

Inverse Kinematics, Kinematic Control and Redundancy Resolution for Chained-Link Robotic Manipulators

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Certificate of Authorship and Originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

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Abstract

This aim of this work is to present a comprehensive review and analysis with experiments and concrete comparison on the methods, algorithms and techniques proposed for the Inverse Kinematics, Kinematic Control and Redundancy Resolution problems in chained-link manipulators. In addition to the review of classic solutions proposed in the literature, this thesis introduces some novel and innovative methods from the author that have not been used for the IK and RR problems prior to this study.

This thesis also presents a targeted layout of experiments in order to evaluate and compare the performance of different solutions and techniques in the IK problem. Various algorithmic factors and settings have been tested on different solution methods for four manipulators with different geometries and degrees of freedom. The tests are designed to find the optimum values for different influential parameters in order to improve the performance step by step so that in the final test, a good performance with almost %100 success rate and reasonable computational cost is achieved.

In addition to the comprehensive review and proposition of novel techniques, this thesis presents two robust software packages named as *Manipulator Generic Inverse Kinematic Solver (MAGIKS)* and *Skilled-PR2 (S-PR2)* which have been used to implement the experiments. The first one is a local Jacobian-based numeric IK solver that can be used for any general chained-link manipulator with no limitation on degree of freedom and number of end-effectors. The second one is an analytic (position-based) IK solver for PR2 with the ability of redundancy optimization for this robot. Both solvers are able to project and generate smooth and feasible trajectories in the joint-space and can be used by researchers and developers working on robot kinematics.