient matrix are uniformly distributed between -10 and 10. The right-hand side of each constraint condition is the sum of the absolute value of the coefficients in the constraint condition. According to the construction method by Calvete, Galé and Mateo (2008), the test problems are classified into three groups (G_1, G_2 and G_3) by the number n of decision variables of the bi-level decision problem, shown in Table 3.8. n_1 and n_2 respectively denote the number of decision variables of the leader and the follower, while m denotes the number of constraint conditions of the bi-level problem. It can be seen from Table 3.8 that there are nine problem types in each test problem group by different combinations of n_1, n_2 and m. In this computational study, 30 test problems are randomly constructed within each problem type; thus, there are $30 \times 9 \times 3 = 810$ bi-level problems randomly generated in total within three test problem groups.

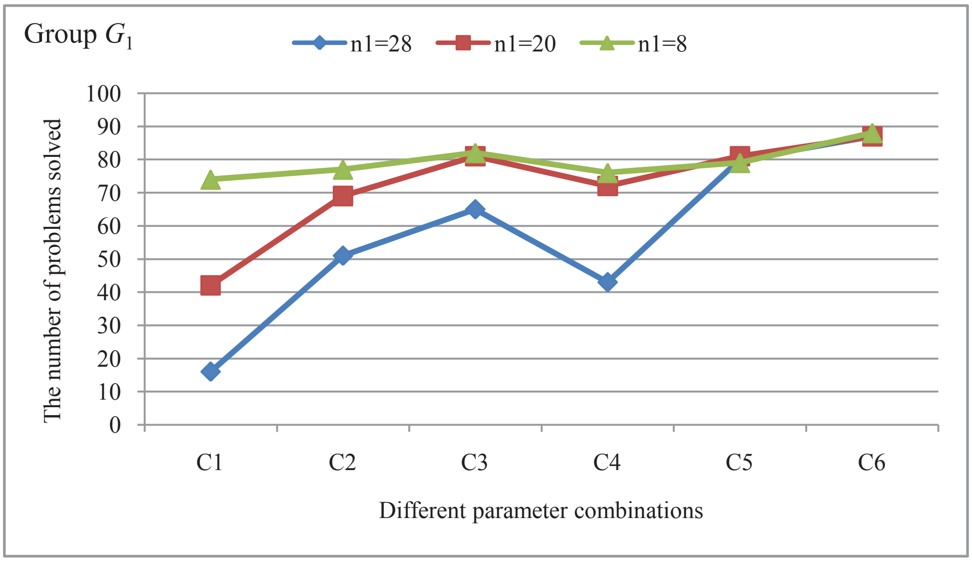
Within the bi-level PSO algorithm, the key parameters involve the inertia weight w, the population size N and the maximum number of iterations *Iter_max*. To explore the influence of the three parameters on the performance of the bi-level PSO algorithm, each test problem is solved under six kinds of parameter combinations of the bi-level PSO algorithm, which involve C_1 (w_{max}=1.0, N=100, *Iter_max*=300), C_2 (w_{max}=0.75, N=100, *Iter_max*=300), C_3 (w_{max}=0.50, N=100, *Iter_max*=300) , C_4 (w_{max}=1.0, N=50, *Iter_max*=500), C_5 (w_{max}=0.75, N=50, *Iter_max*=500) and C_6 (w_{max}=0.50, N=50, *Iter_max*=500). In addition, other parameters within the bi-level PSO algorithm are set as follows: v_{max}=5.0, v_{min}=-5.0, w_{min}=0.01, c_1=c_2=2. For 810 test problems, each of them is carried out 16 runs under each parameter combination. In terms of each test problem, $F^{\min}$ is defined as the best objective value of the leader obtained from all parameter combinations; if the best objective value F of the leader obtained from 16 runs under each parameter combination equals to $F^{\min}$, it can be considered that the bi-level PSO algorithm can find a solution for the test problem under the parameter combination. Table 3.9 displays the number of test problems successfully solved under each parameter combination.

Table 3.9 The number of test problems successfully solved under each parameter combination

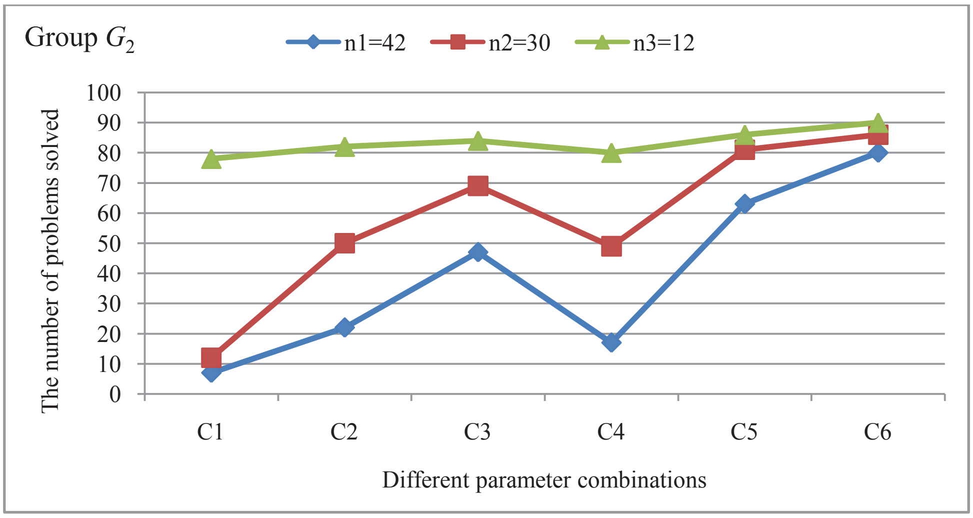
G_1: n=40							G_2: n=60							G_3: n=100						
n_1-n_2-m	C_1	C_2	C_3	C_4	C_5	C_6	n_1-n_2-m	C_1	C_2	C_3	C_4	C_5	C_6	n_1-n_2-m	C_1	C_2	C_3	C_4	C_5	C_6
28-12-12	5	17	23	18	27	28	42-18-18	3	10	15	7	20	26	70-30-30	8	9	17	8	16	25
28-12-20	3	16	20	13	27	29	42-18-30	2	7	18	6	22	26	70-30-50	10	13	22	6	18	23
28-12-32	8	18	22	12	26	30	42-18-48	2	5	14	4	21	28	70-30-80	11	15	22	9	19	24
20-20-12	13	22	25	21	24	29	30-30-18	5	14	19	17	26	27	50-50-30	1	7	17	7	16	26
20-20-20	13	21	27	23	27	28	30-30-30	2	16	24	17	27	29	50-50-50	2	4	15	7	18	27
20-20-32	16	26	29	28	30	30	30-30-48	5	20	26	15	28	30	50-50-80	6	12	17	11	25	25
8-32-12	19	20	23	20	21	29	12-48-18	23	25	26	22	27	30	20-80-30	10	18	22	14	21	28
8-32-20	26	27	29	27	28	29	12-48-30	28	28	29	28	29	30	20-80-50	10	25	28	18	26	29
8-32-32	29	30	30	29	30	30	12-48-48	27	29	29	30	30	30	20-80-80	13	23	29	21	28	30
Total	132	197	228	191	240	262	Total	97	154	200	146	230	256	Total	71	126	189	101	187	237

Table 3.9 clearly shows that different combinations of the inertia weight *w*, the population size *N* and the maximum number of iterations *Iter_max* have significant influences on the performance of the bi-level PSO algorithm. As shown in Table 3.9, most test problems are successfully solved under the parameter combination C_6 within each problem group, which means that the bi-level PSO algorithm shows higher performance under C_6 than other parameter combinations. However, the algorithm performance under C_1 - C_5 becomes more and more close to that under C_6 following the decline of the number n_1 of the leader's decision variables, in particular in groups G_1 and G_2; Figure 3.1 clearly presents these results, which display the total number of test problems that have the same number of the leader's decision variables successfully solved under C_1 - C_6. Also, it is clear in Figure 3.1 that the algorithm performance under each parameter combination experiences a noticeable upward trend along with a decrease in the number n_1 of the leader's decision variables within groups G_1 and G_2. In terms of problem group G_3, it is noticeable that the number of test problems with n_1=70 successfully solved exceeds that with n_1=50, which implies that the increase in the population size of the bi-level PSO algorithm is able to improve its performance in solving these problems when much more decision variables of the leader are involved.

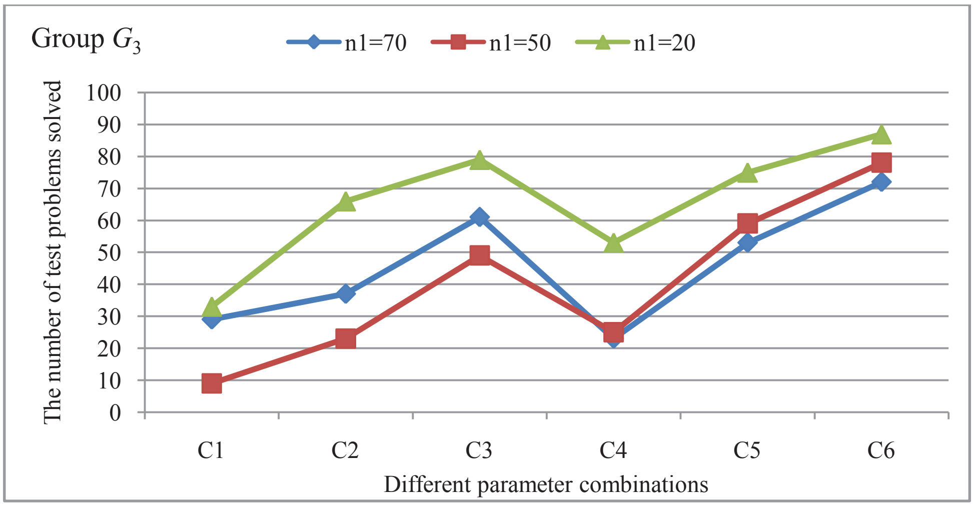
To explore more in depth, the algorithm efficiency of the bi-level PSO algorithm is compared with that of the genetic algorithm based on bases (GABB) developed by Calvete, Galé and Mateo (2008) for solving these test problems randomly constructed. The convergent CPU time and the total CPU time of all iterations completed for both algorithms are examined in Table 3.10. Table 3.10 shows the average of the convergent CPU time (in seconds) and the total CPU time (in seconds) for each problem type using both algorithms. Note that the computational results of the bi-level PSO algorithm are obtained under the parameter combination C_6, while the GABB is performed under its best related parameter combination presented by Calvete, Galé and Mateo (2008).



(a) Group G_1



(b) Group G_2



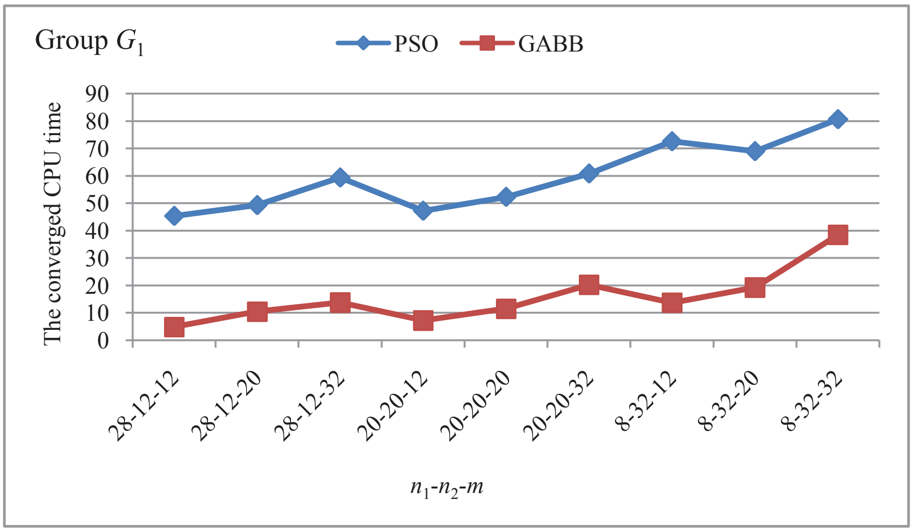
(c) Group G_3

Figure 3.1 The performance of the bi-level PSO algorithm following different parameter combinations

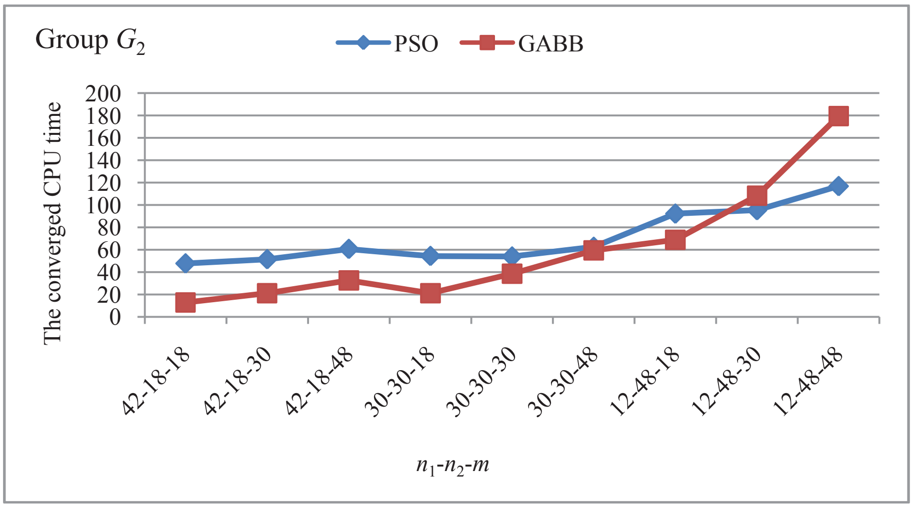
Table 3.10 The computational results respectively obtained by the bi-level PSO algorithm and GABB

Test Problems		PSO			GABB	
Group	n_1-n_2-m	Convergent iteration number	Convergent time (s)	Total time (s)	Convergent time (s)	Total time (s)
G_1	28-12-12	356.03	45.34	74.22	4.92	27.32
	28-12-20	376.38	49.28	64.48	10.49	33.65
	28-12-32	344.50	59.35	84.90	13.85	41.26
	20-20-12	360.97	47.21	66.92	7.31	39.25
	20-20-20	365.39	52.28	72.57	11.58	51.36
	20-20-32	353.80	60.82	87.52	20.29	67.35
	8-32-12	265.48	72.58	136.15	13.75	72.10
	8-32-20	240.28	68.97	137.76	19.34	82.65
	8-32-32	237.50	80.64	169.46	38.52	110.65
G_2	42-18-18	365.38	47.54	63.58	12.68	56.38
	42-18-30	370.81	51.27	69.68	20.96	71.84
	42-18-48	397.96	60.49	74.36	32.29	95.43
	30-30-18	366.52	54.18	73.87	21.03	105.57
	30-30-30	368.03	53.69	73.09	38.35	128.64
	30-30-48	383.47	62.36	81.26	59.31	169.91
	12-48-18	351.53	92.03	129.38	68.62	284.05
	12-48-30	358.03	95.11	135.48	108.24	361.54

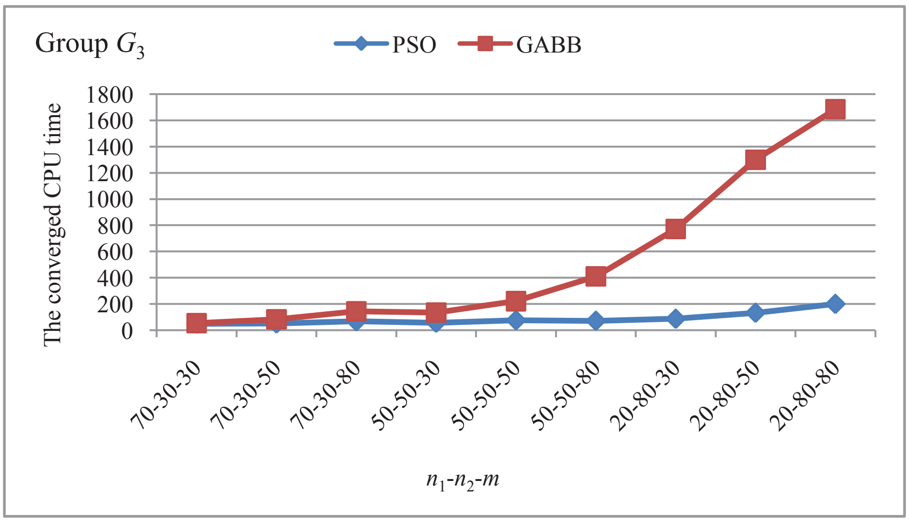
	12-48-48	324.50	116.66	180.28	179.35	440.72
G_3	70-30-30	318.96	48.83	76.69	54.69	115.20
	70-30-50	328.52	51.32	78.05	83.61	159.61
	70-30-80	337.29	67.33	99.80	145.37	231.49
	50-50-30	368.65	56.69	77.16	134.29	331.85
	50-50-50	340.52	75.70	107.85	221.89	428.96
	50-50-80	344.32	71.54	103.76	410.36	630.87
	20-80-30	339.29	86.84	128.72	771.59	1652.19
	20-80-50	396.31	132.91	168.76	1299.76	2238.47
	20-80-80	427.47	200.41	234.34	1684.53	2489.63



(a) Group G_1

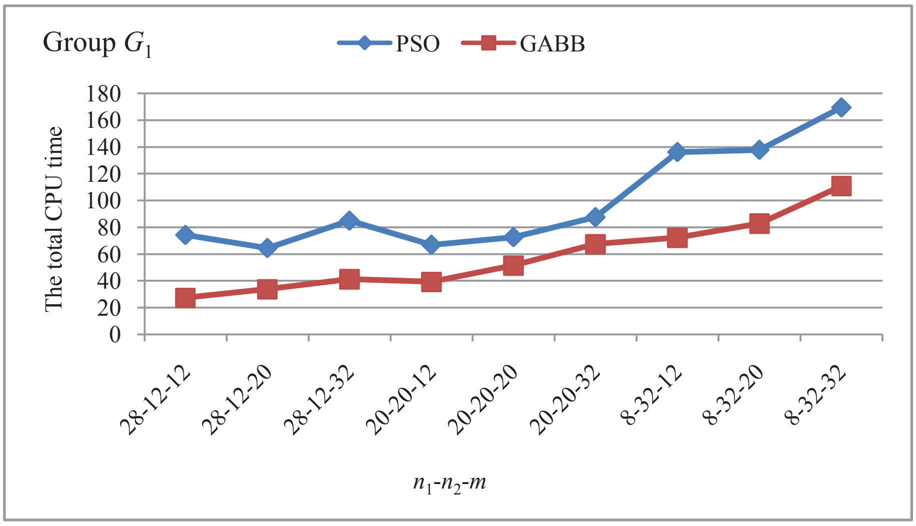


(b) Group G_2

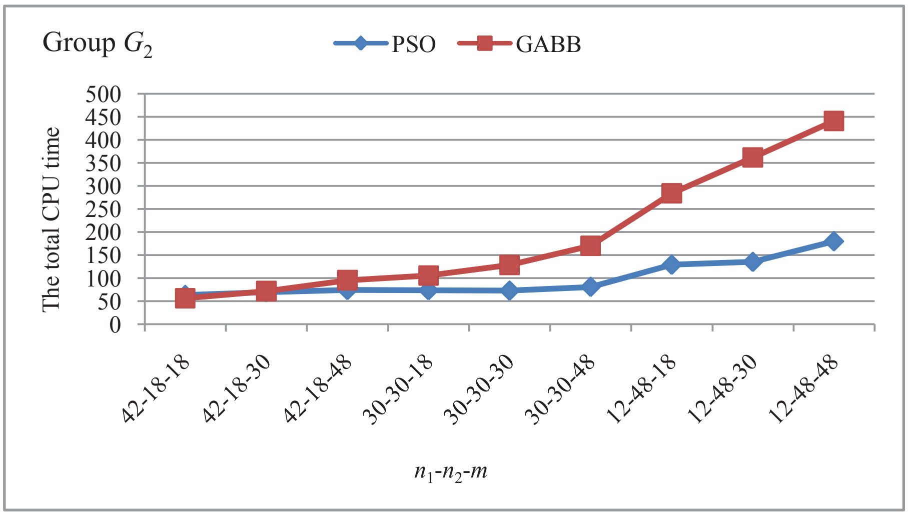


(c) Group G_3

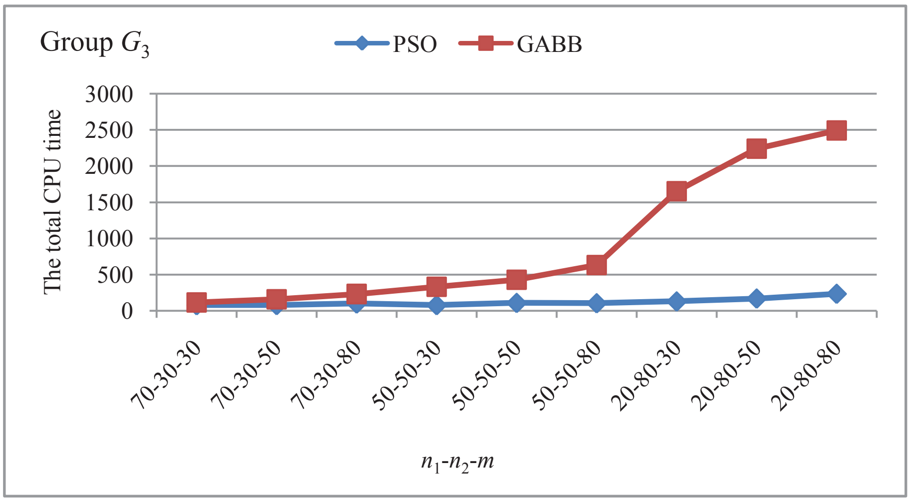
Figure 3.2 The average of the convergent CPU time



(a) Group G_1



(b) Group G_2



(c) Group G_3

Figure 3.3 The average of the total CPU time of all iterations completed

It can be seen from Table 3.10 that the bi-level PSO algorithm spends more CPU times obtaining the best solution and completing all iterations for solving problem group G_1, however, both CPU times of the bi-level PSO algorithm become less and less than GABB following the increase in the number of decision variables within problem groups G_2 and G_3; Figure 3.2 and Figure 3.3 display much more evident results. Figure 3.2 and Figure 3.3 clearly show that both the convergent and total CPU times of GABB increase steeply with the increase in the size of the test problems. In particular in group G_3, GABB takes much more CPU times than the bi-level PSO algorithm to converge to the best solution and complete all the iterations, which implies that the bi-level PSO algorithm has a significant advantage in solving larger-scale problems.

3.5 SUMMARY

In this chapter, a bi-level PSO algorithm is developed to solve nonlinear and large-scale bi-level decision problems, whereas the algorithm is then extended to a tri-level PSO algorithm for solving tri-level decision problems. In the proposed bi-level/tri-level PSO algorithms, the leader's problem and the follower's problem are separated based on related solution concepts for solving conveniently. To handle the complexity of the constraint region of nonlinear and large-scale problems, two methods for constructing the initial population are given. Moreover, the decreasing inertia weight with time is used to control the velocity of particles in the search space at different stages, which aims to improve both search and convergence abilities of the bi-level/tri-level PSO algorithms.

To illustrate the effectiveness of the proposed bi-level/tri-level PSO algorithms, the algorithms are applied to solve 62 benchmark problems from references and 810 large-scale problems which are randomly constructed. The computational results are compared with those obtained by the existing PSO-CST algorithm (Wan, Wang & Sun 2013), evolutionary algorithms (Sinha, Malo & Deb 2014; Wan, Mao & Wang 2014; Wang, Jiao & Li 2005) and genetic algorithm (Calvete, Galé & Mateo 2008).

On the one hand, the computational results of these benchmark bi-level and tri-level decision problems report that the bi-level/tri-level PSO algorithms are able to find much better solutions than the compared algorithms. On the other hand, the computational results of these large-scale problems clearly indicate that the bi-level PSO algorithm shows much better performance in terms of efficiency than the compared algorithms following the problem size becoming larger and larger. In conclusion, the proposed bi-level PSO algorithm provides a practical way to solve nonlinear and large-scale bi-level decision problems; also, it can be extended to a tri-level PSO algorithm for solving tri-level decision problems.

CHAPTER 4 COMPROMISE-BASED FUZZY NONLINEAR BI-LEVEL DECISION MAKING

4.1 INTRODUCTION

An important issue in modeling and solving a bi-level decision problem is that parameters involved are sometimes obtained through experiments or experts' understanding of the nature of the parameters, which are often imprecisely or ambiguously known to decision entities; clearly, it is not reasonable to describe these parameters by precise values (Zhang, Lu & Gao 2015). With this observation, it would be certainly more appropriate to interpret the experts' understanding of such parameters as fuzzy numerical data that can be represented by means of fuzzy set theory. A bi-level decision problem in which the parameters are described by fuzzy values, often characterized by fuzzy numbers, is called a fuzzy bi-level decision problem (Zhang & Lu 2007).

Although numerous solution approaches have been developed to solve fuzzy bi-level decision problems, these solution approaches have the following limitations: (1) limited to handling linear problems involving special fuzzy numbers, such as triangular fuzzy number; (2) limited to solving fuzzy linear decision problems in a special situation where all of the decision entities share the same constraint conditions. Consequently, further investigation into solution approaches is necessary for solving nonlinear bi-level decision problems involving general fuzzy numbers.

This chapter aims to provide an effective algorithm for solving fuzzy nonlinear bi-level decision problems. It first presents a general fuzzy bi-level decision problem which can be transformed into a crisp problem by a commonly used fuzzy number ranking method proposed by Jiménez (1996). The solution to the crisp problem varies with different identifications of fuzzy decision conditions by the leader and follower; in this situation, the leader and follower need to achieve a compromised selection of fuzzy decision conditions to obtain an acceptable optimal solution. Based on rules of compromise between the leader and the follower under fuzziness, the bi-level PSO algorithm in Chapter 3 is then extended to solve the proposed fuzzy nonlinear bi-level decision problem, called the compromise-based PSO algorithm. Lastly, numerical examples are used to illustrate the effectiveness of the proposed solution approach.

This chapter is organized as follows. Following the introduction, Section 4.2 presents related preliminaries of fuzzy set theory which are used in this chapter. In Section 4.3, a general fuzzy nonlinear bi-level decision problem and related theoretical properties are proposed. In Section 4.4, the compromise-based PSO algorithm is developed. The proposed PSO algorithm is used to solve numerical examples in Section 4.5. A summary is given in Section 4.6.

4.2 PRELIMINARIES OF FUZZY SET THEORY

This section presents related notations and definitions of fuzzy set theory that are used in the subsequent sections.

Definition 4.1 (Heilpern 1992) The membership function of a fuzzy number $\tilde{a}$ can be described in the following manner:

$$r = \mu_{\tilde{a}}(x) = \begin{cases} 0 & x \in (-\infty, a_1], \\ f_a(x) & x \in [a_1, a_2], \\ 1 & x \in [a_2, a_3], \\ g_a(x) & x \in [a_3, a_4], \\ 0 & x \in [a_4, \infty), \end{cases}$$

where the function f_a and g_a are called the left and the right side of $\tilde{a}$, and f_a is an increasing and g_a is a decreasing function. The r-cuts are closed and bounded intervals and can be represented by $a_r = [f_a^{-1}(r), g_a^{-1}(r)]$. The membership function can be shown as the following piecewise function in Figure 4.1.

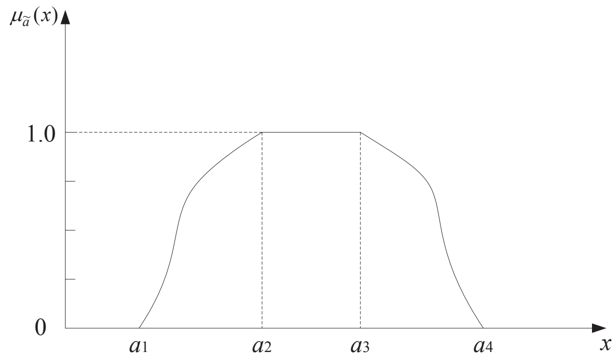


Figure 4.1 The membership function of fuzzy number $\tilde{a}$

Definition 4.2 (Heilpern 1992) The expected interval and the expected value of a continuous fuzzy number $\tilde{a}$ are respectively defined as $EI(\tilde{a})$ and $EV(\tilde{a})$:

$$EI(\tilde{a}) = [E_1^a, E_2^a] = [\int_0^1 f_a^{-1}(r)dr, \int_0^1 g_a^{-1}(r)dr],$$

$$EV(\tilde{a}) = \frac{E_1^a + E_2^a}{2}.$$

Definition 4.3 (Jiménez et al. 2007) $EI(\lambda\tilde{a} + \gamma\tilde{b}) = \lambda EI(\tilde{a}) + \gamma EI(\tilde{b})$,

$$EV(\lambda\tilde{a} + \gamma\tilde{b}) = \lambda EV(\tilde{a}) + \gamma EV(\tilde{b}).$$

Definition 4.4 (Jiménez 1996) For any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is bigger than $\tilde{b}$ is the following:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & E_2^a - E_1^b < 0, \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & 0 \in [E_1^a - E_2^b, E_2^a - E_1^b], \\ 1 & E_1^a - E_2^b > 0, \end{cases}$$

where $[E_1^a, E_2^a]$ and $[E_1^b, E_2^b]$ are the expected intervals of $\widetilde{a}$ and $\widetilde{b}$. When $\mu_M(\widetilde{a},\widetilde{b}) = 0.5$, $\widetilde{a}$ and $\widetilde{b}$ are indifferent. $\mu_M(\widetilde{a},\widetilde{b}) \geq \alpha$ means that $\widetilde{a}$ is bigger than, or equal to $\widetilde{b}$ at least in a degree α and that can be represented by $\widetilde{a} \geq_\alpha \widetilde{b}$.

4.3 General fuzzy bi-level decision problem and theoretical properties

In this section, a general fuzzy bi-level decision problem is first presented. Second, related theoretical properties are discussed based on the fuzzy number ranking method defined by Definition 4.4.

4.3.1 General fuzzy Bi-level decision problem

The general fuzzy bi-level decision problem that is studied in this study is defined as follows.

Definition 4.5 For $x \in X \subset R^p$, $y \in Y \subset R^q$, a general fuzzy bi-level decision problem is defined as:

$$\min_{x \in X} \widetilde{F}(x, y) = \widetilde{c}_1 F(x, y) \quad \text{(Leader)} \tag{4.1a}$$

$$\text{s.t.} \quad \widetilde{G}(x, y) \leq \widetilde{b}_1, \tag{4.1b}$$

where y, for each x fixed, solves the follower's problem (4.1c-4.1d)

$$\min_{y \in Y} \widetilde{f}(x, y) = \widetilde{c}_2 f(x, y) \quad \text{(Follower)} \tag{4.1c}$$

$$\text{s.t.} \quad \widetilde{g}(x, y) \leq \widetilde{b}_2, \tag{4.1d}$$

where x and y are the decision variables of the leader and the follower respectively; $\widetilde{c}_1 \in \Gamma^n(R)$, $\widetilde{c}_2 \in \Gamma^m(R)$, $\widetilde{b}_1 \in \Gamma^s(R)$, $\widetilde{b}_2 \in \Gamma^t(R)$, $F(x, y): R^p \times R^q \to R^n$,

$f(x,y): R^p \times R^q \to R^m$, $\widetilde{F}(x,y): R^p \times R^q \to \Gamma(R)$, $\widetilde{f}(x,y): R^p \times R^q \to \Gamma(R)$, $\widetilde{G}(x,y): R^p \times R^q \to \Gamma^s(R)$, $\widetilde{g}(x,y): R^p \times R^q \to \Gamma^t(R)$, $\Gamma(R)$ is the set of all finite fuzzy numbers.

To find an acceptable optimal solution to the fuzzy bi-level decision problem (4.1), solution concepts in relation to operations of the fuzzy bi-level decision-making process are presented as follows:

Definition 4.6

(1) The constraint region of the fuzzy bi-level decision problem (4.1):

$$S = \{(x,y) \in X \times Y : \widetilde{G}(x,y) \le \widetilde{b}_1, \widetilde{g}(x,y) \le \widetilde{b}_2\}.$$

(2) The feasible set of the follower for each fixed x:

$$S(x) = \{y \in Y : \widetilde{g}(x,y) \le \widetilde{b}_2\}.$$

(3) The rational reaction set of the follower:

$$P(x) = \{y \in Y : y \in \arg\min[\widetilde{f}(x,y) : y \in S(x)]\}.$$

(4) The inducible region of the fuzzy bi-level decision problem (4.1):

$$IR = \{(x,y) : (x,y) \in S, y \in P(x)\}.$$

(5) The optimal solution set of the fuzzy bi-level decision problem (4.1):

$$OS = \{(x,y) : (x,y) \in \arg\min[\widetilde{F}(x,y) : (x,y) \in IR]\}.$$

It is clear from Definition 4.6 that the fuzzy bi-level decision-making process involves uncertain parameters compared with crisp problems. However, the operations of both fuzzy and crisp decision-making process are the same as each other.

4.3.2 RELATED THEORETICAL PROPERTIES

To handle fuzzy parameters involved and develop an effective solution algorithm, this section discusses related theoretical properties of the fuzzy bi-level decision problem (4.1) based on fuzzy set theory.

Definition 4.7 Given a decision vector (x, y), it is said to be feasible in a degree α (α-feasible) to the constraint region *S* if

$$\min\{\mu_M(\widetilde{b}_1, \widetilde{G}(x,y)), \mu_M(\widetilde{b}_2, \widetilde{g}(x,y))\} = \alpha . \quad (4.2)$$

In view of Definition 4.4, the previous expression (4.2) can be written as :

$$\alpha E_2^G + (1-\alpha)E_1^G \le (1-\alpha)E_2^{b_1} + \alpha E_1^{b_1} ,$$

$$\alpha E_2^g + (1-\alpha)E_1^g \le (1-\alpha)E_2^{b_2} + \alpha E_1^{b_2} .$$

The α-feasible constraint region of the fuzzy bi-level decision problem (4.1) can be denoted by

$$_\alpha S = \{(x,y) \in X \times Y : \alpha E_2^G + (1-\alpha)E_1^G \le (1-\alpha)E_2^{b_1} + \alpha E_1^{b_1},$$

$$\alpha E_2^g + (1-\alpha)E_1^g \le (1-\alpha)E_2^{b_2} + \alpha E_1^{b_2}\} .$$

By Definition 4.7, if $\alpha_1 < \alpha_2$, then $_{\alpha_1}S \supset {}_{\alpha_2}S$.

In line with Definition 4.7, let $\min\{\mu_M(\widetilde{b}_1, \widetilde{G}(x,y)\} = \alpha_L$ and $\min\{\mu_M(\widetilde{b}_2, \widetilde{g}(x,y)\} = \alpha_F$, thus, $\alpha = \min\{\alpha_L, \alpha_F\}$. For the fixed x by the leader, y can be said to be α_F-feasible to the feasible set of the follower $S(x)$ under $\min\{\mu_M(\widetilde{b}_2, \widetilde{g}(x,y)\} = \alpha_F$. Accordingly, the feasible set of the follower in relation to all α_F-feasible decision vectors can be denoted by:

$$_{\alpha_F}S(x) = \{y \in Y : \alpha_F E_2^g + (1-\alpha_F)E_1^g \le (1-\alpha_F)E_2^{b_2} + \alpha_F E_1^{b_2}\},$$

and the α_F-feasible rational reaction set of the follower can be written as:

$$_{\alpha_F}P(x) = \{y \in Y : y \in \arg\min[\widetilde{f}(x,y) : y \in_{\alpha_F} S(x)]\}. \quad (4.3)$$

Thus, the α-feasible inducible region of the fuzzy bi-level problem (4.1) is:

$$_{\alpha}IR = \{(x,y) : (x,y) \in_{\alpha} S, y \in_{\alpha_F} P(x)\}.$$

Definition 4.8 For each given x by the leader, y^* is said to be an acceptable optimal solution to the problem $\min\{\widetilde{f}(x,y) : y \in_{\alpha_F} S(x)\}$ if it is verified that:

$$\mu_M(\widetilde{f}(x,y), \widetilde{f}(x,y^*)) \ge 0.5 \quad \text{for} \quad \forall y \in_{\alpha_F} S(x).$$

By Definition 4.4,

$$\widetilde{f}(x,y) \ge_{0.5} \widetilde{f}(x,y^*) \quad \text{for} \quad \forall y \in_{\alpha_F} S(x) \quad (4.4)$$

can be easily obtained, which means that y^* is a better choice of the follower at least in degree 0.5 as opposed to the other feasible solutions in $_{\alpha_F}S(x)$. Using the Definition 4.3, the previous expression (4.4) can be written as:

$$\frac{E_2^{\widetilde{f}(x,y)} - E_1^{\widetilde{f}(x,y^*)}}{E_2^{\widetilde{f}(x,y)} - E_1^{\widetilde{f}(x,y^*)} - (E_1^{\widetilde{f}(x,y)} - E_2^{\widetilde{f}(x,y^*)})} \ge 0.5$$

or

$$\frac{E_2^{\widetilde{f}(x,y)} + E_1^{\widetilde{f}(x,y)}}{2} \ge \frac{E_2^{\widetilde{f}(x,y^*)} + E_1^{\widetilde{f}(x,y^*)}}{2}.$$

In view of Definition 4.2 and Definition 4.3, the expression allows us to set the following proposition:

Proposition 4.1 For each fixed x, y^* is an α_F-acceptable optimal solution to the second-level problem $\min\{\widetilde{f}(x,y): y \in S(x)\}$ if it is an optimal solution to the following crisp problem:

$$\min\{EV(\widetilde{f}(x,y)): y \in_{\alpha_F} S(x)\} = \min\{EV(\widetilde{c}_2) f(x,y): y \in_{\alpha_F} S(x)\}, \tag{4.5}$$

where $EV(\widetilde{c}_2) \in F^m(R)$ is the expected value of the fuzzy vector $\widetilde{c}_2$.

By Proposition 4.1, the expression (4.3) can be written as ${}_{\alpha_F}P(x) = \{y \in Y : y \in \arg\min[EV(\widetilde{c}_2) f(x,y): y \in_{\alpha_F} S(x)]\}$. Similarly, the following Proposition 4.2 is obtained.

Proposition 4.2 (x^o, y^o) is an α-acceptable optimal solution to the fuzzy bi-level decision problem (4.1) if it is an optimal solution to the following crisp problem:

$$\min\{EV(\widetilde{F}(x,y)): (x,y) \in_{\alpha} IR\} = \min\{EV(\widetilde{c}_1) F(x,y): (x,y) \in_{\alpha} IR\}, \tag{4.6}$$

where $EV(\widetilde{c}_1) \in F^n(R)$ is the expected value of the fuzzy vector $\widetilde{c}_1$.

In the light of the proposed definitions and propositions, an optimal solution to the fuzzy bi-level decision problem (4.1) can be found under the minimal feasible degrees α and α_F respectively preferred by the leader and the follower; the solution obtained is considered as at least an α-acceptable optimal solution where $\alpha \le \alpha_F$. However, to find a solution to the fuzzy bi-level decision problem (4.1), two conflicting factors need to be taken into account: the acceptable value for the objective functions and the feasible degree for the constraint conditions. On the one hand, for each fixed *x*, the objective value of the follower will become worse following the increase in the feasible degree α_F. On the other hand, for all solutions

$(x, y) \in_{\alpha} IR$, the objective value of the leader also becomes worse with the feasible degree α going up. Thus, the optimal solution obtained depends on the selection of the minimal feasible degrees α and α_F.

4.4 Compromise-based PSO algorithm

Different selections of α and α_F by the leader and the follower result to changes of the constraint region and inducible region, which will generate different solutions and related objective values. It is the fact that different selections of α and α_F keep the feature of uncertainty of the fuzzy bi-level decision problem, although it has been transformed in to a crisp problem using fuzzy set theory. Within the decision-making process, decision entities can choose values of α and α_F by communicating and consulting with one another in line with different decision environments; this is called compromise-based rules. Under compromise-based rules, decision entities can not only achieve the preferred optimal solution in specific decision situations, but also obtain different solutions due to various decision environments.

This section employs compromise-based rules to deal with the selection α and α_F. Also, the bi-level PSO algorithm presented in Chapter 3 is extended to obtain optimal solutions in relation to different compromised selections of α and α_F. Note that if the leader and the follower share the same constraint conditions in problem (4.1), $\alpha = \alpha_F$ must be made to ensure the leader and the follower having the same identification towards the shared constraint conditions.

In the compromise-based PSO algorithm, related definitions of swarm, particles, *pbest* solution and *gbest* solution are the same as the bi-level PSO algorithm in Chapter 3. In a swarm with the size *N*, the position vector of each particle with index *i*

$(i = 1,2,\ldots,N)$ is denoted as $X_i^t = (x_i^t, y_i^t)$ at iteration *t*, which represents a potential solution to the problem (4.1). For the sake of convenient discussion, let $X_i^t = (x_i^t, y_i^t) = (x_{i1}^t, x_{i2}^t)$. At iteration *t*, each particle *i* moves from X_i^t to X_i^{t+1} in the search space at a velocity $V_i^{t+1} = (v_{i1}^{t+1}, v_{i2}^{t+1})$ along each dimension. Each particle keeps track of its coordinates in hyperspace which are associated with the best solution (fitness), called *pbest* solution ($p_i = (p_{i1}, p_{i2})$). The particle swarm optimizer keeps track of the overall best value, called *gbest* solution ($p_g = (p_{g1}, p_{g2})$).

(1) Initial population

In an initial population of particles with the number N, each particle i $(i = 1,2,\ldots,N)$ can be represented as $X_i^0 = (x_i^0, y_i^0) = (x_{i1}^0, x_{i2}^0)$. An initial population is randomly constructed with the size *N*, where X_i^0 is randomly generated in ${}_{\alpha}S$ by setting $\alpha = 0$.

(2) The updating rules of particles

The updating rules of particles in this compromise-based PSO algorithm are the same as the rules proposed in Section 3.2.2 of Chapter 3.

3) Fitness evaluation

For each particle i at the iteration t $X_i^t = (x_i^t, y_i^t)$, adopt the existing simplex method or interior point method to solve the problem (4.5) under $x = x_i^t$ and $\alpha_F = \alpha_F^*$ specified by the follower using the existing simplex method or interior point method, then obtain the solution (x_i^t, y^*) where $y^* \in_{\alpha_F^*} P(x_i^t)$ and update $X_i^t = (x_i^t, y_i^t) = (x_i^t, y^*)$. If $\min\{\mu_M(\tilde{b}_1, \tilde{G}(x_i^t, y_i^t))\} = \alpha_L^* \geq \alpha^*$ where α^* is specified by the leader, then $y_i^t \in_{\alpha_F^*} P(x_i^t)$ and $(x_i^t, y_i^t) \in_{\alpha^*} S$, which means $(x_i^t, y_i^t) \in_{\alpha^*} IR$;

that is, (x_i^t, y_i^t) is at least a α^*-acceptable feasible solution to the fuzzy bi-level decision problem (4.1). The *pbest* solution is $p_i = (p_{i1}, p_{i2}) = (x_i^t, y_i^t)$ and the exact feasible degree for the constraint region S is $\alpha(p_i) = \min\{\alpha_L^*, \alpha_F^*\} \geq \alpha^*$, if $EV(\widetilde{F}(x_i^t, y_i^t)) \leq EV(\widetilde{F}(p_{i1}, p_{i2}))$ where $p_i = (p_{i1}, p_{i2}) = (x_i^0, y_i^0)$, $EV(\widetilde{F}(p_{i1}, p_{i2})) = +\infty$ and $\alpha(p_i) = 0$ are set at the beginning. The global best solution *gbest* of the swarm is $p_g = (p_{g1}, p_{g2})$ and the corresponding feasible degree for the constraint region *S* is $\alpha = \alpha(p_g)$ where

$$EV(\widetilde{F}(p_{g1}, p_{g2})) = \min\{EV(\widetilde{F}(p_{i1}, p_{i2})), i = 1,2,\ldots,N\}.$$

Clearly, $p_g = (p_{g1}, p_{g2})$ is an α-acceptable optimal solution to the fuzzy bi-level problem (4.1).

(4) Termination criterion

The compromise-based PSO algorithm will be terminated after a maximum number of iterations *Iter_max* or when it achieves a maximum CPU time.

(5) Computational procedures of the compromise-based PSO algorithm

Based on the theoretical basis proposed above, the complete computational procedures of the compromise-based PSO algorithm is presented for solving the fuzzy bi-level decision problem (4.1).

Algorithm 4.1: Compromise-based PSO algorithm

[Begin]

Step 1: *Initialization*.

a) Construct the population size N and generate the initial population of particles $X_i^0 = (x_i^0, y_i^0), i = 1,2,\ldots,N$;

b) Initialize the *pbest* solution $p_i = (p_{i1}, p_{i2}) = (x_i^0, y_i^0)$, the fitness $EV(\widetilde{F}(p_i)) = +\infty$ and the feasible degrees for the constraint conditions $\alpha_F = \alpha_F^*$, $\alpha = \alpha^*$ and $\alpha(p_i) = 0$;

c) Set the maximum and minimum velocity levels $v_{\max}$ and $v_{\min}$, and initialize $v_{ij}^0 = v_{\max}$;

d) Set the upper and lower bounds on the inertia weight $w_{\max}$ and $w_{\min}$, acceleration coefficients c_1 and c_2, and the maximum iteration number *Iter_max*;

e) Set the current iteration number *t*=0 and go to Step 2.

Step 2: *Compute the fitness value and update the pbest solution for each particle*. Set *i*=1 and go to Step 2.1.

Step 2.1: Under $x = x_i^t$, solve the problem (4.5) under $x = x_i^t$ and $\alpha_F = \alpha_F^*$ using the existing simplex method or interior point method, obtain the solution (x_i^t, y^*) and update $X_i^t = (x_i^t, y_i^t) = (x_i^t, y^*)$. Go to Step 2.2.

Step 2.2: If $\min\{\mu_M(\widetilde{b}_1, \widetilde{G}(x_i^t, y_i^t)\} = \alpha_L^* \geq \alpha^*$, go to Step 2.3; otherwise, go to Step 2.4.

Step 2.3: If $EV(\widetilde{F}(x_i^t, y_i^t)) < EV(\widetilde{F}(p_{i1}, p_{i2}))$ or $EV(\widetilde{f}(x_i^t, y_i^t)) < EV(\widetilde{f}(p_{i1}, p_{i2}))$ under $EV(\widetilde{F}(x_i^t, y_i^t)) = EV(\widetilde{F}(p_{i1}, p_{i2}))$, update $p_i = (p_{i1}, p_{i2}) = (x_i^t, y_i^t)$ and $\alpha(p_i) = \min\{\alpha_L^*, \alpha_F^*\}$. Go to Step 2.4.

Step 2.4: If *i*<*N*, set *i*=*i*+1 and go to Step 2.1; otherwise, go to Step 3.

Step 3: *Update the gbest solution*. Set $p_g = (p_{g1}, p_{g2})$ and $\alpha = \alpha(p_g)$ where $EV(\widetilde{F}(p_{g1}, p_{g2})) = \min\{EV(\widetilde{F}(p_{i1}, p_{i2})), i = 1,2,\ldots,N\}$. Go to Step 4.

Step 4: *Termination criterion*. If *t<Iter_max*, go to Step 5; otherwise, stop and $p_g = (p_{g1}, p_{g2})$ is an α-acceptable optimal solution to the fuzzy bi-level decision problem (4.1).

Step 5: *Update the inertia weight, and the velocity and the position of each particle* by the formulas (3.2), (3.3) and (3.4) in Chapter 3. If the current velocity $v_{ij}^{t+1} > v_{\max}$, set $v_{ij}^{t+1} = v_{\max}$; while $v_{ij}^{t+1} = v_{\min}$ if $v_{ij}^{t+1} < v_{\min}$. Set *t*=*t*+1 and go to Step 2.

[End]

4.5 NUMERICAL EXAMPLES

This section first illustrates how the proposed PSO algorithm works through solving a fuzzy nonlinear bi-level decision problem in which the fuzzy numbers are characterized by nonlinear membership functions. Second, the PSO algorithm is used to solve two benchmark problems and the computational results are compared with that obtained by the existing algorithms.

4.5.1 AN ILLUSTRATIVE EXAMPLE

Consider the following fuzzy nonlinear bi-level decision problem (4.7):

$$\min_{x} \tilde{F}(x,y) = -\tilde{1}x_1^2 - \tilde{3}x_2^2 - \tilde{4}y_1 + \tilde{1}y_2^2 \qquad \text{(Leader)} \tag{4.7a}$$

$$\text{s.t.} \quad \tilde{1}x_1^2 + \tilde{2}x_2 \le \tilde{4}, \tag{4.7b}$$

$$x \ge 0, \tag{4.7c}$$

where *y*, for each *x* fixed, solves the follower's problem (4.7d-4.7g)

$$\min_{y} \tilde{f}(x,y) = \tilde{2}x_1^2 + \tilde{1}y_1^2 - \tilde{5}y_2 \qquad \text{(Follower)} \tag{4.7d}$$

$$\text{s.t.} \quad \tilde{1}x_1^2 - \tilde{2}x_1 + \tilde{1}x_2^2 - \tilde{2}y_1 + \tilde{1}y_2 \ge -\tilde{3}, \tag{4.7e}$$

$$\tilde{1}x_2 + \tilde{3}y_1 - \tilde{4}y_2 \geq \tilde{4}, \tag{4.7f}$$

$$y \geq 0. \tag{4.7g}$$

The membership functions of the coefficients in this example are given as follows:

$$\mu_{\tilde{1}}(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq t \leq 1 \\ \dfrac{4-t^2}{3} & 1 \leq t \leq 2 \\ 0 & t > 2 \end{cases}, \quad \mu_{\tilde{2}}(t) = \begin{cases} 0 & t < 1 \\ \dfrac{t^2-1}{3} & 1 \leq t \leq 2 \\ \dfrac{9-t^2}{5} & 2 \leq t \leq 3 \\ 0 & t > 3 \end{cases},$$

$$\mu_{\tilde{3}}(t) = \begin{cases} 0 & t < 2 \\ \dfrac{t^2-4}{5} & 2 \leq t \leq 3 \\ \dfrac{16-t^2}{7} & 3 \leq t \leq 4 \\ 0 & t > 4 \end{cases}, \quad \mu_{\tilde{4}}(t) = \begin{cases} 0 & t < 3 \\ \dfrac{t^2-9}{7} & 3 \leq t \leq 4 \\ \dfrac{25-t^2}{9} & 4 \leq t \leq 5 \\ 0 & t > 5 \end{cases},$$

$$\mu_{\tilde{5}}(t) = \begin{cases} 0 & t < 4 \\ \dfrac{t^2-16}{9} & 4 \leq t \leq 5 \\ \dfrac{36-t^2}{11} & 5 \leq t \leq 6 \\ 0 & t > 6 \end{cases}.$$

Whereas the existing solution approaches cannot be adopted to solve the fuzzy nonlinear bi-level decision problem, the compromise-based PSO algorithm is used to find acceptable optimal solutions for the problem. Based on the PSO procedures developed in Section 4.4, the related parameters involved in the algorithm are initialized in Table 4.1.

Table 4.1 Parameters employed in the compromise-based PSO algorithm for solving problem (4.7)

N	v_{max}	v_{min}	w_{max}	w_{min}	c_1	c_2	$Iter\_max$
30	1.0	-1.0	0.5	0.01	2.0	2.0	60

Table 4.2 The computational results of problem (4.7) under different compromised conditions

α^*	α_F^*	(x, y)	$(EV(\widetilde{F}), EV(\widetilde{f}))$	α	Iterations
0.5	0.5	(0.0014, 1.9669, 1.7003, 0.8241)	(-17.7991, -0.9220)	0.5	39
	0.6	(0.0010, 1.9669, 1.6050, 0.6290)	(-17.7309, -0.2933)	0.5	32
	0.7	(0.0016, 1.9669, 1.5142, 0.4506)	(-17.5797, 0.2870)	0.5	35
	0.8	(0.0002, 1.9669, 1.4276, 0.2872)	(-17.3653, 0.8235)	0.5	39
	0.9	(0.0001, 1.9669, 1.3448, 0.1374)	(-17.1032, 1.3202)	0.5	31
	1.0	(0, 1.9669, 1.2672, 0.0007)	(-16.8121, 1.7809)	0.5	38
0.6	0.6	(0.0003, 1.8307, 1.6050, 0.5952)	(-16.2103, -0.1238)	0.6	33
	0.7	(0.0005, 1.8307, 1.5142, 0.4205)	(-16.0423, 0.4381)	0.6	32
	0.8	(0.0011, 1.8307, 1.4276, 0.2606)	(-15.8149, 0.9571)	0.6	44
	0.9	(0.0009, 1.8307, 1.3448, 0.1141)	(-15.5432, 1.4371)	0.6	37
	1.0	(0.0018, 1.8307, 1.3019, 0)	(-15.3852, 1.8832)	0.6	36
0.7	0.7	(0.0009, 1.7063, 1.5142, 0.3930)	(-14.7354, 0.5761)	0.7	46
	0.8	(0.0039, 1.7063, 1.4276, 0.2363)	(-14.4965, 1.0790)	0.7	29
	0.9	(0.0051, 1.7063, 1.3448, 0.0928)	(-14.2162, 1.5438)	0.7	41
	1.0	(0.0008, 1.7063, 1.3346, 0)	(-14.1848, 1.9790)	0.7	47
0.8	0.8	(0.0004, 1.5924, 1.4276, 0.2140)	(-13.3696, 1.1907)	0.8	36
	0.9	(0.0035, 1.5924, 1.3448, 0.0734)	(-13.0817, 1.6414)	0.8	36
	1.0	(0.0097,1.5924, 1.3646, 0)	(-13.1667, 2.0692)	0.8	46
0.9	0.9	(0.0009, 1.4877, 1.3448, 0.0555)	(-12.1071, 1.7313)	0.9	38
	1.0	(0.0110, 1.4875, 1.3922, 0)	(-12.3000,2.1537)	0.9	30
1.0	1.0	(0.0544, 1.3708, 1.4229, 0)	(-11.4164, 2.2556)	1.0	48

The PSO algorithm is implemented in MATLAB R2014a. The computational results under different compromised selections of α and α_F are reported in Table 4.2. In Table 4.2, the first column α^* and the second column α_F^* are the minimal α and α_F respectively preferred by the leader and the follower. The fifth column α represents the exact feasible degree for constraint conditions under the solution (x, y), which indicates that the solution (x, y) is an α-acceptable optimal solution to the numerical example. The last column shows the iteration number when the PSO algorithm is convergent. In the real world, decision entities can make free choices of their preferred solutions from Table 4.2 in view of various decision situations in relation to their decentralized management problems.

In regard to solving this numerical example (4.7), a pair of α^* and α_F^* is randomly generated in the interval [0.5, 1] and [α^*, 1]. The computational results imply that a convergent solution can be obtained using the PSO algorithm under the parameters shown in Table 4.1. For example, the convergence curves of the expected objective values of the leader and the follower $(EV(\widetilde{F}), EV(\widetilde{f}))$ under $(\alpha^*, \alpha_F^*) = (0.8320, 0.9386)$ are shown in Figure 4.1. It can be seen from Figure 4.1 that the expected objective values of the leader and the follower have converged to $(EV(\widetilde{F}), EV(\widetilde{f})) = (-15.5979, 0.8508)$ since the 30th iteration. With this observation, a *gbest* solution $p_g = (0.0040, 1.8048, 1.4499, 0.2960)$ is obtained for the fuzzy nonlinear bi-level decision problem. Clearly, the compromise-based PSO algorithm provides a practical way to solve nonlinear bi-level decision problems with fuzzy parameters.

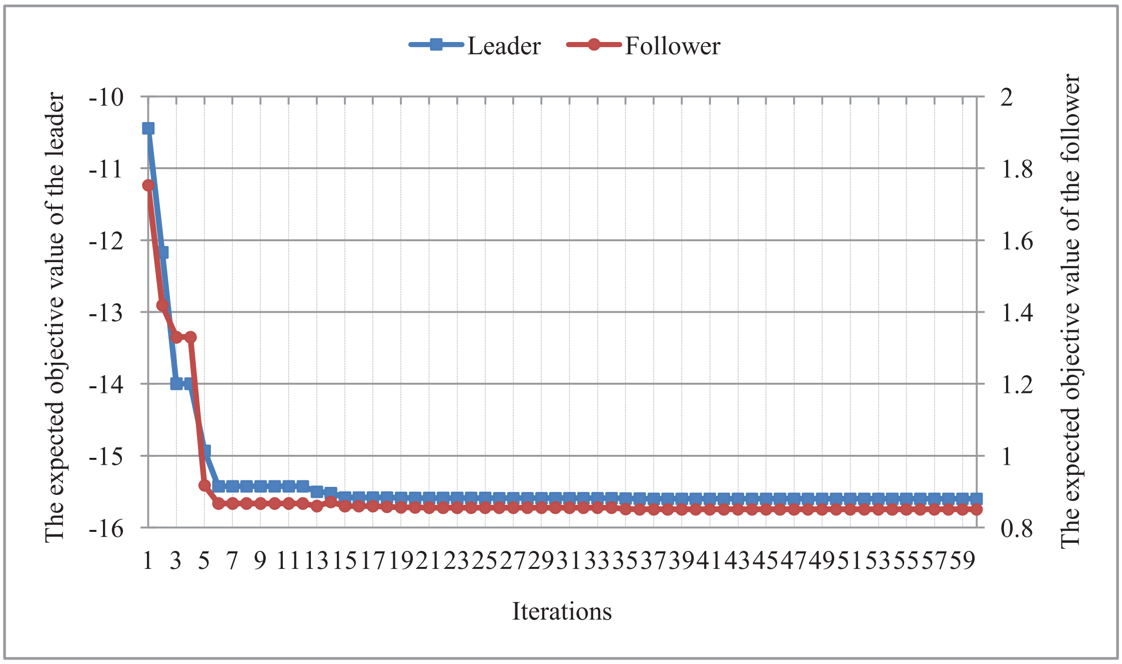


Figure 4.2 The convergence curves of the leader's and the follower's expected objective values

4.5.2 BENCHMARK EXAMPLES

In this section, the compromise-based PSO algorithm is applied to solve two benchmark problems that respectively appear in (Zhang & Lu 2007) and (Zhang & Lu 2005). Also, the computational results obtained by the PSO algorithm are compared with those provided in (Zhang & Lu 2007) and (Zhang & Lu 2005).

Table 4.3 Parameters in the PSO algorithm for solving problems in (Zhang & Lu 2005, 2007)

N	v_{max}	v_{min}	w_{max}	w_{min}	c_1	c_2	*Iter_max*
30	1.0	-1.0	0.5	0.01	2.0	2.0	60

The related parameters involved in the PSO algorithm for solving the problems are initialized as the same, shown in Table 4.3. Table 4.4 and Table 4.5 respectively display the results for the problems in (Zhang & Lu 2007) and (Zhang & Lu 2005) obtained by the PSO algorithm under different compromised α^* and α_F^*.

Decision entities are able to choose their preferred optimal solution from Table 4.4 and Table 4.5 in line with different decision situations. It is noticeable that the solution provided in (Zhang & Lu 2007) is $(x, y) = (0.5, 1.25)$ that is the same as the solution obtained under the compromised condition $\alpha^* = \alpha_F^* = 0.7727$. As well, the solution reported in (Zhang & Lu 2005) is $(x, y) = (0, 0.5)$ and $(EV(\widetilde{F}), EV(\widetilde{f})) = (-1.0, 0.50)$ that satisfies $(x, y) \in_{\alpha^*} IR$ with $\alpha^* = \alpha_F^* = 0.8333$. Under the same decision situation $\alpha^* = \alpha_F^* = 0.8333$, the PSO algorithm can find a better solution $(x, y) = (0.3334, 1.1667)$ and $(EV(\widetilde{F}), EV(\widetilde{f})) = (-2.0, 1.5001)$. Clearly, the compromise-based PSO algorithm provides not only more options of solutions due to different decision environments but also better solutions under the same decision situation for the decision entities.

Table 4.4 The computational results of the problem in (Zhang & Lu 2007)

α^*	α_F^*	(x, y)	$(EV(\widetilde{F}), EV(\widetilde{f}))$	α	Iterations
0.5	0.5	(2.0, 2.0)	(5.0, 2.0)	0.5	23
0.6	0.6	(1.2308, 1.6154)	(5.0, 1.6154)	0.6	22
0.7	0.7	(0.75, 1.375)	(5.0, 1.3750)	0.7	29
0.7727	0.7727	(0.50, 1.25)	(5.0, 1.2500)	0.7727	22
0.8	0.8	(0.421, 1.2105)	(5.0, 1.2105)	0.8	21
0.9	0.9	(0.1818, 1.0909)	(5.0, 1.0909)	0.9	21
1.0	1.0	(0, 1.0)	(5.0, 1.0)	1.0	33

Table 4.5 The computational results of the problem in (Zhang & Lu 2005)

α^*	α_F^*	(x, y)	$(EV(\widetilde{F}), EV(\widetilde{f}))$	α	Iterations
0.5	0.5	(2.0, 2.0)	(-2.0, 4.0)	0.5	23
0.6	0.6	(1.2308, 1.6154)	(-2.0, 2.8462)	0.6	24
0.7	0.7	(0.75, 1.375)	(-2.0, 2.1250)	0.7	31
0.8	0.8	(0.421, 1.2105)	(-2.0, 1.6315)	0.8	28
0.8333	0.8333	(0.3334, 1.1667)	(-2.0, 1.5001)	0.8333	30
0.9	0.9	(0.1818, 1.0909)	(-2.0, 1.2727)	0.9	25
1.0	1.0	(0, 1.0)	(-2.0, 1.0)	1.0	32

4.6 SUMMARY

Existing solution approaches for solving fuzzy bi-level decision problems have been limited to two categories of special problems: (1) linear problems involving special fuzzy numbers, (2) fuzzy linear decision problems in a special situation where all of the decision entities share the same constraint conditions. To overcome these issues, this chapter aims to develop a compromise-based PSO algorithm for solving nonlinear bi-level decision problems with general fuzzy numbers. In the compromise-based PSO algorithm, the leader and follower can choose acceptable decision conditions based on rules of compromise due to different decision environments, which can result in the preferred solution under individual decision situations. To illustrate the effectiveness of the proposed compromise-based PSO algorithm, the algorithm is applied to solve an illustrative example and two benchmark examples. The computational results show that, the compromise-based PSO algorithm can provide not only better solutions under the specific decision situation compared with the existing solution approaches but also different options of solutions due to various decision environments. In conclusion, the compromise-based PSO algorithm provides an effective approach for solving fuzzy nonlinear bi-level decision problems with general fuzzy numbers.

CHAPTER 5 TRI-LEVEL MULTI-FOLLOWER DECISION MAKING

5.1 INTRODUCTION

With respect to a tri-level decision problem, multiple decision entities are often involved at the middle and bottom levels; these entities are called multiple followers. For example, in the tri-level decision-making case presented in Chapter 1, the sales company (the leader) may have several subordinate logistics centers (the middle-level followers) and there may also be several manufacturing factories (the bottom-level followers) attached to each logistics center. Moreover, multiple followers at the same level may have a variety of relationships with one another, which will generate different decision processes.

In general, there are two fundamental issues in supporting this category of tri-level multi-follower (TLMF) decision problems. One is how to use a model to describe the decision-making process, which may manifest different characteristics at the three decision levels, and the other is how to find an optimal solution to the problem. In addition, the optimal solution means a compromised result for a TLMF decision problem, which cannot completely reflect the operations of the complex TLMF decision-making process; that is, it is imprecise or ambiguous for decision entities to evaluate the solution obtained whether or not they desire to in real-world cases. It is necessary to find a practical way to identify the satisfaction of decision entities towards the solution obtained. However, existing tri-level decision-making research has been primarily limited to a specific situation in which one single decision entity is involved at each level.

To handle TLMF decision problems, this chapter first introduces various relationships between multiple followers, gives linear TLMF decision models in line with different relationships, and discusses theoretical properties in relation to the existence and optimality of solutions. A TLMF *K*th-Best algorithm is then developed to find an optimal solution to TLMF decision models. An evaluation method based on fuzzy programming is used to assess the satisfaction of decision entities towards the obtained solution. Lastly, a detailed case study on production-inventory planning illustrates the proposed TLMF decision-making techniques in applications.

This chapter is organized as follows. Following the introduction, general linear TLMF decision models and related theoretical properties are proposed in Section 5.2. In Section 5.3, the TLMF *K*th-Best algorithm is developed. Section 5.4 presents the solution evaluation method based on fuzzy programming. In Section 5.5, the proposed TLMF decision-making techniques are applied to handle the real-world problem in relation to production-inventory planning. A summary is given in Section 5.6.

5.2 TLMF DECISION MODELS AND RELATED THEORETICAL PROPERTIES

This section introduces definitions of three basic relationships (known as uncooperative, cooperative and reference-based relationships) and their hybrid relationships to describe a variety of decision-making situations between multiple followers. In line with different relationships, TLMF decision models and their solution concepts are given. Moreover, related theoretical properties are discussed, which provide the theoretical basis for designing a solution algorithm.

5.2.1 TLMF DECISION MODELS AND SOLUTION CONCEPTS

The organizational structure of the TLMF decision-making hierarchy is shown as Figure 5.1. In this organizational structure, there are one leader, n middle-level followers and m_i bottom-level followers attached to the middle-level follower i ($i = 1,2,\ldots,n$). Different TLMF decision models based on uncooperative, cooperative and reference-based relationships are respectively defined as follows.

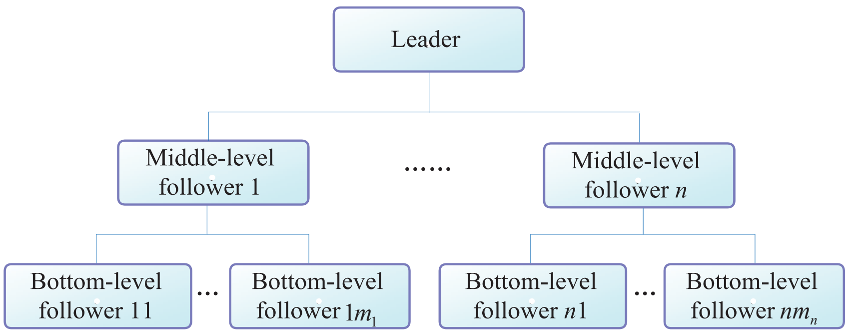


Figure 5.1 The organizational structure of the TLMF decision-making hierarchy

5.2.1.1 UNCOOPERATIVE TLMF DECISION MODEL

Multiple followers at the same level make their respective decisions independently without any information exchange or share; this is known as an uncooperative relationship between multiple followers at the same level. For example, in the tri-level decision-making case given in Chapter 1 and Section 5.1, if logistics centers and manufacturing factories make their respective and individual inventory or production decisions without any information exchange or share, this can be called an uncooperative relationship between the middle-level followers, as well as the bottom-level followers.

Let $x \in X \subset R^k$, $y_i \in Y_i \subset R^{k_i}$, $z_{ij} \in Z_{ij} \subset R^{k_{ij}}$ denote the vectors of decision variables of the leader, the middle-level follower i, and the bottom-level follower ij

respectively where $j = 1,2,\ldots,m_i, i = 1,2,\ldots,n$. Detailed definitions of the uncooperative relationship are given as follows.

Definition 5.1 (Lu et al. 2012) Apart from the decision variables x and $z_{i1},\ldots,z_{im_i}$ respectively controlled by the leader and bottom-level followers, if both the objective function and constraint conditions of the middle-level follower i only involve its own decision variable y_i, this is known as the uncooperative relationship between the middle-level follower i and other middle-level followers.

Definition 5.2 (Lu et al. 2012) Apart from the decision variables x and y_i respectively controlled by the leader and middle-level follower i, if both the objective function and constraint conditions of the bottom-level follower ij only involve its own decision variable z_{ij}, this is known as the uncooperative relationship between the bottom-level follower ij and other bottom-level followers attached to the middle-level follower i.

Based on Definitions 5.1 and 5.2, for $x \in X \subset R^k$, $y_i \in Y_i \subset R^{k_i}$, $z_{ij} \in Z_{ij} \subset R^{k_{ij}}$, $f^{(1)} : X \times Y_1 \times \ldots \times Y_n \times Z_{11} \times \ldots \times Z_{1m_1} \times \ldots \times Z_{nm_n} \to R^1$, $f_i^{(2)} : X \times Y_i \times Z_{i1} \times \ldots \times Z_{im_i} \to R^1$, $f_{ij}^{(3)} : X \times Y_i \times Z_{ij} \to R^1$, $j = 1,2,\ldots,m_i$, $i = 1,2,\ldots,n$, a linear TLMF decision model in which the uncooperative relationship is involved at both middle and bottom levels is defined as follows:

$$\min_{x \in X} f^{(1)}(x, y_1,\ldots, y_n, z_{11},\ldots, z_{1m_1},\ldots, z_{n1},\ldots, z_{nm_n}) = cx + \sum_{i=1}^{n} d_i y_i + \sum_{i=1}^{n}\sum_{j=1}^{m_i} e_{ij} z_{ij} \qquad \text{(Leader) (5.1a)}$$

$$\text{s.t. } Ax + \sum_{i=1}^{n} B_i y_i + \sum_{i=1}^{n}\sum_{j=1}^{m_i} C_{ij} z_{ij} \le b, \qquad (5.1b)$$

where $(y_i, z_{i1},\ldots, z_{im_i})$ $(i = 1,2,\ldots,n)$, for each given x, solves (5.1c-5.1f):

$$\min_{y_i \in Y_i} f_i^{(2)}(x, y_i, z_{i1},\ldots, z_{im_i}) = c_i x + g_i y_i + \sum_{j=1}^{m_i} h_{ij} z_{ij} \qquad \text{(Middle-level follower } i\text{) (5.1c)}$$

s.t. $A_i x + D_i y_i + \sum_{j=1}^{m_i} E_{ij} z_{ij} \le b_i,$ (5.1d)

where z_{ij} $(j = 1,2,\ldots,m_i)$, for each given (x, y_i), solves (5.1e-5.1f):

$$\min_{z_{ij} \in Z_{ij}} f_{ij}^{(3)}(x, y_i, z_{ij}) = c_{ij} x + p_{ij} y_i + q_{ij} z_{ij} \qquad \text{(Bottom-level follower } ij\text{)} \quad (5.1e)$$

s.t. $A_{ij} x + P_{ij} y_i + Q_{ij} z_{ij} \le b_{ij},$ (5.1f)

where $c, c_i, c_{ij} \in R^k$, $d_i, g_i, p_{ij} \in R^{k_i}$, $e_{ij}, h_{ij}, q_{ij} \in R^{k_{ij}}$, $A \in R^{s \times k}$, $A_i \in R^{s_i \times k}$, $A_{ij} \in R^{s_{ij} \times k}$, $B_i \in R^{s \times k_i}$, $D_i \in R^{s_i \times k_i}$, $P_{ij} \in R^{s_{ij} \times k_i}$, $C_{ij} \in R^{s \times k_{ij}}$, $E_{ij} \in R^{s_i \times k_{ij}}$, $Q_{ij} \in R^{s_{ij} \times k_{ij}}$, $b \in R^s$, $b_i \in R^{s_i}$, $b_{ij} \in R^{s_{ij}}$ are matrices of decision coefficients.

To find an optimal solution to the TLMF decision model (5.1), solution concepts in relation to operations of the uncooperative TLMF decision-making process are presented as follows:

Definition 5.3

(1) Constraint region of the TLMF decision model (5.1):

$$S = \{(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}) \in X \times \prod_{i=1}^{n} Y_i \times \prod_{i=1}^{n} \prod_{j=1}^{m_i} Z_{ij} :$$

$$Ax + \sum_{i=1}^{n} B_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m_i} C_{ij} z_{ij} \le b, A_i x + D_i y_i + \sum_{j=1}^{m_i} E_{ij} z_{ij} \le b_i, A_{ij} x + P_{ij} y_i + Q_{ij} z_{ij} \le b_{ij},$$

$$j = 1,2,\ldots,m_i, i = 1,2,\ldots,n\}.$$

(2) For each x given by the leader, feasible set of the middle-level follower i $(i = 1,2,\ldots,n)$ and its bottom-level followers:

$$S_i(x) = \{(y_i, z_{i1}, \ldots, z_{im_i}) \in Y_i \times \prod_{j=1}^{m_i} Z_{ij} : A_i x + D_i y_i + \sum_{j=1}^{m_i} E_{ij} z_{ij} \le b_i,$$

$$A_{ij} x + P_{ij} y_i + Q_{ij} z_{ij} \le b_{ij}, j = 1,2,\ldots,m_i\}.$$

(3) For each (x, y_i) given by the leader and the middle-level follower i, feasible set of the bottom-level follower ij $(j = 1,2,\ldots,m_i, i = 1,2,\ldots,n)$:

$S_{ij}(x, y_i) = \{z_{ij} \in Z_{ij} : A_{ij}x + P_{ij}y_i + Q_{ij}z_{ij} \le b_{ij}\}$.

(4) For each (x, y_i) given by the leader and the middle-level follower i, rational reaction set of the bottom-level follower ij ($j = 1,2,\ldots,m_i, i = 1,2,\ldots,n$):

$P_{ij}(x, y_i) = \{z_{ij} \in Z_{ij} : z_{ij} \in \arg\min[f_{ij}^{(3)}(x, y_i, \hat{z}_{ij}) : \hat{z}_{ij} \in S_{ij}(x, y_i)]\}$.

(5) For each x given by the leader, rational reaction set of the middle-level follower i ($i = 1,2,\ldots,n$) and its bottom-level followers:

$$P_i(x) = \{(y_i, z_{i1},\ldots,z_{im_i}) \in Y_i \times \prod_{j=1}^{m_i} Z_{ij} : (y_i, z_{i1},\ldots,z_{im_i}) \in \arg\min[f_i^{(2)}(x, \hat{y}_i,$$

$$\hat{z}_{i1},\ldots,\hat{z}_{im_i}) : (\hat{y}_i, \hat{z}_{i1},\ldots,\hat{z}_{im_i}) \in S_i(x), \hat{z}_{ij} \in P_{ij}(x, \hat{y}_i), j = 1,2,\ldots,m_i]\} .$$

(6) Inducible region (IR) of model (5.1):

$$IR = \{(x, y_1,\ldots,y_n, z_{11},\ldots,z_{1m_1},\ldots,z_{nm_1},\ldots,z_{nm_n}) : (x, y_1,\ldots,y_n,$$

$$z_{11},\ldots,z_{1m_1},\ldots,z_{nm_1},\ldots,z_{nm_n}) \in S, (y_i, z_{i1},\ldots,z_{im_i}) \in P_i(x), i = 1,2,\ldots,n\} .$$

(7) Optimal solution set of model (5.1):

$$OS = \{(x, y_1,\ldots,y_n, z_{11},\ldots,z_{1m_1},\ldots,z_{nm_1},\ldots,z_{nm_n}) : (x, y_1,\ldots,y_n, z_{11},\ldots,z_{1m_1},\ldots,$$

$$z_{nm_1},\ldots,z_{nm_n}) \in \arg\min[f^{(1)} : (x, y_1,\ldots,y_n, z_{11},\ldots,z_{1m_1},\ldots,z_{nm_1},\ldots,z_{nm_n}) \in IR]\} .$$

5.2.1.2 Cooperative TLMF decision model

Multiple followers at the same level share information and make joint decisions because of common business needs; this is known as a cooperative relationship between multiple followers at the same level. For example, in the tri-level decision-making case given in Chapter 1 and Section 5.1, if the logistics centers share a warehouse, they need to cooperate with each other for making an inventory solution satisfied by every logistics center; this can be called a cooperative relationship between the middle-level followers.

Let $x \in X \subset R^k$, $y \in Y_i \subset R^{k_i}$, $z_i \in Z_{ij} \subset R^{k_{ij}}$ denote the vectors of decision variables of the leader, the middle-level follower *i*, and the bottom-level follower *ij* respectively where $j = 1,2,\dots,m_i, i = 1,2,\dots,n$. Detailed definitions of the cooperative relationship are given as follows.

Definition 5.4 (Lu et al. 2012) Apart from the decision variables x and z_i respectively controlled by the leader and bottom-level followers, if both the objective function and constraint conditions of the middle-level follower *i* involve the decision variable y shared with other middle-level followers, this is known as the cooperative relationship between the middle-level follower *i* and other middle-level followers.

Definition 5.5 (Lu et al. 2012) Apart from the decision variables x and y respectively controlled by the leader and the middle-level follower *i*, if both the objective function and constraint conditions of the bottom-level follower *ij* involve the decision variable z_i shared with other bottom-level followers attached to the middle-level follower *i*, this is known as the cooperative relationship between the bottom-level follower *ij* and other bottom-level followers attached to the middle-level follower *i*.

Based on Definitions 5.4 and 5.5, for $x \in X \subset R^k$, $y \in Y_i \subset R^{k_i}$, $Y = Y_1 \cap \dots \cap Y_n$, $y \in Y \subset R^{k_0}$, $z_i \in Z_{ij} \subset R^{k_{ij}}$, $Z_i = Z_{i1} \cap \dots \cap Z_{im_i}$, $z_i \in Z_i \subset R^{k_{i0}}$, $f^{(1)} : X \times Y \times Z_i \times \dots \times Z_n \to R^1$, $f_i^{(2)} : X \times Y_i \times Z_i \to R^1$, $f_{ij}^{(3)} : X \times Y_i \times Z_{ij} \to R^1$, $j = 1,2,\dots,m_i$, $i = 1,2,\dots,n$, a linear TLMF decision model in which the cooperative relationship is involved at both middle and bottom levels is defined as follows:

$$\min_{x \in X} f^{(1)}(x, y, z_1, \dots, z_n) = cx + dy + \sum_{i=1}^{n} e_i z_i \qquad \text{(Leader) (5.2a)}$$

s.t. $Ax + By + \sum_{i=1}^{n} C_i z_i \le b$, (5.2b)

where (y, z_i) $(i = 1,2,\dots,n)$, for each given x, solves (5.2c-5.2f):

$$\min_{y \in Y_i} f_i^{(2)}(x, y, z_i) = c_i x + d_i y + g_i z_i \quad \text{(Middle-level follower } i\text{)} \quad (5.2c)$$

s.t. $A_i x + B_i y + D_i z_i \le b_i$, (5.2d)

where z_i $(j = 1,2,\dots,m_i)$, for each given (x, y), solves (5.2e-5.2f):

$$\min_{z_i \in Z_{ij}} f_{ij}^{(3)}(x, y, z_i) = c_{ij} x + d_{ij} y + h_{ij} z_i \quad \text{(Bottom-level follower } ij\text{)} \quad (5.2e)$$

s.t. $A_{ij} x + B_{ij} y + E_{ij} z_i \le b_{ij}$, (5.2f)

where $c, c_i, c_{ij} \in R^k$, $d, d_i, d_{ij} \in R^{k_0}$, $e_i, g_i, h_{ij} \in R^{k_{i0}}$, $A \in R^{s \times k}$, $A_i \in R^{s_i \times k}$, $A_{ij} \in R^{s_{ij} \times k}$, $B \in R^{s \times k_0}$, $B_i \in R^{s_i \times k_0}$, $B_{ij} \in R^{s_{ij} \times k_0}$, $C_i \in R^{s \times k_{i0}}$, $D_i \in R^{s_i \times k_{i0}}$, $E_{ij} \in R^{s_{ij} \times k_{i0}}$, $b \in R^s$, $b_i \in R^{s_i}$, $b_{ij} \in R^{s_{ij}}$ are matrices of decision coefficients.

To find an optimal solution to the TLMF decision model (5.2), solution concepts in relation to operations of the cooperative TLMF decision-making process are presented as follows:

Definition 5.6

(1) Constraint region of the TLMF decision model (5.2):

$$S = \{(x, y, z_1, \dots, z_n) \in X \times Y \times \prod_{i=1}^{n} Z_i : Ax + By + \sum_{i=1}^{n} C_i z_i \le b,$$

$$A_i x + B_i y + D_i z_i \le b_i, A_{ij} x + B_{ij} y + E_{ij} z_i \le b_{ij},\ j = 1,2,\dots,m_i, i = 1,2,\dots,n\}.$$

(2) For each x given by the leader, feasible set of the middle-level follower i $(i = 1,2,\dots,n)$ and its bottom-level followers:

$$S_i(x) = \{(y, z_i) \in Y \times Z_i : A_i x + B_i y + D_i z_i \le b_i, A_{ij} x + B_{ij} y + E_{ij} z_i \le b_{ij}, j = 1,2,\dots,m_i\}.$$

(3) For each (*x*, *y*) given by the leader and the middle-level follower *i*, feasible set of the bottom-level follower *ij* $(j=1,2,\ldots,m_i, i=1,2,\ldots,n)$:

$$S_{ij}(x,y)=\{z_i \in Z_i : A_{ij}x+B_{ij}y+E_{ij}z_i \le b_{ij}\}.$$

(4) For each (*x*, *y*) given by the leader and the middle-level follower *i*, rational reaction set of the bottom-level follower *ij* ($j=1,2,\ldots,m_i, i=1,2,\ldots,n$):

$$P_{ij}(x,y)=\{z_i \in Z_i : z_i \in \arg\min[f_{ij}^{(3)}(x,y,\hat{z}_i) : \hat{z}_i \in S_{ij}(x,y_i)]\}.$$

(5) For each *x* given by the leader, rational reaction set of the middle-level follower *i* ($i=1,2,\ldots,n$) and its bottom-level followers:

$$P_i(x)=\{(y,z_i)\in Y\times Z_i : (y,z_i)\in \arg\min[f_i^{(2)}(x,\hat{y},\hat{z}_i):$$

$$(\hat{y},\hat{z}_i)\in S_i(x), \hat{z}_i \in P_{ij}(x,\hat{y}), j=1,2,\ldots,m_i]\}.$$

(6) Inducible region (*IR*) of model (5.2):

$$IR=\{(x,y,z_1,\ldots,z_n):(x,y,z_1,\ldots,z_n)\in S,(y,z_i)\in P_i(x), i=1,2,\ldots,n\}.$$

(7) Optimal solution set of model (5.2):

$$OS=\{(x,y,z_1,\ldots,z_n):(x,y,z_1,\ldots,z_n)\in \arg\min[f^{(1)}:(x,y,z_1,\ldots,z_n)\in IR]\}.$$

5.2.1.3 REFERENCE-BASED TLMF DECISION MODEL

Multiple followers at the same level make their individual decisions independently but exchange information between themselves, which implies that followers consider the decision results of their counterparts as references when making their individual decisions; this situation is known as a reference-based relationship. For example, in the tri-level decision-making case given in Chapter 1 and Section 5.1, the logistics centers and manufacturing factories may reference inventory or production plans determined by their counterparts at the same level when making their individual inventory or production decisions; this can be called a reference-based relationship between the middle-level followers, as well as the bottom-level followers.

Let $x \in X \subset R^k$, $y_i \in Y_i \subset R^{k_i}$, $z_{ij} \in Z_{ij} \subset R^{k_{ij}}$ denote the vectors of decision variables of the leader, the middle-level follower *i*, and the bottom-level follower *ij* respectively where $j = 1,2,\ldots,m_i$, $i = 1,2,\ldots,n$. Detailed definitions of the reference-based relationship are given as follows.

Definition 5.7 (Lu et al. 2012) Apart from its own decision variable y_i and the decision variables $x, z_{i1}, \ldots, z_{im_i}$ determined by the leader and the bottom-level followers, if both the objective function and constraint conditions of the middle-level follower i involve the decision variables $y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n$ controlled by other middle-level followers, this is known as the reference-based relationship between the middle-level follower *i* and other middle-level followers.

Definition 5.8 (Lu et al. 2012) Apart from its own decision variable z_{ij} and the decision variables x and y_i respectively determined by the leader and the middle-level follower *i*, if both the objective function and constraint conditions of the bottom-level follower ij involve the decision variables $z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i}$ controlled other bottom-level followers attached to the same middle-level follower *i*, this is known as the reference-based relationship between the bottom-level follower *ij* and other bottom-level followers attached to the middle-level follower *i*.

Based on Definitions 5.7 and 5.8, for $x \in X \subset R^k$, $y_i \in Y_i \subset R^{k_i}$, $z_{ij} \in Z_{ij} \subset R^{k_{ij}}$, $f^{(1)}: X \times Y_1 \times \ldots \times Y_n \times Z_{11} \times \ldots \times Z_{1m_1} \times \ldots \times Z_{nm_n} \to R^1$, $f_i^{(2)}: X \times Y_1 \times \ldots \times Y_n \times Z_{i1} \times \ldots \times Z_{im_i} \to R^1$, $f_{ij}^{(3)}: X \times Y_i \times Z_{i1} \times \ldots Z_{im_i} \to R^1$, $j = 1,2,\ldots,m_i$, $i = 1,2,\ldots,n$, a linear TLMF decision model in which the reference-based relationship is involved at both middle and bottom levels is defined as follows:

$$\min_{x\in X} f^{(1)}(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}) = cx + \sum_{i=1}^{n} d_i y_i + \sum_{i=1}^{n}\sum_{j=1}^{m_i} e_{ij} z_{ij} \quad \text{(Leader)} \tag{5.3a}$$

$$\text{s.t. } Ax + \sum_{i=1}^{n} B_i y_i + \sum_{i=1}^{n}\sum_{j=1}^{m_i} C_{ij} z_{ij} \le b, \tag{5.3b}$$

where $(y_i, z_{i1}, \ldots, z_{im_i})$ $(i = 1,2,\ldots,n)$, for each given $(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$, solves (5.3c-5.3f):

$$\min_{y_i\in Y_i} f_i^{(2)}(x, y_1, \ldots, y_n, z_{i1}, \ldots, z_{im_i}) = c_i x + \sum_{s=1}^{n} g_{is} y_s + \sum_{j=1}^{m_i} h_{ij} z_{ij} \quad \text{(Middle-level follower } i\text{)} \tag{5.3c}$$

$$\text{s.t. } A_i x + \sum_{s=1}^{n} D_{is} y_s + \sum_{j=1}^{m_i} E_{ij} z_{ij} \le b_i, \tag{5.3d}$$

where z_{ij} $(j = 1,2,\ldots,m_i)$, for each given $(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$, solves (5.3e-5.3f):

$$\min_{z_{ij}\in Z_{ij}} f_{ij}^{(3)}(x, y_i, z_{i1}, \ldots, z_{im_i}) = c_{ij} x + p_{ij} y_i + \sum_{t=1}^{m_i} q_{ijt} z_{it} \quad \text{(Bottom-level follower } ij\text{)} \tag{5.3e}$$

$$\text{s.t. } A_{ij} x + P_{ij} y_i + \sum_{t=1}^{m_i} Q_{ijt} z_{it} \le b_{ij}, \tag{5.3f}$$

where $c, c_i, c_{ij} \in R^k$, $d_i, p_{ij} \in R^{k_i}$, $g_{is} \in R^{k_s}$, $e_{ij}, h_{ij} \in R^{k_{ij}}$, $q_{ijt} \in R^{k_{it}}$, $A \in R^{r\times k}$, $A_i \in R^{r_i\times k}$, $A_{ij} \in R^{r_{ij}\times k}$, $B_i \in R^{r\times k_i}$, $D_{is} \in R^{r_i\times k_s}$, $P_{ij} \in R^{r_{ij}\times k_i}$, $C_{ij} \in R^{r\times k_{ij}}$, $E_{ij} \in R^{r_i\times k_{ij}}$, $Q_{ijt} \in R^{r_{ij}\times k_{it}}$, $b \in R^r$, $b_i \in R^{r_i}$, $b_{ij} \in R^{r_{ij}}$ are matrices of decision coefficients, $t = 1,2,\ldots,m_i$, $s = 1,2,\ldots,n$.

To find an optimal solution to the TLMF decision model (5.3), solution concepts in relation to operations of the reference-based TLMF decision-making process are presented as follows:

Definition 5.9

(1) Constraint region of the TLMF decision model (5.3):

$$S = \{(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}) \in X \times \prod_{i=1}^{n} Y_i \times \prod_{i=1}^{n}\prod_{j=1}^{m_i} Z_{ij} :$$

$$Ax+\sum_{i=1}^{n}B_i y_i+\sum_{i=1}^{n}\sum_{j=1}^{m_i}C_{ij}z_{ij}\le b, A_i x+\sum_{s=1}^{n}D_{is}y_s+\sum_{j=1}^{m_i}E_{ij}z_{ij}\le b_i,$$

$$A_{ij}x+P_{ij}y_i+\sum_{t=1}^{m_i}Q_{ijt}z_{it}\le b_{ij}, j=1,2,\ldots,m_i, i=1,2,\ldots,n\}.$$

(2) For each $(x,y_1,\ldots,y_{i-1},y_{i+1},\ldots,y_n)$ given by the leader and other middle-level followers, feasible set of the middle-level follower i $(i=1,2,\ldots,n)$ and its bottom-level followers:

$$S_i(x,y_1,\ldots,y_{i-1},y_{i+1},\ldots,y_n)=\{(y_i,z_{i1},\ldots,z_{im_i})\in Y_i\times\prod_{j=1}^{m_i}Z_{ij}:$$

$$A_i x+\sum_{s=1}^{n}D_{is}y_s+\sum_{j=1}^{m_i}E_{ij}z_{ij}\le b_i, A_{ij}x+P_{ij}y_i+\sum_{t=1}^{m_i}Q_{ijt}z_{it}\le b_{ij}, j=1,2,\ldots,m_i\}.$$

(3) For each $(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})$ given by the leader, the middle-level follower i and bottom-level followers, feasible set of the bottom-level follower ij $(j=1,2,\ldots,m_i, i=1,2,\ldots,n)$:

$$S_{ij}(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})=\{z_{ij}\in Z_{ij}:A_{ij}x+P_{ij}y_i+\sum_{t=1}^{m_i}Q_{ijt}z_{it}\le b_{ij}\}.$$

(4) For each $(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})$ given by the leader, the middle-level follower i and bottom-level followers, rational reaction set of the bottom-level follower ij ($j=1,2,\ldots,m_i, i=1,2,\ldots,n$):

$$P_{ij}(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})=\{z_{ij}\in Z_{ij}:$$

$$z_{ij}\in\arg\min[f_{ij}^{(3)}(x,y_i,z_{i1},\ldots,z_{i(j-1)},\hat{z}_{ij},z_{i(j+1)},\ldots,z_{im_i}),$$

$$\hat{z}_{ij}\in S_{ij}(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})]\}.$$

(5) For each $(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ given by the leader and other middle-level followers, rational reaction set of the middle-level follower i ($i = 1,2,\ldots,n$) and its bottom-level followers:

$$P_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n) = \{(y_i, z_{i1}, \ldots, z_{im_i}) \in Y_i \times \prod_{j=1}^{m_i} Z_{ij} :$$

$$(y_i, z_{i1}, \ldots, z_{im_i}) \in \arg\min[f_i^{(2)}(x, y_1, \ldots, y_{i-1}, \hat{y}_i, y_{i+1}, \ldots, y_n, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) :$$

$$(\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) \in S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n),$$

$$\hat{z}_{ij} \in P_{ij}(x, \hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{i(j-1)}, \hat{z}_{i(j+1)}, \ldots, \hat{z}_{im_i}), j = 1,2,\ldots,m_i]\} .$$

(6) Inducible region (IR) of model (5.3):

$$IR = \{(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{nm_1}, \ldots, z_{nm_n}) :$$

$$(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{nm_1}, \ldots, z_{nm_n}) \in S,$$

$$(y_i, z_{i1}, \ldots, z_{im_i}) \in P_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n), i = 1,2,\ldots,n\} .$$

(7) Optimal solution set of model (5.3):

$$OS = \{(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{nm_1}, \ldots, z_{nm_n}) : (x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots,$$

$$z_{nm_1}, \ldots, z_{nm_n}) \in \arg\min[f^{(1)} : (x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{nm_1}, \ldots, z_{nm_n}) \in IR]\} .$$

In summary, this study presents three categories of TLMF decision models in line with uncooperative, cooperative and reference-based relationships. Apart from these three categories of basic decision models, a new category of decision models can be established based on the hybrid of three aforementioned relationships. Decision models based on hybrid relationships can be clustered into two main categories. One is that there exist different relationships at different decision levels. For example, in the tri-level decision-making case given in Chapter 1 and Section 5.1, the logistics centers at the middle level have the cooperative relationship whereas there is the uncooperative relationship between the manufacturing factories at the bottom level. The relevant TLMF decision model can be built based on Definitions 4.4 and 4.2. The other is that there exist different relationships at the same decision level. For example, there exist both the uncooperative and cooperative relationships between the logistics centers at the middle level, which implies that the objective function and constraint conditions involve not only individual decision variables but also decision variables shared by multiple logistics centers. Since the modeling process based on hybrid relationships is similar to the TLMF decision models (5.1-5.3), this chapter will not present hybrid-relationship-based TLMF decision models in detail.

In addition, it can be seen from solution concepts of TLMF decision models (5.1-5.3) that the reference-based TLMF decision model (5.3) is the most complex and representative model, thus, this study consider this model as a general TLMF decision model. This chapter will discuss related theoretical properties of the reference-based TLMF model (5.3) and develop a solution algorithm for solving this model.

5.2.2 RELATED THEORETICAL PROPERTIES

To develop an effective solution algorithm for solving the reference-based TLMF decision model (5.3), this section discusses related theoretical properties based on Definition 5.9.

Based on related solution concepts, it can be concluded that the reference-based TLMF decision model (5.3) has the following features:

(1) The leader has the priority to determine its decision variable x to optimize its objective function under the constraint region S.

(2) The middle-level follower i then determines its individual decision variable y_i under the feasible set $S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ to react to the given decision $(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ from the leader and other middle-level followers.

(3) The bottom-level follower *ij* determines its decision variable z_{ij} under its feasible set $S_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$ to respond to the decision $(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$ made by the leader, the middle-level follower *i* and other bottom-level followers attached to the middle-level follower *i*.

(4) Since each decision entity seeks to optimize its own objective function, the decision variable selection of the bottom-level follower *ij* must be involved in its rational reaction set $P_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$, which ensures an optimal solution to problem (5.3e-5.3f) under the given decision $(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$.

(5) Under the given decision (x, y_i) by the leader and the middle-level follower *i*, an optimal solution $(z_{i1}, \ldots, z_{im_i})$ to all the bottom-level followers attached to the middle-level follower *i* must be involved in

$$P_i(x, y_i) = \{(z_{i1}, \ldots, z_{im_i}) \in \prod_{j=1}^{m_i} Z_{ij} : z_{ij} \in P_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i}),$$

$$j = 1, 2, \ldots, m_i\}.$$

(6) As the decision of the middle-level follower *i* is affected by actions of its bottom-level followers, it must consider implicit reactions of its bottom-level followers when making its own decisions, thus, an optimal solution $(y_i, z_{i1}, \ldots, z_{im_i})$ to the middle-level follower *i* and its bottom-level followers must occur in their rational reaction set $P_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$, which can also be considered as an optimal solution to problem (5.3c-5.3f) under the given decision $(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$.

(7) An optimal solution to all the followers under the given *x* by the leader must be involved in

$$P(x)=\{(y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,z_{n1},\ldots,z_{nm_n}):(y_i,z_{i1},\ldots,z_{im_i})\in P_i(x,y_1,\ldots,y_{i-1},y_{i+1},\ldots,y_n),$$

$$i=1,2,\ldots,n\}.$$

(8) As the leader should consider implicit reactions of all the followers when making its own decisions, an optimal solution to model (5.3) must occur over the *IR* and the optimal solution set is expressed by *OS*.

To ensure that the model (5.3) is well posed, it is necessary to make the following assumptions as the basis for the existence of solutions.

Assumption 5.1 *S* is nonempty and compact.

Assumption 5.2 *IR* is nonempty.

Assumption 5.3 $P_i(x,y_i)$ and $P(x)$ have at most one solution respectively for each parameter (x,y_i) and *x*, where $i=1,2,\ldots,n$.

Theorem 5.1 If the above Assumptions 5.1-5.3 hold, there exists an optimal solution to the TLMF decision model (5.3).

Proof. Since both *S* and *IR* are not empty, there is at least one parameter value x^* and $P(x^*)\neq\varnothing$. Consider a sequence $\{(x^t,y_1^t,\ldots,y_n^t,z_{11}^t,\ldots,z_{1m_1}^t,\ldots,z_{n1}^t,\ldots,z_{nm_n}^t)\}_{t=1}^{\infty}\subseteq IR$ converging to $(x^*,y_1^*,\ldots,y_n^*,z_{11}^*,\ldots,z_{1m_1}^*,\ldots,z_{n1}^*,\ldots,z_{nm_n}^*)$. Then, since $(y_1^t,\ldots,y_n^t,z_{11}^t,\ldots,z_{1m_1}^t,\ldots,z_{n1}^t,\ldots,z_{nm_n}^t)\in P(x^t)$, let $P_0(x^t)=(y_1^t,\ldots,y_n^t,z_{11}^t,\ldots,z_{1m_1}^t,\ldots,z_{n1}^t,\ldots,z_{nm_n}^t)$ and $P_0(x^t)\in P(x^t)$. Because of the continuity properties of linear parametric optimization proposed in (Dempe 2002) and $\lim_{t\to\infty}x^t=x^*$, $\lim_{t\to\infty}P_0(x^t)=P_0(x^*)$ is obtained. Since $\lim_{t\to\infty}P_0(x^t)=(y_1^*,\ldots,y_n^*,z_{11}^*,\ldots,z_{1m_1}^*,\ldots,z_{n1}^*,\ldots,z_{nm_n}^*)$ and $P_0(x^*)\in P(x^*)$, $(y_1^*,\ldots,y_n^*,z_{11}^*,\ldots,z_{1m_1}^*,\ldots,z_{n1}^*,\ldots,z_{nm_n}^*)\in P(x^*)$. Hence, *IR* is closed. Also, *IR* is bounded by $IR\subseteq S$ and Assumption 5.1. Therefore, the inducible region *IR* is a nonempty and compact set. Furthermore, by Assumption 5.3, the solution $(z_{i1}^*,\ldots,z_{im_i}^*)$ to the bottom-level followers attached to the middle-level follower i ($i=1,2,\ldots,n$) and the solution $(y_1^*,\ldots,y_n^*,z_{11}^*,\ldots,z_{1m_1}^*,\ldots,z_{n1}^*,\ldots,z_{nm_n}^*)$ to all the followers are uniquely determined respectively for the given value (x^*,y_i^*) and x^*, which implies that the leader must optimize its objective over *IR*. According to the optimal solution set *OS* in Definition 5.9, finding an optimal solution to model (5.3) is equivalent to solving the following problem:

$$\min\{f^{(1)}(x,y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,z_{n1},\ldots,z_{nm_n}):$$
$$(x,y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,z_{n1},\ldots,z_{nm_n})\in IR\}. \tag{5.4}$$

Clearly, problem (5.4) consists of minimizing a continuous function over a nonempty and compact set *IR*, which implies that there exists an optimal solution to the problem (5.4) that is also an optimal solution to the TLMF decision model (5.3). □

It is noticeable from the Theorem 2.1 that if the followers have multiple optimal solutions to respond to the parameter value *x* of the leader, it will be difficult for the leader to realize its objective function value prior to the determination of the optimal solution taken by the followers (Mersha & Dempe 2006). In this case, if the followers cannot select the solution preferred by the leader, the leader may achieve its optimal solution outside *IR*, which implies that the TLMF decision model (5.3) may not have an optimal solution. Therefore, to avoid this situation in the presentation of solution algorithms, the Assumption 5.3 is necessary.

Theorem 5.2 The *IR* can be expressed equivalently as a piecewise linear equality constraint comprised of supporting hyperplanes of *S*.

Proof. First, for $j=1,2,\ldots,m_i, i=1,2,\ldots,n$, define

$$F_{ij}(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})$$
$$=\min\{q_{ijj}z_{ij}:z_{ij}\in S_{ij}(x,y_i,z_{i1},\ldots,z_{i(j-1)},z_{i(j+1)},\ldots,z_{im_i})\},$$

$$F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$$

$$= \min\{g_{ii}\hat{y}_i + \sum_{j=1}^{m_i} h_{ij}\hat{z}_{ij} : (\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) \in S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n),$$

$$q_{ijj}\hat{z}_{ij} = F_{ij}(x, \hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{i(j-1)}, \hat{z}_{i(j+1)}, \ldots, \hat{z}_{im_i}), j = 1,2,\ldots,m_i\}.$$

Since $F_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$ can be seen as a linear programming problem with parameters x, y_i and $z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i}$, the dual problem of $F_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$ can be written as:

$$\max\{(A_{ij}x + P_{ij}y_i + \sum_{t=1, t\neq j}^{m_i} Q_{ijt}z_{it} - b_{ij})u_{ij} : Q_{ijj}u_{ij} \geq -q_{ijj}, u_{ij} \geq 0\}, j = 1,2,\ldots,m_i, i = 1,2,\ldots,n. \quad (5.5)$$

If both $F_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$ and problem (5.5) have feasible solutions, according to the dual theorem of linear programming, both of them have optimal solutions and the same optimal objective function value. A solution to problem (5.5) occurs at a vertex of its constraint region $U_{ij} = \{(u_{ij} : Q_{ijj}u_{ij} \geq -q_{ijj}, u_{ij} \geq 0\}$. Adopting $u_{ij}^1, u_{ij}^2, \ldots, u_{ij}^{k_{ij}}$ to express all the vertices of U_{ij}, problem (5.5) can be written as:

$$\max\{(A_{ij}x + P_{ij}y_i + \sum_{t=1, t\neq j}^{m_i} Q_{ijt}z_{it} - b_{ij})u_{ij} :$$

$$u_{ij} \in \{u_{ij}^1, u_{ij}^2, \ldots, u_{ij}^{k_{ij}}\}, j = 1,2,\ldots,m_i, i = 1,2,\ldots,n. \quad (5.6)$$

Clearly, $F_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$ is a piecewise linear function according to problem (5.6).

In the next step, $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ needs to be proved to be a piecewise linear function. Suppose that $(z_{i1}^1, z_{i2}^1, \ldots, z_{im_i}^1), \ldots, (z_{i1}^{s_i}, z_{i2}^{s_i}, \ldots, z_{im_i}^{s_i})$ are solutions to the problem $\{F_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i}), j = 1,2,\ldots,m_i\}$ for all $i = 1,2,\ldots,n$. For each fixed i and a solution $(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})$ where $t_i = 1,2,\ldots,s_i$,

$F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ becomes a programming problem with parameters $x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n$ and $(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})$, and there are s_i parameterized programming problems such as $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)|_{(z_{i1}^1, z_{i2}^1, \ldots, z_{im_i}^1)}, \ldots,$ $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)|_{(z_{i1}^{s_i}, z_{i2}^{s_i}, \ldots, z_{im_i}^{s_i})}$. Considering different combinations of $(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})$ for all $i = 1,2,\ldots,n$, therefore, there are $\prod_{i=1}^{n} s_i$ parameterized programming problems $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)|_{(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})}$. Similarly, $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)|_{(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})}$ is also a piecewise linear function as $F_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})$. Lastly, according to the above definition of $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$, the *IR* can be rewritten as:

$$IR = \{(x, y_1, \ldots, y_n, z_{11}^{t_1}, \ldots, z_{1m_1}^{t_1}, \ldots, z_{n1}^{t_n}, \ldots, z_{nm_n}^{t_n}) \in S : g_{ii} y_i + \sum_{j=1}^{m_i} h_{ij} z_{ij}^{t_i} =$$

$$F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)|_{(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})}, t_i = 1,2,\ldots,s_i, i = 1,2,\ldots,n\}$$

$$= \{(x, y_1, \ldots, y_n, z_{11}^{t_1}, \ldots, z_{1m_1}^{t_1}, \ldots, z_{n1}^{t_n}, \ldots, z_{nm_n}^{t_n}) \in S : g_{ii} y_i =$$

$$F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)|_{(z_{i1}^{t_i}, z_{i2}^{t_i}, \ldots, z_{im_i}^{t_i})} - \sum_{j=1}^{m_i} h_{ij} z_{ij}^{t_i} = 0, t_i = 1,2,\ldots,s_i, i = 1,2,\ldots,n\}, \quad (5.7)$$

and it can be seen as a piecewise linear equality constraint for problem (5.4). □

In line with Theorem 5.2, the Corollary 5.1 is easily obtained.

Corollary 5.1 The TLMF decision model (5.3) is equivalent to optimizing $f^{(1)}$ over a feasible region comprised of a piecewise linear equality constraint.

Corollary 5.2 An optimal solution to the TLMF decision model (5.3) occurs at a vertex of *IR*.

Proof. According to the equivalent form (5.4) of the TLMF decision model (5.3) and since $f^{(1)}(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n})$ is linear, an optimal solution to the problem must occur at a vertex of *IR* if it exists. □

Theorem 5.3 An optimal solution $(x^*, y_1^*, \ldots, y_n^*, z_{11}^*, \ldots, z_{1m_1}^*, \ldots, z_{n1}^*, \ldots, z_{nm_n}^*)$ to the TLMF decision model (5.3) occurs at a vertex of *S*.

Proof. Let $(x^1, y_1^1, \ldots, y_n^1, z_{11}^1, \ldots, z_{1m_1}^1, \ldots, z_{n1}^1, \ldots, z_{nm_n}^1), \ldots, (x^t, y_1^t, \ldots, y_n^t, z_{11}^t, \ldots, z_{1m_1}^t, \ldots, z_{n1}^t, \ldots, z_{nm_n}^t)$ express the distinct vertices of *S*. Since any point in *S* can be written as a convex combination of these vertices,

$$(x^*, y_1^*, \ldots, y_n^*, z_{11}^*, \ldots, z_{1m_1}^*, \ldots, z_{n1}^*, \ldots, z_{nm_n}^*) = \sum_{r=1}^{\bar{t}} \delta_r (x^r, y_1^r, \ldots, y_n^r, z_{11}^r, \ldots, z_{1m_1}^r, \ldots, z_{n1}^r, \ldots, z_{nm_n}^r)$$

is obtained, where $\sum_{r=1}^{\bar{t}} \delta_r = 1,\ \ \delta_r > 0,\ \ r = 1,2,\ldots,\bar{t}$ and $\bar{t} \le t$. By the convexity of $F_i(x^*, y_1^*, \ldots, y_{i-1}^*, y_{i+1}^*, \ldots, y_n^*)$, let us write the constraints of model (5.3) in the piecewise linear form (5.7) discussed in Theorem 5.2:

$$\begin{aligned}
0 &= F_i(x^*, y_1^*, \ldots, y_{i-1}^*, y_{i+1}^*, \ldots, y_n^*)\,|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} - g_{ii} y_i^* - \sum_{j=1}^{m_i} h_{ij} z_{ij}^* \\
&= F_i(\sum_{r=1}^{\bar{t}} \delta_r (x^r, y_1^r, \ldots, y_{i-1}^r, y_{i+1}^r, \ldots, y_n^r))\,|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} - g_{ii} \sum_{r=1}^{\bar{t}} \delta_r y_i^r - \sum_{j=1}^{m_i} h_{ij} (\sum_{r=1}^{\bar{t}} \delta_r z_{ij}^r) \\
&\le \sum_{r=1}^{\bar{t}} \delta_r F_i(x^r, y_1^r, \ldots, y_{i-1}^r, y_{i+1}^r, \ldots, y_n^r)\,|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} - \sum_{r=1}^{\bar{t}} \delta_r g_{ii} y_i^r - \sum_{r=1}^{\bar{t}} \delta_r (\sum_{j=1}^{m_i} h_{ij} z_{ij}^r) \\
&= \sum_{r=1}^{\bar{t}} \delta_r (F_i(x^r, y_1^r, \ldots, y_{i-1}^r, y_{i+1}^r, \ldots, y_n^r)\,|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} - g_{ii} y_i^r - \sum_{j=1}^{m_i} h_{ij} z_{ij}^r), i = 1,2,\ldots,n. \quad (5.8)
\end{aligned}$$

By the definition of $F_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)\,|_{(z_{i1}^{s_i}, z_{i2}^{s_i}, \ldots, z_{im_i}^{s_i})}$,

$$F_i(x^r, y_1^r, \ldots, y_{i-1}^r, y_{i+1}^r, \ldots, y_n^r)\,|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} =$$

$$\min(g_{ii} y_i + \sum_{j=1}^{m_i} h_{ij} z_{ij}^{t_i}) \le g_{ii} y_i^r + \sum_{j=1}^{m_i} h_{ij} z_{ij}^r, r = 1,2,\ldots,\bar{t}, i = 1,2,\ldots,n.$$

Thus,

$$F_i(x^r, y_1^r, \ldots, y_{i-1}^r, y_{i+1}^r, \ldots, y_n^r)|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} - g_{ii} y_i^r - \sum_{j=1}^{m_i} h_{ij} z_{ij}^r \leq 0, r = 1,2,\ldots,\bar{t}, i = 1,2,\ldots,n.$$

Because the preceding expression (5.8) must be held with $\delta_r > 0$, $r = 1,2,\ldots,\bar{t}$, there must exist $F_i(x^r, y_1^r, \ldots, y_{i-1}^r, y_{i+1}^r, \ldots, y_n^r)|_{(z_{i1}^*, z_{i2}^*, \ldots, z_{im_i}^*)} - g_{ii} y_i^r - \sum_{j=1}^{m_i} h_{ij} z_{ij}^r = 0, r = 1,2,\ldots,\bar{t}$, $i = 1,2,\ldots,n$. These statements imply that $(x^r, y_1^r, \ldots, y_n^r, z_{11}^r, \ldots, z_{1m_1}^r, \ldots, z_{n1}^r, \ldots, z_{nm_n}^r) \in IR$, $r = 1,2,\ldots,\bar{t}$, and that $(x^*, y_1^*, \ldots, y_n^*, z_{11}^*, \ldots, z_{1m_1}^*, \ldots, z_{n1}^*, \ldots, z_{nm_n}^*)$ can be denoted as a convex combination of the points in the *IR*. Since $(x^*, y_1^*, \ldots, y_n^*, z_{11}^*, \ldots, z_{1m_1}^*, \ldots, z_{n1}^*, \ldots, z_{nm_n}^*)$ is a vertex of the *IR* according to Corollary 5.2, there must exist $\bar{t} = 1$, which implies that $(x^*, y_1^*, \ldots, y_n^*, z_{11}^*, \ldots, z_{1m_1}^*, \ldots, z_{n1}^*, \ldots, z_{nm_n}^*)$ is a vertex of *S*. □

Corollary 5.3 If $(x^*, y_1^*, \ldots, y_n^*, z_{11}^*, \ldots, z_{1m_1}^*, \ldots, z_{n1}^*, \ldots, z_{nm_n}^*)$ is a vertex of the *IR*, it is also a vertex of *S*.

5.3 TLMF *K*TH-BEST ALGORITHM AND A NUMERICAL EXAMPLE

This section first presents the TLMF *K*th-Best algorithm for solving the TLMF decision model (5.3) based on related theoretical properties. A numerical example is then used to illustrate how the TLMF *K*th-Best algorithm works.

5.3.1 TLMF *K*TH-BEST ALGORITHM DESCRIPTION

Theorem 5.3 and Corollary 5.3 imply that an optimal solution to the TLMF decision model (5.3) can be found by enumerating vertices (also called extreme points) of the constraint region *S*, which clearly provide an appropriate way to develop the following TLMF *K*th-Best algorithm to solve the problem. According to the notations and theoretical foundation respectively defined and demonstrated in Section 5.2, the main principle of the TLMF *K*th-Best algorithm is proposed as follows.

To begin, consider the following linear programming problem:

$$\min\{cx+\sum_{i=1}^{n}d_i y_i+\sum_{i=1}^{n}\sum_{j=1}^{m_i}e_{ij}z_{ij}:(x,y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,z_{n1},\ldots,z_{nm_n})\in S\}. \tag{5.9}$$

Let the vertices

$$(x^1,y_1^1,\ldots,y_n^1,z_{11}^1,\ldots,z_{1m_1}^1,\ldots,z_{n1}^1,\ldots,z_{nm_n}^1),\ldots,(x^N,y_1^N,\ldots,y_n^N,z_{11}^N,\ldots,z_{1m_1}^N,\ldots,z_{n1}^N,\ldots,z_{nm_n}^N)$$

of the constraint region S denote the N-ranked basic feasible solutions to problem (5.9), such that

$$cx^k+\sum_{i=1}^{n}d_i y_i^k+\sum_{i=1}^{n}\sum_{j=1}^{m_i}e_{ij}z_{ij}^k\le cx^{k+1}+\sum_{i=1}^{n}d_i y_i^{k+1}+\sum_{i=1}^{n}\sum_{j=1}^{m_i}e_{ij}z_{ij}^{k+1}, k=1,2,\ldots,N-1.$$

Solving the equivalent problem (5.4) of model (5.3) is then equivalent to finding the index

$$K^*=\min\{k\in\{1,2,\ldots,N\}:(x^k,y_1^k,\ldots,y_n^k,z_{11}^k,\ldots,z_{1m_1}^k,\ldots,z_{n1}^k,\ldots,z_{nm_n}^k)\in IR\},$$

which ensures that $(x^{K^*},y_1^{K^*},\ldots,y_n^{K^*},z_{11}^{K^*},\ldots,z_{1m_1}^{K^*},\ldots,z_{n1}^{K^*},\ldots,z_{nm_n}^{K^*})$ is an optimal solution to model (5.3). Therefore, it needs to be verified that whether or not $(x^{K^*},y_1^{K^*},\ldots,y_n^{K^*},z_{11}^{K^*},\ldots,z_{1m_1}^{K^*},\ldots,z_{n1}^{K^*},\ldots,z_{nm_n}^{K^*})\in IR$ under the condition $(x^{K^*},y_1^{K^*},\ldots,y_n^{K^*},$ $z_{11}^{K^*},\ldots,z_{1m_1}^{K^*},\ldots,z_{n1}^{K^*},\ldots,z_{nm_n}^{K^*})\in S$. If $(y_i^{K^*},z_{i1}^{K^*},\ldots,z_{im_i}^{K^*})\in P_i(x^{K^*},y_1^{K^*},\ldots,y_{i-1}^{K^*},y_{i+1}^{K^*},\ldots,y_n^{K^*})$ for all $i=1,2,\ldots,n$ that means $(y_i^{K^*},z_{i1}^{K^*},\ldots,z_{im_i}^{K^*})$ is an optimal solution to the problem (5.3c-5.3f) under the fixed $x=x^{K^*},y_1=y_1^{K^*},\ldots,y_{i-1}=y_{i-1}^{K^*},$ $y_{i+1}=y_{i+1}^{K^*},\ldots,y_n=y_n^{K^*}$ for $i=1,2,\ldots,n$, there exists $(y_1^{K^*},\ldots,y_n^{K^*},z_{11}^{K^*},\ldots,z_{1m_1}^{K^*},\ldots,$ $z_{n1}^{K^*},\ldots,z_{nm_n}^{K^*})\in P(x^{K^*})$, thus, $(x^{K^*},y_1^{K^*},\ldots,y_n^{K^*},z_{11}^{K^*},\ldots,z_{1m_1}^{K^*},\ldots,z_{n1}^{K^*},\ldots,z_{nm_n}^{K^*})\in IR$ by Definition 5.9. As this requires finding the K^*th best vertex of S to obtain an optimal solution to model (5.3), the algorithm is named the TLMF *K*th-Best algorithm.

Second, it needs to be verified that whether or not $(y_i^{K^*}, z_{i1}^{K^*}, \ldots, z_{im_i}^{K^*}) \in P_i(x^{K^*}, y_1^{K^*}, \ldots, y_{i-1}^{K^*}, y_{i+1}^{K^*}, \ldots, y_n^{K^*})$ through solving problem (5.3c-5.3f). For $i = 1,2,\ldots,n$ and fixing the given $x = x^{K^*}, y_1 = y_1^{K^*}, \ldots, y_{i-1} = y_{i-1}^{K^*}, y_{i+1} = y_{i+1}^{K^*}, \ldots, y_n = y_n^{K^*}$, problem (5.3c-5.3f) can be seen as the problem (5.10) by Definition 5.9:

$$\min\{c_i x + \sum_{s=1}^{n} g_{is} y_s + \sum_{j=1}^{m_i} h_{ij} z_{ij} : (y_i, z_{i1}, \ldots, z_{im_i}) \in P_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)\}. \qquad (5.10)$$

For the given $x = x^{K^*}, y_1 = y_1^{K^*}, \ldots, y_{i-1} = y_{i-1}^{K^*}, y_{i+1} = y_{i+1}^{K^*}, \ldots, y_n = y_n^{K^*}$, consider the following linear programming problem (5.11):

$$\min\{c_i x + \sum_{s=1}^{n} g_{is} y_s + \sum_{j=1}^{m_i} h_{ij} z_{ij} : (y_i, z_{i1}, \ldots, z_{im_i}) \in S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)\} \qquad (5.11)$$

and the vertices $(y_i^1, z_{i1}^1, \ldots, z_{im_i}^1), \ldots, (y_i^{N_i}, z_{i1}^{N_i}, \ldots, z_{im_i}^{N_i})$ of $S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ become the ranked basic feasible solutions to problem (5.11), such that

$$c_i x^{K^*} + \sum_{\substack{s=1,\\ s\neq i}}^{n} g_{is} y_s^{K^*} + g_{ii} y_i^{k_i} + \sum_{j=1}^{m_i} h_{ij} z_{ij}^{k_i} \le c_i x^{K^*} + \sum_{\substack{s=1,\\ s\neq i}}^{n} g_{is} y_s^{K^*} + g_{ii} y_i^{k_i+1} + \sum_{j=1}^{m_i} h_{ij} z_{ij}^{k_i+1},$$

$$k_i = 1,2,\ldots,N_i - 1,\ i = 1,2,\ldots,n\,.$$

Solving problem (5.10) is then equivalent to finding the index $K_i^* = \min\{k_i \in \{1,2,\ldots,N_i\} : (y_i^{k_i}, z_{i1}^{k_i}, \ldots, z_{im_i}^{k_i}) \in P_i(x^{K^*}, y_1^{K^*}, \ldots, y_{i-1}^{K^*}, y_{i+1}^{K^*}, \ldots, y_n^{K^*})\}$, which ensures that $(y_1^{K_i^*}, z_{i1}^{K_i^*}, \ldots, z_{im_i}^{K_i^*})$ is an optimal solution to (5.3c-5.3f) where i=1,2,...,n. If $(y_1^{K_i^*}, z_{i1}^{K_i^*}, \ldots, z_{im_i}^{K_i^*}) = (y_i^{K^*}, z_{i1}^{K^*}, \ldots, z_{im_i}^{K^*})$, $(y_i^{K^*}, z_{i1}^{K^*}, \ldots, z_{im_i}^{K^*}) \in P_i(x^{K^*}, y_1^{K^*}, \ldots, y_{i-1}^{K^*}, y_{i+1}^{K^*}, \ldots, y_n^{K^*})$ is obtained.

Before the detailed procedures of the TLMF *K*th-Best algorithm are presented, the notations used in the algorithm are explained in Table 5.1.

Table 5.1 Notations used in the TLMF *K*th-Best algorithm

Notation	Explanation
k	Current iteration number for solving the TLMF decision model (5.3)
T	The feasible vertices set of S that has been searched for solving model (5.3)
W	The feasible vertices set of S that needs to be searched for solving model (5.3)
i	The ith middle-level follower
n	The total number of middle-level followers
W_k	The adjacent vertices set of the current vertex $(x^k, y_1^k, \ldots, y_n^k, z_{11}^k, \ldots, z_{1m_1}^k, \ldots, z_{n1}^k, \ldots, z_{nm_n}^k)$ over S
K^*	The iteration number when finding an optimal solution to model (5.3)
k_i	Current iteration number for solving problem (5.3c-5.3f) involving the ith middle-level follower and its bottom-level followers
T_i	The feasible vertices set of $S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ that has been searched for solving problem (5.3c-5.3f)
W_i'	The feasible vertices set of $S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ that needs to be searched for solving problem (5.3c-5.3f)
j	The jth bottom-level follower attached to the ith middle-level follower
m_i	The total number of bottom-level followers attached to the ith middle-level follower
W_{k_i}	The adjacent vertices set of the vertex $(y_i^{ik_i}, z_{i1}^{ik_i}, \ldots, z_{im_i}^{ik_i})$ over $S_i(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$
K_i^*	The iteration number when finding an optimal solution to problem (5.3c-5.3f)

Algorithm 5.1: TLMF *K*th-Best algorithm

[Begin]

Step 1: Set k=1, adopt the simplex method to obtain an optimal solution $(x^1, y_1^1, \ldots, y_n^1, z_{11}^1, \ldots, z_{1m_1}^1, \ldots, z_{n1}^1, \ldots, z_{nm_n}^1)$ to the linear programming problem (5.9). Let $T = \varnothing$ and $W = \{(x^1, y_1^1, \ldots, y_n^1, z_{11}^1, \ldots, z_{1m_1}^1, \ldots, z_{n1}^1, \ldots, z_{nm_n}^1)\}$. Set i=1 and go to Step 2.

Step 2: Put $x = x^k, y_1 = y_1^k, \ldots, y_{i-1} = y_{i-1}^k, y_{i+1} = y_{i+1}^k, \ldots, y_n = y_n^k$, solve the problem (5.3c-5.3f) or problem (5.10) and obtain an optimal solution $(\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i})$ using the following subroutine Step 2.1-Step 2.5. Then go to Step 3.

Step 2.1: Set $x = x^k$ and $k_i = 1$, adopt the simplex method to obtain the optimal solution $(y_i^{i1}, z_{i1}^{i1}, \ldots, z_{im_i}^{i1})$ to the linear programming problem (5.11). Within the subroutine, let $T_i = \varnothing$ and $W_i' = \{(y_1^{i1}, z_{i1}^{i1}, \ldots, z_{im_i}^{i1})\}$. Set *j*=1 and go to Step 2.2.

Step 2.2: Put $x = x^k, y_i = y_i^{ik_i}, z_{i1} = z_{i1}^{ik_i}, \ldots, z_{i(j-1)} = z_{i(j-1)}^{ik_i}, z_{i(j+1)} = z_{i(j+1)}^{ik_i}, \ldots,$ $z_{im_i} = z_{im_i}^{ik_i}$ and adopt the simplex method to solve the problem (5.12):

$$\min\{c_{ij}x + p_{ij}y_i + \sum_{t=1}^{m_i} q_{ijt}z_{it} :$$

$$z_{ij} \in S_{ij}(x, y_i, z_{i1}, \ldots, z_{i(j-1)}, z_{i(j+1)}, \ldots, z_{im_i})\} \tag{5.12}$$

and obtain the optimal solution $\tilde{z}_{ij}$.

Step 2.3: If $\tilde{z}_{ij} \neq z_{ij}^{ik_i}$, go to Step 2.4. If $\tilde{z}_{ij} = z_{ij}^{ik_i}$ and $j \neq m_i$, set *j*=*j*+1 and go to Step 2.2. If $\tilde{z}_{ij} = z_{ij}^{ik_i}$ and $j = m_i$, stop the subroutine, $K_i^* = k_i$ and go to Step 2 with $(\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) = (y_i^{ik_i}, z_{i1}^{ik_i}, \ldots, z_{im_i}^{ik_i})$.

Step 2.4: Let W_{k_i} denote the set of adjacent vertices of $(y_i^{ik_i}, z_{i1}^{ik_i}, \ldots, z_{im_i}^{ik_i})$ that $(y_i, z_{i1}, \ldots, z_{im_i}) \in W_{k_i}$ implies

$$c_i x^k + \sum_{\substack{s=1,\\ s\neq i}}^{n} g_{is}y_s^k + g_{ii}y_i + \sum_{j=1}^{m_i} h_{ij}z_{ij} \geq c_i x^k + \sum_{\substack{s=1,\\ s\neq i}}^{n} g_{is}y_s^k + g_{ii}y_i^{ik_i} + \sum_{j=1}^{m_i} h_{ij}z_{ij}^{ik_i} .$$

Let $T_i = T_i \cup \{(y_i^{ik_i}, z_{i1}^{ik_i}, \ldots, z_{im_i}^{ik_i})\}$ and $W_i' = (W_i' \cup W_{k_i}) \setminus T_i$. Go to Step 2.5.

Step 2.5: Set $k_i = k_i + 1$ and choose $(y_i^{ik_i}, z_{i1}^{ik_i}, \ldots, z_{im_i}^{ik_i})$ such that

$$c_i x^k + \sum_{\substack{s=1,\\ s\neq i}}^{n} g_{is}y_s^k + g_{ii}y_i^{ik_i} + \sum_{j=1}^{m_i} h_{ij}z_{ij}^{ik_i}$$

$$= \min\{c_i x^k + \sum_{\substack{s=1,\\ s\neq i}}^{n} g_{is} y_s^k + g_{ii} y_i + \sum_{j=1}^{m_i} h_{ij} z_{ij} : (y_i, z_{i1}, \ldots, z_{im_i}) \in W_i'\}.$$

Set j=1 and go to Step 2.2.

Step 3: If $(\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) \neq (y_i^k, z_{i1}^k, \ldots, z_{im_i}^k)$, go to Step 4. If $(\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) = (y_i^k, z_{i1}^k, \ldots, z_{im_i}^k)$ and $i \neq n$, set i=i+1 and go to Step 2. If $(\hat{y}_i, \hat{z}_{i1}, \ldots, \hat{z}_{im_i}) = (y_i^k, z_{i1}^k, \ldots, z_{im_i}^k)$ and $i = n$, stop and $(x^k, y_1^k, \ldots, y_n^k, z_{11}^k, \ldots, z_{1m_1}^k, \ldots, z_{n1}^k, \ldots, z_{nm_n}^k)$ is an optimal solution to the TLMF decision model (5.3) and $K^* = k$.

Step 4: Let W_k denote the set of adjacent vertices of $(x^k, y_1^k, \ldots, y_n^k, z_{11}^k, \ldots, z_{1m_1}^k, \ldots, z_{n1}^k, \ldots, z_{nm_n}^k)$ such that $(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots,$ $z_{n1}, \ldots, z_{nm_n}) \in W_k$ implies

$$cx + \sum_{i=1}^{n} d_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m_i} e_{ij} z_{ij} \geq cx^k + \sum_{i=1}^{n} d_i y_i^k + \sum_{i=1}^{n} \sum_{j=1}^{m_i} e_{ij} z_{ij}^k .$$

Let $T = T \cup \{(x^k, y_1^k, \ldots, y_n^k, z_{11}^k, \ldots, z_{1m_1}^k, \ldots, z_{n1}^k, \ldots, z_{nm_n}^k)\}$ and $W = (W \cup W_k) \setminus T$. Go to Step 5.

Step 5: Set k=k+1 and choose $(x^k, y_1^k, \ldots, y_n^k, z_{11}^k, \ldots, z_{1m_1}^k, \ldots, z_{n1}^k, \ldots, z_{nm_n}^k)$ such that

$$cx^k + \sum_{i=1}^{n} d_i y_i^k + \sum_{i=1}^{n} \sum_{j=1}^{m_i} e_{ij} z_{ij}^k$$

$$= \min\{cx + \sum_{i=1}^{n} d_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m_i} e_{ij} z_{ij} : (x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}) \in W\}.$$

Set i=1 and go to Step 2.

[End]

Within the TLMF *K*th-Best algorithm, Step 2 and its subroutine (Step 2.1-2.5) are adopted to obtain an optimal solution to problem (5.3c-5.3f) of the ith middle-level follower and its bottom-level followers under the given decision

$(x, y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$ from the leader and other middle-level followers. Step 3 is repeatedly performed to see whether or not the current vertex is an element involved in the *IR*. If the current vertex occurs outside *IR*, the algorithm will go to Step 4 in which the adjacent vertices of the current vertex will be found and added to the vertices set *W* that needs to be searched. Step 5 is developed to choose a vertex from the vertices set *W* to optimize the objective function of problem (5.9) and prepare for the next iteration to verify whether or not the vertex is an element of the *IR*.

5.3.2 A NUMERICAL EXAMPLE

A simple numerical example is used to illustrate how the TLMF *K*th-Best algorithm works. Consider that the example involves one leader, two middle-level followers and two bottom-level followers attached to each middle-level follower, which means that $n = 2$, $m_1 = m_2 = 2$ in model (5.3). For $X = \{x : x \geq 0\}$, $Y_i = \{y_i : y_i \geq 0\}$, $Z_{ij} = \{z_{ij} : z_{ij} \geq 0\}$, $j = 1,2,\ i = 1,2$, coefficients of the decision variables in model (5.3) are shown in Table 5.2.

Table 5.2 Coefficients of model (5.3)

Decision entity	Coefficients of model (5.3)	
	Coefficients of objective functions	Coefficients of constraint conditions
Leader	$c = -1, d_1 = 1, d_2 = -2, e_{11} = 3,$ $e_{12} = 1, e_{21} = -1, e_{22} = -1$	$A = (1,1)^T, B_1 = (1,0)^T, B_2 = (1,0)^T, C_{11} = (1,0)^T,$ $C_{12} = (2,0)^T, C_{21} = (2,0)^T, C_{22} = (1,0)^T, b = (14,1.5)^T$
Follower 1	$c_1 = 1, g_{11} = -1, g_{12} = 1, h_{11} = 1, h_{12} = 1$	$A_1 = (2,0)^T, D_{11} = (1,1)^T, D_{12} = (1,0)^T, E_{11} = (1,0)^T,$ $E_{12} = (1,0)^T, b_1 = (8.5,1)^T$
Follower 2	$c_2 = 1, g_{21} = -1, g_{22} = 2, h_{21} = 1, h_{22} = 1$	$A_2 = (1,0)^T, D_{21} = (-1,-1)^T, D_{22} = (1,-1)^T, E_{21} = (1,0)^T,$ $E_{22} = (1,0)^T, b_2 = (3,-1.5)^T$
Follower 11	$c_{11} = 1, p_{11} = 1, q_{111} = 2, q_{112} = 1$	$A_{11} = (-1,0)^T, P_{11} = (-1,0)^T, Q_{111} = (-1,1)^T,$ $Q_{112} = (-2,0)^T, b_{11} = (-10,3)^T$
Follower 12	$c_{12} = 2, p_{12} = 1, q_{121} = 1, q_{122} = 1$	$A_{12} = (-1,1)^T, P_{12} = (-1,0)^T, Q_{121} = (-1,1)^T,$ $Q_{122} = (-1,1)^T, b_{12} = (-7,6)^T$
Follower 21	$c_{21} = 1, p_{21} = 1, q_{211} = -3, q_{212} = 1$	$A_{21} = (-1,0)^T, P_{21} = (-1,1)^T, Q_{211} = (-1,1)^T,$ $Q_{212} = (-2,0)^T, b_{21} = (-5.5,1.5)^T$
Follower 22	$c_{22} = 1, p_{22} = 2, q_{221} = 2, q_{222} = -1$	$A_{22} = (1,0)^T, P_{22} = (1,0)^T, Q_{221} = (1,1)^T,$ $Q_{222} = (1,1)^T, b_{22} = (4,2.5)^T$

Detailed procedures of the TLMF *K*th-Best algorithm that are executed to solve the numerical example are shown as follows.

Iteration 1

Step 1: Set k=1 and adopt the simplex method to obtain an optimal solution to the following linear programming problem (5.13) in the format (5.9):

$$\min\{cx+\sum_{i=1}^{2}d_i y_i+\sum_{i=1}^{2}\sum_{j=1}^{2}e_{ij}z_{ij}:(x,y_1,y_2,z_{11},z_{12},z_{21},z_{22})\in S\}. \tag{5.13}$$

The optimal solution to problem (5.13) is $(x^1,y_1^1,y_2^1,z_{11}^1,z_{12}^1,z_{21}^1,z_{22}^1)=$ $(1,1,0.5,0.5,4.5,0,2)$ and now $T=\varnothing$, W={(1,1,0.5,0.5,4.5,0,2)}. Set i=1 and go to Step 2.

Step 2: Put $x=x^1=1$, $y_2=y_2^1=0.5$ and solve the problem (5.14) of the middle-level follower i(=1) and its bottom-level followers in the format (5.10):

$$\min\{c_1x+\sum_{s=1}^{2}g_{1s}y_s+\sum_{j=1}^{2}h_{1j}z_{1j}:(y_1,z_{11},z_{12})\in P_1(x,y_2)\}. \tag{5.14}$$

An optimal solution $(\hat{y}_1,\hat{z}_{11},\hat{z}_{12})=(1,2,3)$ can be obtained by Steps 2.1-2.5 of the TLMF *K*th-Best algorithm and go to Step 3.

Step 3: $(\hat{y}_1,\hat{z}_{11},\hat{z}_{12})\neq(y_1^1,z_{11}^1,z_{12}^1)$ and go to Step 4.

Step 4: Find the set W_1 of adjacent vertices of $(x^1,y_1^1,y_2^1,z_{11}^1,z_{12}^1,z_{21}^1,z_{22}^1)$ and now $W_1=\{(1,1,0.5,1,4,0,2.5),(1,1,0.5,2,3,1,1.5),(1,1,0.5,2,3,0,2),(0,1,1.5,2.5,3.5,0,2),(0,1,0.5,2,4,0,2.5)\}$, $T=T\cup\{(1,1,0.5,0.5,4.5,0,2)\}=\{(1,1,0.5,0.5,4.5,0,2)\}$, $W=W\cup W_1\setminus T=W_1$. Go to Step 5.

Step 5: Set k=k+1=2 and choose $(x^2,y_1^2,y_2^2,z_{11}^2,z_{12}^2,z_{21}^2,z_{22}^2)=(1,1,0.5,1,4,0,2.5)$ from the vertices set W such that

$$f^{(1)}(x^2, y_1^2, y_2^2, z_{11}^2, z_{12}^2, z_{21}^2, z_{22}^2)$$
$$= \min\{cx + \sum_{i=1}^{2} d_i y_i + \sum_{i=1}^{2}\sum_{j=1}^{2} e_{ij} z_{ij} : (x, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22}) \in W\},$$

set i=1 and go to Step 2, and the second iteration will begin.

Iteration 2

Step 2: Put $x = x^2 = 1$, $y_2 = y_2^2 = 0.5$ and solve the problem (5.14). An optimal solution $(\hat{y}_1, \hat{z}_{11}, \hat{z}_{12}) = (1,2,3)$ can be obtained by Steps 2.1-2.5 of the TLMF *K*th-Best algorithm and go to Step 3.

Step 3: $(\hat{y}_1, \hat{z}_{11}, \hat{z}_{12}) \neq (y_1^2, z_{11}^2, z_{12}^2)$ and go to Step 4.

Step 4: Find the set W_2 of adjacent vertices of $(x^2, y_1^2, y_2^2, z_{11}^2, z_{12}^2, z_{21}^2, z_{22}^2)$ and now $W_2 = \{(1,1,0.5,2,3,0,2.5),(0,1,1.5,3,3,0,2.5)\}$, $T = T \cup \{(1,1,0.5,1,4,0,2.5)\} = \{(1,1,0.5,0.5,4.5,0,2),(1,1,0.5,1,4,0,2.5)\}$, $W = W \cup W_2 \setminus T = \{(1,1,0.5,2,3,1,1.5),(1,1,0.5,2,3,0,2),(0,1,1.5,2.5,3.5,0,2),(0,1,0.5,2,4,0,2.5),(1,1,0.5,2,3,0,2.5),(0,1,1.5,3,3,0,2.5)\}$. Go to Step 5.

Step 5: Set k=k+1=3 and choose $(x^3, y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) = (1,1,0.5,2,3,1,1.5)$ from the vertices set W such that

$$f^{(1)}(x^3, y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) =$$
$$\min\{cx + \sum_{i=1}^{2} d_i y_i + \sum_{i=1}^{2}\sum_{j=1}^{2} e_{ij} z_{ij} : (x, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22}) \in W\},$$

set i=1 and go to Step 2, and the third iteration will begin.

Iteration 3

Step 2: Put $x = x^3 = 1$, $y_2 = y_2^3 = 0.5$ and solve the problem (5.14). An optimal solution $(\hat{y}_1, \hat{z}_{11}, \hat{z}_{12}) = (1,2,3)$ is obtained by Steps 2.1-2.5 of the TLMF *K*th-Best algorithm and go to Step 3.

Step 3: Clearly, $(\hat{y}_1, \hat{z}_{11}, \hat{z}_{12}) = (y_1^3, z_{11}^3, z_{12}^3) = (1,2,3)$, and $i \neq n$, set i=i+1=2 and go to Step 2.

Step 2: Put $x = x^3 = 1$, $y_1 = y_1^3 = 1$ and solve the problem (5.15) of the middle-level follower i(=2) and its bottom-level followers in the format (5.10):

$$\min\{c_2 x + \sum_{s=1}^{2} g_{2s} y_s + \sum_{j=1}^{2} h_{2j} z_{2j} : (y_1, z_{21}, z_{22}) \in P_2(x, y_1)\}. \tag{5.15}$$

An optimal solution $(\hat{y}_2, \hat{z}_{21}, \hat{z}_{22}) = (0.5,1,1.5)$ is obtained by Steps 2.1-2.5 of the TLMF *K*th-Best algorithm and go to Step 3.

Step 3: $(\hat{y}_2, \hat{z}_{21}, \hat{z}_{22}) = (y_2^3, z_{21}^3, z_{22}^3) = (0.5,1,1.5)$ and i=n=2, stop and $(x^3, y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) = (1,1,0.5,2,3,1,1.5)$ is an optimal solution to the Example 5.1 and the iteration number $K^* = k = 3$.

An optimal solution is finally obtained to the numerical example through three iterations, which means that three vertices are enumerated to get an optimal solution. The objective function values of all decision entities are $f^{(1)} = 5.5$, $f_1^{(2)} = 5.5$, $f_2^{(2)} = 3.5$, $f_{11}^{(3)} = 9.0$, $f_{12}^{(3)} = 8.0$, $f_{21}^{(3)} = 0$, $f_{22}^{(3)} = 2.5$. Thus, the TLMF *K*th-Best algorithm provides a convenient way to solve linear TLMF decision problems.

5.4 SOLUTION EVALUATION

The proposed TLMF *K*th-Best algorithm is able to find an optimal solution to the TLMF decision model (5.3). However, it is difficult to illustrate the operations of the complex TLMF decision-making process by the optimal solution defined by Definition 5.9 because the solution only represents the decision result rather than the decision-making process. In this section, a fuzzy programming approach is used to evaluate the solution obtained and illustrate why decision entities have to achieve and accept the final result during the TLMF decision-making process.

Within a TLMF decision-making process, each decision entity seeks to optimize its own objective but its decision is affected by actions of others, thus, decision entities achieve a compromised result with a possible relaxation rather than their individual best solutions as desired. Since it is imprecise or ambiguous for decision entities to identify a compromised result whether or not they desire it, the objective functions can be transformed into fuzzy goals using an imprecise aspiration level. $f^{\min}$ and $f^{\max}$ are used to denote the individual best and worst results respectively that a decision entity may achieve. Finally, the compromised objective value of the decision entity must be involved in the interval $[f^{\min}, f^{\max}]$. Therefore, membership functions $\mu(f)$ can be elicited to characterize fuzzy goals over the domain $[f^{\min}, f^{\max}]$ for the objective functions, which can also be adopted to describe the satisfactory degree of decision entities towards a solution or an objective value. For example, a decision entity specifies the objective value f^0 such that the satisfactory degree is 0, that is $\mu(f^0) = 0$, while the value f^1 of the objective function such that $\mu(f^1) = 1$ means that the satisfactory degree is 1. Clearly, if an objective value f is undesired (larger) than f^0, it is defined that $\mu(f) = 0$; whereas $\mu(f) = 1$ if an objective value f is desired (smaller) than f^1. In this study, for the sake of simplicity, f^0 and f^1 are specified as $f^0 = f^{\max}$ and $f^1 = f^{\min}$, and that the membership functions are linear versions shown as Figure 5.2 although they do not always need to be linear. Also, note that in this research the satisfactory degree $\mu(f) = 1$ if there exists $f^{\min} = f^{\max}$.

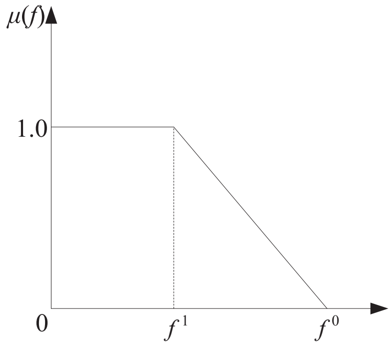


Figure 5.2 Linear membership function

(1) The membership function of the leader

The individual best objective value of the leader is:

$$f^1 = f^{\min} = \min\{f^{(1)}(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}):$$

$$(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}) \in S\}.$$

The individual worst objective value is:

$$f^0 = f^{\max} = \max\{f^{(1)}(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}):$$

$$(x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{n1}, \ldots, z_{nm_n}) \in S\}.$$

The corresponding linear membership function $\mu(f^{(1)})$ is defined as:

$$\mu(f^{(1)}) = \begin{cases} 0, & f^{(1)} \geq f^0, \\ \dfrac{f^{(1)} - f^0}{f^1 - f^0}, & f^1 < f^{(1)} < f^0, \\ 1, & f^{(1)} \leq f^1. \end{cases} \tag{5.16}$$

$\mu(f^{(1)})$ can be used to denote the satisfactory degree of the leader towards an objective value $f^{(1)}$. $\mu(f^0) = 0$ implies that the satisfactory degree of the leader is 0 when the objective value $f^{(1)} = f^0$, while the objective value $f^{(1)} = f^1$ such that $\mu(f^1) = 1$ means that the satisfactory degree of the leader becomes 1.

(2) The membership function of the middle-level follower *i*

The middle-level follower i makes its decision under the given decisions $(x^*, y_1^*, \ldots, y_{i-1}^*, y_{i+1}^*, \ldots, y_n^*)$ from the leader and other middle-level followers, thus, its individual best objective value is:

$$f_i^1 = f_i^{\min} = \min\{f_i^{(2)}(x^*, y_1^*, \ldots, y_{i-1}^*, y_i, y_{i+1}^*, \ldots, y_n^*, z_{i1}, \ldots, z_{im_i}):$$

$$(y_i, z_{i1}, \ldots, z_{im_i}) \in S_i(x^*, y_1^*, \ldots, y_{i-1}^*, y_{i+1}^*, \ldots, y_n^*)\}.$$

The individual worst objective value is:

$$f_i^0 = f_i^{\max} = \max\{f_i^{(2)}(x^*, y_1^*, \ldots, y_{i-1}^*, y_i, y_{i+1}^*, \ldots, y_n^*, z_{i1}, \cdots, z_{im_i}):$$

$$(y_i, z_{i1}, \ldots, z_{im_i}) \in S_i(x^*, y_1^*, \ldots, y_{i-1}^*, y_{i+1}^*, \ldots, y_n^*)\}.$$

The corresponding linear membership function $\mu_i(f_i^{(2)})$ is defined as:

$$\mu_i(f_i^{(2)}) = \begin{cases} 0, & f_i^{(2)} \geq f_i^0, \\ \dfrac{f_i^{(2)} - f_i^0}{f_i^1 - f_i^0}, & f_i^1 < f_i^{(2)} < f_i^0, \\ 1, & f_i^{(2)} \leq f_i^1. \end{cases} \tag{5.17}$$

$\mu_i(f_i^{(2)})$ can be used to denote the satisfactory degree of the middle-level follower i towards an objective value $f_i^{(2)}$. $\mu_i(f_i^0) = 0$ implies that the satisfactory degree of the middle-level follower i is 0 when the objective value $f_i^{(2)} = f_i^0$, whereas the objective value $f_i^{(2)} = f_i^1$ such that $\mu_i(f_i^1) = 1$ means that the satisfactory degree becomes 1.

(3) The membership function of the bottom-level follower *ij*

The bottom-level follower *ij* makes its decision under the given decision $(x^*, y_i^*, z_{i1}^*, \ldots, z_{i(j-1)}^*, z_{i(j+1)}^*, \ldots, z_{im_i}^*)$ from the leader, the middle-level follower *i* and its own counterparts, thus, its individual best objective value is:

$$f_{ij}^1 = f_{ij}^{\min} = \min\{f_{ij}^{(3)}(x^*, y_i^*, z_{i1}^*, \ldots, z_{i(j-1)}^*, z_{ij}, z_{i(j+1)}^*, \ldots, z_{im_i}^*):$$
$$z_{ij} \in S_{ij}(x^*, y_i^*, z_{i1}^*, \ldots, z_{i(j-1)}^*, z_{i(j+1)}^*, \ldots, z_{im_i}^*)\}.$$

The individual worst objective value is:

$$f_{ij}^0 = f_{ij}^{\max} = \max\{f_{ij}^{(3)}(x^*, y_i^*, z_{i1}^*, \ldots, z_{i(j-1)}^*, z_{ij}, z_{i(j+1)}^*, \ldots, z_{im_i}^*):$$
$$z_{ij} \in S_{ij}(x^*, y_i^*, z_{i1}^*, \ldots, z_{i(j-1)}^*, z_{i(j+1)}^*, \ldots, z_{im_i}^*)\}.$$

The corresponding linear membership function $\mu_{ij}(f_{ij}^{(3)})$ is defined as:

$$\mu_{ij}(f_{ij}^{(3)}) = \begin{cases} 0, & f_{ij}^{(3)} \geq f_{ij}^0, \\ \dfrac{f_{ij}^{(3)} - f_{ij}^0}{f_{ij}^1 - f_{ij}^0}, & f_{ij}^1 < f_{ij}^{(3)} < f_{ij}^0, \\ 1, & f_{ij}^{(3)} \leq f_{ij}^1. \end{cases} \tag{5.18}$$

$\mu_{ij}(f_{ij}^{(3)})$ can be used to denote the satisfactory degree of the bottom-level follower *ij* towards an objective value $f_{ij}^{(3)}$. $\mu_{ij}(f_{ij}^0) = 0$ implies that the satisfactory degree is 0 when the objective value $f_{ij}^{(3)} = f_{ij}^0$, while the objective value $f_{ij}^{(3)} = f_{ij}^1$ such that $\mu_{ij}(f_{ij}^1) = 1$ means that the satisfactory degree is 1.

The proposed evaluation method based on fuzzy programming is applied to assess the solution to the numerical example in Section 5.3.2. For the leader, $f^1 = f^{\min} = f^{(1)}(1,1,0.5,0.5,4.5,0,2) = 3$ and $f^0 = f^{\max} = f^{(1)}(0,1,0.5,3,3,0,2.5) = 9.5$, thus, by the formula (5.16) the leader's satisfactory degrees are 1.0, 0.92 and 0.62 respectively towards the vertices $(x^1, y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (1,1,0.5,0.5,4.5,0,2)$,

$(x^2, y_1^2, y_2^2, z_{11}^2, z_{12}^2, z_{21}^2, z_{22}^2) = (1,1,0.5,1,4,0,2.5)$ and $(x^3, y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) = (1,1,0.5,2,3,1,1.5)$. By the formulas (5.17) and (5.18), the satisfactory degrees of multiple followers towards each solution are presented in Table 5.3.

Table 5.3 Objective values and corresponding satisfactory degrees of decision entities in Example 5.1

Vertex	Leader		Follower 1		Follower 2		Follower 11		Follower 12		Follower 21		Follower 22	
	$f^{(1)}$	$\mu(f^{(1)})$	$f_1^{(2)}$	$\mu_1(f_1^{(2)})$	$f_2^{(2)}$	$\mu_2(f_2^{(2)})$	$f_{11}^{(3)}$	$\mu_{11}(f_{11}^{(3)})$	$f_{12}^{(3)}$	$\mu_{12}(f_{12}^{(3)})$	$f_{21}^{(3)}$	$\mu_{21}(f_{21}^{(3)})$	$f_{22}^{(3)}$	$\mu_{22}(f_{22}^{(3)})$
(1,1,0.5,0.5,4.5,0,2)	3.0	1.0	5.5	1.0	3.0	1.0	7.5	0.83	8.0	1.0	3.5	0	0	0.80
(1,1,0.5,1,4,0,2.5)	3.5	0.92	5.5	1.0	3.5	0.67	8.0	0.67	8.0	1.0	4.0	0	-0.5	1.0
(1,1,0.5,2,3,1,1.5)	5.5	0.62	5.5	1.0	3.5	0.67	9.0	1.0	8.0	1.0	0	1.0	2.5	1.0

As shown in Table 5.3, the vertex $(x^1, y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (1,1,0.5, 0.5,4.5,0,2)$ is the individual best solution to the leader such that the satisfactory degree is 1.0, thus the leader anticipates that the followers can select $(y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22}) = (1,0.5,0.5,4.5,0,2)$ to respond to its own decision $x=1$. Under the decision $x=1$ given by the leader, the middle-level followers make their decisions $(y_1, y_2) = (1,0.5)$ as desired by the leader, and they also desire that the bottom-level followers can react to the given decision $(x, y_1, y_2) = (1,1,0.5)$ by determining $(z_{11}, z_{12}, z_{21}, z_{22}) = (0.5,4.5,0,2)$ because their satisfactory degrees are both 1.0 under the solution. However, in view of the given decision by the leader and the middle-level follower 1, the bottom-level followers 11 and 12 will not choose the decision $(z_{11}, z_{12}) = (0.5,4.5)$ that are desired by the leader and the middle-level follower 1 since they still have space to optimize their objectives and improve their satisfactory degrees. Thus, $(z_{11}, z_{12}) = (0.5,4.5)$ is not an optimal solution to the bottom-level followers 11 and 12 and they will select $(z_{11}, z_{12}) = (2,3)$ to achieve the highest satisfactory degree 1.0 under the decision made by the leader and the middle-level follower 1. Similarly, the bottom-level followers 21 and 22 will make the decision $(z_{21}, z_{22}) = (1,1.5)$ to respond to the leader and the middle-level follower

2. The leader and the middle-level followers have to reduce their individual satisfactory degrees to bend to the increase in the satisfactory degrees of the bottom-level followers throughout the TLMF decision-making process. In this way, the decision entities finally achieve an optimal solution $(x^3, y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) = (1,1,0.5,2,3,1,1.5)$, under which the satisfactory degrees of all the bottom-level followers go up to 1.0.

Although the satisfactory degrees of the leader and the middle-level follower 2 drop to 0.62 and 0.67 respectively, the numbers become the highest satisfactory degrees for them under the reference-based relationship between all decision entities. In real-world cases, the situation indicates that higher satisfactory degrees of the leader and the middle-level follower 2 cannot be achieved under the current decision conditions unless they may persuade the bottom-level followers to cooperate with them and to reduce the corresponding satisfactory degrees. For example, if the bottom-level followers 11, 21 and 22 are willing to accept their respective satisfactory degrees 0.83, 0 and 0.80, the solution to the numerical example in Section 5.3.2 would be $(x^1, y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (1,1,0.5,0.5,4.5,0,2)$, which ensures that the corresponding satisfactory degrees of the leader and the bottom-level follower 2 rise up to 1.0. Otherwise, they have to adjust the current decision context through changing objective functions or constraint conditions to generate a new round of the decision-making process. The evaluation criterion will be applied to deal with a production-inventory planning problem in the following Section 5.5.

5.5 CASE STUDY

In this section, a case study on production-inventory planning is handled using the proposed TLMF decision-making techniques.

5.5.1 CASE DESCRIPTION

Nowadays, manufacturers usually work in a distributed or decentralized manner in a complex supply chain network comprising of suppliers, sales and logistics companies, customers and other specialized service functions (Chan & Chan 2010). Researchers as well as practitioners in manufacturing industries have placed importance on developing production and inventory control capabilities to enhance their market position in supply chain management (Sana 2011), which demands that manufacturing enterprises have to make right decisions on scheduling of their production and allocation of inventory to satisfy market requirements, shorten delivery time and reduce total production costs (Askin, Baffo & Xia 2014; Chang, Li & Chiang 2014; Tan, Lee & Goh 2012). Therefore, it is increasingly important to have efficient and easily-applicable models and solution methods to describe and solve related production-inventory decision problems (Berling & Marklund 2014; Ho & Hsiao 2014) although modeling deception in a real-world conflict situation is usually difficult (Li & Cruz Jr 2009).

In this section, the proposed TLMF decision techniques are applied to handle a production-inventory planning problem within a real-world conglomerate enterprise. The conglomerate is composed of a sales company, two logistics centers and two manufacturing factories attached to each logistics center, which are distributed throughout a three-stage hierarchical supply chain. The three-level hierarchical structure of the conglomerate is shown in Figure 5.3. Specifically, the sales company covers products marketing of the enterprise and has an independent products warehouse to satisfy market demand and shorten time-to-market. Both logistics centers also hold a certain amount of products inventory to respond to market requirements and reduce the inventory pressure of the sales company. According to market requirements and the holding inventories of the sales company and the logistics centers, the manufacturing factories are responsible for the production organization involving making detailed production plans and executing production activities.

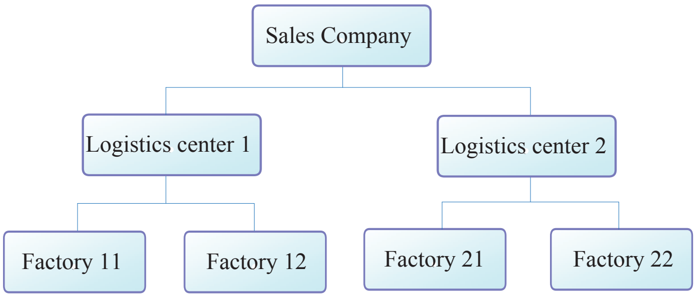


Figure 5.3 Hierarchical structure of the conglomerate enterprise

The decision-making situation is described as follows. During a peak season of products sales, the market requirements exceed the normal supply capacity of the enterprise so that four manufacturing factories have to organize overtime production. The sum of overtime outputs produced by four factories and safety stocks held by the sales company and two logistics centers are demanded to satisfy the exceeded market requirements. The more safety stocks imply the fewer overtime production outputs, but also mean the more inventory holding costs. Under the given market requirements, the decision entities distributed throughout the three-level hierarchy try to minimize their individual costs by considering their own constraints and implicit decisions made by other decision entities. More specifically, the sales company at the top level has the priority to determine its safety stock to minimize its inventory holding cost by considering the given market requirements and implicit reactions of other decision entities. In view of the decision made by the sales company, the logistics centers at the middle level then determine their individual safety stocks to minimize their own inventory holding costs by considering their own constraints and implicit reactions of their subordinate factories. Finally, each manufacturing factory at the bottom level makes overtime production plans in the light of the inventories held by the top and middle levels.

Furthermore, to reduce the total cost of the conglomerate, the conglomerate anticipates that decision entities whose inventory holding cost or overtime production cost is lower are able to keep more inventories or manufacture more production outputs. Thus, the conglomerate makes some management strategies to intervene and reconcile the decision process of its subordinate decision entities. For example, the conglomerate claims that each logistics center should take the inventory determined by the other logistics center as a reference when making its own decisions. If the inventory of a logistics center is less than the other, it means the less inventory holding cost but implies that the logistics center is demanded to undertake an opportunity cost for its own decision on holding less inventory. Also, each factory needs to reference the production plans made by other counterparts attached to the same logistics center when making its own production plans. If the production outputs of a factory are less than the other, it means the less overtime production costs but implies that the factory needs to cover an opportunity cost for its own decision on less production outputs. In addition, the sales company is demanded to afford the marketing cost and backlogging cost of the conglomerate. However, to reduce the total cost to the sales company, the conglomerate demands that both logistics centers must share part of the inventory holding cost and compensate for the marketing cost of the sales company. Similarly, to reduce the pressure of overtime production, the factories at the bottom level are also demanded to compensate the inventory holding cost of their superior logistics center to encourage it to keep more safety stocks. Therefore, under the current decision situation, the decision entities will try to minimize their individual overall costs by making their individual decisions, and the decision processes are executed sequentially, interactively and repeatedly within the tri-level hierarchy until the equilibrium is achieved among them.

This case clearly describes a TLMF decision process which includes one leader (the sales company), two middle-level followers (the logistics centers) and two bottom-level followers (the manufacturing factories) attached to each middle-level follower. The leader, the middle-level followers and the bottom-level followers make their individual decisions in sequence, and each decision entity cannot control decisions of the others but is affected by their reactions. It is noticeable that the multiple middle-level and bottom-level followers also consider decisions made by their counterparts as references, which implies the reference-based relationship between multiple followers at both the middle and bottom levels. This case can thus be considered as a reference-based TLMF decision problem.

Table 5.4 Notations for decision variables and parameters employed

Notation	Explanation
x	The safety stock controlled by the sales company
y_i	The independent safety stock determined by the logistics center *i*
z_{ij}	The overtime production plan determined by the factory *ij*
a, a_i	The inventory holding cost per unit of the sales company and the logistics center *i*
a_{ij}	The overtime production cost per unit afforded by the factory *ij*
b, b_i	The marketing cost per unit of the sales company and the compensation cost per unit that the logistics center *i* has to pay for the marketing cost of the sales company
c	The products backlogging cost per unit that is paid by the sales company
d_i, d_{ij}	The opportunity cost per unit of the logistics center *i* and the factory *ij*
e, e_i, e_{ij}	The proportion of the inventory holding cost of the sales company that is respectively shared by the sales company, the logistics center *i* and the factory *ij*
e_i^{pw}, e_{ij}^{pw}	The proportion of the inventory holding cost of the logistics center *i* that is respectively shared by itself and the factory *ij*
p	The exceeded products requirements of the market
q	The minimum inventories sum of all safety stocks anticipated by the sales company
q_i	The logistics center *i* must hold that the products sum of its own safety stock, the overtime production outputs of its lower-level factories and the safety stock of the sales company does not exceed q_i
q_{ij}	The factory *ij* must satisfy that the products sum comprised of its own and its counterparts' production outputs and the safety stocks of the sales company and the logistics center *i* exceeds q_{ij}
r, r_i	The maximum safety stock of the sales company and the logistics center *i*
r_{ij}	The maximum overtime production outputs of the factory *ij*

In the light of the above problem description, let *n*=2 be the number of logistics centers, and *i* be the index for logistics centers, $i = 1,2$; while let $m_i = 2$ be the number of manufacturing factories attached to the logistics center *i*, and *j* be the index for manufacturing factories, $j = 1,2$. To model the problem conveniently, related notations of decision variables and some key parameters in the scenario are shown in Table 5.4.

5.5.2 Model building

Based on the above decision conditions and strategies, the TLMF decision model of this case is established as follows in the form of the general model (5.3) proposed in Section 5.2.1.3.

(1) The decision problem of the sales company (top-level leader)

$$\min_{x \in X} f^{(1)}(x, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22}) =$$

$$aex + bp + c(x + \sum_{i=1}^{2} y_i + \sum_{i=1}^{2}\sum_{j=1}^{2} z_{ij} - p) - \sum_{i=1}^{2} b_i (y_i + \sum_{j=1}^{2} z_{ij}) \tag{5.19a}$$

$$\text{s.t.} \quad x + \sum_{i=1}^{2} y_i + \sum_{i=1}^{2}\sum_{j=1}^{2} z_{ij} \geq p, \tag{5.19b}$$

$$x + \sum_{i=1}^{2} y_i \geq q, \tag{5.19c}$$

$$0 \leq x \leq r. \tag{5.19d}$$

The sales company's objective function (5.19a) involves its safety stock's inventory holding cost *aex*, the marketing cost *bp*, the backlogging cost $c(x + \sum_{i=1}^{2} y_i + \sum_{i=1}^{2}\sum_{j=1}^{2} z_{ij} - p)$ and the minus marketing compensation cost $\sum_{i=1}^{2} b_i (y_i + \sum_{j=1}^{2} z_{ij})$ derived from the logistics centers. Constraint condition (5.19b) means the upper bound of the products sum of all safety stocks and overtime production outputs of all manufacturing factories, while constraint condition (5.19c) implies the upper bound of the sum of all safety stocks. Constraint condition (5.19d) represents the lower and upper limits to the sales company's safety stock.

(2) The decision problem of the logistics center (middle-level follower) i

$$\min_{y_i \in Y_i} f_i^{(2)}(x, y_1, y_2, z_{i1}, z_{i2}) = a_i e_i^{pw} y_i + d_i \Delta y_i + a e_i x + b_i (y_i + \sum_{j=1}^{2} z_{ij}) \tag{5.19e}$$

$$\text{s.t. } x + \sum_{i=1}^{2} y_i + \sum_{j=1}^{2} z_{ij} \le q_i, \tag{5.19f}$$

$$0 \le y_i \le r_i. \tag{5.19g}$$

The objective function (5.19e) of the logistics center i involves its safety stock holding cost $a_i e_i^{pw} y_i$, the opportunity cost $d_i \Delta y_i$, the shared inventory holding cost $a e_i x$ of the sales company's safety stock and the marketing compensation cost $b_i (y_i + \sum_{j=1}^{2} z_{ij})$ paid to the sales company. Note that $\Delta y_1 = y_2 - y_1$ and $\Delta y_2 = y_1 - y_2$. Constraint condition (5.19f) reflects the upper bound of the products sum consisting of all safety stocks and the overtime production outputs of the manufacturing factories attached to the middle-level follower i. Constraint condition (5.19g) represents the lower and upper limits to the safety stock of the logistics center i.

(3) The decision problem of the manufacturing factory (bottom-level follower) ij

$$\min_{z_{ij} \in Z_{ij}} f_{ij}^{(3)}(x, y_i, z_{i1}, z_{i2}) = a_{ij} z_{ij} + d_{ij} \Delta z_{ij} + a e_{ij} x + a_i e_{ij}^{pw} y_i \tag{5.19h}$$

$$\text{s.t. } x + y_i + \sum_{j=1}^{2} z_{ij} \ge q_{ij}, \tag{5.19i}$$

$$0 \le z_{ij} \le r_{ij}. \tag{5.19j}$$

The objective function (5.19h) of the manufacturing factory ij involves its overtime production cost $a_{ij} z_{ij}$, the opportunity cost $d_{ij} \Delta z_{ij}$, and the shared inventory holding cost $a e_{ij} x$ and $a_i e_{ij}^{pw} y_i$ respectively for the safety stocks of the sales company and the logistics center i. Note that $\Delta z_{i1} = z_{i2} - z_{i1}$ and $\Delta z_{i2} = z_{i1} - z_{i2}$. Constraint condition (5.19i) reflects the upper bound of the products sum consisting of overtime production outputs of the manufacturing factories attached to the logistics center i and safety stocks of the sales company and the logistics center i. Constraint condition (5.19j) represents the lower and upper limits to the overtime production outputs of the manufacturing factory ij.

This TLMF decision model (5.19) describes the real-world production and inventory decision problem which is a concretization of the general model (5.3) proposed in Section 5.2.1.3. The TLMF *K*th-Best algorithm is then adopted to solve the model by a numerical experiment. Also, the fuzzy programming approach is used to evaluate the solutions obtained.

5.5.3 NUMERICAL EXPERIMENT AND RESULTS ANALYSIS

This section shows the computational results achieved by the proposed TLMF *K*th-Best algorithm and the fuzzy programming approach. The experimental data employed for the model (5.19) is provided in Tables 5.5-5.7.

Table 5.5 Data for the sales company

a	b	c	e	p	q	r
5.0	2.0	2.0	0.20	9.0	3.5	2.5

Table 5.6 Data for the logistics centers

i	a_i	b_i	d_i	e_i	e_i^{pw}	q_i	r_i
1	4.0	1.0	4.0	0.20	0.50	8.0	1.0
2	4.0	3.0	4.0	0.20	0.50	6.0	0.50

Table 5.7 Data for the manufacturing factories

i	j	a_{ij}	d_{ij}	e_{ij}	e_{ij}^{pw}	q_{ij}	r_{ij}
1	1	1.0	2.0	0.10	0.25	7.0	3.0
1	2	3.0	2.0	0.10	0.25	7.0	3.0
2	1	2.0	3.0	0.10	0.25	4.0	1.0
2	2	4.0	3.0	0.10	0.25	4.0	2.0

The detailed computing process driven by the TLMF *K*th-Best algorithm is shown in Table 5.8 which includes related data and parameters generated in the computing process. Specifically, Table 5.8 presents the vertex s^k that is searched in the current iteration *k*, and the adjacent vertices set W_k of the current vertex s^k. *T* represents the set of vertices that have been searched in the past iterations while *W* is the set of vertices that are needed to verify whether or not an optimal solution occurs inside in the following iteration. Following procedures of the TLMF *K*th-Best algorithm, an optimal solution is finally obtained after 12 iterations. Note that $W_7 = \varnothing$, $W_8 = \varnothing$ and $W_{10} = \varnothing$ in Table 5.8 do not mean that adjacent vertices of s^7, s^8 and s^{10} do not exist, but imply that their adjacent vertices have been found in previous iterations and have been involved in *W*.

Table 5.9 displays the objective values of all decision entities respectively towards each solution enumerated by the TLMF *K*th-Best algorithm, while Table 5.10 shows the corresponding satisfactory degrees that are computed by the formulas (5.16), (5.17) and (5.18). It can be seen from Table 5.9 that, under the current decision context within the conglomerate enterprise, $(x^1, y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (2,1,0.5,1,3,0.5,2)$, $(x^2, y_1^2, y_2^2, z_{11}^2, z_{12}^2, z_{21}^2, z_{22}^2) = (2,1,0.5,3,1,0.5,2)$, $(x^3, y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) = (2,1,0.5,1,3,1,1.5)$ and $(x^4, y_1^4, y_2^4, z_{11}^4, z_{12}^4, z_{21}^4, z_{22}^4) = (2,1,0.5,3,1,1,1.5)$ are the individual best solutions to the leader (the sales company), which implies that the leader anticipates that the middle-level and bottom-level followers (the logistics centers and manufacturing factories) can choose $(y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (1,0.5,1,3,0.5,2)$, $(y_1^2, y_2^2, z_{11}^2, z_{12}^2, z_{21}^2, z_{22}^2) = (1,0.5,3,1,0.5,2)$, $(y_1^3, y_2^3, z_{11}^3, z_{12}^3, z_{21}^3, z_{22}^3) = (1,0.5,1,3,1,1.5)$ or $(y_1^4, y_2^4, z_{11}^4, z_{12}^4, z_{21}^4, z_{22}^4) = (1,0.5,3,1,1,1.5)$ to respond to itself after it determined x=2. However, it can be seen from Table 5.10 that the middle-level follower 2 and the bottom-level followers 11, 21, and 22 cannot always achieve individual best satisfactory degrees if they make the decisions desired by the leader. In the reference-based decision situation, the followers will choose $(y_1^{12}, y_2^{12}, z_{11}^{12}, z_{12}^{12}, z_{21}^{12}, z_{22}^{12}) = (1,0.5,3,1,1,0.5)$ to react to the leader's decision x=2 such that their satisfactory degrees all grow up to 1.0.

Table 5.8 The detailed computing process of the TLMF *K*th-Best algorithm

Iteration k	Vertex s^k	W_k	T	W
1	(2,1,0.5,1,3,0.5,2)	{(2.5,1,0,0.5,3,0.5,2), (2,1,0.5,1,3,0,2), (2.5,1,0.5,0.5,3,0,2), (2,1,0.5,3,1,0.5,2), (2,1,0.5,1,3,1,1.5), (2.5,0.5,0.5,1,3,0.5,2)}	$\{s^1\}$	W_1
2	(2,1,0.5,3,1,0.5,2)	{(2.5,1,0,3,0.5,0.5,2), (2,1,0.5,3,1,0,2), (2.5,1,0.5,3,0.5,0,2), (2,1,0.5,3,1,1,1.5), (2.5,0.5,0.5,3,1,0.5,2)}	$\{s^1,s^2\}$	$(W \cup W_2) \setminus T$
3	(2,1,0.5,1,3,1,1.5)	{(2.5,1,0,0.5,3,1,1.5), (2.5,0.5,0.5,1,3,1,1.5), (2.5,1,0.5,0.5,3,1,1), (2,1,0.5,1,3,1,0.5)}	$\{s^1,s^2,s^3\}$	$(W \cup W_3) \setminus T$
4	(2,1,0.5,3,1,1,1.5)	{(2.5,1,0,3,0.5,1,1.5), (2.5,0.5,0.5,3,1,1,1.5), (2.5,1,0.5,3,0.5,1,1), (2,1,0.5,3,1,1,0.5)}	$\{s^1,s^2,s^3,s^4\}$	$(W \cup W_4) \setminus T$
5	(2,1,0.5,1,3,0,2)	{(2.5,1,0,0.5,3,0,2), (2.5,0.5,0.5,1,3,0,2), (2,1,0.5,1,3,0,1.5)}	$\{s^1,s^2,s^3,s^4,s^5\}$	$(W \cup W_5) \setminus T$
6	(2,1,0.5,3,1,0,2)	{(2.5,1,0,3,0.5,0,2), (2.5,0.5,0.5,3,1,0,2), (2,1,0.5,3,1,0,1.5)}	$\{s^1,s^2,s^3,s^4,s^5,s^6\}$	$(W \cup W_6) \setminus T$
7	(2.5,0.5,0.5,1,3,0.5,2)	$\varnothing$	$\{s^1,s^2,s^3,s^4,s^5,s^6,s^7\}$	$(W \cup W_7) \setminus T$
8	(2.5,0.5,0.5,3,1,0.5,2)	$\varnothing$	$\{s^1,s^2,s^3,s^4,s^5,s^6,s^7,s^8\}$	$(W \cup W_8) \setminus T$
9	(2.5,0.5,0.5,1,3,1,1.5)	{(2.5,0.5,0.5,1,3,1,0.5)}	$\{s^1,s^2,s^3,s^4,s^5,s^6,s^7,s^8,s^9\}$	$(W \cup W_9) \setminus T$
10	(2,1,0.5,1,3,1,0.5)	$\varnothing$	$\{s^1,s^2,s^3,s^4,s^5,s^6,s^7,s^8,s^9,s^{10}\}$	$(W \cup W_{10}) \setminus T$
11	(2.5,0.5,0.5,3,1,1,1.5)	{(2.5,0.5,0.5,3,1,1,0.5)}	$\{s^1,s^2,s^3,s^4,s^5,s^6,s^7,s^8,s^9,s^{10},s^{11}\}$	$(W \cup W_{11}) \setminus T$
12	(2,1,0.5,3,1,1,0.5)	---	---	---

Table 5.9 Solutions and objective values of decision entities

Iteration k	Vertex s^k	$f^{(1)}$	$f_1^{(2)}$	$f_2^{(2)}$	$f_{11}^{(3)}$	$f_{12}^{(3)}$	$f_{21}^{(3)}$	$f_{22}^{(3)}$
1	(2,1,0.5,1,3,0.5,2)	8.0	7.0	14.0	7.0	7.0	7.0	5.0
2	(2,1,0.5,3,1,0.5,2)	8.0	7.0	14.0	1.0	9.0	7.0	5.0
3	(2,1,0.5,1,3,1,1.5)	8.0	7.0	14.0	7.0	7.0	5.0	6.0
4	(2,1,0.5,3,1,1,1.5)	8.0	7.0	14.0	1.0	9.0	5.0	6.0
5	(2,1,0.5,1,3,0,2)	8.5	7.0	12.5	7.0	7.0	7.5	3.5
6	(2,1,0.5,3,1,0,2)	8.5	7.0	12.5	1.0	9.0	7.5	3.5
7	(2.5,0.5,0.5,1,3,0.5,2)	9.0	8.0	12.5	6.75	6.75	7.25	5.25
8	(2.5,0.5,0.5,3,1,0.5,2)	9.0	8.0	12.5	0.75	8.75	7.25	5.25
9	(2.5,0.5,0.5,1,3,1,1.5)	9.0	8.0	12.5	6.75	6.75	5.25	6.25
10	(2,1,0.5,1,3,1,0.5)	9.0	7.0	11.0	7.0	7.0	2.0	5.0
11	(2.5,0.5,0.5,3,1,1,1.5)	9.0	8.0	12.5	0.75	8.75	5.25	6.25
12	(2,1,0.5,3,1,1,0.5)	9.0	7.0	11.0	1.0	9.0	2.0	5.0

More specifically, for the given decision $(x, y_1) = (2,1)$ by the leader and the middle-level follower 1, the bottom-level followers 11 and 12 achieve a Nash equilibrium solution $(z_{11}, z_{12}) = (3,1)$ to respond to the leader and the middle-level follower 1. Similarly, for the given decision $(x, y_2) = (2,0.5)$ by the leader and the middle-level follower 2, the bottom-level followers 21 and 22 achieve a Nash equilibrium solution $(z_{21}, z_{22}) = (1,0.5)$ to respond to the leader and the middle-level follower 2. Therefore, $(y_1, z_{11}, z_{12}) = (1,3,1)$ and $(y_2, z_{21}, z_{22}) = (0.5,1,0.5)$ are the optimal solutions respectively for the middle-level follower i (i=1,2) and its bottom-level followers under the given decision $x = 2$ by the leader. Also, for the given decision x=2 by the leader, $(y_1, y_2) = (1,0.5)$ is the optimal solution for the middle-level followers while taking into account implicit reactions of their respective bottom-level followers. Therefore, $(x^{12}, y_1^{12}, y_2^{12}, z_{11}^{12}, z_{12}^{12}, z_{21}^{12}, z_{22}^{12}) = (2,1,0.5,3,1,1,0.5)$ is an optimal solution to the production-inventory planning problem.

Table 5.10 The satisfactory degree of decision entities towards solutions

Iteration	Leader			Follower 1			Follower 2			Follower 11			Follower 12			Follower 21			Follower 22		
k	f^1	f^0	$\mu(f^{(1)})$	f_1^1	f_1^0	$\mu_1(f_1^{(2)})$	f_2^1	f_2^0	$\mu_2(f_2^{(2)})$	f_{11}^1	f_{11}^0	$\mu_{11}(f_{11}^{(3)})$	f_{12}^1	f_{12}^0	$\mu_{12}(f_{12}^{(3)})$	f_{21}^1	f_{21}^0	$\mu_{21}(f_{21}^{(3)})$	f_{22}^1	f_{22}^0	$\mu_{22}(f_{22}^{(3)})$
1	8.0	11.5	1.0	7.0	9.5	1.0	11.0	15.0	0.25	5.0	7.0	0	7.0	7.0	1.0	6.5	7.5	0.5	4.0	5.0	0
2	8.0	11.5	1.0	7.0	9.5	1.0	11.0	15.0	0.25	1.0	1.0	1.0	9.0	11.0	1.0	6.5	7.5	0.5	4.0	5.0	0
3	8.0	11.5	1.0	7.0	9.5	1.0	11.0	15.0	0.25	5.0	7.0	0	7.0	7.0	1.0	5.0	6.0	1.0	5.0	6.5	0.33
4	8.0	11.5	1.0	7.0	9.5	1.0	11.0	15.0	0.25	1.0	1.0	1.0	9.0	11.0	1.0	5.0	6.0	1.0	5.0	6.5	0.33
5	8.0	11.5	0.86	7.0	9.5	1.0	11.0	15.0	0.63	5.0	7.0	0	7.0	7.0	1.0	6.5	7.5	0	3.0	3.5	0
6	8.0	11.5	0.86	7.0	9.5	1.0	11.0	15.0	0.63	1.0	1.0	1.0	9.0	11.0	1.0	6.5	7.5	0	3.0	3.5	0
7	8.0	11.5	0.71	7.0	9.5	0.6	8.0	13.5	0.18	4.75	6.75	0	6.75	6.75	1.0	6.75	7.75	0.5	3.75	5.25	0
8	8.0	11.5	0.71	7.0	9.5	0.6	8.0	13.5	0.18	0.75	0.75	1.0	8.75	10.75	1.0	6.75	7.75	0.5	3.75	5.25	0
9	8.0	11.5	0.71	7.0	9.5	0.6	8.0	13.5	0.18	4.75	6.75	0	6.75	6.75	1.0	5.25	6.25	1.0	4.75	6.75	0.25
10	8.0	11.5	0.71	7.0	9.5	1.0	11.0	15.0	1.0	5.0	7.0	0	7.0	7.0	1.0	2.0	2.0	1.0	5.0	6.5	1.0
11	8.0	11.5	0.71	7.0	9.5	0.6	8.0	13.5	0.18	0.75	0.75	1.0	8.75	10.75	1.0	5.25	6.25	1.0	4.75	6.75	0.25
12	8.0	11.5	0.71	7.0	9.5	1.0	11.0	15.0	1.0	1.0	1.0	1.0	9.0	11.0	1.0	2.0	2.0	1.0	5.0	6.5	1.0

Although the leader's satisfactory degree has dropped to 0.71 under the solution $(x^{12}, y_1^{12}, y_2^{12}, z_{11}^{12}, z_{12}^{12}, z_{21}^{12}, z_{22}^{12}) = (2,1,0.5,3,1,1,0.5)$, the leader cannot obtain a better objective value or a higher satisfactory degree by moving away from the vertex over the *IR*. Therefore, within the real-world case study, the solution $(x^{12}, y_1^{12}, y_2^{12}, z_{11}^{12}, z_{12}^{12}, z_{21}^{12}, z_{22}^{12}) = (2,1,0.5,3,1,1,0.5)$ is the optimal solution to the TLMF decision model (5.19), which means a final compromised result among all decision entities under the current decision context in the conglomerate enterprise. This TLMF hierarchical decision situation indicates that the leader may not achieve an individual optimal solution under the constraint region even though it has priority in making decisions, since its decisions are determined by implicit reactions of the followers. Moreover, the decision process and results of an TLMF decision problem are affected by the reference-based relationship between multiple followers at the same level. In summary, the proposed TLMF decision techniques provide an effective way to model and solve real-world TLMF decision problems and to recognize the satisfactory degree of decision entities towards solutions.

Furthermore, by the optimal solution, it can be analyzed that whether or not the conglomerate employed practical and effective management strategies to balance the production-inventory planning among its subordinate sales company, logistics centers and manufacturing factories. Based on the given experimental data in Tables 5.5-5.7, the contrastive analysis between the upper limits to the holding inventory or overtime production capacity of each decision entity and the final solution is shown as Figure 5.4. It can be seen from Figure 5.4 that the holding inventories of the logistics centers peak at their respective upper limits. Also, the production outputs of the manufacturing factories 11 and 21 reach their maximum overtime production capacities respectively. In contrast, the holding inventory or overtime production outputs of other decision entities are less or much less than their corresponding upper limits. These results indicate that decision entities whose inventory holding cost or overtime production cost is lower prefer to keep more inventories or manufacture more production outputs under the current decision context, which is exactly desired by the conglomerate as presented in Section 5.5.1. Therefore, the current management strategy implemented by the conglomerate is an available way to balance the production-inventory planning throughout the three-stage supply chain with conflicting objectives of decision entities.

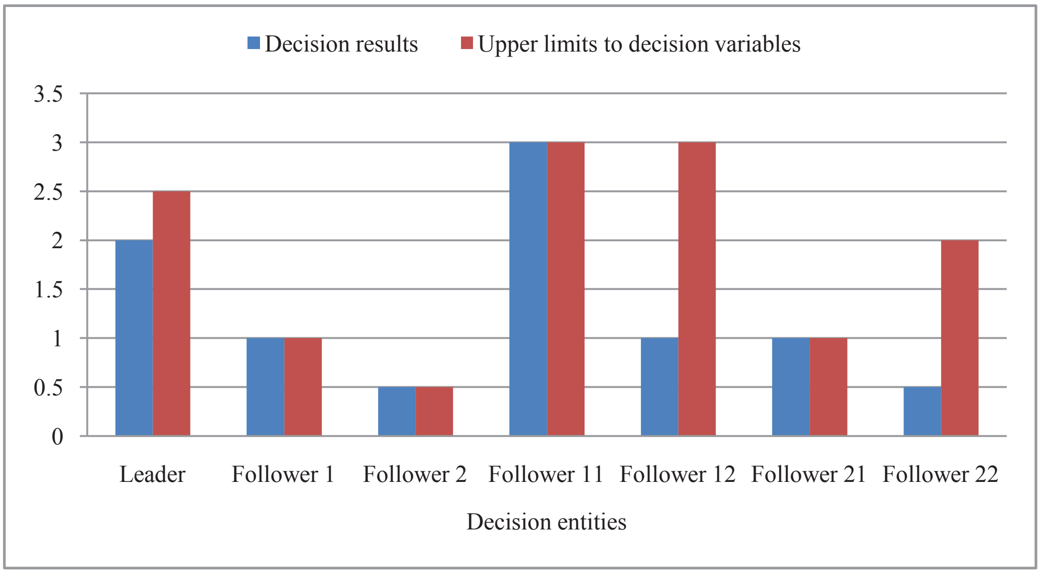


Figure 5.4 The contrastive analysis of results

5.5.4 FURTHER DISCUSSIONS

This section discusses in depth characteristics of the TLMF *K*th-Best algorithm and the evaluation criterion defined by fuzzy programming. Also, limitations to this research are analyzed.

Table 5.8 clearly shows that an optimal solution is finally found by completing the enumeration of 12 vertices, of which most (8 in 12) are accompanied by the same decision made by the leader and the middle-level followers, which implies that the search approach of the TLMF *K*th-Best algorithm is easily convergent. Also, only a few data involving W_k, T, and W are necessary to write down within the algorithm operation. The features of the algorithm can be also observed through computing Example 5.1 in Section 5.3. Thus, the TLMF *K*th-Best algorithm can be carried out efficiently because each successive pair of points is adjacent. Moreover, note that the other 11 vertices searched, apart from the optimal vertex 12, are all feasible solutions to the TLMF decision problem even if they cannot be an optimal solution. The property gives us another advantage of the TLMF *K*th-Best algorithm in that the upper and lower bounds on an optimal solution are generated by the procedure even if storage or computational limits are reached before convergence. However, when plenty of followers are involved at the middle and bottom levels or a large number of decision variables and constraints exist, the execution efficiency of the algorithm may experience a steep decline as superabundant vertices need to be completed the search. The efficiency performance of the algorithm in solving large-scale TLMF problems will be explored through a real-world problem in Chapter 6.

It is noticeable from Tables 5.9 and 5.10 that the middle-level follower 2 obtains the same individual objective value 12.5 at the vertices s^5 and s^7; however, following this, the decision entity achieves two different satisfactory degrees 0.63 and 0.18 respectively. Also, note Table 5.3 in Section 5.4 that the objective value of the bottom-level follower 11 in Example 5.1 becomes worse from 7.5 to 9.0; however, following this, the corresponding satisfactory degree increases from 0.83 to 1.0. Evidently, it is not a positive correlation between the objective value and the corresponding satisfactory degree for followers. In this case study, the situation means that the feasible set and the rational set of the middle-level follower 2 are changed as the leader and the middle-level follower 1 change their decisions $(x, y_1) = (2,1)$ to $(x, y_1) = (2.5,0.5)$. Therefore, the satisfactory degree can be considered as a relative but not an absolute evaluation criterion as individual best and worst objective values of each decision entity would vary with the changing externalities determined by others, which clearly reflects the characteristic of the TLMF hierarchical decision-making process.

In this study, the reference-based relationship is considered within a three-stage supply chain comprised of one leader and multiple followers. All decision entities have to achieve a compromised solution under the current decision conditions. Thus, under the decision-making situation, decision entities have to adjust the current decision context through changing objective functions or constraint conditions to generate a new round of decision-making processes if they desire to improve their respective satisfactory degrees. However, all decision entities that are distributed throughout a conglomerate enterprise may have chances to cooperate with each other and achieve an agreement on their decisions in the real world. For example, if the leader desires to improve its own satisfactory degree, it may persuade the middle-level follower 2 and the bottom-level follower 22 to react to others' decisions $(x, y_1, z_{11}, z_{12}, z_{21}) = (2,1,1,3,1)$ by determining their own decisions $(y_2, z_{22}) = (0.5,1.5)$ such that the leader can achieve its individual best solution, yielding the solution $(x^4, y_1^4, y_2^4, z_{11}^4, z_{12}^4, z_{21}^4, z_{22}^4) = (2,1,0.5,1,3,1,1.5)$. Thus, definitions of the satisfactory degree provide a practical way in finding some possibly satisfactory solutions to a TLMF decision case in the real world, because the satisfactory degree can be considered as an evaluation criterion that can be adopted to recognize a solution whether or not decision entities desire it. Also, the evaluation criterion provides an available approach to solve a TLMF decision problem without an optimal solution. As discussed above, if decision entities are willing to cooperate with each other, a satisfactory solution can be found through recognizing the satisfactory degree of decision entities.

5.6 SUMMARY

To handle TLMF decision problems, this chapter first introduces the definitions of relationships between multiple followers at the same level, such as uncooperative, cooperative, reference-based relationships and their hybrid relationships; gives linear TLMF decision models in line with different decision relationships; analyzes the operations of TLMF decision-making process using solution concepts; and discusses related theoretical properties of TLMF decision models. Second, based on theoretical properties discussed, this chapter develops a TLMF *K*th-Best algorithm for solving TLMF decision problems. Moreover, a fuzzy programming approach is proposed to evaluate the satisfaction of decision entities towards solutions obtained. Lastly, a real-world case study on production-inventory planning in SCM illustrates the effectiveness of the proposed TLMF decision techniques in handling such problems in applications. In conclusion, this study provides the theoretical foundation for TLMF decision-making research and overcomes the lack of solution algorithms for solving TLMF decision problems.

CHAPTER 6 APPLICATION IN DECENTRALIZED VENDOR-MANAGED INVENTORY

6.1 INTRODUCTION

The operations in supply chain management (SCM) are part of today's most important economic activities as they remain to be vital tools for business firms to remain competitive. Inventory control, one of the key factors in SCM (Kumar & Chandra 2002; Sadeghi et al. 2013), can be considered as the major driver of a supply chain owing to its strong influence on supply-chain performance (Chandra & Grabis 2005; Chopra & Meindl 2007). High or undesirable inventory levels are often the result of poor cash flow, the amount of space available to store goods, and the high risk of dealing with obsolete goods. In recent years, an increasing number of companies in retail business and manufacturing industry have identified the vendor-managed inventory (VMI) policy as one strategy for reducing inventory, speeding up the supply chain (Holweg et al. 2005) and eliminating the bullwhip effect in SCM (Dong, Xu & Dresner 2007). Practical and academic works have implied that implementing VMI programs has resulted in significant benefits and cost reduction for both vendor and buyer (Dong & Xu 2002), and has increased flexibility in production scheduling and decision-making on distribution (Claassen, van Weele & van Raaij 2008; Lee & Cho 2014; Ryu et al. 2013).

VMI is defined as a concept for planning and control of inventory based on the fact that the vendor (or supplier) has access to the buyer's (or retailer's) demand data and is responsible for maintaining the appropriate inventory level and determining replenishment policies (Govindan 2013; Marquès et al. 2010). An important issue in designing a VMI system is how to ensure optimal inventory planning, such as decision-making at the inventory level and replenishment frequency. The majority of research on VMI has been focused on centralized inventory control that features inventory decisions only managed from the vendor (Pasandideh, Niaki & Roozbeh Nia 2010; Zavanella & Zanoni 2009); for example, this situation often appears in consignment and retail business (Gümüş, Jewkes & Bookbinder 2008; Lee & Cho 2014). However, centralized VMI cannot be applied to handle the inventory management in which both the vendor and buyer are manufacturers and try their best to achieve the inventory as small as possible or even at zero inventory. Moreover, business firms are often distributed throughout a multi-echelon supply chain network of three levels or more in today's global market, in particular with the development of third-party logistics (Aguezzoul 2014; Ivanov, Sokolov & Dolgui 2013; Kumar, Singh & Kumari 2012; Pal, Sana & Chaudhuri 2013). In this situation, each company is usually concerned with its own profit and costs when making inventory decisions, thus, VMI coordination becomes a challenging task in SCM.

This chapter considers a VMI coordination problem in a three-echelon supply chain network comprised of one vendor, multiple distributors and multiple buyers, which are distributed across three hierarchical levels. In contrast to centralized inventory control that features operations managed from a single point, each decision entity is given the power to make its own optimal inventory decision based on local inventory conditions and decisions (or implicit decisions) of other decision entities under this VMI arrangement; this can be therefore regarded as a decentralized VMI coordination scenario.

To identify the optimal inventory level and replenishment frequency under VMI coordination, this chapter handles the decentralized VMI scenario using multilevel decision-making techniques. This chapter discusses how to improve the individual performance of each decision entity and balance the total cost sharing in a decentralized VMI hierarchy. Moreover, it displays how to design a manufacturer-manufacturer (vendor-buyer) VMI system where third-party logistics are involved, and seeks to show how the vendor and buyers can achieve a minimal to zero inventory.

This chapter is organized as follows. Following the introduction, this chapter describes and analyzes this decentralized VMI coordination problem in Section 6.2. In Section 6.3, an analytical model is then applied to describe the problem. The resulting model is a linear TLMF decision problem addressed in Chapter 5, which allows us to examine how the decision entities coordinate with each other on the decentralized VMI. Lastly, the TLMF *K*th-Best algorithm presented in Chapter 5 is used to solve the TLMF decision problem; a computational study is conducted in Section 6.4 to illustrate how to apply the multilevel decision-making techniques to handle the VMI coordination problem. A summary is given in Section 6.5.

6.2 Problem Statement

This study considers a vendor supplying a single product to multiple buyers under a VMI arrangement. The buyers are MTO (make-to-order) manufacturing enterprises that produce different types of end products but make use of the same raw material, which is the product manufactured and supplied by the vendor. Since both the vendor and buyers are manufacturing enterprises that consider production-manufacturing to be the core competence rather than logistics distribution and inventory management, some third-party logistics companies are selected as the distributors responsible for the raw material distribution and inventory of the buyers. The raw material distribution for buyers that are located in the same industrial park or city is managed by the same third-party logistics company. Thus, the VMI system appears in a three-echelon supply chain network consisting of one vendor, multiple distributors and multiple buyers. To generalize the scenario, The three-echelon supply chain consists of one vendor, n distributors and m_i buyers attached to the *i*th distributor,

$i = 1,2,\ldots,n$. The hierarchical structure of the three-echelon supply chain network is shown in Figure 6.1.

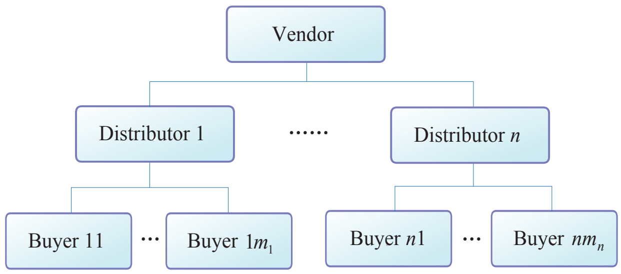


Figure 6.1 The organizational structure of the three-echelon supply chain

Under the VMI system, the vendor and distributors are responsible for raw material supply for the buyers by determining appropriate inventory levels and replenishment policies. To achieve inventory cost reduction and improved responsiveness, each distributor is willing to store the raw material using a VMI hub that is geographically close to its downstream buyers. The VMI hub is a private warehouse owned by the distributor or a public warehouse hired by the distributor. In addition, it is a requirement that the vendor and each buyer must hold a certain amount of back-up inventory using their own warehouses in order to reduce stock-out risk and respond to fluctuations in their production. It is noticeable that the vendor can have access to the buyers' material requirement planning (MRP) information during a production period of the buyers. Thus, the vendor controls the total raw material quantity every time it replenishes to the supply chain network, which is the total inventory held by all decision entities, based on the buyers' MRP information and its own production capacity. Clearly, the replenishment quantity determines the frequency that the vendor should replenish the raw material to the supply chain network during the production period of the buyers. Also, the minimum replenishment quantity of the vendor must account for a certain proportion of the MRP demanded by the buyers; that is, the replenishment capacity of the vendor can be measured by the minimum proportion between the total raw material inventory and the MRP demanded by the buyers.

In the decision-making process of coordinating the VMI, all decision entities are able to determine their respective inventory levels under the agreed replenishment policy with the aim of minimizing their individual inventory holding costs. Apart from the inventory holding cost, the vendor covers the cost of stock-out risk in the VMI supply chain, whereas the distributors bear the raw material transportation cost. For the sake of motivating the distributors to hold more inventory of the VMI hubs to maintain the raw material supply, the vendor adopts an incentive contract with the distributors to reduce its total cost, in which the vendor proportionally shares part of the distributors' transportation cost. In addition, to prevent the high stock-out risk caused by insufficient buyer back-up inventory, the distributors use a penalty contract with the buyers in which each buyer must compensate for the quantity variance between its own back-up inventory and the total inventory of its upstream decision entities.

As a result of information sharing in the VMI system, the vendor has full knowledge of the inventory costs, demand data and policies of the distributors and buyers, as well as the related production decision of each buyer. When planning the VMI, the vendor gives priority to the decision on its inventory level to minimize its total cost while still considering the optimal decision-making processes and reactions of its downstream decision entities based on the shared information. In the light of the decision made by the vendor, the distributors determine their respective optimal hub inventory levels taking into account the implicit reactions of their downstream buyers. Lastly, the buyers make the best possible decisions to respond to the decisions made by the upstream decision entities. In this way, the decision entities distributed throughout the tri-level hierarchical VMI system make their decisions on inventory planning in sequence with the aim of minimizing their individual total costs; however, the decision-making process of upstream decision entities must take into account the implicit reactions of downstream decision entities.

Table 6.1 Notations of decision variables and parameters employed

Notation	Explanation
x_{ij}	The inventory controlled by the vendor that will be distributed to the buyer *ij*
y_{ij}	The independent VMI-hub inventory determined by the distributor *i* that will be distributed to the buyer *ij*
z_{ij}	The independent inventory respectively determined by the buyer *ij*
a	The transportation cost per unit of the raw material
b_{ij}	The risk cost per unit of the raw material stock-out for the buyer *ij*
c	The raw material inventory holding cost per unit of the vendor
c_i	The raw material inventory holding cost per unit of the hired VMI-hub by the distributor *i*
c_{ij}	The raw material inventory holding cost per unit of the buyer *ij*
d	The upper limit to the inventory controlled by the vendor
d_i	The upper limit to the inventory controlled by the distributor *i*
d_{ij}	The upper limit to the inventory controlled by the buyer *ij*
e_{ij}	The raw material demand of the buyer *ij* according to its production capacity
p_i	The proportion of the *i*th distributor's transportation cost covered by the vendor in the incentive contract
r	The lower limit to the proportion between the raw material supply and the MRP demanded by the buyers
r^{VD}	The lower limit to the proportion between the inventory sum of the vendor and distributors and the MRP demanded by the buyers
r_i	The upper limit to the proportion between the raw material supply and the MRP demanded by the buyers attached to the distributor *i*
r_{ij}	The lower limit to the proportion between the raw material supply and the MRP demanded by the buyer *ij*
s_{ij}	The penalty cost per unit by which the buyer *ij* must compensate its upstream distributor *i* for quantity variance between its own inventory and the total inventory of its upstream decision entities

According to the relevant background of multilevel decision-making, this problem can be recognized as a tri-level decision problem that includes one leader (vendor), *n* middle-level followers (distributors) and m_i bottom-level followers (buyers) attached to the *i*th middle-level follower. In the light of the above problem description, let n be the number of distributors, and *i* be the index for distributors, $i = 1,2,\ldots,n$; while let m_i be the number of buyers attached to the distributor *i*, and *j* be the index for buyers, $j = 1,2,\ldots,m_i$. To model the problem conveniently, related notations of decision variables and key parameters in the scenario are shown in Table 6.1. It can be easily found in Table 6.1 that x_{ij} $(i=1,2,\ldots,n,\ j=1,2,\ldots,m_i)$ are the decision variables controlled by the vendor; y_{ij} $(j=1,2,\ldots,m_i)$ are the decision variables controlled by the distributor i ($i=1,2,\ldots,n$); and z_{ij} is the decision variable controlled by the buyer ij ($i=1,2,\ldots,n,\ j=1,2,\ldots,m_i$). For the sake of convenient argument in the following sections, let $x=(x_{11},\ldots,x_{1m_1},\ldots,x_{n1},\ldots,x_{nm_n})$ denote the vector of the vendor's decision variables, whereas let $y_i=(y_{i1},\ldots,x_{im_i})$ be the vector of decision variables controlled by the distributor i for $i=1,2,\ldots,n$.

6.3 ANALYTICAL MODEL

Based on the above problem statement and related notations, the tri-level decision model for the decentralized VMI coordination scenario is established as follows.

$$\min_{x\in X} f^{(1)}(x,y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,z_{n1},\ldots,z_{nm_n})$$

$$=(a+c)\sum_{i=1}^{n}\sum_{j=1}^{m_i}x_{ij}+\sum_{i=1}^{n}\sum_{j=1}^{m_i}ap_i(y_{ij}+z_{ij})+\sum_{i=1}^{n}\sum_{j=1}^{m_i}b_{ij}(e_{ij}-x_{ij}-y_{ij}-z_{ij}) \qquad \text{(Vendor)} \quad (6.1a)$$

$$\text{s.t.}\quad \sum_{i=1}^{n}\sum_{j=1}^{m_i}(x_{ij}+y_{ij}+z_{ij})\geq\sum_{i=1}^{n}\sum_{j=1}^{m_i}re_{ij}, \qquad (6.1b)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}(x_{ij}+y_{ij})\geq\sum_{i=1}^{n}\sum_{j=1}^{m_i}r^{VD}e_{ij}, \qquad (6.1c)$$

$$0\leq\sum_{i=1}^{n}\sum_{j=1}^{m_i}x_{ij}\leq d, \qquad (6.1d)$$

where, for each $x_i=(x_{i1},\ldots,x_{im_i})$ given, $(y_i,z_{i1},\ldots,z_{im_i})$ solve the problem (6.1e-6.1j):

$$\min_{y_i\in Y_i,} f_i^{(2)}(x_{i1},\ldots,x_{im_i},y_i,z_{i1},\ldots,z_{im_i})$$

$$=c_i\sum_{j=1}^{m_i}y_{ij}+a(1-p_i)\sum_{j=1}^{m_i}(y_{ij}+z_{ij})-\sum_{j=1}^{m_i}s_{ij}(x_{ij}+y_{ij}-z_{ij}) \qquad \text{(Distributor } i\text{)} \quad (6.1e)$$

$$\text{s.t.} \quad \sum_{j=1}^{m_i}(x_{ij}+y_{ij}+z_{ij}) \le \sum_{j=1}^{m_i} r_i e_{ij}, \tag{6.1f}$$

$$0 \le \sum_{j=1}^{m_i} y_{ij} \le d_i, \tag{6.1g}$$

where, for each (x_{ij}, y_{ij}) given, z_{ij} solves the problem (6.1h-6.1j):

$$\min_{z_{ij}\in Z_{ij}} f_{ij}^{(3)}(x_{ij}, y_{ij}, z_{ij}) = c_{ij}z_{ij} + s_{ij}(x_{ij}+y_{ij}-z_{ij}) \qquad \text{(Buyer } ij\text{)} \tag{6.1h}$$

$$\text{s.t.} \quad x_{ij}+y_{ij}+z_{ij} \ge r_{ij}e_{ij}, \tag{6.1i}$$

$$0 \le z_{ij} \le d_{ij}. \tag{6.1j}$$

The vendor's objective function (6.1a) involves the inventory and transportation cost $(a+c)\sum_{i=1}^{n}\sum_{j=1}^{m_i} x_{ij}$, the incentive payment $\sum_{i=1}^{n} ap_i[\sum_{j=1}^{m_i}(y_{ij}+z_{ij})]$ for the distributors' transportation cost, and the risk cost $\sum_{i=1}^{n}\sum_{j=1}^{m_i} b_{ij}(e_{ij}-x_{ij}-y_{ij}-z_{ij})$ of stock-out. The constraint condition (6.1b) implies that the lower limit to the total inventory of the VMI system must satisfy the proportion r of the raw material demand of all buyers, which reflects the replenishment policy anticipated by the vendor. (6.1c) implies that the lower limit to the total inventory of the vendor and the distributors is $\sum_{i=1}^{n}\sum_{j=1}^{m_i} r^{VD} e_{ij}$. (6.1d) represents the lower and upper limits to the inventory holding capacity of the vendor.

The objective function (6.1e) of the distributor i involves the raw material inventory holding cost $c_i \sum_{j=1}^{m_i} y_{ij}$ of its hired VMI hub, the transportation cost $a(1-p_i)\sum_{j=1}^{m_i}(y_{ij}+z_{ij})$ and the subtractive penalty term $\sum_{j=1}^{m_i} s_{ij}(x_{ij}+y_{ij}-z_{ij})$ derived from its downstream buyers' compensation payments for the inventory variance. The constraint condition (6.1f) means the replenishment policy anticipated by the distributor i. (6.1g) reflects the lower and upper limits to the inventory holding capacity of the distributor i.

The buyer's objective function (6.1h) involves the raw material inventory holding cost $c_{ij}z_{ij}$ and the penalty cost $s_{ij}(x_{ij}+y_{ij}-z_{ij})$ paid to its upstream distributor. The constraint condition (6.1i) represents the desired replenishment policy by the buyer ij. (6.1j) means the lower and upper limits to the inventory of the buyer ij.

According to the organizational structure of the VMI system, the analytical model (6.1) can be considered as a tri-level decision problem with multiple followers at the middle and bottom levels, which is called a tri-level multi-follower (TLMF) problem as proposed in Chapter 5. In the TLMF problem (6.1), the priority of the vendor is to optimize its objective function (6.1a) under the constraint region S comprised of the constraint conditions (6.1b-6.1d), (6.1f-6.1g), and (6.1i-6.1j) with $j=1,2,\ldots,m_i, i=1,2,\ldots,n$. For each fixed $x_i=(x_{i1},\ldots,x_{im_i})$ by the vendor, the distributor i aims to optimize its objective function (6.1e) under the feasible set $S_i(x_i)$ consisting of the constraint conditions (6.1f-6.1g) together with problem (6.1h-6.1j) and $j=1,2,\ldots,m_i$. In view of the given (x_{ij},y_{ij}) from the upstream vendor and distributor, each buyer optimizes its objective function (6.1h) under the feasible set $S_{ij}(x_{ij},y_{ij})$ consisting of the constraint conditions (6.1i-6.1j). The optimal sets (or the rational reaction sets) of the buyer ij and the distributor i for the fixed (x_{ij},y_{ij}) and x_i are therefore respectively defined as

$$P_{ij}(x_{ij},y_{ij})=\{z_{ij}:z_{ij}\in\arg\min\{f_{ij}^{(3)}:z_{ij}\in S_{ij}(x_{ij},y_{ij})\}\},$$

$$P_i(x_i)=\{(y_i,z_{i1},\ldots,z_{im_i}):(y_i,z_{i1},\ldots,z_{im_i})\in\arg\min\{f_i^{(2)}:(y_i,z_{i1},\ldots,z_{im_i})\in S_i(x_i)\}\}.$$

Thus, the inducible region of the TLMF problem (6.1) can be written as

$$IR=\{(x,y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,z_{nm_1},\ldots,z_{nm_n}):(x,y_1,\ldots,y_n,z_{11},\ldots,z_{1m_1},\ldots,$$

$$z_{nm_1}, \ldots, z_{nm_n}) \in S,\ (y_i, z_{i1}, \ldots, z_{im_i}) \in P_i(x_i), i = 1,2,\ldots,n\}. \tag{6.2}$$

The TLMF problem (6.1) can be equivalently formulated as

$$\min\{f^{(1)} : (x, y_1, \ldots, y_n, z_{11}, \ldots, z_{1m_1}, \ldots, z_{nm_1}, \ldots, z_{nm_n}) \in IR\}. \tag{6.3}$$

This decision problem (6.1) can be considered as a concretization of the TLMF models (5.1) and (5.3) proposed in Chapter 5. Consequently, the TLMF *K*th-Best algorithm presented in Chapter 5 is able to be adopted to solve the problem (6.1).

6.4 COMPUTATIONAL STUDY

In this section, detailed computational experiments are implemented to illustrate how the proposed multilevel decision-making approach works, and to conduct the sensitivity analysis for key parameters and evaluate their influence on the VMI arrangement. Moreover, the efficiency performance of the TLMF *K*th-Best algorithm is examined by solving 540 large-scale instances. All numerical experiments in this study are operated in Java programs performed on a 3.47GHz Inter Xeon W3690 CPU with 12G of RAM under Red Hat Enterprise Linux Workstation.

6.4.1 AN ILLUSTRATIVE INSTANCE

This section adopts an illustrative instance to show how the TLMF *K*th-Best algorithm works on the TLMF problem (6.1). This study considers a decentralized VMI system consisting of one vendor, two distributors and two buyers attached to each distributor as the illustrative instance, which means that $n = 2, m_1 = m_2 = 2$ in the TLMF problem (6.1). Some related parameters involved in the problem are shown in Tables 6.2-6.4 as the experimental data.

From the experimental data, it can be derived that the main features of this instance are as follows: (1) the inventory costs per unit of both the vendor and buyers are higher than that of the distributors; (2) the lower limit to the replenishment capacity of the vendor is r=50.0% of the total demand of the whole supply chain network; (3) the vendor adopts an incentive contract with the first distributor ($p_1 = 100\%$) while the vendor does not use an incentive contract with the second distributor ($p_2 = 0$); (4) both distributors use a penalty contract with their respective downstream buyers and the penalty cost ($s_{ij} = 0.50$) is much smaller than the inventory holding cost.

Table 6.2 Data for the vendor

a	c	d	r	r^{VD}
1.0	3.0	25	50.0%	35%

Table 6.3 Data for the distributors

i	c_i	d_i	p_i	r_i
1	2.0	50	100%	90.0%
2	2.0	50	0	80.0%

Table 6.4 Data for the buyers

i	j	b_{ij}	c_{ij}	d_{ij}	e_{ij}	r_{ij}	s_{ij}
1	1	2.0	3.0	10	50	80.0%	0.50
	2	2.0	3.0	20	50	80.0%	0.50
2	1	2.0	3.0	10	50	70.0%	0.50
	2	2.0	3.0	15	50	70.0%	0.50

The TLMF *K*th-Best algorithm is used to solve the problem and the detailed computing process is shown in Table 6.5 which includes related data and parameters generated in the computing process. Table 6.5 presents the vertex $s^k = (x^k, y_1^k, y_2^k, z_{11}^k, z_{12}^k, z_{21}^k, z_{22}^k)$ that is searched in the current iteration *k*, and the adjacent vertices set W_k of the current vertex s^k. *T* represents the set of vertices that have been searched in past iterations while *W* is the set of vertices that are required to verify whether or not an optimal solution occurs inside in the following iteration. Following the procedures of the TLMF *K*th-Best algorithm, an optimal solution is finally obtained after five iterations. Note that W_k in Table 6.5 does not involve the adjacent vertices of s^k that have been found in previous iterations and have been involved in W.

Table 6.5 The detailed computing process of the TLMF *K*th-Best algorithm

Iteration k	Vertex s^k	W_k	T	W
1	(0,0,0,0,30,20,30,20,10,20,10,15)	{(0,0,5,0,30,20,30,20,10,20,10,15), (0,0,0,5,30,20,35,15,10,20,10,15), (0,0,0,0,30,20,25,20,10,20,10,15), (0,0,0,0,30,20,30,20,10,20,5,15), (0,0,0,0,30,20,25,25,10,20,10,10), (0,0,0,0,30,20,25,25,10,20,10,15)}	$\{s^1\}$	W_1
2	(0,0,0,0,30,20,25,25,10,20,10,15)	{(0,0,5,0,30,20,20,30,10,20,10,15), (0,0,0,5,30,20,25,25,10,20,10,15)}	$\{s^1,s^2\}$	$(W \cup W_2) \setminus T$
3	(0,0,5,0,30,20,30,20,10,20,10,15)	{(0,0,15,0,30,20,30,20,10,20,0,15), (0,0,20,0,30,20,15,35,10,20,10,0), (0,0,25,0,30,20,10,20,10,20,10,15)}	$\{s^1,s^2,s^3\}$	$(W \cup W_3) \setminus T$
4	(0,0,0,5,30,20,35,15,10,20,10,15)	{(0,0,0,15,30,20,45,5,10,20,0,15), (0,0,0,20,30,20,35,0,10,20,10,15), (0,0,0,20,30,20,35,15,10,20,10,0)}	$\{s^1,s^2,s^3,s^4\}$	$(W \cup W_4) \setminus T$
5	(0,0,0,0,30,20,25,20,10,20,10,15)	---	--	---

Table 6.6 displays the objective values of all decision entities respectively towards each solution enumerated by the TLMF *K*th-Best algorithm. As it can be seen from Table 6.6, under the current VMI decision-making context in the supply chain network, $s^1 = (x^1, y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (0,0,0,0,30,20,30,20,10,20,10,15)$ is the individual best solution to the vendor, which implies that the vendor anticipates that the distributors and buyers can choose $(y_1^1, y_2^1, z_{11}^1, z_{12}^1, z_{21}^1, z_{22}^1) = (30,20,30,20,10,20,10,15)$ to respond to the vendor after it has determined x=(0,0,0,0). However, the distributors and buyers prefer to choose $(y_1^5, y_2^5, z_{11}^5, z_{12}^5, z_{21}^5, z_{22}^5) = (30,20,25,20,10,10,10,15)$ to respond to the vendor to optimize their own objective functions. For the fixed decision x=(0,0,0,0) by the vendor, $y_1 = (30,20)$ and $y_2 = (25,20)$ are the best solution for the first and second distributor respectively while considering the implicit

reactions of their downstream buyers. In view of the given decision $(x_{11}, y_{11}) = (0,30)$ and $(x_{12}, y_{12}) = (0,20)$ from the vendor and the first distributor, buyer 11 and buyer 12 achieve an optimal solution $(z_{11}, z_{12}) = (10,20)$ to respond to the vendor and the first distributor. Similarly, for the given decision $(x_{21}, y_{21}) = (0,25)$ and $(x_{22}, y_{22}) = (0,20)$ from the vendor and the second distributor, buyer 21 and buyer 22 achieve an optimal solution $(z_{21}, z_{22}) = (10,15)$ to respond to the vendor and the second distributor. Clearly, $(y_{11}, y_{12}, z_{11}, z_{12}) = (30,20,10,20)$ and $(y_{21}, y_{22}, z_{21}, z_{22}) = (25,20,10,15)$ are the best solutions respectively for the distributor i(i=1,2) and its buyers to react to the given decision x=(0,0,0,0) by the vendor. Therefore, $(x^5, y_1^5, y_2^5, z_{11}^5, z_{12}^5, z_{21}^5, z_{22}^5) = (0,0,0,0,30,20,25,20,10,10,10,15)$ is an optimal solution to the decentralized VMI coordination problem.

Table 6.6 Solutions and objective values of decision entities

Iteration k	Vertex s^k	$f^{(1)}$	$f_1^{(2)}$	$f_2^{(2)}$	$f_{11}^{(3)}$	$f_{12}^{(3)}$	$f_{21}^{(3)}$	$f_{22}^{(3)}$
1	(0,0,0,0,30,20,30,20,10,20,10,15)	170.0	90.0	162.5	40.0	60.0	40.0	47.5
2	(0,0,0,0,30,20,25,25,10,20,10,15)	170.0	90.0	162.5	40.0	60.0	37.5	50.0
3	(0,0,5,0,30,20,30,20,10,20,10,15)	180.0	90.0	160.0	40.0	60.0	42.5	47.5
4	(0,0,0,5,30,20,35,15,10,20,10,15)	180.0	90.0	160.0	40.0	60.0	42.5	47.5
5	(0,0,0,0,30,20,25,20,10,20,10,15)	180.0	90.0	150.0	40.0	60.0	37.5	47.5

Although the vendor's total cost has become worse by increasing from 170.0 to 180.0 during the decision process from s^1 to s^5, the vendor cannot obtain a better objective value by moving away from the vertex over the *IR*, because some individuals try to obtain better results for themselves under the VMI coordination, yielding the solution s^5. Therefore, the vertex s^5 is an optimal solution to the VMI planning problem, which means a final coordinated result for decision entities in the current VMI decision context. The coordinated result indicates that the vendor may not achieve its individual optimal solution under the constraint region even though it has priority in making decisions, since its decisions are implicitly determined by reactions of its downstream decision entities. Clearly, the TLMF *K*th-Best algorithm provides a practical way to solve the TLMF optimization problem (6.1). The optimal solution to the TLMF problem (6.1) means a compromised equilibrium under VMI coordination.

6.4.2 SENSITIVITY ANALYSIS

In this section, the sensitivity analysis is conducted for key parameters of the VMI system involving the distributor's and buyer's inventory upper limits, the distributor's transportation cost, and the incentive and penalty mechanism. This section analyzes their influence on the coordinated VMI results and the individual performance of each decision entity.

6.4.2.1 EFFECT OF THE DISTRIBUTOR'S AND BUYER'S INVENTORY UPPER LIMITS

This section conducts one group of numerical experiments to analyze the effect of the distributor's and buyer's inventory upper limits on the VMI performance while considering the illustrative instance in Section 6.4.1 as the base problem. Under VMI coordination, with the upper limits to the first distributor's inventory and its first buyer's inventory respectively increasing from 30 to 60 and decreasing from 10 to 0, the corresponding optimal solutions and coordinated results are given in Table 6.7.

It can be seen from Table 6.7 that the vendor prefers to reduce its own inventory when the distributor's inventory upper limit increases under certain inventory upper limit of the buyers. The cost of the distributor increases while that of the vendor declines during the period. The much higher upper limit to the distributor's inventory will not result in an extended reduction of the vendor's inventory until the vendor's inventory equals to zero. Table 6.7 also shows that the distributor will increase its inventory level following the decrease in its downstream buyer's inventory upper limit.

After the distributor's inventory upper limit is full filled, the vendor will keep higher inventory level to maintain the raw material supply for the buyer. The buyer's cost experiences a downward trend following the decrease in its own inventory, while that of the vendor grows gradually. However, the experiment is performed in which the number d_{11} is set smaller than 5 under $d_1 = 30$ such that an optimal solution to the TLMF problem (6.1) cannot be found, which implies that the much smaller upper limit to the buyer's inventory is not permitted in the VMI system if the vendor and the distributor do not possess sufficient inventory upper limit. In contrast, it is noticeable in case 10 that, if the distributor’s inventory upper limit is large enough, both the vendor and the buyer are able to reduce their own inventory level to zero; meanwhile, their costs both decreases. Thus, for the sake of achieving zero inventory, the vendor and buyers, as the manufacturing enterprises, should try to persuade the distributors to improve their individual inventory upper limits when making the VMI contract.

Table 6.7 The experimental results based on the distributor's and buyer's inventory upper limits changes

Experiments			The optimal solutions and coordinated results								Computational performance	
Case	d_1	d_{11}	$(x, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22})$	$f^{(1)}$	$f_1^{(2)}$	$f_2^{(2)}$	$f_{11}^{(3)}$	$f_{12}^{(3)}$	$f_{21}^{(3)}$	$f_{22}^{(3)}$	Iteration	Time (s)
1	30	10	(0,20,0,0,30,0,25,20,10,20,10,15)	240.0	50.0	150.0	40.0	60.0	37.5	47.5	6	0.221
2		5	(5,20,0,0,30,0,25,20,5,20,10,15)	255.0	45.0	150.0	30.0	60.0	37.5	47.5	5	0.159
3		0	Not Applicable								0	0.015
4	40	10	(0,10,0,0,30,10,25,20,10,20,10,15)	210.0	70.0	150.0	40.0	60.0	37.5	47.5	7	0.236
5		5	(0,15,0,0,35,5,25,20,5,20,10,15)	225.0	65.0	150.0	30.0	60.0	37.5	47.5	8	0. 251
6		0	(0,20,0,0,40,0,25,20,0,20,10,15)	240.0	60.0	150.0	20.0	60.0	37.5	47.5	6	0.189
7	50	10	(0,0,0,0,30,20,25,20,10,20,10,15)	180.0	90.0	150.0	40.0	60.0	37.5	47.5	5	0.144
8		5	(0,5,0,0,35,15,25,20,5,20,10,15)	195.0	85.0	150.0	30.0	60.0	37.5	47.5	7	0.236
9		0	(0,10,0,0,40,10,25,20,0,20,10,15)	210.0	80.0	150.0	20.0	60.0	37.5	47.5	7	0.235
10	60	10	(0,0,0,0,30,20,25,20,10,20,10,15)	180.0	90.0	150.0	40.0	60.0	37.5	47.5	7	0.285
11		5	(0,0,0,0,35,20,25,20,5,20,10,15)	180.0	95.0	150.0	30.0	60.0	37.5	47.5	7	0.208
12		0	(0,0,0,0,40,20,25,20,0,20,10,15)	180.0	100.0	150.0	20.0	60.0	37.5	47.5	7	0.220

6.4.2.2 Effect of the penalty mechanism

To identify the effect of the penalty mechanism between each distributor and its downstream buyers, this section analyzes the coordinated results for the penalty cost of the second buyer attached to the second distributor (called buyer 22) increasing from 0 to 4.0. The experimental results, which consider case 7 in the above section as the basic problem, are shown in Table 6.8.

As shown in Table 6.8, when the penalty cost is smaller (e.g. $s_{22}=0$, $s_{22}=0.5$ and $s_{22}=1.0$), the second distributor's cost amounts is higher in value. In this situation, the second distributor does not prefer to increase its inventory level and buyer 22 has to keep a certain amount of back-up inventory to service the production demand. With the penalty cost increasing from 1.5 to 3.0, the second distributor is willing to increase its inventory to its upper limit to reduce its total cost while the buyer 22 need not to keep so many inventory but has to compensate the higher penalty cost for the second distributor. However, when the penalty cost is higher than the inventory cost (e.g. $s_{22}=3.5$ and $s_{22}=4.0$), buyer 22 will increase its holding inventory to reduce the penalty cost, even though the second distributor prefers to hold the largest inventory. Table 6.8 also clearly shows that the total cost of the second distributor reaches the bottom when the penalty cost is equal to the inventory cost. Thus, the second distributor can price the penalty cost at $s_{22}=3.0$ in designing the penalty VMI contract with the buyer. During the same period, the vendor's cost decreases from 180.0 to 170.0, which means that the vendor can persuade distributors to increase the penalty cost with the buyers in order to motivate the buyers to hold more inventory if the vendor seeks to improve its individual performance.

Table 6.8 The experimental results based on the penalty cost changes

Experiments		The optimal solutions and coordinated results								Computational performance	
Case	s_{22}	$(x, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22})$	$f^{(1)}$	$f_1^{(2)}$	$f_2^{(2)}$	$f_{11}^{(3)}$	$f_{12}^{(3)}$	$f_{21}^{(3)}$	$f_{22}^{(3)}$	Iteration	Time (s)
13	0	(0,0,0,30,20,25,20,10,20,10,15)	180.0	90.0	152.5	40.0	60.0	37.5	45.0	5	0.149
7	0.5	(0,0,0,30,20,25,20,10,20,10,15)	180.0	90.0	150.0	40.0	60.0	37.5	47.5	5	0.144
14	1.0	(0,0,0,30,20,25,20,10,20,10,15)	180.0	90.0	147.5	40.0	60.0	37.5	50.0	5	0.143
15	1.5	(0,0,0,30,20,25,25,10,20,10,10)	180.0	90.0	140.0	40.0	60.0	37.5	52.5	7	0.221
16	2.0	(0,0,0,30,20,25,25,10,20,10,10)	180.0	90.0	132.5	40.0	60.0	37.5	60.0	7	0.212
17	2.5	(0,0,0,30,20,25,25,10,20,10,10)	180.0	90.0	125.0	40.0	60.0	37.5	67.5	7	0.225
18	3.0	(0,0,0,30,20,25,25,10,20,10,10)	180.0	90.0	117.5	40.0	60.0	37.5	75.0	7	0.220
19	3.5	(0,0,0,30,20,25,25,10,20,10,15)	170.0	90.0	132.5	40.0	60.0	37.5	80.0	2	0.063
20	4.0	(0,0,0,30,20,25,25,10,20,10,15)	170.0	90.0	127.5	40.0	60.0	37.5	85.0	2	0.047

Table 6.9 The experimental results based on the distributor's transportation cost changes

Experiments			The optimal solutions and coordinated results								Computational performance	
Case	p_2	a	$(x, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22})$	$f^{(1)}$	$f_1^{(2)}$	$f_2^{(2)}$	$f_{11}^{(3)}$	$f_{12}^{(3)}$	$f_{21}^{(3)}$	$f_{22}^{(3)}$	Iteration	Time (s)
17	0	1.0	(0,0,0,0,30,20,25,25,10,20,10,10)	180.0	90.0	125.0	40.0	60.0	37.5	67.5	7	0.225
21		1.5	(0,0,0,0,30,20,25,20,10,20,10,15)	220.0	90.0	160.0	40.0	60.0	37.5	67.5	5	0.159
22		2.0	(0,0,0,0,30,20,25,20,10,20,10,15)	260.0	90.0	195.0	40.0	60.0	37.5	67.5	5	0.172
23		2.5	(0,0,0,0,30,20,25,20,10,20,10,15)	300.0	90.0	230.0	40.0	60.0	37.5	67.5	5	0.126
24	100%	1.0	(0,0,0,0,30,20,25,25,10,20,10,10)	250.0	90.0	55.0	40.0	60.0	37.5	67.5	5	0. 134
25		1.5	(0,0,0,0,30,20,25,25,10,20,10,10)	325.0	90.0	55.0	40.0	60.0	37.5	67.5	5	0.141
26		2.0	(0,0,0,0,30,20,25,25,10,20,10,10)	400.0	90.0	55.0	40.0	60.0	37.5	67.5	3	0.078
27		2.5	(0,0,0,0,30,20,25,25,10,20,10,10)	475.0	90.0	55.0	40.0	60.0	37.5	67.5	3	0.078

6.4.2.3 EFFECT OF THE TRANSPORTATION COST AND THE INCENTIVE MECHANISM

In this section, case 17 is considered as the basic problem based on the related discussion in Section 6.4.2.2 to ensure that the reaction space of the distributors is well positioned. The experimental results for the transportation cost ranging from 1.0 to 2.5 are reported in Table 6.9 and can be classified into two groups according to whether or not the incentive mechanism by $p_2 = 0$ and $p_2 = 100\%$ is considered between the vendor and the second distributor.

The obtained results in Table 6.9 without considering the incentive mechanism ($p_2 = 0$) show that, when the transportation cost is increased, the second distributor's inventory decreases during the same period while the inventory of buyer 22 increases. Since the buyer will react to the decrease in the second distributor's inventory by improving its own inventory level, the second distributor anticipates the reduction in its transportation cost by reducing its own inventory level. In contrast, when the incentive contract is considered ($p_2 = 100\%$), the inventory of the second distributor remains stable at a higher level. The results imply that the increase in the transportation cost results in the inventory reduction of the distributor when the incentive contract is not considered.

Table 6.9 reports that the vendor's and the second distributor's costs respectively experience an upward trend without the incentive mechanism, while the first distributor's cost remains stable at 90.0 because of the incentive mechanism. When the incentive contract is considered, the total cost of the vendor is higher than the same number under $p_2 = 0$, while the second distributor's cost becomes smaller than the same number under $p_2 = 0$. In addition, following the transportation cost increasing under $p_2 = 100\%$, the cost of the second distributor remains stable as does that of the first distributor, and only the total cost of the vendor increases. The above analysis reflects that the incentive mechanism contributes to a better individual performance of the distributor during the period that the transportation cost rises up, thus, the vendor can use a incentive contract to encourage the distributor to keep more inventory for maintaining the raw material supply following the increase in the transportation cost. The results also imply that the upstream decision entities should take into account the implicit reactions of its downstream decision entities when making its own decisions under VMI coordination.

6.4.3 ASSESSING THE EFFICIENCY PERFORMANCE OF THE TLMF *K*TH-BEST ALGORITHM

The experimental results in Tables 6.7-6.9 show that very little computing time is spent on solving each experimental case, which implies that the TLMF *K*th-Best algorithm can be carried out efficiently. However, those previously mentioned cases 1-27 are considered as small-scale instances. The efficiency performance of the algorithm may declines rapidly with the increase in the mass of distributors and buyers. To assess the algorithm performance in depth, the proposed TLMF *K*th-Best algorithm is used to solve large-scale instances in this section.

The TLMF decision techniques for modeling decentralized VMI coordination problem have not been considered in literatures, therefore, there are no benchmark instances. In this study, large-scale instances are randomly generated to cover a wide range of problem structures based on the sensitivity analysis in Section 6.4.2. The test instances are classified into 18 different problem sets following the number of distributors and buyers becoming larger and larger, which are displayed in Table 6.10. As shown in Table 6.10, n and m are the number of distributors and buyers respectively, whereas the number (m_i) of buyers that are attached to the same distributor i ($i = 1,2,\ldots,n$) is randomly determined under $\sum_{i=1}^{n} m_i = m$ and $m_i \neq 0$. Table 6.10 also shows the numbers of decision variables and constraint conditions, respectively known as *Num*(v) and *Num*(con). It can be seen from model (6.1) that *Num*(v) is determined by the number of buyers, whereas *Num*(con) is determined by the total number of distributors and buyers. In this computational study, 30 test problems are randomly constructed within each problem set; thus, there are $30 \times 18 = 540$ different instances randomly generated as a whole within 18 test problem sets.

Table 6.10 Test instances randomly generated

Problem set	n	m	*Num*(v)	*Num*(con)	Problem set	n	m	*Num*(v)	*Num*(con)
P_{01}	2	8	24	23	P_{10}	10	30	90	83
P_{02}	4	8	24	27	P_{11}	15	30	90	93
P_{03}	6	8	24	31	P_{12}	20	30	90	103
P_{04}	4	15	45	41	P_{13}	15	40	120	113
P_{05}	6	15	45	45	P_{14}	20	40	120	123
P_{06}	10	15	45	53	P_{15}	30	40	120	143
P_{07}	6	20	60	55	P_{16}	20	50	150	143
P_{08}	10	20	60	63	P_{17}	30	50	150	163
P_{09}	15	20	60	73	P_{18}	50	50	150	203

In terms of parameters in model (6.1), buyer demand e_{ij} is randomly chosen from the uniform distribution on [40, 60], and buyer inventory upper limit d_{ij} is generated from uniform random numbers between 5 and 20. For the sake of ensuring the problem is well posed, distributor inventory upper limit d_i is randomly chosen from the interval $[\sum_{j=1}^{m_i}(e_{ij}r_{ij} - d_{ij}) - 10, \sum_{j=1}^{m_i}(e_{ij}r_{ij} - d_{ij}) + 10]$, whereas vendor inventory upper limit d is uniformly distributed on $[\sum_{i=1, i\neq s}^{n}(\sum_{j=1}^{m_i}(e_{ij}r_{ij} - d_{ij}) - d_i), \sum_{i=1, i\neq s}^{n}(\sum_{j=1}^{m_i}(e_{ij}r_{ij} - d_{ij}) - d_i) + 10]$ where $\sum_{j=1}^{m_s}(e_{sj}r_{sj} - d_{sj}) - d_s < 0$. The remainder parameters in model (6.1) keep the same as those in Section 6.4.1 apart from setting $p_i = 100\%$, $r_i = 90\%$, $r_{ij} = 80\%$ and $s_{ij} = 1.5$.

Table 6.11 The experimental results of the randomly generated test problems

Problem set	*n-m-Num(v)-Num(con)*	*Num(best)*	Iteration	Time (*s*)	Problem set	*n-m-Num(v)-Num(con)*	*Num(best)*	Iteration	Time (*s*)
P_{01}	2-8-24-23	30	17.77	17.953	P_{10}	10-30-90-83	27	579.52	387.541
P_{02}	4-8-24-27	30	36.57	23.117	P_{11}	15-30-90-93	26	558.81	401.259
P_{03}	6-8-24-31	30	32.50	24.812	P_{12}	20-30-90-103	27	564.67	431.524
P_{04}	4-15-45-41	30	147.27	39.798	P_{13}	15-40-120-113	22	768.82	694.258
P_{05}	6-15-45-45	30	196.40	57.814	P_{14}	20-40-120-123	20	737.05	729.654
P_{06}	10-15-45-53	30	164.53	65.237	P_{15}	30-40-120-143	24	727.50	758.216
P_{07}	6-20-60-55	29	491.69	185.642	P_{16}	20-50-150-143	19	873.53	991.852
P_{08}	10-20-60-63	28	427.82	208.858	P_{17}	30-50-150-163	22	843.45	1036.248
P_{09}	15-20-60-73	30	383.23	221.341	P_{18}	50-50-150-203	18	819.00	1123.982

The TLMF *K*th-Best algorithm is used to solve these 540 test problems randomly generated. In terms of solving each problem, the maximal iteration number is 1000 and each problem is carried out 10 runs to collect the computing CPU time. If the algorithm can find an optimal solution for a problem within 1000 iterations, it is considered as successful computation. Table 6.11 shows the number of test problems successfully solved within each problem set, denoted by *Num*(*best*). Table 6.11 also displays the average of the iteration number and the computing CPU time (in seconds) for each problem set.

It is clear in Table 6.11 that the number of problems successfully solved becomes smaller following the increase in the number of distributors and buyers. The reason is that more and more test problems have no solutions or the computational iterations exceeds 1000 during the experimental process. To explore more in depth, the algorithm efficiency is examined, shown as the iterations and computing time in Table 6.11. As shown in Table 6.11, both the number of iterations and computing time experience a sharp rise with the problem size being larger and larger. Notice that, when the number of buyers remains stable, the number of problems successfully solved, as well as the algorithm efficiency, does not shows substantial change with the number of distributors increasing. Thus, the number of buyers has a significant influence on the performance of the TLMF *K*th-Best algorithm; Figure 6.2 displays much more evident results. The results mean that the number of decision variables, rather than constraint conditions, determines the characteristics of the TLMF problem (6.1) and the efficiency performance of the TLMF *K*th-Best algorithm.

Since the TLMF problems have not been solved by other algorithms in literatures, the algorithm performance cannot be compared with others. In this study, the efficiency performance of the TLMF *K*th-Best algorithm is assessed using the growth trend of the computing CPU time for solving these test problems. It can be found in Figure 6.3 that the growth trend of CPU time coincides with a fitted quadratic polynomial curve rather than a exponential curve. Clearly, although the computational load of the TLMF *K*th-Best experiences a steep upward trend in response to the problem size becoming larger, these test problems can be solved by the algorithm in a quadratic polynomial time, which is a reasonable computing time for solving NP-hard problems. Therefore, the TLMF *K*th-Best algorithm can be considered as a feasible and an effective approach for solving the proposed decentralized VMI coordination problems.

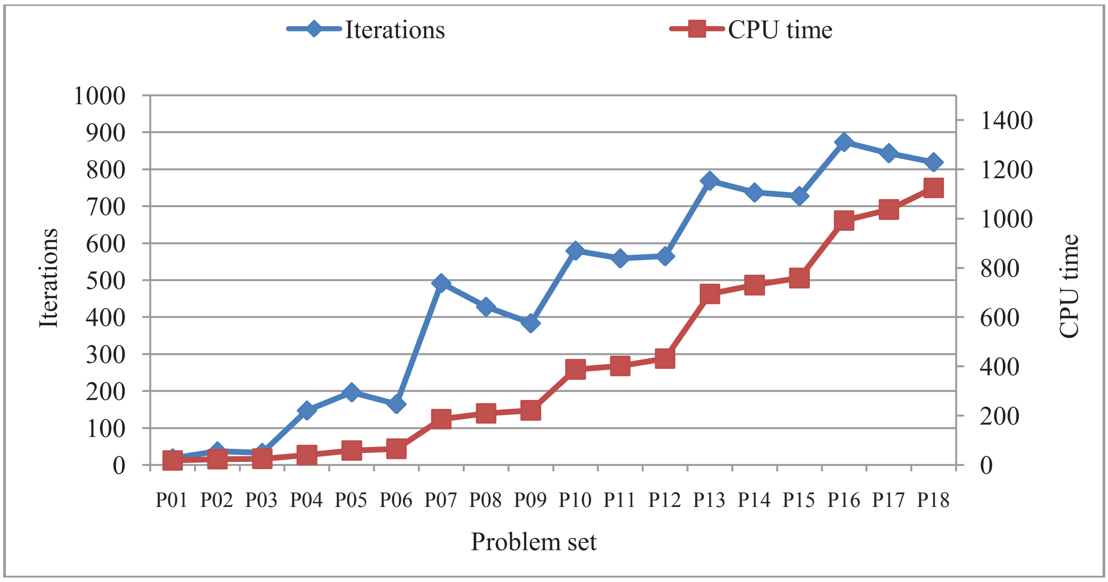


Figure 6.2 The average of iterations and CPU time for solving the test problems

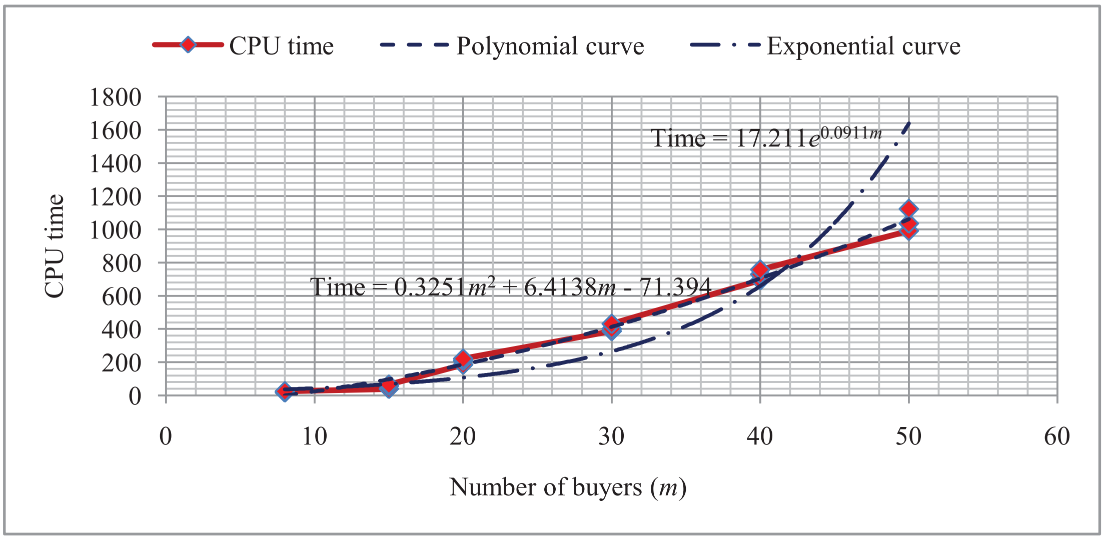


Figure 6.3 The fitted curve of CPU time following the number of buyers change

6.5 SUMMARY

This chapter considers a decentralized VMI coordination problem in a three-echelon hierarchical supply chain network consisting of a vendor, multiple third-party logistics companies as distributors, and multiple buyers. To deal with the inventory coordination among the decision entities, this chapter first proposes a linear TLMF decision model to describe the VMI problem. The TLMF *K*th-Best algorithm developed in Chapter 5 is then used to solve the resulting TLMF decision model. The sensitivity analysis for key VMI parameters is conducted to examine the operations multilevel decision-making process in which decision entities coordinate with one another on the decentralized VMI.

The computational results imply that an optimal solution to the proposed VMI problem results in a compromised equilibrium for decision entities under inventory coordination; it may not be the best solution in respect to the vendor, although it has priority in making decisions. The sensitivity analysis reports that decision entities can achieve different coordinated results by adjusting related VMI parameters, which have different influences on the individual performance of each decision entity. The experimental results also clearly show that the performance improvement of one individual does not mean the cost reduction of all decision entities, which implies that decision entities are only concerned with their own cost reduction rather than the performance improvement of others under VMI coordination. Under VMI coordination, decision entities should prefer to cooperate with one another in designing a VMI contract, if they seek to improve their individual performances; for example, the vendor may persuade distributors to increase their inventory upper limits to improve the VMI performance, as well, the incentive and penalty mechanism can be adopted to balance the total cost sharing between decision entities. Moreover, the computational study shows that large-scale problems can be solved in a reasonable computing time using the proposed TLMF *K*th-Best algorithm.

In conclusion, this chapter provides a practical way to handle the decentralized VMI coordination problem in a three-echelon supply chain network. This chapter shows how to improve the individual performance of each decision entity and balance the total cost sharing in a decentralized VMI hierarchy. Of particular importance, this chapter displays how to design a manufacturer-manufacturer (vendor-buyer) VMI system where third-party logistics are involved, and through which the vendor and buyers can achieve their holding inventory as small as possible or even at zero inventory.

CHAPTER 7 CONCLUSIONS AND FURTHER STUDY

This chapter concludes the whole thesis and provides some further research directions for the topic.

7.1 CONCLUSIONS

Multilevel decision-making techniques have been widely applied to handle decentralized decision problems in the real world. The latest developments of multilevel decision-making typically display three features: (1) large-scale - high-dimensional decision variables make multilevel decision problems large-scale; (2) uncertainty - uncertain information is always involved in related decision parameters and conditions, which become imprecisely or ambiguously known to decision entities; (3) diversification - there may exist multiple decision entities at each decision level, in which multiple decision entities at the same level have a variety of relationships with one another. To support large-scale, uncertain and diversified multilevel decision-making, this research develops related decision models and/or solution approaches to handle three categories of unsolved multilevel decision problems, involving large-scale nonlinear bi-level and tri-level decision problems, fuzzy nonlinear bi-level decision problems and TLMF decision problems. Moreover, these proposed multilevel decision-making techniques are applied to deal with decentralized production and inventory operational problems in SCM.

The main contributions of this study are as follows:

1) It proposes a bi-level PSO algorithm to solve large-scale nonlinear bi-level decision problems; the bi-level PSO algorithm is then extended to a tri-level PSO algorithm for solving tri-level decision problems. (To achieve Objective 1)

In the proposed bi-level/tri-level PSO algorithms, the leader's problem and the follower's problem are separated based on related solution concepts for convenient solving. To handle the complexity of the constraint region of nonlinear and large-scale problems, two methods for constructing the initial population are given. Moreover, the decreasing inertia weight with time is used to control the velocity of particles in the search space at different stages, which can improve both the search and convergence abilities of the bi-level/tri-level PSO algorithms. The computational results report that the bi-level/tri-level PSO algorithms are able to find much better solutions than other algorithms and show much better performance in terms of efficiency than other algorithms which follow the problem size by becoming larger and larger. Consequently, the proposed bi-level PSO algorithm provides a practical way to solve large-scale nonlinear bi-level decision problems; moreover, it can be extended to a tri-level PSO algorithm for solving tri-level decision problems.

2) It proposes a compromise-based PSO algorithm for solving fuzzy nonlinear bi-level decision problems. (To achieve Objective 2)

The compromise-based PSO algorithm can be used to solve nonlinear bi-level decision problems with general fuzzy numbers. With regard to the compromise-based PSO algorithm, the leader and follower can choose acceptable decision conditions based on rules of compromise due to uncertain decision environments, which can result in the preferred solution under individual decision situations. The computational results show that the compromise-based PSO algorithm can not only provide better solutions to the specific decision situation compared with the existing solution approaches but also present different options in terms of solutions due to uncertain decision environments.

3) It proposes different TLMF decision models to describe various relationships between multiple followers at the same level, and discusses theoretical properties in relation to the existence and optimality of solutions. (To achieve Objective 3)

The TLMF decision models can be applied to describe different decision-making processes motivated by various relationships between multiple followers at the same level. Related solution concepts analyze the operations of the TLMF decision-making process. The theoretical properties give sufficient conditions for the optimal solution existence and the geometry of the solution space, which provides the theoretical foundation for designing an effective solution algorithm for solving TLMF decision problems.

4) It proposes a TLMF *K*th-Best algorithm for solving TLMF decision problems and a evaluation method to assess the solution obtained. (To achieve Objective 4)

The TLMF *K*th-Best algorithm ranks the vertices of the constraint region by the leader's objective value and verifies these vertices in sequence whether or not each vertex is an optimal solution. The algorithm is not terminated until the *K*th vertex is found that can be considered as an optimal solution. This algorithm can find an optimal solution to TLMF decision problems; moreover, it is able to solve large-scale problems in a reasonable and an acceptable time. The TLMF *K*th-Best algorithm overcomes the lack of solution algorithms for solving TLMF decision problems. Since it is imprecise or ambiguous for decision entities to identify a solution whether or not they desire it, the solution evaluation method can assess the satisfaction of decision entities by transforming the objective functions into fuzzy goals. The solution evaluation method, as a relative criterion, can be applied to analyze how the TLMF decision-making process varies with changing decision externalities and provide decision support for decision entities.

5) It applies multilevel decision-making techniques to handle a decentralized VMI coordination problem in SCM. (To achieve Objective 5)

The decentralized VMI coordination problem is modeled as a TLMF decision problem, which examines how the decision entities coordinate with each other on the decentralized VMI. The TLMF *K*th-Best algorithm developed in Chapter 5 is then used to solve the resulting TLMF decision model. The computational results indicate that this research provides a practical way to handle the decentralized VMI coordination problem in a three-echelon supply chain network. This research shows how to improve the individual performance of each decision entity and balance the total cost sharing in a decentralized VMI hierarchy. Even more important, this research displays how to design a manufacturer-manufacturer (vendor-buyer) VMI system where third-party logistics are involved, and through which the vendor and buyers can achieve their holding inventory as low as possible or even at zero inventory.

7.2 FURTHER STUDY

There are still some limitations in relation to the current study:

1) Many multilevel decision problems may have no optimal solutions based on existing solution concepts. How to find a usable or satisfactory solution to these multilevel decision problems is an emerging research topic with respect to computational complexity.

2) The fully fuzzy bi-level decision problems, in which both coefficients and variables are characterized by fuzzy numbers, is also an emerging research topic and need to be examined in depth.

3) Multilevel decision problems nowadays often appear in highly complex and diversified decision environments where decision makers sometimes need to make an optimal or a wise decision from big data with uncertainty. This requires further research on how to wisely model such problems and implement data-driven decision-making in the current age of big data by means of a multilevel decision support system.

4) Multilevel decision-making techniques are mainly limited to mathematical programming modeling and solving. However, many multilevel decision problems in the real world cannot be modeled as mathematical programming formulations. It is another challenge to break through mathematical programs and integrate the principle of multilevel decision-making with other decision-making techniques to handle a much wider range of decision problems.

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Abbreviations

AC	Alternating current
BLMF	Bi-level multi-follower
BLML	Bi-level multi-leader
BLMLMF	Bi-level multi-leader and multi-follower
BLMO	Bi-level multi-objective
BLMOMF	Bi-level multi-objective multi-follower
EIP	Eco-industrial park
GABB	Genetic algorithm based on bases
IR	Inducible region
MFCQ	Manasarian-Fromowitz constraint qualification
MRP	Material requirement planning
PSO	Particle swarm optimization
PSO-CST	Particle swarm optimization with chaos searching technique
SCM	Supply chain management
TLMF	Tri-level multi-follower
VMI	Vendor-managed inventory