# DIVERSIFICATION PHILOSOPHY AND BOOSTING TECHNIQUE FOR TRADE EXECUTION STRATEGY

By

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Submitted in fulfillment of the requirements for the degree of

**Doctor of Philosophy**

University of Technology, Sydney

March 2006

# CERTIFICATE OF AUTHORSHIP/ORIGINALITY

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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*To My Wife Ying Liu and My Parents*

# Abstract

This thesis explores the rationale and effectiveness of diversification across time and strategies, which is an important philosophy for risk management in practice, in the framework of developing trade execution strategies. In this thesis, the strategies are defined as making a series of decisions based on real-time state variables over a fixed period to achieve high reward and low risk with given resources. Trade execution strategies are to make a series of decisions on how to place an order in markets based on real-time market information over a fixed period to fill the order with low cost and risk in the end.

In the 1st part, this thesis explores diversification across time. The research of trade execution has shown that although limit order strategy achieves lower cost than market order strategy does, it may incur nonexecution risk and miss trading opportunities. This thesis proposes a strategy that reflects the idea of diversification across time to improve the limit order strategy. In the 2nd part, this thesis explores diversification across strategies. Techniques for implementing this idea have been proposed to acquire strategies from a candidate strategy set and determine their weights. For those techniques, the candidate strategy set normally only contains finite strategies and the risk that they reduce is only measured by one specific standard. This thesis proposes a technique that overcomes those drawbacks. In the 3rd part, the proposed technique is applied to improve trade execution strategies.

The strategy proposed in the 1st part is called DF (dynamic focus) strategy, which incorporates a series of small market orders with different volume into the limit

order strategy and dynamically adjusts each market order volume based on two real-time state variables: inventory and order book imbalance. The sigmoid function is adopted to map the variables to the market order volume. Experiments show that the DF strategy achieves lower cost and risk than the limit order strategy does.

The technique proposed in the 2nd part extends the key idea of the AdaBoost (adaptive boosting) technique, which is discussed mostly in the supervised learning field. It is named DAB (diversification based on AdaBoost) in this thesis. The DAB technique adaptively updates the probability distribution on training examples in the learning process, acquires strategies from a candidate strategy set and determines their weights. Resources (e.g. money or an order) are allocated to each acquired strategy in proportion with its weight and all acquired strategies are then executed in parallel with their allocated resources. The DAB technique allows the candidate strategy set to contain infinite strategies. Analysis shows that as the learning steps increase, the DAB technique lowers the candidate strategy set's risk, which can be measured by different standards, and limits the decrease in its reward.

The DAB technique is applied in the 3rd part to acquire DF strategies from a candidate DF strategy set and determine their weights. The entire order is allocated to each acquired DF strategy in proportion with its weight and all acquired DF strategies are then executed in parallel to fill their allocated order. In this thesis, this parallel execution is called BONUS (boosted dynamic focus) strategy. Experiments support theoretical analysis and show that the BONUS strategy achieves lower risk and cost than the optimal DF strategy and two simple diversification techniques do.

This thesis is contributed to both finance and computer science fields from the theoretical and empirical perspectives. First, the proposed DF strategy verifies the effectiveness of diversification across time through improving the existing trade execution strategies. Second, the proposed DAB technique provides a flexible way for implementing diversification across strategies to complement the existing diversification techniques and enrich the research of the AdaBoost technique. Third, the proposed DAB technique and BONUS strategy provide a flexible way to improve trade execution strategies.

# Acknowledgements

First of all, I am full of gratitude to Prof. Chengqi Zhang and Prof. Jue Wang. Prof. Chengqi Zhang not only introduced me to the interdiscipline of computer science and finance but also provided guidance to my thesis during my PhD study. Prof. Jue Wang introduced me to the machine learning field during my Master's study.

I greatly appreciate the following researchers' help because my thesis absolutely benefits from the discussions with them: Prof. Mike Aitken, Prof. Alex Frino, A/Prof. Graham Partington, Dr. Richard Coggins, Mr. Arnold Bruce, Mr. Marcus Lim, Mr. Kevin Lo, Ms. Aelee Jun, Mr. Kelvin Cheng and Mr. Stuart Jones from Capital Market Cooperative Research Center; Mr. Yang Xia and Mr. Edmund Ching from Credit Suisse Group; Mr. Andreas Ruf, Mr. Michael O'Brian, Ms. Lisa Haertsch and Mr. Tristan Blaker from Capital Markets Surveillance Services.

Here, I express my thanks to several computer scientists for their helpful suggestions: Dr. Mengjie Zhang, Dr. Kai Ming Ting, Dr. Hongxing He, Dr. Jeffery Xu Yu, Prof. Xindong Wu and Prof. Hongjun Lu. I also thank my friends in the data mining team at the University of Technology, Sydney for their kindly help during my PhD study: Ms. Li Liu, Dr. Shichao Zhang, Dr. Xiaowei Yan, Dr. Qingfeng Chen, Dr. Chunsheng Li, Dr. Longbing Cao, Mr. Li Lin, Mr. Zhenxing Qin, Mr. Jiarui Ni, Mr. Yanchang Zhao and Mr. Wanli Chen.

Finally, I deeply thank my parents and my wife because I could not complete my thesis without their encouragement and support.

# List of Publications

Below listed are my research papers that have been finished during my PhD study at the University of Technology, Sydney and published in the international journals or conferences:


1. Wang, J. Q. and Zhang, C. Q. (2006) Dynamic Focus Strategies for Electronic Trade Execution in Limit Order Markets, In *Proceedings of the 8th IEEE International Conference on E-Commerce Technology*.

2. Wang, J. Q., Zhang, C. Q., Wu, X. D., Qi, H. W. and Wang, J. (2006) SVM-OD: SVM Method to Detect Outliers, *Foundations and Novel Approaches in Data Mining (Eds. Lin, T. Y., Ohsuga, S., Liau, C. J. and Hu, X.)*, Springer-Verlag, 129-141.

3. Wang, J. Q., Wu, X. D. and Zhang, C. Q. (2005) Support Vector Machines based on K-Means Clustering for Real-time Business Intelligence Systems, *International Journal of Business Intelligence and Data Mining*, 1 (1), 54-64.

4. Wang, J. Q. and Zhang, C. Q. (2004) KBSVM: KMeans-based SVMs for Business Intelligence, In *Proceedings of the 10th American Conference on Information Systems*, 1889-1893.

5. Wang, J. Q. and Zhang, C. Q. (2004) Support Vector Machines Based on Set Covering, In *Proceedings of the 2nd International Conference on Information Technology and Applications*, 181-184.

6. Wang, J. Q., Zhang, C. Q., Wu, X. D., Qi, H. W. and Wang, J. (2003) SVM-OD:

a New SVM Algorithm for Outlier Detection, In *Proceedings of ICDM'03 Workshop on Foundations and New Directions of Data Mining*, 203-209.

7. Cao, L. B., Wang, J. Q., Lin, L. and Zhang, C. Q. (2004) Agent Services-Based Infrastructure for Online Assessment of Trading Strategies, In *Proceedings of the 2004 IEEE/WIC/ACM International Conference on Intelligent Agent Technology*, 345-349.

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9. Cao, L. B., Ni, J. R., Wang, J. Q. and Zhang, C. Q. (2004) Agent Services-Driven Plug-and-Play in F-TRADE, In *Proceedings of the 17th Australian Joint Conference on Artificial Intelligence*, 917-922.

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# Chapter 1

# Introduction

This thesis explores the rationale and effectiveness of diversification in the framework of developing trade execution strategies from the theoretical and empirical perspectives. In the 1st part, this thesis proposes DF (dynamic focus) strategy, which reflects the idea of diversification across time, to achieve lower cost and risk than the existing trade execution strategies do. In the 2nd part, this thesis extends the AdaBoost (adaptive boosting) technique in the machine learning field to propose DAB (diversification based on AdaBoost) technique for implementing diversification across strategies. Analysis shows that the DAB technique lowers a candidate strategy set's risk, which can be measured by different standards, and meanwhile limits the decrease in its reward. In the 3rd part, this thesis applies the DAB technique to trade execution and proposes BONUS (boosted dynamic focus) strategy, which reflects the idea of diversification across DF strategies. The BONUS strategy reduces a candidate DF strategy set's risk, which can be measured by different standards, and limit the increase in its cost. This chapter is composed of four sections. The 1st section describes the research motivation and goals of this thesis. The 2nd section illustrates the research issues and methodologies of this thesis. The 3rd section clarifies the research contributions of this thesis. The 4th section outlines the organization and structure of this thesis.

# 1.1 Research Motivation and Goals

This thesis explores the rationale and effectiveness of diversification across time and strategies in the framework of developing trade execution strategies. In this thesis, the strategies are defined as making a series of decisions based on real-time state variables over a fixed period to achieve high reward and low risk with given input resources. This general definition covers five key elements related to a strategy: a fixed period, input resources, real-time state variables, decisions, an objective of achieving high reward and low risk. The five elements involved in the definition have their specific meanings in applications.

In a financial investment strategy, for example, the fixed period can be several days, several months or several years; the input resources are referred to as capital; the real-time state variables as real-time market information like price; the decisions as how to buy and/or sell financial products; the objective of achieving high reward and low risk as achieving high investment return and low investment risk. A financial investment strategy is to make a series of decisions on how to buy and/or sell financial products based on real-time market information over a fixed period to achieve high investment return and low investment risk.

In a trade execution strategy, the fixed period can be several minutes, several hours or several days; the input resources are referred to as a buy or sell order (e.g. purchasing or liquidating 1 million shares of one stock); the real-time state variables as real-time market information like price; the decisions as how to place the order in markets; the objective of achieving high reward and low risk as achieving low execution cost and risk. A trade execution strategy is to make a series of decisions on how to place the order in markets based on real-time market information over a fixed period to fill the order with low execution cost and risk in the end.

Practitioners have been trying to design efficient trade execution strategies since economic activities came into being. With the development of modern financial markets, the research on trade execution strategies has attracted more attention from academia and industry because execution cost and risk can significantly affect the return in investment especially with high turnover. It was reported that the expected annual return during the period of 1979 – 1991 was around 26.2% for the stocks recommended by Value Line (a financial firm) whereas the fund that invested in these stocks had an actual annual return of 16.1% only. The 10% difference between the actual and expected annual returns was lost to transaction cost (Perold 1988). It was also estimated that transaction cost per annum could be up to $120 billion in the $12 trillion U.S. equity market. Research institutions and financial firms are actively involved in developing efficient strategies to reduce trade execution cost and risk.

Diversification across time and strategies is an important philosophy for risk management in practices and its principal idea can be expressed as a saying "do not put all eggs in one basket". It has been emphasized in practice for a long time. Diversification across time is to allocate the input resources to different time points instead of a single one. Diversification across strategies is to allocate the input resources to various strategies instead of a single one. For example, rational investors allocate their money to different investment periods of time and/or various investment products such as stocks and bonds to hedge investment risk. Note that investing in one financial product can be regarded as a strategy as per the general definition of strategy in this section. The major objective of this thesis is to explore the rationale and effectiveness of diversification across time and strategies in the framework of developing trade execution strategies and apply it to improve the existing trade execution strategies from the theoretical and empirical perspectives. Following the major objective, three sub-objectives are respectively illustrated in the three paragraphs below.

In the 1st part, this thesis explores diversification across time in developing trade execution strategies. Various trade execution strategies have been proposed to fill a buy or sell order over a fixed period with low cost and risk. Two typical strategies are called market order strategy and limit order strategy. The market order strategy can immediately fill the entire order without any restrictions on price while the limit order strategy normally does not fill the entire order immediately by assigning a price to the order. Through assigning a price to the order, the limit order strategy could capture trading opportunities as the market evolves and limit execution cost to an expected range. The research of trade execution has shown that limit order strategy achieves lower execution cost than market order strategy does. However, the limit order strategy could not fill the entire order (particularly a large order) at the end of execution (this is called nonexecution risk) and might miss trading opportunities in the period of execution because it assigns a price to the order. This thesis proposes a strategy to solve the problems that the limit order strategy faces. The proposed strategy reflects the idea of diversification across time. This thesis theoretically analyzes why the proposed strategy can improve the limit order strategy and empirically evaluates its effectiveness.

In the 2nd part, this thesis explores diversification across strategies, which aims to lower the strategies' risk and limit the decrease in their average reward. In the finance field, the techniques for implementing diversification across strategies have been proposed to acquire strategies from a candidate strategy set and determine their weights. There are two limitations in the existing techniques. First, the candidate strategy set normally only contains finite strategies (e.g. investing in 50 stocks). Second, they are normally designed to reduce risk, which is only measured by one specific standard (e.g. the variance of returns). But in practice, a candidate strategy set can contain infinite strategies if the strategy parameter takes values in a range of real number. Besides, various standards have been proposed to measure risk in the finance field such as the variance of returns (Markowitz 1952), the k-order lower partial moment (k-LPM) of returns and the minimum of returns (Bawa 1975, Balzer 1994). Investors expect to achieve higher average return and lower risk, which can be measured by different standards (e.g. the variance of returns, the k-LPM of returns and the minimum of returns). This thesis proposes a technique for implementing diversification across strategies that overcomes the drawbacks of the existing techniques. This thesis also theoretically analyzes the proposed technique's advantages and statistical properties.

In the 3rd part, this thesis applies the proposed technique for implementing diversification across strategies to trade execution to improve the strategy proposed in the 1st part. The proposed technique acquires strategies from a candidate strategy set, which is allowed to contain infinite strategies proposed in the 1st part, and determines their weights. This thesis theoretically analyzes its statistical property in trade execution, i.e. it lowers the candidate trade execution strategy set's risk that can be measured by different standards and limits the increase in cost. This thesis also empirically evaluates its effectiveness.

# 1.2 Research Issues and Methodologies

The market order strategy avoids nonexecution risk and captures the current trading opportunity by immediately filling the entire order, but it incurs large execution cost particularly when it fills a large order. Although the limit order strategy achieves lower execution cost than the market order strategy does, it may incur nonexecution risk and miss the trading opportunities in the period of execution since it does not immediately fill the entire order by assigning a price to the order. In the 1st part, this thesis proposes a strategy to solve the problems that the limit order strategy faces. The proposed strategy is called DF strategy, which reflects the idea of diversification across time (also see Wang and Zhang 2006).

The DF strategy does not only passively assign a price to the order but also aggressively fills part of the entire order at different time points over the period of execution. In other words, the DF strategy incorporates a series of small market orders with different volume into the limit order strategy and dynamically adjusts each market order volume based on real-time state variables. Dynamic volume adjustment of each small market order is expected to solve the problems that the market order strategy and the limit order strategy face, i.e. reducing execution cost, avoiding nonexecution risk and capturing trading opportunities. In the 1st part, this thesis poses three questions as *Q1*, *Q2* and *Q3*. The brief answers are given in the paragraph immediately following each question and the details will be illustrated in Chapter 3.

*(Q1) – Which real-time state variables over the period of execution can be used to dynamically adjust the volume of each market order from the DF strategy?*

In the DF strategy, dynamic volume adjustment depends on two real-time state variables over the period of execution: inventory and order book imbalance. Here, inventory is represented as the ratio of the unexecuted volume of the order to the remaining time. Inventory changes over the period of execution and the unexecuted volume decreases as time goes by. The increase in inventory indicates that there is more unexecuted volume in less remaining time. In this case, the DF strategy assigns more volume to the market order for submission to reduce nonexecution risk. The decrease in inventory indicates that there is less unexecuted volume in more remaining time. In this case, the DF strategy assigns less volume to the market order for submission to wait for favorable price movement in future. Order book imbalance represents the relationship between supply and demand in financial products. When supply is stronger (or weaker) than demand, it indicates that prices would go down (or up). If order book imbalance forecasts that prices would move toward the adverse direction, the DF strategy will assign more volume to the market order for submission to capture the current trading opportunity. Otherwise, the DF strategy will assign less volume to the market order for submission to wait for favorable price movement in future.

*(Q2) – How to quantitatively describe the relationship between the real-time state variables and the volume of each market order from the DF strategy?*

The quantitative model should satisfy three conditions: 1) the volume of each market order for submission should not exceed the volume of the entire order; 2) the volume of each market order for submission should not be less than zero as it is assumed that for buying (or selling), no sell (or buy) order be submitted in the period of execution; 3) the quantitative model should be represented as an increasing function based on the qualitative description of dynamic volume adjustment. This thesis suggests the sigmoid function as the quantitative model to map the real-time state variables (inventory and order book imbalance) to the volume of each market order since it satisfies all the above three conditions. A parameter is involved in the sigmoid function and this means that there is actually a set of DF strategies with different parametric values. So an optimized parametric value needs to be determined through in-sample test before the DF strategy is applied to out-of-sample test.

*(Q3) – How well does the DF strategy outperform the limit order strategy through in-sample and out-of-sample test based on real-life data?*

This thesis benefits from a unique advantage of accessing full order and trade records of all stocks from the Australian Stock Exchange (ASX). This advantage makes it possible to build a simulator to execute and testify trade execution strategies. The DF strategy is optimized and backtested through in-sample and out-of-sample test based on 80 datasets, which comprise 5-month order and trade records of 20 stocks from the stock index "ASX20". The in-sample and out-of-sample test show that the DF strategy achieves lower cost and risk than the limit order strategy does.

In the existing techniques for implementing diversification across strategies, the candidate strategy set normally only contains finite strategies. In practice, the candidate strategy set can contain infinite strategies when the strategy parameters take values in a range of real number. Besides, the existing techniques are normally designed to lower risk, which is only measured by one specific standard. Since various standards have been proposed to measure risk in the finance field, it would be better for practitioners if a technique for implementing diversification across strategies could lower the candidate strategies' risk and the risk could be measured by different standards. In the 2$^{nd}$ part, thus, this thesis poses three questions as *Q4*, *Q5* and *Q6*. The brief answers are given in the paragraph immediately following each question and the details will be illustrated in Chapter 4.

*(Q4) – Is there a technique for implementing diversification across strategies that allows the candidate strategy set to contain infinite strategies and lowers the candidate strategy set's risk that can be measured by different standards?*

The AdaBoost technique, which is discussed mostly in the supervised learning field, essentially reflects the idea of diversification across different learning models. The supervised learning model is to identify an object as exactly as possible. The AdaBoost technique acquires models from a candidate model set and determines their weights. An object is then identified based on the ensemble of the acquired learning models instead of each individual one. The research has pointed out that the ensemble achieves higher identification accuracy than each individual one does. In the AdaBoost technique, the candidate model set is allowed to contain finite or infinite learning models. Theoretical analysis has shown that for the ensemble of the acquired models, its misidentification rate decreases and its worst identification result on training examples (also called margin) is improved as the learning steps increase. The misidentification rate and the worst identification result can be regarded as different risk measures. These characteristics of the AdaBoost technique are what to be expected in *Q4*. But it cannot be applied directly to acquire strategies and determine their weights for implementing diversification across strategies. The AdaBoost technique trains supervised learning models based on the examples with labels. To acquire a strategy, however, the training examples cannot be marked with labels before a candidate strategy set is given. Chapters 2 and 4 will explain the difference between acquiring a supervised learning model and acquiring a strategy in details.

*(Q5) – Can the AdaBoost technique, which is applicable to training supervised learning models, be extended to acquire strategies and determine their weights for implementing diversification across strategies?*

The success in the AdaBoost technique results from its key idea – adaptively updating the probability distribution on training examples at each learning step. After the AdaBoost technique acquires a model, higher probability is assigned to the training examples misidentified by the acquired model. At the next learning step, this update makes the AdaBoost technique acquire a new model, which is more possible to correctly identify the training examples misidentified by the previous model. In the end, each acquired model can identify part of training examples well. The key idea of adaptively updating the probability distribution could be extended to implement diversification across strategies as below. After a strategy is acquired, higher probability is assigned to the training examples, on which the acquired strategy achieves lower reward. At the next learning step, it is more possible to acquire a new strategy, which achieves high reward on those training examples. In the end, each acquired strategy can achieve high reward on part of training examples. Here, the training examples are not marked with labels and the probability distribution is updated based on the reward, which is achieved by the acquired strategy on the training example. The above extension composes the core of the DAB technique for implementing diversification across strategies proposed in this thesis.

*(Q6) – What are the advantages, statistical properties and theoretical foundation of the DAB technique?*

The DAB technique inherits the advantages of the AdaBoost technique. It allows the candidate strategy set to contain infinite strategies. Theoretical analysis shows that as the learning steps increase, the DAB technique lowers the candidate strategy set's risk that can be measured by different standards (e.g. the k-LPM and minimum of rewards) and limits the decrease in its average reward. From the perspective of the candidate strategy set's efficient frontier, the DAB technique moves it toward the favorable direction. The efficient frontier represents a set of optimal strategies in terms of achieving high reward and low risk. The DAB technique, as a meta-type method, can be generally used for implementing diversification across trade execution strategies. This thesis further applies the DAB technique to acquire DF strategies from a candidate DF strategy set and determines their weights. The entire order is allocated to each acquired DF strategy in proportion with its weight. All acquired DF strategies are executed in parallel to fill their allocated order. In this thesis, the parallel execution of all acquired DF strategies to fill their allocated order is called BONUS (boosted dynamic focus) strategy. In the 3$^{rd}$ part, this thesis poses two questions as *Q7* and *Q8*. The brief answers are given in the paragraph immediately following each question and the details will be introduced in Chapter 5.

*(Q7) – What are the advantages, statistical properties and theoretical foundation of the BONUS strategy based on the DAB technique?*

The DAB technique uses a DF strategy set as the candidate strategy set, which is allowed to contain infinite DF strategies. Theoretical analysis shows that as the learning steps increase, the BONUS strategy based on the DAB technique lowers the candidate DF strategy set's risk that can be measured by different standards and limits the increase in its cost. From the perspective of the candidate DF strategy set's efficient frontier, the BONUS strategy based on the DAB technique moves it toward the favorable direction. The efficient frontier represents a set of optimal DF strategies in terms of achieving low cost and risk. The above theoretical analysis is further verified through the empirical study in this thesis.

*(Q8) – How well does the BONUS strategy outperform the DF strategy through in-sample and out-of-sample test based on real-life data?*

The in-sample and out-of-sample tests are conducted on the 80 datasets, which comprise 5-month order and trade records of 20 stocks. The in-sample test strongly supports the theoretical analysis on the BONUS strategy based on the DAB technique. The out-of-sample test shows that the BONUS strategy outperforms the optimal DF strategy and two simple diversification techniques. To sum up, this section has briefly described the key research issues listed in 8 questions and the methodology for each research issue. The following chapters will illustrate them in details from the theoretical and empirical perspectives.

# 1.3 Research Contributions

This thesis is contributed to both finance and computer science fields. This thesis explores diversification in the framework of developing trade execution strategies. Diversification is an important philosophy, which has been emphasized in finance and computer science. The 3 research parts of this thesis are linked together through the philosophy of diversification. Each part composes one major contribution of this thesis. In the 1st part, this thesis develops the DF strategy, which reflects the idea of diversification across time, to improve the existing trade execution strategies. In the 2nd part, this thesis extends the AdaBoost technique to develop the DAB technique for implementing diversification across strategies, which overcomes the limitation of the existing diversification techniques. In the 3rd part, this thesis applies the DAB technique to trade execution and develops the BONUS strategy, which reflects the idea of diversification across the DF strategies, to improve the DF strategy. Figure 1.1 outlines the 3 major contributions and their relationship as below:

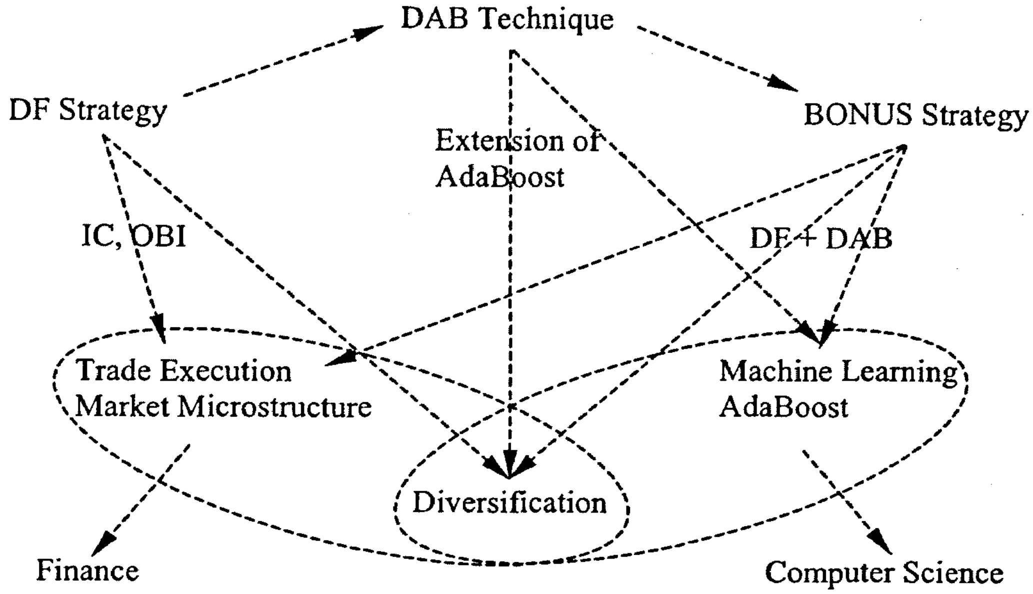


**Figure 1.1 Research contributions of this thesis**

The DF strategy incorporates a series of small market orders with different volume into the limit order strategy and dynamically adjusts the volume of each market order based on two real-time state variables: inventory and order book imbalance. The sigmoid function is adopted to map the real-time state variables to the market order volume. A lot of experiments show that the DF strategy achieves lower cost and risk than the limit order strategy does. This research verifies the effectiveness of diversification across time in trade execution and enriches the family of trade execution strategies.

The DAB technique extends the key idea of the AdaBoost technique to acquire strategies from a candidate strategy set and determines their weights. The resources are allocated to each acquired strategy in proportion with its weight and all acquired strategies are then executed in parallel with their allocated resources. The DAB technique allows the candidate strategy set to contain infinite strategies. This thesis also builds the statistical foundation for the DAB technique. Theoretical analysis shows that as the learning steps increase, the DAB technique lowers the candidate strategy set's risk, which can be measured by different standards, and limits the decrease in its average reward. This research provides a flexible way for implementing diversification across strategies, complements the existing diversification techniques and enriches the research of AdaBoost.

The DAB technique is further applied to acquire DF strategies from a candidate DF strategy set and determine their weights. The entire order is allocated to each acquired DF strategy in proportion with its weight and all acquired DF strategies are then executed in parallel to fill their allocated order. The BONUS strategy is referred to as the parallel execution. A lot of experiments support the theoretical conclusions about the DAB technique and the BONUS strategy. The experiments also show that the BONUS strategy based on the DAB technique outperforms the optimal DF strategy and two simple diversification techniques. This research provides a flexible way for implementing diversification across trade execution strategies and improves the existing trade execution strategies.

## 1.4 Organization and Structure

This thesis is composed of six chapters. Chapter 1 introduces research motivation and goals, key research issues and methodologies, major research contributions, organization and structure. Chapter 2 reviews the work related to this thesis from both finance and computer science fields: trade execution and strategy analysis, diversification and risk management, supervised learning and AdaBoost. Chapter 3 proposes the DF strategy, which reflects the idea of diversification across time, to achieve lower cost and risk than the limit order strategy does. Chapter 4 proposes the DAB technique, which reflects the idea of diversification across strategies, to lower the candidate strategy set's risk that can be measured by different standards and limits the decrease in its average reward. Chapter 5 applies the DAB technique to trade execution and proposes the BONUS strategy, which reflects the idea of diversification across the DF strategies, to lower the candidate DF strategy set's risk that can be measured by different standards and limit the increase in its cost. Chapter 6 draws conclusions, summarizes major research contributions and discusses future research for this thesis.

# Chapter 2

# Background and Literature Review

The research issues and methodologies discussed in this thesis are closely related to several research subjects in both finance and computer science fields: trade execution and strategy analysis, risk management and diversification, supervised learning and the AdaBoost technique. This chapter introduces the basic concepts involved in these subjects and illustrates the limitations of the existing theories and methods. In addition, this chapter also describes the simulator for executing trade execution strategies and the experimental datasets for empirically evaluating the DF strategy, the DAB technique and the BONUS strategy. The simulator and datasets introduced in this chapter will be used consistently in the experiments in the following chapters. This chapter is composed of four sections. The 1$^{st}$ section introduces the background of trade execution and strategy analysis and clarifies the drawbacks of the existing trade execution strategies. The 2$^{nd}$ section introduces the background of risk management and diversification and clarifies the limitations of the existing diversification techniques. The 3$^{rd}$ section introduces the background of supervised learning and the AdaBoost technique and explains the difference between training the strategy defined in this thesis and supervised learning. The 4$^{th}$ section summarizes the related work.

# 2.1 Trade Execution and Strategy Analysis

Investors implement their investment plan through submitting, amending and/or canceling their orders in modern financial markets like ASX. Their orders are accumulated in an order book, which evolves as time goes by. Figure 2.1 shows two real-life order books of one stock at two different time points $t_1$ and $t_2$. Each order book contains both buy (marked as "B") and sell (marked as "S") orders in sequence of prices, e.g. from \$3.16 to \$3.26 in the left book. There may be more than one order at the same price level. For example, "B-\$3.17-x8694" in the left book represents that investors are going to buy 8,694 shares of the stock. The 8694 shares can be either the volume of one buy order or the total volume of several buy orders at the price of \$3.17. If there is more than one order at the same price level, they are placed in sequence of time when they enter the market. In an order book, the highest buy price and the lowest sell price is respectively called best bid and best ask. There is normally a spread between best bid and best ask. In the left book, for example, the spread between the best bid of \$3.17 and the best ask of \$3.18 is one cent. The volume accumulated at price levels of the buy (or sell) side is called bid (or ask) depth.

| *12th March, 10:09:40 Trading Price: \$3.17* | *25th March, 15:33:11 Trading Price: \$3.25* |
|---|---|
| ... | ... |
| S – \$3.26 – x110,000 | S – \$3.26 – x100,000 |
| S – \$3.25 – x103,000 | S – \$3.25 – x468,297 |
| S – \$3.24 – x11,000 | B – \$3.24 – x118,105 |
| S – \$3.23 – x3,000 | B – \$3.23 – x135,832 |
| S – \$3.22 – x1,000 | B – \$3.22 – x25,000 |
| S – \$3.21 – x31,153 | B – \$3.21 – x18,500 |
| S – \$3.20 – x32,750 | B – \$3.20 – x54,106 |
| S – \$3.19 – x234,740 | B – \$3.19 – x7,550 |
| S – \$3.18 – x40,300 | B – \$3.18 – x60,915 |
| B – \$3.17 – x8,694 | B – \$3.17 – x222,900 |
| B – \$3.16 – x406,850 | B – \$3.16 – x157,633 |
| ... | ... |

Execution Price 3.210883

3.170000 Decision Price

Decision Price 3.250000

3.211581 Execution Price

**Figure 2.1 Two real-life order books of one stock at two time points**

Trades happen when buy and sell orders exist in the order book at the same time and the prices of buy orders are higher than or equal to the prices of sell orders. Suppose that a new order of buying 50,300 shares at \$3.19 enter the market at time $t_1$ (the left book). In this case, trades then happen. According to the market rule of price priority, the new buy order will firstly hits all sell orders (40,300 shares) at the lowest sell price of \$3.18. Then there are still 10,000 shares unfilled in the new buy order. So it will keep hitting sell orders with the higher price of \$3.19. According to the market rule of time priority, the new buy order with 10,000 shares unfilled will firstly hit the sell orders with earliest time stamp at \$3.19. The new buy order with 10,000 shares unfilled will be filled at \$3.19 since there are 234,740 shares to be sold at \$3.19. Should there be less than 10,000 shares to be sold at \$3.19, some shares in the new buy order still would have not been filled. In this case, the unfilled shares would remain in the book because all sell orders with prices lower than or equal to \$3.19 (the new buy order price) would have been used up. In the order book, the best bid (and ask) would move up to \$3.19 (and \$3.20).

"Trading Price: \$3.17" in the left book represents that a trade happens at \$3.17 at the time $t_1$. Investors can make decision on buying or selling the stock according to the change in trading prices. For example, an investor makes a decision to buy (and sell) 500,000 shares of the stock when the trading price of \$3.17 (and \$3.25) occurs in the left book at time $t_1$ (and the right book at the time $t_2$). In this case, the investor can make a profit of \$40,000 from buying at the lower price and selling at the higher price. For the total capital, the return in this investment can be up to around 2.5% over 10 trading days (12th – 25th March 2002).

It is assumed in the above investment plan that 500,000 shares of the stock could be bought at \$3.17 and sold at \$3.25. However, executing an investment plan in real markets may be much more complicated. For example, there is not any sell order at the same or lower prices (see the left book of Figure 2.1) though the investor plans to buy 500,000 shares of the stock at \$3.17 at the time $t_1$. In order to acquire 500,000 shares immediately at $t_1$, the investor has to place a buy order to hit the sell orders with prices higher than \$3.17 in the book. In this case, the actual average buy price is \$3.210883 per share. The similar case occurs for selling 500,000 shares of the stock at \$3.25 at the time $t_2$ (see the right book of Figure 2.1) and the actual average sell price is \$3.211581 per share. The actual average buy/sell prices are called execution prices, which may be much worse than the investor's decision prices (the buy price of \$3.17 and the sell price of \$3.25). The expected profit of \$40,000 and expected return of 2.5% respectively decrease to \$349 and 0.022% only (see Table 2.1). The big difference between the actual profit/return and the expected profit/return is lost to trade execution cost.

**Table 2.1 Investment plan and actual trade execution**

| | Buy Price (\$) | Sell Price (\$) | Profit (\$) | Return (%) |
|---|---|---|---|---|
| Investment Plan | 3.17 | 3.25 | 40,000 | 2.5 |
| Actual Execution | 3.210883 | 3.211581 | 349 | 0.022 |

Transaction costs are normally divided into explicit and implicit costs. Compared to explicit costs such as commission fee and tax fee, it is much more difficult to analyze and manage implicit cost, which is also called market impact cost (i.e. trade execution cost in the above example). Market impact cost is a key factor that affects the performance of trade execution particularly for a large order. It is referred to as the difference between the execution price and an expected (or benchmark) price. In the above example, the investor adopts a market order strategy, which immediately fills the entire order without any restrictions on price, to capture the current trading opportunity ($t_1$ and $t_2$). In the market order strategy, however, the order does not only cross the spread but also hits more orders in the book at worse prices. So the investor has to pay for significant market impact cost in this strategy.

Theoretical and empirical researches have seen the evidence that limit order strategy achieves lower market impact cost than market order strategy does (Biais et al. 1995, Handa and Schwartz 1996, Hasbrouck and Harris 1996, Nevmyvaka et al. 2005). The limit order strategy limits the market impact cost by assigning a price to the order. In the above example, the investor can assign the price of \$3.17 in executing the buy order at the time $t_1$. This means that the buy order does not hit the sell orders with prices higher than \$3.17 and so it normally would not be filled immediately. As time goes by, other market participants might submit their sell orders with prices lower than or equal to \$3.17 to hit the investor's buy order. If prices (e.g. the best bid/ask) move toward the favorable direction, the limit order strategy would capture better trading opportunities in future since it does not fill the entire order immediately.

However, the limit order strategy is a two-edged sword for trade execution. When a buy (or sell) order is required to fill in a fixed period, the limit order strategy may not fill the entire order at the end of execution because it assigns a price to the order, which other market participants may not provide enough sell (or buy) orders to hit the buy (or sell) order in the period of execution. This is called nonexecution risk, which means that in order to fill the entire order, the limit order strategy has to submit a market order at the end of execution regardless of price impact to fill the unexecuted order. Besides, the limit order strategy would miss trading opportunities if prices move toward the adverse direction. Dynamic price adjustment has been proposed to solve the above problems (Nevmyvaka et al. 2005, 2006). Instead of assigning a fixed price to the order, the idea of dynamic price adjustment is to dynamically adjust the price based on real-time market information, e.g. inventory and order book imbalance.

Inventory control has been discussed in the research of dealers' behaviors. If dealers accommodate more orders from other market participants, their inventory position will exceed the expected level. Theoretical and empirical researches have shown that dealers always optimize their inventory position to hedge the risk of holding extra inventory. This type of risk is incurred for two reasons: 1) it is unknown how long the extra inventory will be held; 2) future price movement is uncertain after dealers hold the extra inventory. The dealers can control their inventory and resume it to the expected level by dynamically adjusting their buy and sell order prices. For example, they can move their order prices downward (or upward) to attract more buy (or sell) orders from other market participants when their longer (or shorter) inventory position exceeds the expected level. This is called dynamic price adjustment based on inventory. It is described as below when the above idea is applied to improve the limit order strategy for trade execution. In trade execution, inventory is referred to as the ratio of the unexecuted volume to the remaining time. As inventory increases and exceeds an expected level, the buy (or sell) order price is moved upward (or downward) to actively hit the sell (or buy) orders from other market participants and to reduce the nonexecution risk. As inventory decreases and exceeds an expected level, the buy (or sell) order price is moved downward (or upward) to avoid hitting the sell (or buy) orders from other market participants and to wait for favorable price movement.

Order book imbalance is measured by the difference between the bid depth and the ask depth, e.g. 8694 shares to be bought at $ 3.17 vs. 40300 shares to be sold at the $3.18 (see the left book in Figure 2.1). It represents the relationship between demand (buy side) and supply (sell side) in stocks. Theoretical and empirical researches have shown that order book imbalance is closely related to short-term price movement (Harris and Panchapagesan 2005). The information of order book imbalance is valuable to investors in a general sense that prices would increase (or decrease) if the buy side of a book were heavier (or thinner) than the sell side. This information can be used to dynamically adjust the order price in trade execution since it is related to price movement. The idea of dynamic price adjustment based on order book imbalance is described as below. When order book imbalance forecasts that prices would move toward the favorable direction, the buy (or sell) order price is moved downward (or upward) to avoid hitting the sell (or buy) orders from other market participants and wait for favorable price movement. When order book imbalance forecasts that prices would move toward the adverse direction, the buy (or sell) price is moved upward (or downward) to actively hit the sell (or buy) orders from other market participants and capture the current trading opportunity.

As two most important attributes of an order, price and volume have similar functions to respond the real-time market information and overcome the drawbacks of the limit order strategy. The DF strategy proposed in this thesis actively submits a series of small market orders with different volume in the period of execution and dynamically adjusts the volume of each market order based on inventory and order book imbalance. Dynamic volume adjustment in the DF strategy reflects the idea of diversification across time. The details will be illustrated in Chapter 3.

Trade execution cost is measured by the difference between the execution price and the expected (or benchmark) price. Several benchmark prices have been proposed and used in practice such as Volume Weighted Average Price (VWAP) (Berkowitz et al. 1988), open/closing prices (Barnea and Logue 1978, Beebower and Priest 1980) and arrival price (Perold 1988). The arrival price is normally referred to as the midpoint price between the best bid and the best ask at the time when a buy or sell decision is made. It has become more and more popular since it is closer to investors' expectation in contrast to other benchmark prices. In the above example (see Figure 2.1 and Table 2.1), the arrival price at the time t1 (and t2) is \$3.175 (and \$3.245) while the decision price at t1 (and t2) is \$3.17 (and \$3.25). Given the arrival price and the execution price, trade execution cost is measured as below (Treynor 1981, Perold 1988):

$$c = \mathrm{sgn} \times 10000 \times \frac{p_E - p_A}{p_A},$$

where sgn = 1 (or −1) for buying (or selling), $p_E$ is the execution price per share, $p_A$ is the arrival price, c is called implementation shortfall (or shortfall) and its unit is called basis points (BPS). In addition, trade execution risk is measured by the standard deviation of shortfalls (Almgren and Chriss, 1997).

In this thesis, the empirical study benefits from a unique advantage of accessing full order and trade records of all stocks from the ASX. This makes it possible to build a simulator for executing trade execution strategies and backtesting them. In simulation, the artificial orders from a backtested strategy are executed based on two market rules: price priority and time priority. In the rule of price priority, a buy (or sell) order with higher (or lower) price is executed prior to other orders with lower (or higher) prices. In the rule of time priority, the buy (or sell) order with an earlier time-stamp is executed prior to other buy (or sell) orders if they are placed at the same price level. Since the historical data available contains the detailed information of each order (price, volume, time stamp), the simulator in this thesis can identify whether the limit order from a backtested strategy would be filled in the period of execution according to the above two market rules. In simulation, the execution of a market order from a backtested strategy follows the assumption in

(Coggins et al. 2003) that the market order does not influence future order flows and the simulator only takes account of instantaneous market impact. In addition, the simulator in this thesis does not consider any market participant's reaction to the artificial orders from a backtested strategy due to the limitation of historical data. This is different from real markets where the participants would react to any event. The existing studies (Nevmyvaka 2005, 2006, Coggins 2003) also had to make this assumption for backtesting strategies due to the limitation of historical data. Nevertheless, this type of simulation still makes sense because the practitioners could have basic understanding for a new strategy so that they can further improve it with penetration.

This thesis sets one specific example of trade execution for the experiments – buying 40,000 shares of one stock in 10 minutes. The experiments are conducted on the historical order and trade records during the period of 10:10:00 – 16:00:00 in each trading day of the ASX. So 35 time-interval examples (or time-series) can be used in one trading day of each stock. Each experimental dataset contains these types of examples in a 2-month test period of one stock and it is divided into an in-sample test period and an out-of-sample test period (see Table 2.2). This thesis evaluates the proposed technique/strategy on 80 datasets, which comprise four 2-month test periods of 20 stocks from the stock index "ASX20". The raw data used in this thesis is provided through the Capital Market Cooperative Research Center in Australia.

**Table 2.2 Four in-sample and out-of-sample test periods of one stock**

| | In-sample Test | Out-of-sample Test |
|---|---|---|
| Test Period 1 | January 2002 | February 2002 |
| Test Period 2 | February 2002 | March 2002 |
| Test Period 3 | March 2002 | April 2002 |
| Test Period 4 | April 2002 | May 2002 |

# 2.2 Risk Management and Diversification

Risk is referred to as the statistical measure of uncertain loss. This description covers two elementary components of risk: loss and its uncertainty. Various measure standards have been proposed in the investment field, where loss is referred to as the returns lower than a target return and so the uncertainty of loss results from the uncertainty of returns. Originally, the variance of investment returns is suggested to measure investment risk (Fisher 1906). It was then used in the portfolio theory for investment as the risk measure (Markowitz 1952). Let $\Phi$ denote an example set, in which each example $\phi$ represents an investment period. Let $r_\phi$ denote the return on the example $\phi$. The variance of returns is calculated as below:

$$\sum_{\phi\in\Phi}\left(r_\phi - \frac{1}{|\Phi|}\sum_{\phi\in\Phi} r_\phi\right)^2,$$

where $|\Phi|$ is the number of examples in the example set $\Phi$. In the variance of returns, the average of returns is regarded as the target return. Thus, the variance of returns actually contains both two parts of the returns: those lower and higher than the target return. If the returns follow the normal distribution, it is reasonable that the variance of returns is used to measure risk because of the symmetry of the normal distribution. But if the returns follow a non-normal distribution with fat tail, the variance of returns is not the most appropriate measure of risk.

The above problem is further illustrated through an intuitive example in the first row charts of Figure 2.2, where the horizontal coordinates represent returns and the vertical coordinates their probability. In the left chart, the returns, which are achieved by the investment strategies $s_1$ (the solid curve) and $s_2$ (the dash curve), follow the normal distribution. The variance of returns indicates that $s_1$ achieves lower risk than $s_2$ does. If the risk measure only includes the returns lower than the target return (see the straight line), the same conclusion that $s_1$ achieves lower risk than $s_2$ does can be drawn. In the right chart, the returns, which are achieved by the investment strategies $s_3$ (the solid curve) and $s_4$ (the dash curve), follow the non-normal distribution with fat tail. Calculation shows that the variance of returns by $s_3$ is a bit smaller than that by $s_4$. Thus, the variance of returns indicates that $s_3$ achieves lower risk than $s_4$ does. However, if the risk measure only includes the returns lower than the target return (see the straight line), an opposite conclusion that $s_3$ achieves higher risk than $s_4$ does is drawn.

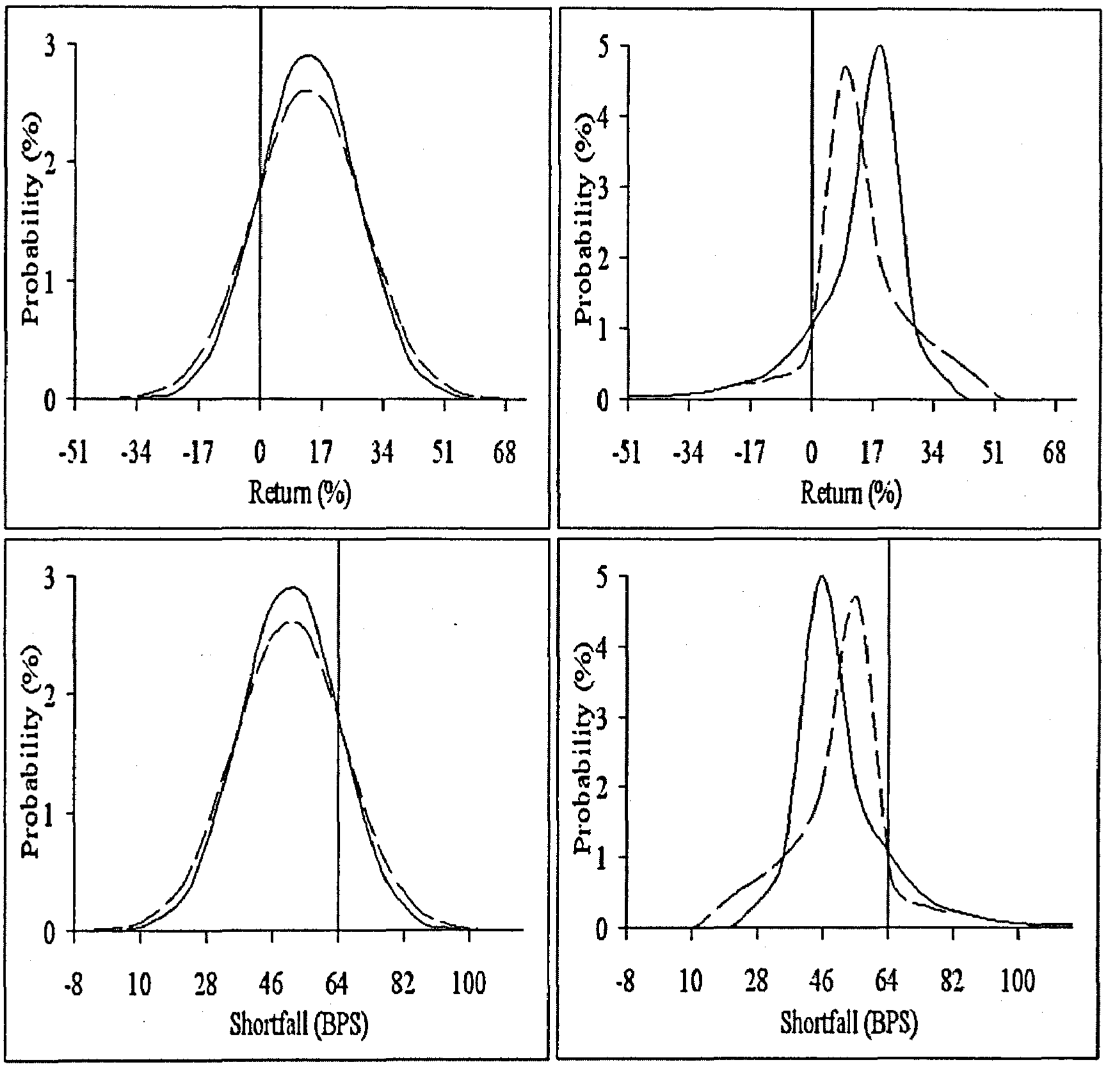


**Figure 2.2 Risk and variance**

To solve the above problem, the semi-variance of returns was suggested as the investment risk measure (Markowitz 1959). The semi-variance of returns is calculated as below:

$$\sum_{\phi\in\Phi}\left(max\left(0,\frac{1}{|\Phi|}\times\sum_{\phi\in\Phi}r_\phi-r_\phi\right)\right)^2.$$

The k-LPM of returns was further suggested to measure investment risk (Bawa 1975). The k-LPM of returns is calculated as below:

$$\sum_{\phi\in\Phi}\left(max\left(0,r_\theta-r_\phi\right)\right)^k,$$

where $r_\theta$ is the target return and k is a nonnegative integral number. In this measure of risk (k-LPM of returns), the returns lower than the target return $r_\theta$ are regarded as loss. The k-LPM of returns is a more flexible risk measure than the semi-variance of returns in that 1) the value of k can represent the degree of risk aversion or tolerance; 2) the target return $r_\theta$ is not only the average of returns but also can be determined by investors. The risk measured by those types of standards such as the semi-variance or the k-LPM is called downside risk. In addition, the minimum of returns $\min_{\phi\in\Phi} r_\phi$, which represents the worst case in the investment periods, is also used as an investment risk measure is (Balzer 1994). A brief story of investment risk measures can be found in (Nawrocki 1999, Balzer 1994).

In the trade execution field, the variance of shortfalls has been proposed to measure trade execution risk (Almgren and Chriss 1997). Let $\Phi$ denote an example set, in which each example $\phi$ represents a trade execution period. Let $c_\phi$ denote the return on the example $\phi$. The variance of shortfalls is calculated as below:

$$\sum_{\phi\in\Phi}\left(c_\phi-\frac{1}{|\Phi|}\sum_{\phi\in\Phi}c_\phi\right)^2,$$

In evaluating trade execution strategies, loss is referred to as the shortfalls higher than a target shortfall. When the variance of shortfalls is used to measure trade execution risk, there exists the similar problem that the variance of returns faces (see the charts 3 and 4 of Figure 2.2). Inspired by the research of investment risk measures, this thesis also suggests the k-order upper partial moment (UPM) of shortfalls and the maximum of shortfalls as the trade execution risk measures. The k-UPM of shortfalls and the maximum of shortfalls are respectively calculated as below:

$$\sum_{\phi\in\Phi}\left(max\left(0, c_\phi - c_\theta\right)\right)^k,$$

$$\max_{\phi\in\Phi} c_\phi,$$

where $c_\theta$ is a target shortfall. The variance of shortfalls, the k-UPM of shortfalls and the maximum of shortfalls will be used to measure the trade execution risk in the empirical studies of this thesis.

Diversification across time and strategies is an important philosophy for risk management in practice. Its principal idea can be expressed as a saying "don't put eggs in one basket". The importance of diversification and its effect on risk have been paid attention for a long time since economic activities came into being. In Shakespeare's "The Merchant of Venice", for example, the merchant Antonio says:

"… I thank my fortune for it,
My ventures are not in one bottom trusted,
Nor to one place; nor is my whole estate.
Upon the fortune of this present year;
Therefore, my merchandise makes me not sad …".

The merchant Antonio clearly understood that he could benefit from diversification. In Bernoulli's article about St. Petersburg Paradox, he pointed out "… it is advisable to divide goods which are exposed to some small danger into several portions rather than to risk them all together" (Bernoulli 1738). The above examples show that risk-aversion investors prefer to diversify their investment across time and strategies.

Because the uncertainty of market evolution results in the uncertainty of returns (or shortfalls), risk-aversion investors (or traders) adopt the idea of diversification. The researchers have done a lot of work about diversification and risk management for a long time (Fisher 1906, Hicks 1935, Marschak 1938, Williams 1938, Leavens 1945). A milestone of the research on diversification is the portfolio theory for investment (Markowitz 1952), which points out that diversification would not generally eliminate investment risk though it could reduce it. Different from the earlier work prior to 1952, the portfolio theory illustrates the relationship between diversification and correlated risks, clarifies the difference between an efficient portfolio and an inefficient portfolio, and emphasizes the tradeoff between investment risk and investment return in designing a portfolio (Markowitz 1952, 1959, 1991, 1999). The portfolio theory firstly formalizes the idea of diversification in investment from the mathematical point of view. Diversification can reduce (but generally not eliminate) risk and limit the decrease in return.

The concept of efficient frontier was proposed in the portfolio theory to evaluate the performance of investment strategies (Markowitz 1952). It was also applied in the research of trade execution to evaluate the performance of trade execution strategies (Almgren and Chriss 1997). Figure 2.3 gives an intuitive example about efficient frontier. In Figure 2.3, the horizontal coordinates represent risk that investment strategies (left chart) and trade execution strategies (right chart) bring respectively, and the vertical coordinates represent average return that investment strategies bring (left chart) and average shortfall that trade execution strategies bring (right chart) respectively. For the strategy set (see the shade area), the arcs in the left and right charts are respectively called efficient investment frontier (EIF) and efficient trading frontier (ETF). The efficient frontier represents a set of optimal strategies because any other strategy in the shade area cannot achieve 1) higher average return (left chart) or lower average shortfall (right chart) than that on the arc does given the same risk; or 2) lower risk than that on the arc does given the same average return (left chart) or average shortfall (right chart).

It is not easy for investors (or traders) to find a strategy on the efficient frontier because the strategies on it represent either higher average return (lower average shortfall) but higher risk or lower risk but lower average return (higher average shortfall). In this case, the investors (or traders) normally find a strategy based on the degree of their risk aversion or tolerance. Given the degree of risk aversion or tolerance, the optimization objective is represented as the tradeoff between risk and average return (or average shortfall). Let $\gamma$ ($\gamma \geq 0$) denote the tradeoff factor in the optimization objective and $\Omega$ denote a strategy set. The optimization objective for determining the investment strategy or the optimal trade execution strategy is respectively represented as below:

$$\min_{s\in\Omega}\left(-\frac{1}{|\Phi|}\sum_{\phi\in\Phi} r_{s,\phi} + \gamma \times \text{risk}\right),$$

$$\min_{s\in\Omega}\left(\frac{1}{|\Phi|}\sum_{\phi\in\Phi} c_{s,\phi} + \gamma \times \text{risk}\right),$$

where $r_{s,\phi}$ and $c_{s,\phi}$ respectively represent the return and the shortfall brought by the strategy s ($s\in\Omega$) on the example $\phi$. When $\gamma$ is assigned with a greater (or smaller) value, it represents higher risk aversion (or tolerance). When $\gamma = 0$, it means that only the average return (or the average shortfall) is considered in the optimization objective but the variable "risk" is ignored. When $\gamma \to \infty$, it means that only the variable "risk" is considered in the optimization objective but the average return (or the average shortfall) is ignored. After a strategy is determined by optimizing the above objective in in-sample test, the strategy is applied to out-of-sample test for evaluation.

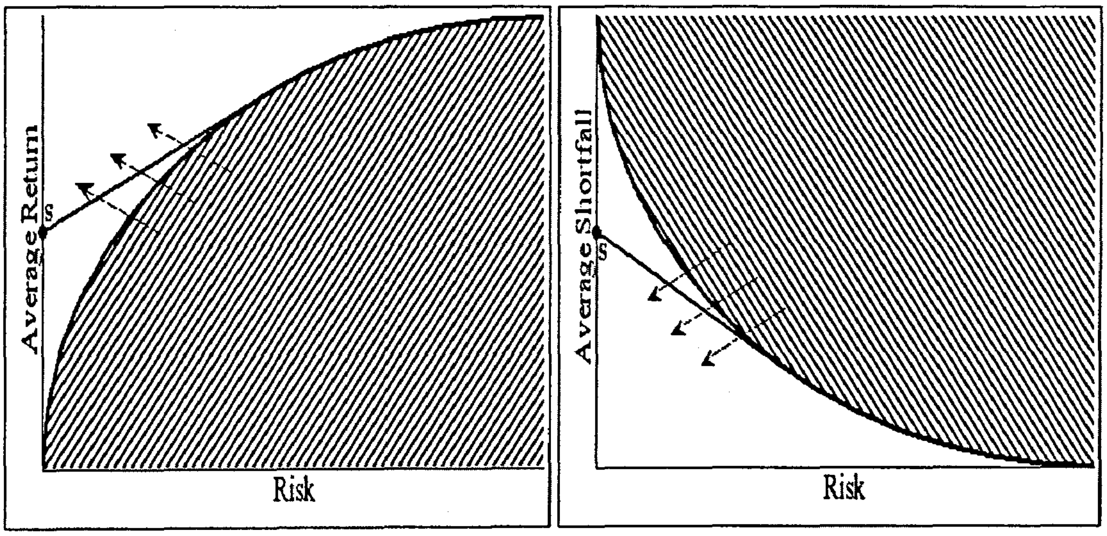


**Figure 2.3 Efficient investment (left chart) and trading (right chart) frontier**

Following the concept of efficient frontier, it was further pointed out that the EIF (the arc in left chart) can be moved to the straight line called Capital Market Line (CML) in the left chart by diversifying between an equity portfolio (e.g. the arc in left chart) and a risk-free asset such as a bond (e.g. the point s in left chart) (Sharpe 1964). It was also pointed out that the ETF (the arc in right chart) can be moved to the straight line called Capital Trade Line (CTL) in the right chart by diversifying between a set of agency trading strategies (e.g. the arc in right chart) and a risk-free principal trading strategy (e.g. the point s in right chart) (Kissell and Glantz 2003). Thus, a strategy set's efficient frontier is expected to move toward the favorable direction (see the arrows in Figure 2.3) by diversifying across strategies so that risk decreases and meanwhile the decrease in average reward (the decrease in average return or the increase in average shortfall) is limited. Numerical techniques have been proposed to acquire strategies from a candidate strategy set and determine their weights for implementing diversification across strategies.

But the existing techniques have two limitations. First, the candidate strategy set normally only contains finite strategies. For example, a portfolio of 10 stocks and their weights are to be determined for diversification based on a set of 50 stocks. In practice, the candidate strategy set may contain not only finite but also infinite strategies when the parameters in the strategies take values in a range of real number. The DAB technique for implementing diversification across strategies, which is proposed in this thesis, allows the candidate strategy set to contain infinite strategies. Second, the existing techniques are designed to reduce risk that is only measured by one specific standard such as the variance of returns (Markowitz 1956), the variance of shortfalls (Kissell and Glantz 2003) or the k-LPM of returns (Hogan and Warren 1974). The DAB technique inherits the key idea of the AdaBoost technique so that it lowers risk that can be measured by different standards and limits the decrease in reward. The DAB technique will be illustrated in Chapter 4 and 5 from the theoretical and empirical perspectives. The next section will introduce the AdaBoost technique and supervised learning.

# 2.3 Supervised Learning and AdaBoost

This thesis extends the AdaBoost technique to propose the DAB technique for several reasons: 1) it reflects the idea of diversification across learning models; 2) it allows the candidate model set to contain infinite models; 3) its statistical properties can be described in the DAB technique as lowering risk, which can be measured by different standards. This section introduces the AdaBoost technique and its background in the supervised learning field.

The problem of learning has been discussed in the artificial intelligence (AI) and statistics inference fields. Many empirical methods such as Neural Networks and Decision Trees have been proposed to deal with this problem (Duda et al. 2000). A learning problem can be generally described as "a process that modifies a system as to improve, more or less irreversibly, its subsequent performance of the same task or of tasks drawn from the same population" (Simon and Langley 1981). In the statistical inference field, it is described as below: a learning model is trained based on an example set and it not only fits in the training examples well but also identifies the unseen examples from an independently identical distribution as accurately as possible. The learning model $f$ is referred to as the map from an input example $\phi$, which is extracted from an independently identical probability distribution $P_I(\phi)$, to the example $\phi$'s label $\varphi$, which follows another probability distribution $P_O(\varphi)$. It is mathematically represented as $\varphi = f(\phi)$. Let $L(\varphi, f(\phi))$ denote the loss, which the model $f$ brings on the example $\phi$. Given a candidate learning model set (also called a hypothesis space) F, the objective of learning is to minimize the following expectation:

$$\mathrm{E}(f) = \int L(\varphi, f(\phi))\, \mathrm{d}P(\phi, \varphi),$$

where $P(\phi,\varphi)$ is a joint probability distribution $\phi$ and $\varphi$. The expectation is also called risk functional in the statistical learning field.

Let $\Phi$ denote an (input) example set, which comprises finite examples. In the example set $\Phi$, each example $\phi$ follows the probability distribution $P_I(\phi)$ and it is marked with a label $\varphi$ that follows the probability distribution $P_O(\phi)$. In supervised learning, a model, which maps the example $\phi$ to its label $\varphi$, is trained based on the example set $\Phi$ with labels. The model is expected to identify both the training examples in $\Phi$ and the unseen examples that follow the joint probability distribution $P(\phi,\varphi)$ as accurately as possible. Each example $\phi$ is normally described by several attributes, each of which takes symbolic or continuous values. The statistical foundation has been built to deal with the problem of learning and it is called statistical learning theory (Vapnik 1979, Vapnik and Chervonenkis 1971, 1974, 1981, Vapnik and Stefanyuk 1978). A probably approximately correct (PAC) model was proposed to incorporate some requirements on computational complexity into the statistical learning theory (Valiant 1984). Proposing the PAC model is a pioneering attempt to introduce computational learning theory to the AI community. In the PAC model, the question on whether strong and weak learnability are equivalent was posed (Kearns and Valiant 1988, 1989). This question is also described as whether a weak learner can be boosted to a strong one. A weak learner is only required to output the model that performs slightly better than random guessing does while a strong learner achieves low identification error with high confidence.

The strength of weak learnability was regarded as an open question until a constructive polynomial-time boosting algorithm was proposed (Schapire 1990). More efficient boosting algorithms were then developed (Freund 1990, 1993 and 1995). These early boosting algorithms have been testified based on optical character recognition (OCR) (Drucker et al. 1993). Proposing the AdaBoost technique is a milestone in the research of boosting because it overcomes several drawbacks of the earlier boosting algorithms (Freund and Schapire 1995, 1997). For a problem of two-class classification (a typical supervised learning problem), the AdaBoost technique initially sets a probability for each training example. Based on the probability distribution, it acquires a classifier from a candidate classifier set and calculates a weight for the acquired classifier based on its classification accuracy. Then the probability distribution is updated and higher probability is assigned to the examples misidentified by the acquired classifier. At the next learning step, a new classifier is acquired based on the updated probability distribution. At the end of learning, the classifiers with weights are acquired and the classification decision depends on the weighted average of the acquired classifiers' outputs. Figure 2.4 shows how the AdaBoost technique deals with a typical classification problem.

There are four points (examples), each of which has two attributes, in the left-top corner of Figure 2.4. In the four examples, two squares $\phi_2$, $\phi_4$ are categorized into the 1st class (e.g. face images) and two diamonds $\phi_1$, $\phi_3$ into the 2nd class (e.g. non-face images). The problem of learning is to construct a classifier to classify the four examples to two different classes as accurately as possible. This is called two-dimensional XOR problem. It is clear in Figure 2.4 that a nonlinear classifier (e.g. the ellipse in the left-top corner) can correctly classify them but a linear classifier (i.e. the dot line in the left-top corner) fails. In the learnable point of view, the linear classifier is a weak classifier while the nonlinear classifier is a strong classifier. The AdaBoost technique can boost a set of weak classifiers to reach the learnability of a strong classifier.

Suppose that the candidate classifier set comprises the vertical and horizontal straight lines. The AdaBoost technique initially assigns an equally probability to each example and acquires a classifier $c_1$ according to the probability distribution at the 1st learning step. According to $c_1$'s classification error rate $\varepsilon_1$, the AdaBoost technique calculates a weight $w_1$ for $c_1$ and updates the probability distribution on the training examples. The misclassified example $\phi_1$ is assigned with a higher probability in updating the probability distribution. The AdaBoost technique then acquires a new classifier and its weight according to the updated probability distribution. After four learning steps, the AdaBoost technique acquires four classifiers ($c_1$, $c_2$, $c_3$ and $c_4$) and their weights ($w_1$, $w_2$, $w_3$ and $w_4$). The classification decision depends on the weighted average of four classifiers' outputs, i.e. $sign(w_1 \times c_1 + w_2 \times c_2 + w_3 \times c_3 + w_4 \times c_4)$, where $sign(\bullet)=1$ if $\bullet>0$ and $sign(\bullet)=-1$ if $\bullet<0$. In the end, the ensemble of four classifiers is able to classify all four examples $\phi_1$, $\phi_2$, $\phi_3$ and $\phi_4$ correctly.

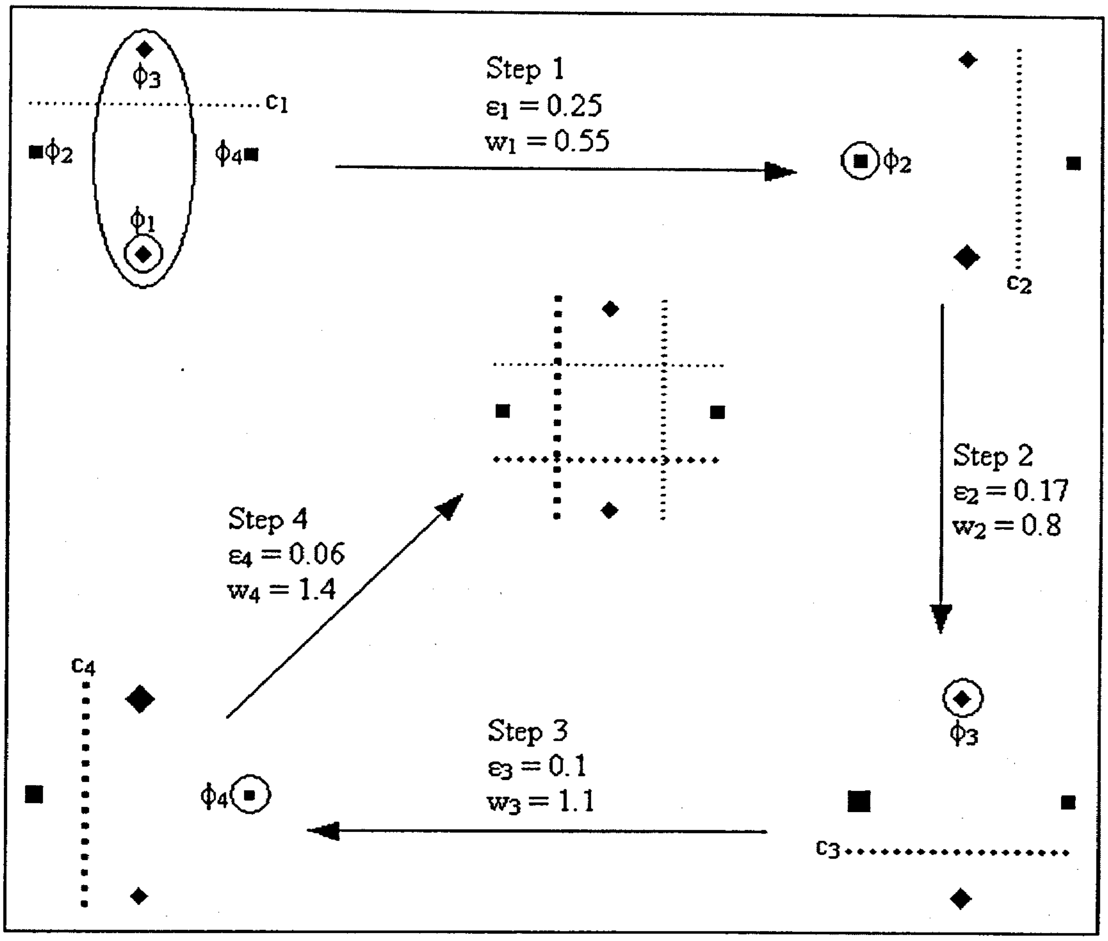


**Figure 2.4 The AdaBoost technique for two-class classification**

The AdaBoost technique has been successfully applied to solve many practical problems in application such as face detection (Viola and Jones 2004) and text categorization (Schapire and Singer 2000). Moreover, its statistical foundation has been built (Freund and Schapire 1997). For a two-class classification problem, as the learning steps increase, the classification error rate of the ensemble of classifiers acquired by the AdaBoost technique drops exponentially fast as long as the classifier, which achieves classification accuracy that is slightly higher than random guessing does, can be consistently found. It was also pointed out that as the learning steps increase, the AdaBoost technique increases the margin of an ensemble of classifiers, i.e. improves the worst classification performance of the ensemble of classifiers on the training examples (Schapire et al. 1998, Rätsch and Warmuth 2002). In addition, the AdaBoost technique allows the candidate model set (e.g. a candidate classifier set) to contain infinite models (e.g. all linear classifiers). The DAB technique is expected to inherit those advantages of the AdaBoost technique.

But the AdaBoost technique cannot be directly applied to acquire strategies from a candidate strategy set for diversification across strategies. In the AdaBoost technique, updating the probability distribution on training examples depends on the model's identification accuracy, which is calculated by the model's output and the label of each example. In supervised learning (e.g. classification), each training example is normally marked with a label before the candidate model set is given. For training a strategy defined in this thesis, however, the training example cannot be marked with a label before a candidate strategy set is given. This difference is further illustrated through several research subjects: 1) face detection, 2) horse racing, 3) playing go and 4) stock trading.

Face detection is a typical classification problem. Chart 1 in Figure 2.5 shows some face images (from the MIT+CMU test set) and some background pictures. The goal of face detection is to distinguish between face images and non-face images as accurately as possible. The frames in Chart 1 are the results from (Viola and Jones 2004), in which only one face image is missed and one football image is misidentified. To train a classifier for face detection, a number of images like those in Chart 1 are collected. The images have their attributes (e.g. colors and textures) and are marked with labels (e.g. face or non-face). Various techniques of supervised learning can be used to build the classifier, which makes judgment on whether an image is face or non-face according its attributes. When the AdaBoost technique is applied, the model is represented as the ensemble of multiple classifiers with their weights (Viola and Jones 2004).

Horse racing (see Chart 2 in Figure 2.5) can be regarded as the problem of supervised learning. Its goal is to achieve high return by making wise judgment on which horse would win the game of racing to. To train a model for this judgment, the historical records of each horse such as its status are collected as the attributes of a training example and the result of racing as the example's label. The model is built based on the training examples with attributes and label. Similar to face detection, the AdaBoost technique can be used to acquire several different models and their weights for horsing racing. Then the judgment on whether a horse can win the game of racing depends on the ensemble of different models with their weights. An alternative method is to allocate money to those horses, which are recommended by different models as the potential winners, in proportion with the weights. This method reflects the idea of diversification and differs from face detection in that there are input resources such as money for horse racing. This is called dynamic allocating problem and an algorithm named Hedge has been proposed to deal with it (Freund and Schapire 1997). But Freund and Schapire pointed out that the Hedge algorithm does not possess the advantage of the AdaBoost technique (the exponential decreasing bound of misidentification rate).

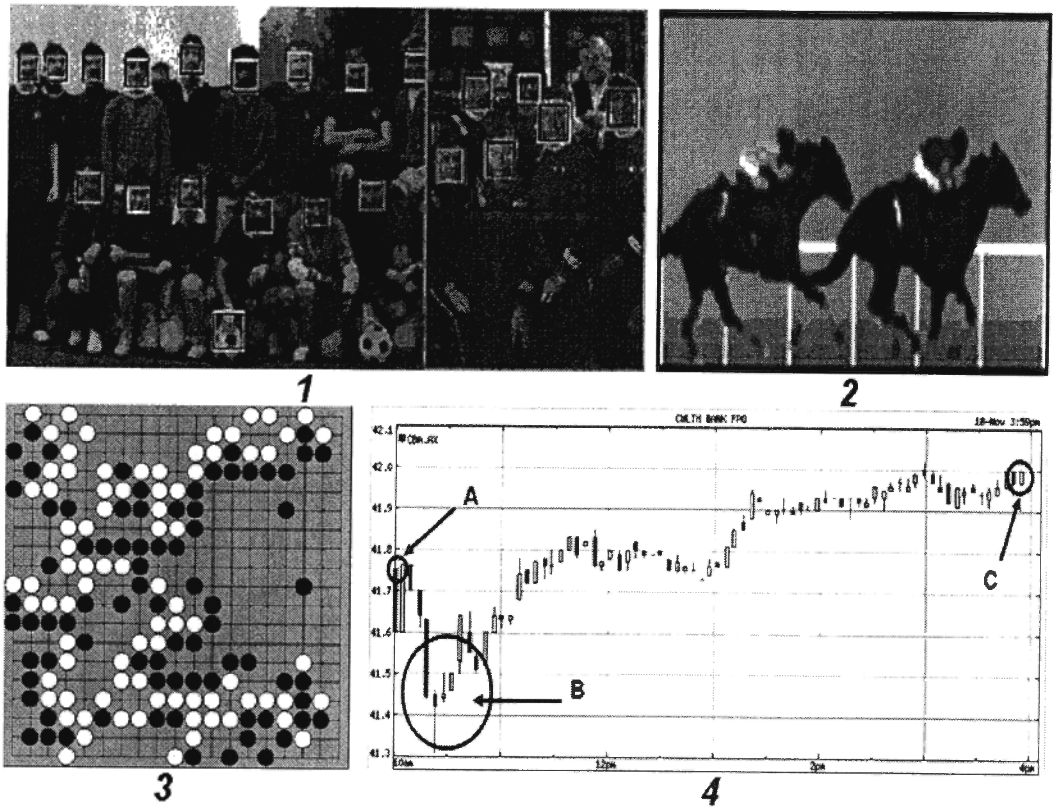


**Figure 2.5 Face detection, horse racing, playing go and stock trading**

Playing go (see Chart 3 in Figure 2.5 from http://weiqi.sports.tom.com) differs from the above two examples in that all decisions in the process of playing game interact with each other and so a correct label does not exist for each individual decision. The training example cannot be marked with a label as what has been done in supervised learning. In stock trading (see Chart 4 in Figure 2.5 from http://au.finance.yahoo.com), the main goal is to make profit as much as possible by buying stocks at lower prices and selling them at higher prices. For example, the stock is bought in the area "A" and sold in the area "C". Further, higher profit can be made through selling the stock in the area "A" and buying it in the area "B". The difference between the above two decisions shows 1) the absolutely accurate prediction of stock prices is impossible due to their uncertainty; 2) all decisions in a process of stock trading interact with each other and each decision depends on the previous and next decisions so that a correct label actually does not exist for this type of training example (e.g. the area "A") if a training example is referred to as information at each time point. Suppose that the attributes of a training example be all information in the whole process of stock trading and its label be the profit. In this case, the profit actually does not exist before a strategy is given since its existence depends on a specific strategy.

From the above explanation, the two examples of both playing go and stock trading cannot be regarded as the subject of supervised learning. The research of reinforcement learning has had deep insights into this type of problem (Sutton and Barto 1998). In addition, there is difference between playing go and stock trading. Different from the example of playing go, there are input resources in the example of stock trading, i.e. money for trading stocks. This is similar to the case of horse racing. While the AdaBoost technique is discussed mostly in supervised learning, the DAB technique proposed in this thesis extends the key idea of adaptively updating the probability distribution in the AdaBoost technique so that it is not only applicable to supervised learning. The DAB technique can acquire strategies from a candidate strategy set and determined their weights. The input resources are allocated to the acquired strategies in proportion with their weights and the acquired strategies are then executed in parallel with their allocated resources.

# 2.4 Summary

This thesis covers several research subjects from both finance and computer science fields: trade execution and strategy analysis, risk management and diversification, supervised learning and AdaBoost. This chapter introduces the basic concepts about these subjects and illustrates the limitation of existing theories and methods. In addition, this chapter also describes the simulator for executing trade execution strategies and the experimental datasets for empirically evaluating the DF strategy, the DAB technique and the BONUS strategy. The following chapters will illustrate the proposed technique/strategies in details from the theoretical and empirical perspectives.

# Chapter 3

# Dynamic Focus Strategy

The market order strategy can capture the current trading opportunity and avoid the nonexecution risk by immediately filling the entire order without any restrictions on price. But it may incur large market impact cost. Theoretical and empirical research have shown that by assigning a price to the order, the limit order strategy can achieve lower cost than the market order strategy does. But it may miss the good trading opportunities and incur the nonexecution risk since it normally does not fill the entire order immediately. This chapter proposes the DF strategy, which reflects the idea of diversification across time, to solve the problems that the limit order strategy faces. The DF strategies incorporate a series of small market orders into the limit order strategy and dynamically adjust the market order volume based on two real-time state variables: inventory and order book imbalance. The sigmoid function is adopted to map the state variable to the market order volume. The empirical results on 80 datasets show that the DF strategy achieves lower cost and risk than the limit order strategy does. This chapter is composed of 3 sections. The 1st section describes the DF strategy. The 2nd section empirically evaluates the DF strategy. The 3rd section summarizes the DF strategy.

# 3.1 DF Strategy and Analysis

This chapter discusses one type of limit order strategy, which assigns the best bid (or ask) price to the buy (or sell) order in the period of execution and fills the unexecuted volume at the end of execution. In this type of limit order strategy, the best bid (or ask) price is assigned to the buy (or sell) order for two reasons. One is that in order to avoid adverse price selection, the limit order strategy does not place the buy (or sell) order at a higher (or lower) price level than all limit orders from other market participants. The other is that in order to reduce nonexecution risk, the limit order strategy does not place the buy (or sell) order far away from the best bid (or ask) price. The reasonability of this type of limit order strategy also can be referred to (Harris and Panchapagesan 2005). It is called NPA (naïve price adjustment) strategy in this chapter because it naïvely follows the best bid or ask price but does not respond any other real-time market information.

The DF strategy, which reflects the idea of diversification across time, does not only passively assigns the best bid (or ask) price to the order but also aggressively fill part of the entire order at different time points over the period of execution. In other words, the DF strategies incorporate a series of small market orders with different volume into the NPA strategy and dynamically adjust the volume of each market order based on two real-time state variables: inventory and order book imbalance. The DF strategy is expected to achieve lower cost and risk than the NPA strategy does.

Inventory control is one of key issues discussed in the finance investment area. Traders' inventory position will exceed the expected level if they accommodate more orders from other market participants. In this case, the traders will achieve higher return if the market moves toward a favorable direction, but meanwhile they will face higher risk of holding extra inventory if the market moves toward an adverse direction. Theoretical and empirical studies show that in practice, the traders always need to optimize their long or short inventory position as per their risk aversion or tolerance. When the inventory position deviates from the expected level, the traders can resume it to the expected level by adjusting their buy or/and sell order prices. For example, they can move their order prices downward (upward) to attract more buyers (sellers) when their inventory position is longer (shorter) than the expected level. This price adjustment is regarded as inventory control.

Actually, the above concern on inventory control also exists in trade execution. Traders have to face nonexecution risk and adverse price movement when they execute a given order by using the limit order strategy. The traders normally do not know for sure when a limit order can be filled or if it can be completely fulfilled at the end of execution. In addition, due to the uncertainty of short-term price movement in the period of execution, the limit order strategy has both the advantage of waiting for favorable price movement and the disadvantage of missing good trading opportunities. The following example helps understand the relationship between real-time inventory control and nonexecution risk. Suppose that there are two cases for purchasing 10,000 shares of one stock in 10 minutes: in the first case, there is the unexecuted volume of 9,000 shares in the last 1 minute whereas in the second case, there is the unexecuted volume of 1,000 shares in the remaining 9 minutes. Intuitively, the first case would incur higher nonexecution risk than the second case. To reduce the nonexecution risk in the first case, the strategy should resume the inventory to the expected level as soon as possible by taking a more aggressive action. Risk management in trade execution may benefit from real-time inventory control. From the above example, the inventory control in trade execution is related to two factors: unexecuted volume and remaining time. Based on the two factors, the inventory i(t) at the time t ($t_s \leq t < t_e$) is defined as below:

$$i(t) = \frac{v_u(t)}{t_e - t},$$

where $v_u(t)$ represents the unexecuted volume at the time t, $t_s$ and $t_e$ are respectively the start time and the end time of execution.

Dynamic price adjustment has been applied in trade execution to respond real-time update of inventory (Nevmyvaka et al. 2006). Different from the dynamic price adjustment, the DF strategy controls inventory by dynamically adjusting the market order volume. The DF strategy assigns higher volume to the market order to reduce the nonexecution risk if the unexecuted volume in the remaining time exceeds the expected level. Otherwise, the DF strategy assigns lower (or zero) volume to the market order to wait for favorable price movement. The DF strategy, which dynamically adjusts the market order volume based on the deviation of the real-time unexecuted volume from the expected level, is called DF-IC strategy in this chapter.

Order book imbalance is closely related to short-term price movement as illustrated in the research of market microstructure such as (Harris and Panchapagesan 2005). Two types of traders who prefer limit orders are pre-committed traders and value-motivated traders. The former aims to reduce transaction costs whereas the latter trades only at the acceptable price given the value estimates. They often place limit orders, which are close to the best bid-ask prices, to increase the possibility of their orders to be filled so as to fulfill their commitment or capture profitable opportunities. Their behaviors are one of the main sources to aggregate order book imbalance, which may indicate price movement. The pre-committed traders (or the value-motivated traders) will place more aggressive orders that affect prices such as market orders if they face the pressure of nonexecution risk (or their profitable information is being impounded into prices). Information indicated by order book imbalance may be valuable to traders in a general sense that prices would increase (or decrease) when the buy side of the order book is heavier (or thinner) than the sell side. Let $v_{bb}(t)$ and $v_{ba}(t)$ denote the volume respectively at the best bid and ask price levels at the time t ($t_s \le t < t_e$). The order book imbalance obi(t), which represents the difference between the best bid volume and the best ask volume at the time t, is defined as below:

$$\mathrm{obi}(t) = ln(v_{bb}(t)) - ln(v_{ba}(t)).$$

If obi(t) is positive, it means that the buy side of the order book is heavier than the sell side. Otherwise, it means that the buy side of the order book is thinner than the sell side.

Order book imbalance has been testified in the study of dynamic price adjustment for trade execution (Nevmyvaka et al. 2006). If order book imbalance forecasts that the market would move toward the adverse direction, the order will be placed at a more aggressive price level to capture the current trading opportunity. Otherwise, the order is placed at a more passive price level to wait for favorable price movement. Different from dynamic price adjustment, the DF strategy responds the forecast of order book imbalance by dynamically adjusting the market order volume. The DF strategy assigns higher volume to the market order to capture the current trading opportunity if order book imbalance indicates that the market would move toward the adverse direction. Otherwise, the DF strategy assigns lower (or zero) volume to the market order to wait for favorable price movement. The DF strategy, which dynamically adjusts the market order volume based on the order book imbalance, is called DF-OBI strategy in this chapter.

The DF-IC strategy will assigns higher volume to the market order if the unexecuted volume in the remaining time exceeds the expected level. The DF-OBI strategy will assign higher volume to the market order if order book imbalance indicates that the market would move toward the adverse direction. The next question is how to quantitatively implement these ideas, in other words, how much volume should be assigned to the market order based on the real-time state variables: i(t) and obi(t). According to requirements in trade execution and the above qualitative description, the quantitative model should satisfy the following three conditions: 1) the market order volume should not exceed the entire order volume in trade execution; 2) the market order volume should not be negative since it is assumed that for buying (or selling), no sell (or buy) order be submitted during the execution period; and 3) the quantitative model should be represented by an increasing function according to the principle on dynamic volume adjustment illustrated above. The thesis suggests the sigmoid function as the quantitative model to describe the relationship between the state variable and the dynamically adjusted volume since it satisfies all of the three conditions. Let $v_p(t)$ denote the ratio of the market order volume at the time t to the entire order volume. The quantitative model based on the sigmoid function is represented as below:

$$v_p(t) = \frac{1}{1+exp\left(-\lambda \times x(t)\right)},$$

where x(t) represents the state variable at the time t and $\lambda$ is a parameter.

A family of sigmoid functions is given with different values of $\lambda$. The sigmoid functions with different values of $\lambda$ represent various types of dynamic volume adjustment. Three sigmoid functions are shown in Figure 3.1, where the horizontal and vertical coordinates respectively represent x(t) and $v_p(t)$. When $\lambda$ takes a large value (e.g. 50), there is a highly sensitive area, where the sigmoid function's output $v_p(t)$ will significantly change as a slight change in the state variable x(t) occurs (see the dash curve in the shade area of Figure 3.1). When $\lambda$ takes a small value (e.g. 0.5), the sigmoid function's output $v_p(t)$ changes in a smooth manner as the state variable x(t) changes (see the dot curve in Figure 3.1). This characteristic of the sigmoid function indicates the fact that as $\lambda$ increases, dynamic volume adjustment in the DF strategy could be more significantly affected by a slight change in the state variable such as inventory or order book imbalance.

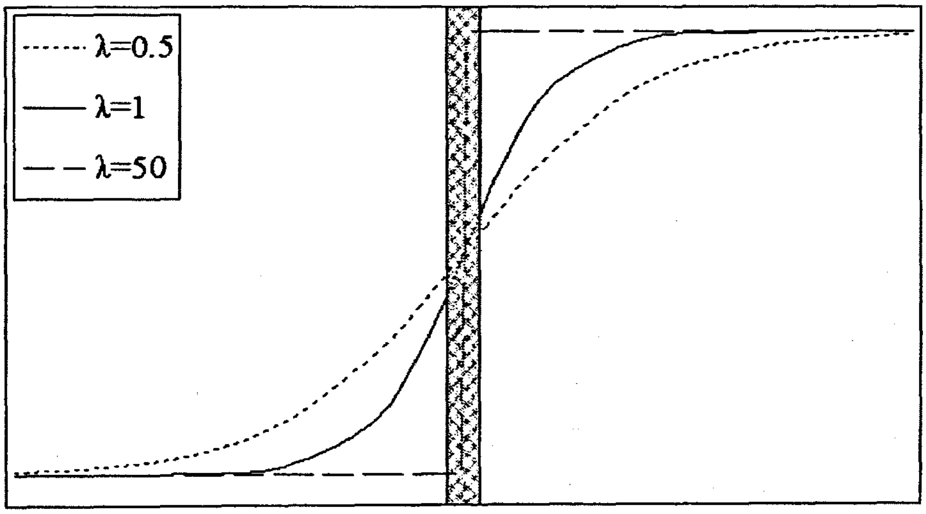


**Figure 3.1 Sigmoid function**

When the sigmoid function is used in the DF-IC strategy, the state variable x(t) is defined as below:

$$x(t) = \frac{i(t) - i(t_s)}{i(t_s)} - b,$$

where b is a bias, i(t) and $i(t_s)$ are the inventory respectively at the time t and the start time $t_s$. When the DF-IC strategy determines to submit a market order, the ratio $v_p(t)$ is calculated as below:

$$v_p(t) = \frac{1}{1+exp\left(-\lambda\times\left(\frac{i(t)-i(t_s)}{i(t_s)}-b\right)\right)}.$$

This definition means that the DF-IC strategy assigns higher volume to the market order to reduce nonexecution risk as the inventory i(t) increases while it assigns lower volume to the market order to wait for favorable price movement as the inventory i(t) decreases. Moreover, this definition guarantees that the market order volume never exceeds the entire order volume and is always nonnegative.

When the sigmoid function is used in the DF-OBI strategy, the state variable x(t) is defined as below:

$$x(t) = obi(t).$$

Let $obi_U$ and $obi_L$ denote respectively the upper and lower bounds of obi(t). So $v_p(t)$ is calculated as below when the DF-OBI strategy determines to submit a market order:

$$v_p(t) = \begin{cases} 0 & obi_L \le obi(t) \le obi_U \\ \dfrac{1}{1+exp(-\lambda\times obi(t))} & \text{others} \end{cases}$$

where $obi_L < 0$ and $obi_U > 0$. This definition means that the DF-OBI strategy does not take action in the insensitive area between the lower bound $obi_L$ and the upper bound $obi_U$ since the insensitive area represents the weak signal for short-term price movement. This definition also means that the DF-OBI strategy will assign higher (or lower) volume to the market order to capture (or wait for) trading opportunities if order book imbalance indicates that the market would move toward the adverse (or favorable) direction. Moreover, this definition guarantees that the market order volume never exceeds the entire order volume and is always nonnegative.

The trading volume is one of the important standards that measure one stock's liquidity over a period of time. If there is higher trading volume over a period of time for one stock, it shows that the stock's liquidity is higher in this period of time. Let $v_{10Min}$ and $v_{Daily}$ respectively denote average 10-minute trading volume and average daily trading volume. Table 3.1 lists the average 10-minute volume and the average daily volume during 4 testing periods of 20 stocks. It is clear in Table 3.1

that liquidity is significantly different for different stocks while small liquidity difference exists for different testing periods of the same stock. The second section will compare the DF strategy to the NPA strategy. The dynamic volume adjustment in the DF strategy, which reflects the idea of diversification across time, will be empirically verified through in-sample test and out-of-sample test on 80 datasets.

**Table 3.1 Average trading volume (AMP – WPL)**

| | January – February 2002 | | February – March 2002 | | March – April 2002 | | April – May 2002 | |
|---|---|---|---|---|---|---|---|---|
| | $V_{10Min}$ | $V_{Daily}$ | $V_{10Min}$ | $V_{Daily}$ | $V_{10Min}$ | $V_{Daily}$ | $V_{10Min}$ | $V_{Daily}$ |
| AMP | 40,143 | 1,404,990 | 38,736 | 1,355,754 | 34,500 | 1,207,513 | 45,668 | 1,598,391 |
| ANZ | 63,309 | 2,215,816 | 53,949 | 1,888,201 | 63,800 | 2,233,008 | 70,117 | 2,454,108 |
| BHP | 217,666 | 7,618,326 | 194,566 | 6,809,801 | 161,872 | 5,665,532 | 176,310 | 6,170,859 |
| BIL | 77,844 | 2,724,549 | 84,879 | 2,970,757 | 69,128 | 2,419,491 | 66,253 | 2,318,847 |
| CBA | 65,306 | 2,285,713 | 59,951 | 2,098,276 | 48,547 | 1,699,147 | 48,914 | 1,711,994 |
| CML | 29,878 | 1,045,727 | 47,434 | 1,660,182 | 45,877 | 1,605,680 | 72,104 | 2,523,632 |
| FGL | 170,611 | 5,971,368 | 163,752 | 5,731,336 | 105,967 | 3,708,848 | 111,281 | 3,894,820 |
| NAB | 63,711 | 2,229,896 | 60,333 | 2,111,670 | 65,288 | 2,285,070 | 65,645 | 2,297,571 |
| NCP | 147,053 | 5,146,872 | 149,535 | 5,233,733 | 109,636 | 3,837,266 | 122,642 | 4,292,458 |
| NCPDP | 82,757 | 2,896,490 | 93,512 | 3,272,903 | 71,726 | 2,510,409 | 70,814 | 2,478,502 |
| PBL | 28,698 | 1,004,417 | 24,082 | 842,864 | 16,974 | 594,099 | 16,589 | 580,606 |
| RIO | 26,458 | 926,026 | 26,870 | 940,452 | 26,936 | 942,776 | 27,048 | 946,693 |
| SGB | 11,348 | 397,187 | 7,662 | 268,174 | 9,872 | 345,518 | 11,876 | 415,652 |
| TLS | 374,556 | 13,109,451 | 392,153 | 13,725,356 | 391,610 | 13,706,358 | 535,237 | 18,733,302 |
| WBC | 61,650 | 2,157,750 | 64,692 | 2,264,231 | 80,638 | 2,822,338 | 92,149 | 3,225,200 |
| WES | 6,946 | 243,121 | 6,582 | 230,366 | 7,524 | 263,342 | 9,534 | 333,698 |
| WFT | 79,515 | 2,783,036 | 86,972 | 3,044,012 | 91,104 | 3,188,645 | 82,414 | 2,884,476 |
| WMC | 56,371 | 1,972,984 | 50,638 | 1,772,324 | 51,884 | 1,815,931 | 63,290 | 2,215,149 |
| WOW | 47,014 | 1,645,499 | 46,102 | 1,613,575 | 46,701 | 1,634,536 | 45,587 | 1,595,551 |
| WPL | 34,297 | 1,200,411 | 32,385 | 1,133,492 | 29,941 | 1,047,938 | 26,778 | 937,245 |

# 3.2 Empirical Evaluation

The DF strategy has been proposed in the previous section. The limit order strategy sets a limit order at the start time of execution and fulfills the unexecuted volume at the end of execution. The DF strategy applies the idea of diversification across time to hedge nonexecution risk and adverse price movement, which the limit order strategy may have to face. The DF strategy submits a series of small market orders and dynamically adjusts the market order volume according to the real-time state variables (inventory and order book imbalance) during the execution period. Different values of the parameter $\lambda$, which is involved in the DF strategy, determine various characteristic of dynamic volume adjustment. This section discusses the in-sample test on the DF strategies with different values of $\lambda$ and compares them to the NPA strategy in terms of cost, risk and efficient frontier.

The details of experimental setup can be found in the first section of Chapter 2. In-sample test is designed to check the cost and risk, which are brought by the DF strategies with different values of $\lambda$, and compare them to those by the NPA strategy. Here, the cost is measured by the average shortfall $c_\mu$ and the risk is measured by the standard deviation of shortfalls $c_\sigma$. Let $c_E(s,1,\phi)$ denote the shortfall, which is brought by the strategy s when it executes the order $\rho$ on the example $\phi$. Given an example set $\Phi$, the average shortfall $c_\mu$ and the standard deviation of shortfalls $c_\sigma$ are respectively calculated as below:

$$c_\mu = \frac{1}{|\Phi|}\sum_{\phi\in\Phi} c_E(s,1,\phi),$$

$$c_\sigma = \sqrt{\frac{1}{|\Phi|-1}\times\sum_{\phi\in\Phi}\left(c_E(s,1,\phi)-c_\mu\right)^2}\,.$$

The average trading volume of the stock WFT is around twice of that of the stock FGL in the 1st and 2nd test periods, while there is not much difference between them in the 3rd and 4th test periods. So this section chooses WFT and FGL to discuss the in-sample results on the four test periods, which are respectively shown in two column charts of Figures 3.2, 3.3, 3.4 and 3.5. In those figures, the horizontal coordinates in the first and second row charts represent the different values of $\lambda$ and the vertical coordinates respectively represent $c_\sigma$ and $c_\mu$. The third row charts show the study of efficient frontier, and its horizontal and vertical coordinates represent $c_\sigma$ and $c_\mu$ respectively. The dot and dash curves respectively represent the in-sample results of the DF-IC strategy and the DF-OBI strategy. The straight line and the triangle being circled represent the in-sample results of the NPA strategy.

All of the first row charts clearly show that 1) the DF strategy with small value of $\lambda$ (this means that a smoother sigmoid function is used) results in lower volatility of shortfalls (measured by $c_\sigma$); and 2) the DF strategy with the optimal value of $\lambda$ can achieve smaller standard deviation than the NPA strategy does. Most of the second row charts show that neither extreme small value of $\lambda$ nor extreme large value of $\lambda$ helps the DF strategy to achieve low average shortfall. All of the second row charts show that the DF-IC strategy with the optimal value of $\lambda$ achieves lower average shortfall than the NPA strategy does. Except for the $4^{th}$ test period, all in-sample results on WFT show that the DF-OBI strategy with the optimal value of $\lambda$ achieves lower average shortfall than the NPA strategy does. But all in-sample results on FGL show that the DF-OBI strategy with the optimal value of $\lambda$ cannot achieve lower average shortfall than the NPA strategy does. Based on the above discoveries, the DF strategy improves the NPA strategy more significantly on WFT than on FGL. A reasonable explanation is that WFT is more illiquid than FGL and the DF strategy is more applicable to improve the performance of trade execution on illiquid stocks. In addition, the fact that the DF-IC strategy outperforms the DF-OBI strategy shows that inventory control is a more important factor for trade execution than order book imbalance. The effectiveness of the DF strategy is also verified from the perspective of strategies' efficient frontier. It is clear in all of the $3^{rd}$ row charts that the NPA strategy is not located on the efficient frontier. In the view of efficient frontier, the optimal solution is achieved by the DF strategy rather than the NPA strategy. The effectiveness of the DF strategy will be further verified by the statistical summary of out-of-sample test results on all 80 datasets.

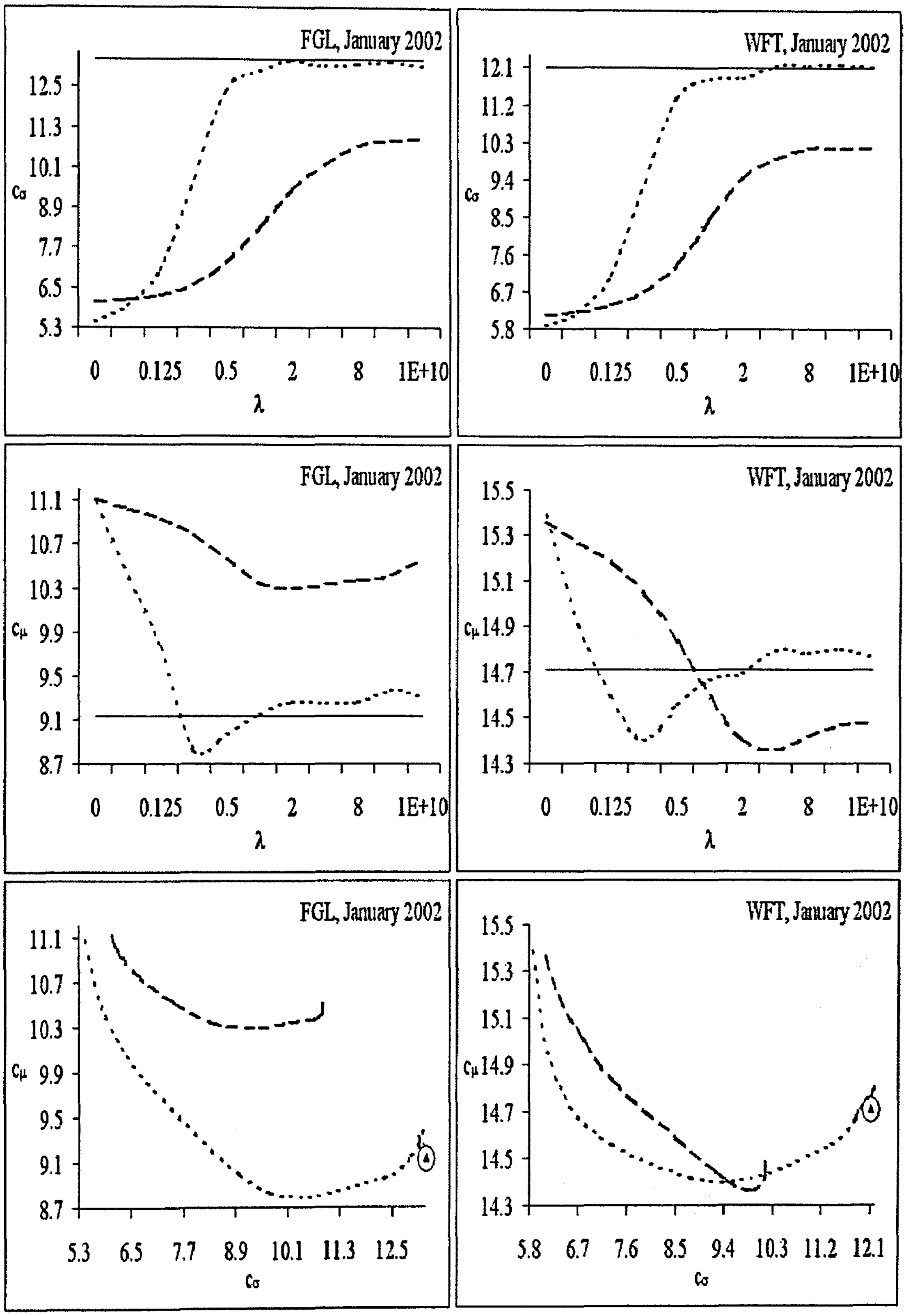


**Figure 3.2 In-sample test on the DF strategy (January 2002)**

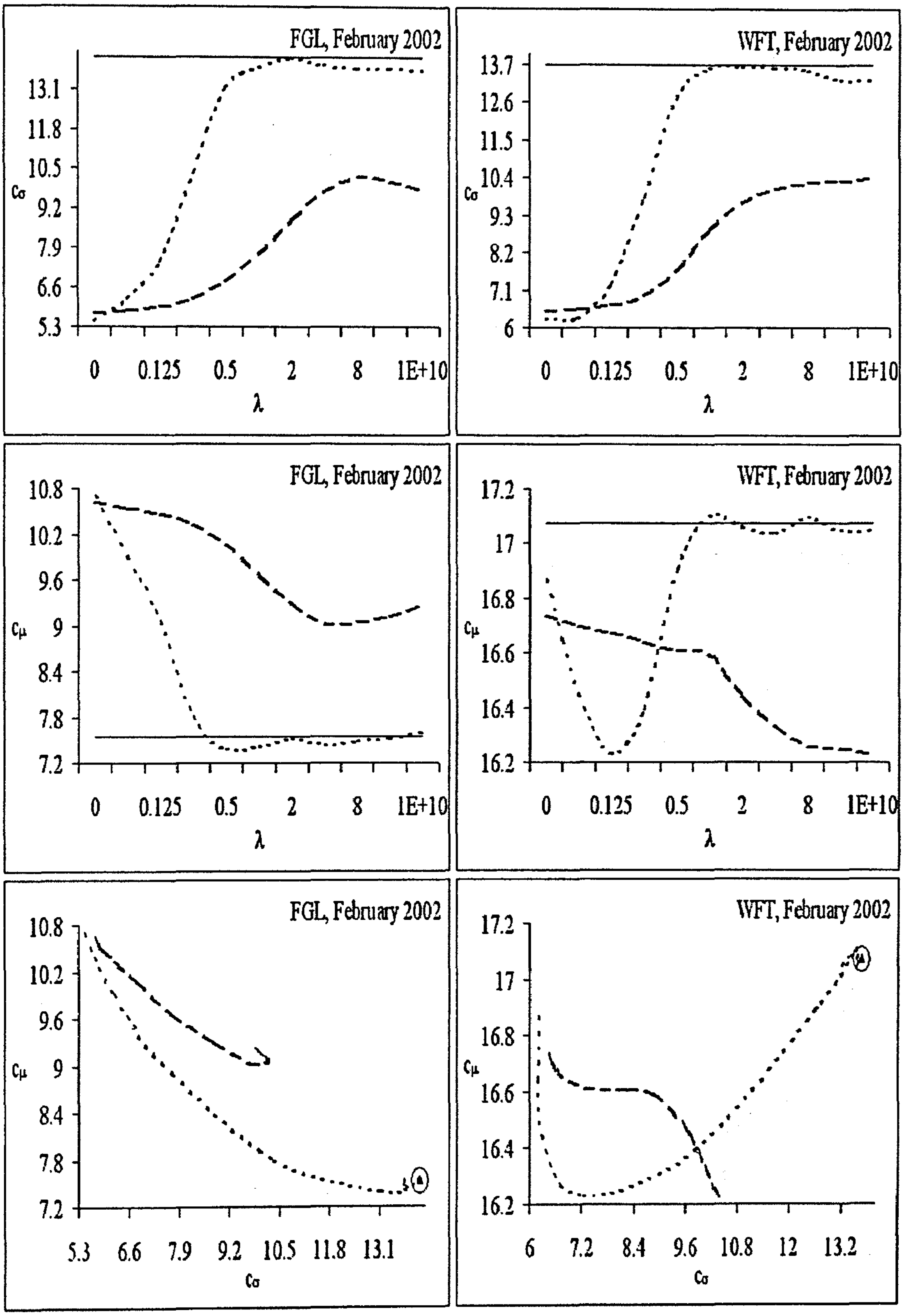


**Figure 3.3 In-sample test on the DF strategy (February 2002)**

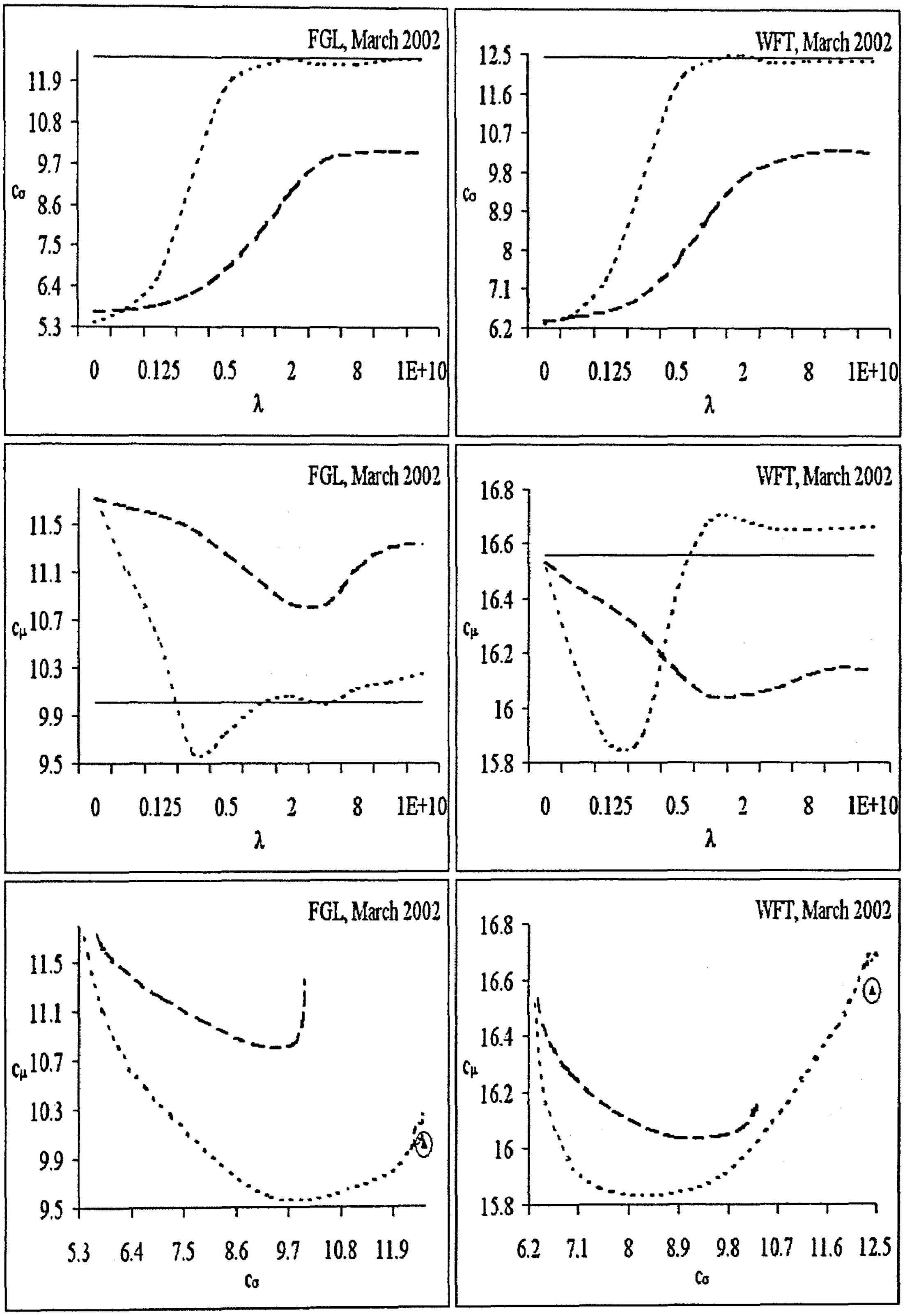


**Figure 3.4 In-sample test on the DF strategy (March 2002)**

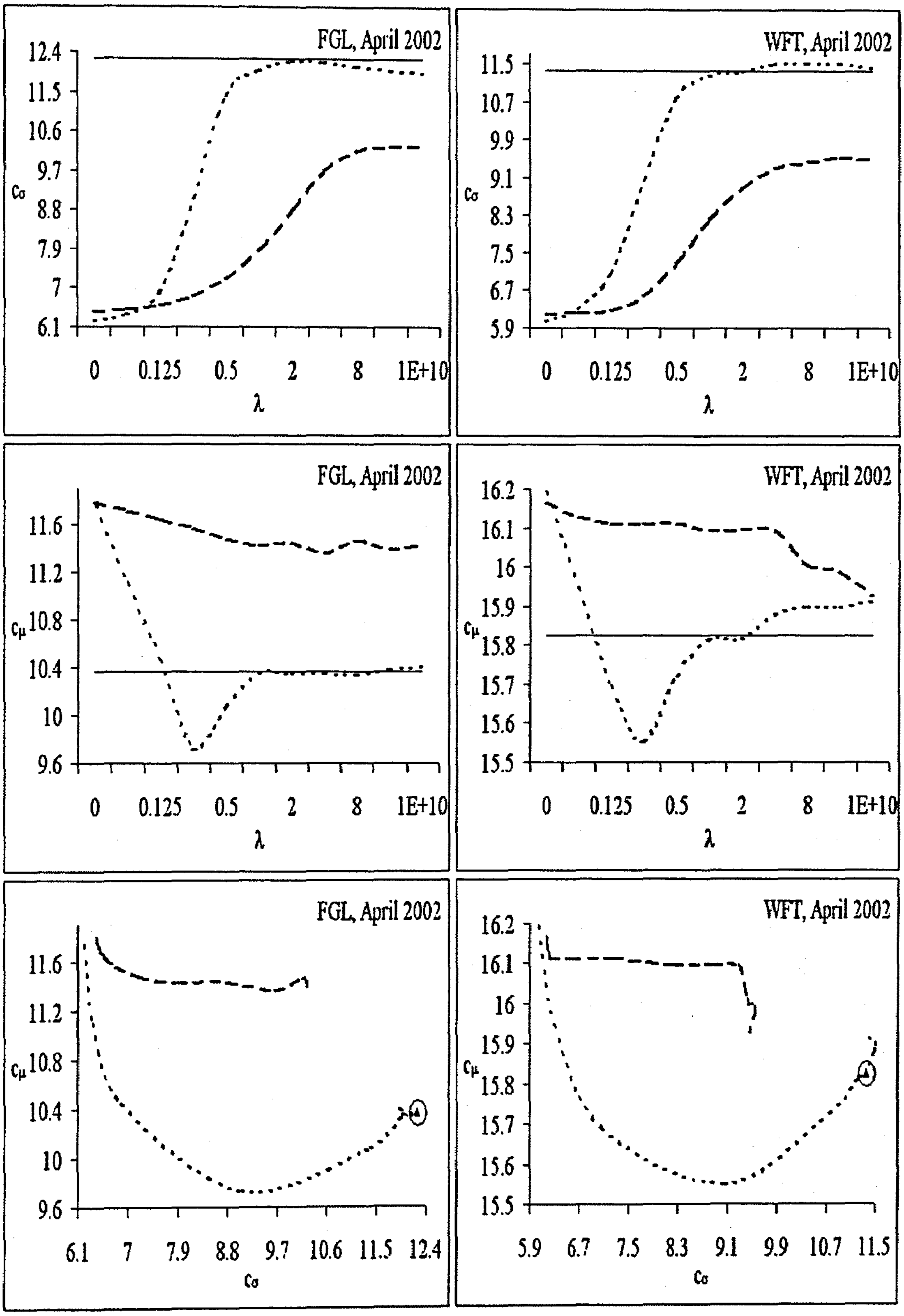


**Figure 3.5 In-sample test on the DF strategy (April 2002)**

In in-sample test, the parameter $\lambda$ is optimized based on different degrees of investors' risk aversion or tolerance. The tradeoff between cost (the average of shortfalls) and risk (the standard deviation of shortfalls) is represented as below:

$$c_\mu + \gamma \times c_\sigma$$

where $\gamma$ is the tradeoff factor that represents the degree of risk aversion or tolerance in trade execution. Greater (or smaller) $\gamma$ represents higher risk aversion (or tolerance) in trade execution. When $\gamma = 0$, it means that only average shortfall $c_\mu$ is involved in the optimization objective but risk (standard deviation of shortfalls $c_\sigma$) is ignored. When $\gamma \to \infty$, it means that only risk (standard deviation of shortfalls $c_\sigma$) is involved in the optimization objective but average shortfall $c_\mu$ is ignored. This section testifies two values 0 and 1 for the tradeoff factor $\gamma$. With the given value of $\gamma$, the DF strategy is optimized in in-sample test by minimizing $c_\mu + \gamma \times c_\sigma$. Then the optimized DF strategy is applied to out-of-sample test and is compared to the NPA strategy in terms of $c_\mu$ and $c_\sigma$. Let IC0, IC1, OBI0 and OBI1 respectively denote the optimized DF-IC and DF-OBI strategies by setting $\gamma = 0$ and 1.

The tables 3.2, 3.3, 3.4 and 3.5 list the out-of-sample results on 80 datasets, which strongly support the DF strategy' effectiveness. The DF strategy achieves lower average shortfall than the NPA strategy does on 75 datasets. The DF-IC strategy achieves lower average shortfall than the DF-OBI strategy does on 79 datasets. According to the research of market microstructure, order book imbalance is useful as it could forecast short-term price movement. But its forecast may not be very accurate due to high price volatility in markets. So the out-of-sample results show that inventory control is a more important factor for trade execution than order book imbalance. This conclusion was also implied in (Nevmyvaka et al. 2006). The out-of-sample results on all 80 datasets show that the lower standard deviation of shortfalls is achieved by setting $\gamma = 1$ than by setting $\gamma = 0$. The out-of-sample results on 79 datasets show that the lower average shortfall is achieved by setting $\gamma = 0$ than by setting $\gamma = 1$. This supports the analysis about the effect of $\gamma$ on the optimization objective.

**Table 3.2 Out-of-sample test on DF strategies (AMP – CBA)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| AMP | NPA | 15.96 | 18.99 | 20.62 | 19.95 | 19.08 | 18.96 | 17.00 | 23.35 |
| | IC0 | **7.70** | **8.21** | **8.49** | 9.83 | **8.74** | 9.90 | **7.23** | 11.66 |
| | IC1 | **7.70** | **8.21** | 8.65 | **7.51** | 8.97 | **7.70** | 8.36 | **9.31** |
| | OBI0 | 13.91 | 11.49 | 15.66 | 11.46 | 15.59 | 12.04 | 15.31 | 13.12 |
| | OBI1 | 13.91 | 11.49 | 15.50 | 11.10 | 15.59 | 12.04 | 15.39 | 12.99 |
| ANZ | NPA | 10.29 | 14.70 | 9.80 | 15.84 | 7.06 | 12.65 | 8.43 | 12.83 |
| | IC0 | **5.32** | 8.23 | **5.03** | 8.58 | **4.04** | 7.83 | **4.66** | 7.54 |
| | IC1 | 6.10 | **6.06** | 5.94 | **6.98** | 4.70 | **5.93** | 5.55 | **5.80** |
| | OBI0 | 10.25 | 7.91 | 10.18 | 9.92 | 7.38 | 6.19 | 9.05 | 7.44 |
| | OBI1 | 10.30 | 7.84 | 10.14 | 9.64 | 7.41 | 6.12 | 9.17 | 7.54 |
| BHP | NPA | 2.16 | 9.28 | 3.61 | 9.83 | 4.57 | 10.53 | 3.23 | 8.94 |
| | IC0 | **1.98** | 8.80 | **3.23** | 9.02 | **4.18** | 8.25 | **3.13** | 6.79 |
| | IC1 | 3.66 | 5.62 | 4.51 | 5.70 | 4.87 | 6.49 | 5.04 | 4.48 |
| | OBI0 | 5.52 | 5.17 | 6.56 | 5.46 | 6.20 | 5.73 | 6.27 | 4.42 |
| | OBI1 | 5.51 | **4.60** | 6.57 | **5.40** | 6.23 | **5.64** | 6.21 | **4.26** |
| BIL | NPA | 9.47 | 17.22 | 9.82 | 18.48 | 12.78 | 17.05 | 10.21 | 16.45 |
| | IC0 | **5.98** | 10.09 | **6.13** | 12.06 | **7.49** | 9.90 | **6.41** | 10.03 |
| | IC1 | 6.58 | **7.30** | 7.19 | **9.25** | 7.91 | **7.29** | 7.28 | **7.99** |
| | OBI0 | 9.28 | 8.34 | 10.66 | 11.28 | 12.11 | 9.93 | 10.88 | 9.95 |
| | OBI1 | 9.28 | 8.34 | 10.72 | 10.96 | 11.91 | 9.55 | 10.88 | 9.95 |
| CBA | NPA | 8.65 | 15.43 | 10.24 | 15.97 | 10.24 | 14.28 | 13.60 | 16.57 |
| | IC0 | **4.32** | 8.54 | **4.81** | 8.59 | **4.88** | 7.56 | **5.96** | 8.51 |
| | IC1 | 5.53 | **6.60** | 6.17 | **6.89** | 5.55 | **5.49** | 6.54 | **6.76** |
| | OBI0 | 10.04 | 9.29 | 11.23 | 9.56 | 9.63 | 7.57 | 11.70 | 9.67 |
| | OBI1 | 10.07 | 9.14 | 11.28 | 9.51 | 9.63 | 7.57 | 11.77 | 9.60 |

**Table 3.3 Out-of-sample test on DF strategies (CML – NCPDP)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| CML | NPA | 25.23 | 28.01 | 16.85 | 22.37 | 18.50 | 20.30 | 13.00 | 19.91 |
| | IC0 | **13.29** | **12.11** | **10.92** | **11.52** | **9.68** | 12.23 | **8.42** | 12.45 |
| | IC1 | **13.29** | **12.11** | **10.92** | **11.52** | 11.51 | **10.17** | 9.93 | **9.87** |
| | OBI0 | 20.84 | 16.25 | 16.48 | 13.76 | 17.80 | 12.41 | 14.15 | 10.98 |
| | OBI1 | 20.81 | 15.99 | 16.61 | 13.75 | 17.81 | 12.30 | 14.21 | 10.83 |
| FGL | NPA | **7.55** | 14.13 | 10.01 | 12.56 | 10.37 | 12.24 | 10.77 | 13.76 |
| | IC0 | 7.80 | 10.37 | **9.74** | 11.71 | **9.73** | 9.06 | **9.74** | 9.90 |
| | IC1 | 9.89 | 6.22 | 11.09 | 5.82 | 11.11 | **6.37** | 10.55 | 7.20 |
| | OBI0 | 9.29 | 8.81 | 10.83 | 9.81 | 11.36 | 9.68 | 11.95 | 10.48 |
| | OBI1 | 10.55 | **5.83** | 11.72 | **5.68** | 11.79 | 6.44 | 11.74 | **6.28** |
| NAB | NPA | 9.87 | 15.93 | 9.61 | 16.81 | 9.15 | 15.51 | 9.98 | 14.83 |
| | IC0 | **4.71** | 9.23 | **4.16** | 8.39 | **4.76** | 8.71 | **5.06** | 8.06 |
| | IC1 | 5.87 | **6.92** | 5.33 | **6.42** | 5.56 | **6.71** | 6.03 | **6.23** |
| | OBI0 | 10.48 | 8.16 | 10.29 | 9.19 | 9.46 | 8.90 | 10.12 | 8.11 |
| | OBI1 | 10.48 | 8.16 | 10.29 | 9.19 | 9.33 | 8.53 | 10.19 | 8.12 |
| NCP | NPA | 6.17 | 15.92 | 6.58 | 15.19 | 7.49 | 16.30 | 6.81 | 15.44 |
| | IC0 | **4.71** | 11.63 | **4.40** | 10.06 | **4.95** | 10.54 | **4.65** | 10.92 |
| | IC1 | 6.08 | 9.08 | 5.54 | 7.64 | 6.76 | 8.18 | 6.25 | 8.43 |
| | OBI0 | 9.36 | 9.18 | 9.11 | 7.58 | 10.89 | 8.13 | 10.01 | 7.90 |
| | OBI1 | 9.37 | **8.55** | 9.16 | **7.42** | 10.96 | **7.86** | 10.10 | **7.79** |
| NCPDP | NPA | 15.68 | 35.14 | 14.91 | 31.03 | 35.18 | 105.58 | 19.11 | 33.98 |
| | IC0 | **9.04** | 19.18 | **7.64** | 15.10 | **24.58** | 96.03 | **9.73** | 18.36 |
| | IC1 | 10.51 | **14.03** | 8.47 | **11.29** | 25.41 | **91.80** | 11.02 | **13.51** |
| | OBI0 | 17.30 | 17.51 | 14.15 | 13.72 | 33.16 | 92.48 | 19.66 | 20.42 |
| | OBI1 | 17.30 | 17.51 | 14.24 | 13.74 | 33.16 | 92.48 | 19.67 | 20.18 |

**Table 3.4 Out-of-sample test on DF strategies (PBL – WBC)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| PBL | NPA | 47.62 | 47.54 | 49.31 | 38.81 | 53.45 | 54.08 | 58.63 | 44.65 |
| | IC0 | **15.79** | **17.70** | **21.06** | **18.65** | **16.19** | **14.84** | **18.58** | **15.97** |
| | IC1 | **15.79** | **17.70** | **21.06** | **18.65** | **16.19** | **14.84** | **18.58** | **15.97** |
| | OBI0 | 27.69 | 26.37 | 33.31 | 24.95 | 29.69 | 30.09 | 31.55 | 24.42 |
| | OBI1 | 27.19 | 25.30 | 33.31 | 24.95 | 29.69 | 30.09 | 31.49 | 23.94 |
| RIO | NPA | 35.97 | 41.40 | 25.26 | 32.64 | 26.37 | 34.55 | 26.70 | 28.19 |
| | IC0 | **11.92** | **12.50** | **8.47** | **11.00** | **9.21** | **10.18** | **9.16** | **9.17** |
| | IC1 | **11.92** | **12.50** | **8.47** | **11.00** | **9.21** | **10.18** | **9.16** | **9.17** |
| | OBI0 | 24.06 | 23.35 | 16.79 | 20.82 | 17.16 | 20.60 | 17.74 | 17.18 |
| | OBI1 | 24.06 | 23.35 | 16.79 | 20.82 | 17.36 | 20.76 | 17.67 | 16.94 |
| SGB | NPA | 65.52 | 37.16 | 61.78 | 42.12 | 66.42 | 55.71 | 53.05 | 36.01 |
| | IC0 | **20.11** | **15.48** | **22.00** | **20.22** | **29.01** | **33.33** | **18.19** | **14.71** |
| | IC1 | **20.11** | **15.48** | **22.00** | **20.22** | **29.01** | **33.33** | **18.19** | **14.71** |
| | OBI0 | 34.88 | 24.74 | 36.79 | 30.08 | 42.74 | 41.00 | 31.61 | 23.17 |
| | OBI1 | 34.88 | 24.74 | 36.79 | 30.08 | 42.91 | 40.93 | 31.61 | 23.17 |
| TLS | NPA | **4.21** | 11.44 | **4.68** | 11.98 | **6.20** | 11.14 | **6.50** | 11.91 |
| | IC0 | 4.29 | 11.38 | **4.68** | 11.98 | 6.21 | 11.15 | 6.51 | 11.88 |
| | IC1 | 8.82 | **4.32** | 8.53 | **4.75** | 9.66 | **4.38** | 9.71 | **4.37** |
| | OBI0 | 5.92 | 8.96 | 6.60 | 8.85 | 7.71 | 8.37 | 7.24 | 9.36 |
| | OBI1 | 8.71 | 4.64 | 8.44 | 5.27 | 9.51 | 4.77 | 9.62 | 4.95 |
| WBC | NPA | 13.61 | 20.75 | 9.31 | 18.22 | 8.12 | 17.80 | 7.73 | 13.51 |
| | IC0 | **7.00** | 12.09 | **5.21** | 9.47 | **5.49** | 13.39 | **4.41** | 7.84 |
| | IC1 | 8.01 | **9.25** | 6.18 | **7.42** | 6.33 | **12.11** | 5.24 | **6.09** |
| | OBI0 | 12.71 | 11.27 | 10.19 | 10.17 | 9.44 | 12.84 | 8.47 | 7.88 |
| | OBI1 | 12.79 | 11.26 | 10.19 | 10.17 | 9.47 | 12.70 | 8.61 | 7.92 |

**Table 3.5 Out-of-sample test on DF strategies (WES – WPL)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| WES | NPA | 84.40 | 47.47 | 90.16 | 40.89 | 111.83 | 75.44 | 93.39 | 50.09 |
| | IC0 | **33.88** | **26.22** | **33.47** | **25.54** | **44.90** | **64.44** | **28.51** | **24.23** |
| | IC1 | **33.88** | **26.22** | **33.47** | **25.54** | **44.90** | **64.44** | **28.51** | **24.23** |
| | OBI0 | 58.01 | 36.76 | 57.36 | 34.48 | 75.15 | 72.00 | 56.65 | 37.09 |
| | OBI1 | 58.01 | 36.76 | 57.36 | 34.48 | 76.50 | 70.39 | 56.70 | 35.71 |
| WFT | NPA | 17.07 | 13.69 | 16.56 | 12.43 | 15.83 | 11.36 | 15.89 | 13.01 |
| | IC0 | 16.39 | 9.85 | **15.86** | 7.50 | **15.69** | 7.08 | **15.40** | 9.94 |
| | IC1 | 16.45 | **6.28** | 16.12 | 6.57 | 15.95 | 6.30 | 16.13 | **5.66** |
| | OBI0 | **16.33** | 10.06 | 16.13 | 10.24 | 16.09 | 8.23 | 16.34 | 11.94 |
| | OBI1 | 16.73 | 6.45 | 16.53 | **6.36** | 16.17 | **6.17** | 16.34 | 7.06 |
| WMC | NPA | 18.35 | 25.69 | 14.15 | 21.51 | 12.16 | 16.90 | 14.26 | 19.06 |
| | IC0 | **9.66** | **10.53** | **8.13** | 12.66 | **6.98** | 9.22 | **8.35** | 11.15 |
| | IC1 | **9.66** | **10.53** | 8.45 | **9.52** | 7.32 | **7.03** | 8.42 | **8.17** |
| | OBI0 | 14.80 | 13.62 | 12.59 | 11.06 | 11.38 | 9.94 | 12.47 | 10.62 |
| | OBI1 | 14.80 | 13.62 | 12.67 | 11.03 | 11.37 | 9.86 | 12.47 | 10.62 |
| WOW | NPA | 14.99 | 20.94 | 18.52 | 26.72 | 18.66 | 21.33 | 16.82 | 24.28 |
| | IC0 | **8.55** | **9.16** | **9.62** | 15.99 | **10.06** | 12.36 | **7.85** | 12.87 |
| | IC1 | **8.55** | **9.16** | 9.78 | **12.58** | 10.17 | **9.90** | 8.13 | **9.27** |
| | OBI0 | 13.33 | 11.09 | 15.50 | 15.23 | 14.96 | 12.36 | 13.83 | 11.83 |
| | OBI1 | 13.41 | 10.95 | 15.62 | 15.20 | 14.96 | 12.36 | 13.89 | 11.65 |
| WPL | NPA | 20.24 | 22.17 | 22.17 | 22.46 | 33.40 | 37.53 | 41.03 | 39.48 |
| | IC0 | **9.94** | **10.11** | **10.72** | 13.28 | 12.70 | 17.16 | **12.26** | **11.96** |
| | IC1 | **9.94** | **10.11** | 10.99 | **10.59** | **11.80** | **12.25** | **12.26** | **11.96** |
| | OBI0 | 16.15 | 12.33 | 17.25 | 13.11 | 23.02 | 20.99 | 23.86 | 22.00 |
| | OBI1 | 16.39 | 12.33 | 17.16 | 12.71 | 23.39 | 21.13 | 23.96 | 21.54 |

Let $c_{\mu,NPA}$ and $c_{\mu,DF}$ denote the average shortfall brought respectively by the NPA strategy and the optimized DF strategy in out-of-sample test. The horizontal and vertical coordinates in Figure 3.6 respectively represent $ln(v_{10Min})$ and the difference between $c_{\mu,NPA}$ and $c_{\mu,DF}$ ($c_{\mu,NPA}-c_{\mu,DF}$). The four charts respectively represent the out-of-sample results on IC0, IC1, OBI0 and OBI1 and the points in the charts are the out-of-sample results on 80 datasets. Figure 3.6 shows that the DF strategy more significantly improves the NPA strategy on the less liquid stock datasets (i.e. lower $v_{10Min}$). A reasonable explanation for this conclusion is that the DF strategy is designed to reduce nonexecution risk, which is caused more easily on the less liquid stock datasets. Figure 3.6 also shows that IC0 and IC1 outperforms OBI0 and OBI1. This indicates that inventory control is a more important factor, which affects the reduction in trade execution cost, than order book imbalance.

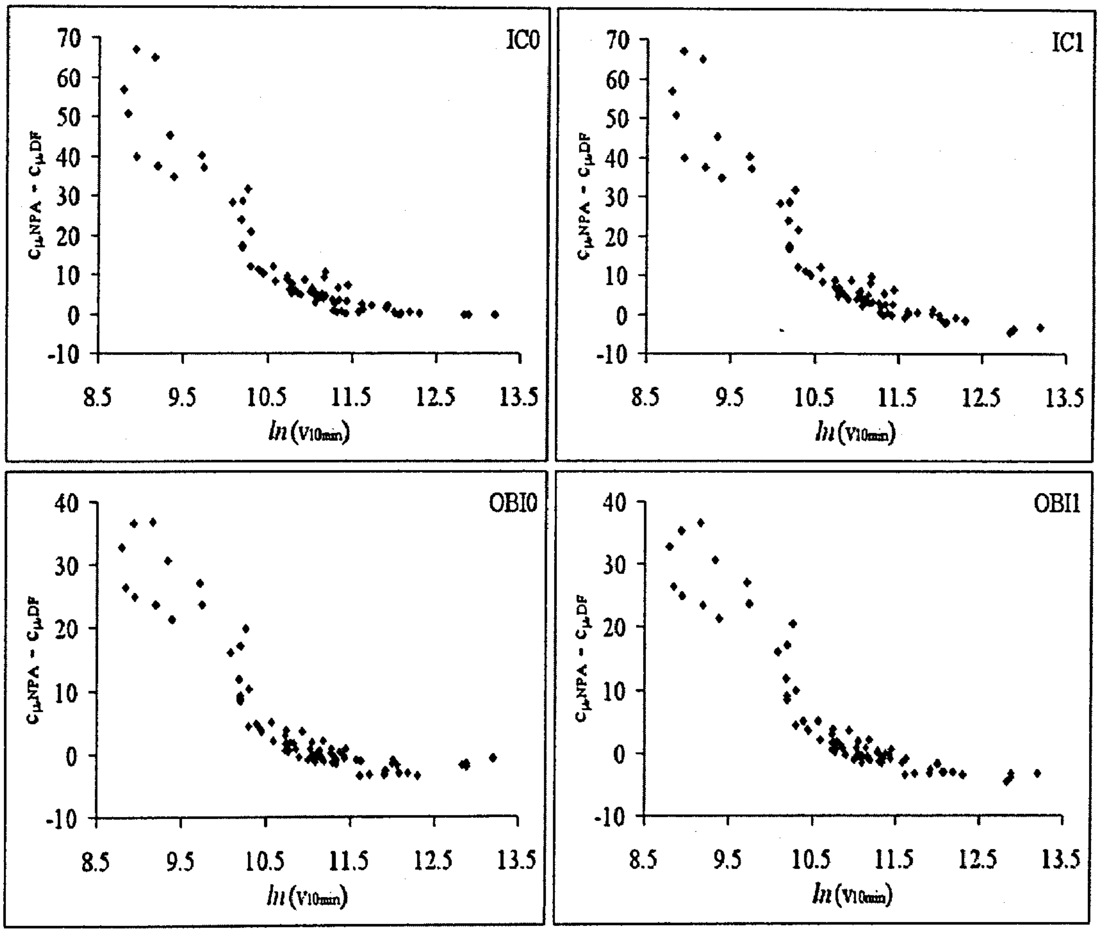


**Figure 3.6 Comparison of IC0/IC1/OBI0/OBI1 to NPA**

# 3.3 Summary

This chapter has proposed the DF strategy, which applies the idea of diversification across time to trade execution. The DF strategy incorporates a series of small market orders into the limit order strategy and dynamically adjusts the market order volume to hedge nonexecution risk and adverse price movement during the execution period. Dynamic volume adjustment in the DF strategy depends on two real-time state variables: inventory and order book imbalance. The sigmoid function is adopted as the quantitative model. The in-sample and out-of-sample test results on 80 datasets show that the DF strategy, which reflects the idea of diversification across time, outperforms the limit order strategy for trade execution. Moreover, the DF strategy is more effective for improving the limit order strategy on the less liquid stock datasets. In addition, the empirical results show that inventory control is a more important factor for trade execution than order book imbalance. The DF strategy can be further improved from several aspects: 1) combining dynamic price adjustment with dynamic volume adjustment; and 2) dynamically adjusting the limit order volume in trade execution.

# Chapter 4

# Diversification Based on AdaBoost

Diversification is an important philosophy for risk management in various practices. Its rationale has been illustrated by the classical portfolio theory for investment and it was recently applied in the trade execution field. The idea of diversification is also reflected in the AdaBoost technique, which is discussed mostly in the supervised learning field. This chapter extends the basic ideas in the AdaBoost technique to propose the DAB technique, which reflects the idea of diversification across strategies. The DAB technique acquires strategies from a candidate strategy set and determines their weights. Then the resources are allocated to each acquired strategy in proportion with its weight and all acquired strategies are executed in parallel with their allocated resources. This chapter theoretically analyzes several advantages of the DAB technique over the existing diversification techniques: 1) it allows the candidate strategy set to contain finite or infinite strategies; 2) as the learning steps increase, it lowers risk that can be measured by different standards (e.g. the k-LPM of rewards and the minimum of rewards) and meanwhile limits the decrease in average reward. This chapter is composed of three sections. The 1st section describes the DAB technique. The 2nd section theoretically analyzes its advantages. The 3rd section summarizes the DAB technique.

# 4.1 DAB Technique

Diversification is an important philosophy for risk management in various practices. Chapter 3 verifies that the idea of diversification across time can improve the existing trade execution strategies. This chapter discusses diversification across strategies, which can also be regarded as diversification across space in contrast with time. An investment example is that investors allocate their money across various financial products such as stocks and bonds to hedge investment risk. The classical portfolio theory has illustrated the rationale of diversification across various investment products (Markowitz 1952, Sharpe 1964). Note that investing on a product during a period of time can be regarded as a strategy since it satisfies the definition of strategies in this thesis. More recently, the idea of diversification across strategies was applied in the trade execution field to reduce transaction cost and risk (Almgren and Chriss 1997, Kissell and Glantz 2003).

In order to implement the idea of diversification across strategies, analytical or numeric techniques have been proposed to acquire strategies and their weights from a candidate strategy set (Markowitz 1956, Sharpe 1964, Hogan and Warren 1974, Kissell and Glantz 2003). In the existing techniques for diversification, the candidate strategy set normally contains finite strategies. For example, 10 stocks and their weights are determined for diversification from a candidate set of 50 stocks. But in practice, the candidate strategy set can contain infinite strategies. For example, a DF strategy set contains infinite DF strategies when its parameter $\lambda$ takes values in a range of real number. In addition, the existing techniques for diversification were designed to lower risk that is measured by some specific standard such as either variance or k-LPM of rewards. It will be better for risk management if the technique for diversification across strategies can lower risk that can be measured by different standards such as the variance of rewards, the k-LPM of rewards and the minimum of rewards.

The philosophy of diversification is also reflected in the AdaBoost technique, which is discussed mostly in the supervised learning field, e.g. classification

(Schapire et al. 1998), regression (Duffy and Helmbold 2002) and conditional density estimation (Stone et al. 2003). In the AdaBoost technique, it is emphasized that the ensemble of models (e.g. classifiers) can outperform each individual model. In the ensemble of models, the decision is made by the models altogether rather than each individual model does. Here, the AdaBoost technique is explained in the subject of two-class classification. The research about this subject is to build a classifier (or an ensemble of classifiers) to map an example with input attributes X to its label y (1 or −1) based on a training example set. The classifier is regarded as correctly identifying the example if its output is consistent with the example label. The classifier is expected to achieve high identification accuracy on both training examples with labels and unseen examples from an independently identical distribution. Let $\Phi$ denoted a training example set and it is represented as $\Phi = \{\phi | \phi = \langle X, y \rangle\}$, where each attribute of X can be a real number or a symbolic value. Table 4.1 shows the pseudo codes to describe the AdaBoost technique for two-class classification.

**Table 4.1 The AdaBoost technique for two-class classification**

---

LEARNING INPUT:

1. A training example set $\Phi$;
2. A probability distribution $P_0$ on $\Phi$;
3. A classifier set $\Omega$;
4. A base learner BL;
5. Maximum number of learning steps $t_M$.

LEARNING INITIALIZATION:

1. $P_1 \leftarrow P_0$;
2. $T \leftarrow \varnothing$;
3. $C \leftarrow \varnothing$;
4. $W \leftarrow \varnothing$.

LEARNING PROCESS: for $t = 1 \ldots t_M$ do

1. Call BL to acquire a classifier $c_t$ from $\Omega$;
2. Calculate the edge $e_t \leftarrow \sum_{\phi\in\Phi} p_{t,\phi} \times y(\phi) \times c_t(X(\phi))$;
3. If $e_t \leq 0$ or $e_t = 1$, then break;
4. Calculate the weight $w_t \leftarrow 0.5 \times (ln(1+e_t) - ln(1-e_t))$;
5. Update the probability distribution

$$P_{t+1} = \left\{ P_{t+1,\phi} \middle| p_{t+1,\phi} = \frac{p_{t,\phi} \times exp(-w_t \times y(\phi) \times c_t(X(\phi)))}{p_t}, \phi \in \Phi \right\},$$

where $p_t = \sum_{\phi\in\Phi} p_{t,\phi} \times exp(-w_t \times y(\phi) \times c_t(X(\phi)))$;

6. Let $T \leftarrow T \cup \{t\}$, $C \leftarrow C \cup \{c_t\}$ and $W \leftarrow W \cup \{w_t\}$;

LEARNING OUTPUT:

1. An index set T;
2. A classifier set $C = \{c_t \mid t \in T\}$;
3. A weight set $\overline{W} = \left\{ \overline{w}_t \middle| \overline{w}_t = \frac{w_t}{w}, w = \sum_{t\in T} w_t, w_t \in W, t \in T \right\}$;
4. A confidence $Cnf(\phi) = y(\phi) \times \sum_{t\in T} \overline{w}_t \times c_t(X(\phi))$;
5. A classifier $c(X(\phi)) = sign\left(\sum_{t\in T} \overline{w}_t \times c_t(X(\phi))\right)$,

   where $sign(\bullet) = \begin{cases} 1 & \bullet > 0 \\ -1 & \bullet < 0 \end{cases}$.

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The AdaBoost technique comprises four parts: learning input, learning initialization, learning process and learning output (see Table 4.1). The AdaBoost technique initially assigns a probability for each training example $\phi$ in the training example set $\Phi$. The base learner BL acquires a classifier $c_t$ from the candidate classifier set $\Omega$, where the acquired classifier $c_t$ achieves higher classification accuracy on the training example set $\Phi$ than other classifiers in the candidate classifier set $\Omega$ do. Here, the classification accuracy is calculated based on the probability distribution $P_t$ (i.e. the edge in Table 4.1). It is more possible for the acquired classifier $c_t$ to correctly classify the training examples that are assigned with higher probability. The AdaBoost technique then updates the probability distribution $P_t$ based on the acquired classifier $c_t$'s edge $e_t$, its output $c_t(X(\phi))$ on each training example $\phi$ and the training example $\phi$'s label $y(\phi)$. In updating the probability distribution $P_t$, higher probability is assigned for the examples that are misidentified by the acquired classifier $c_t$. This update mechanism leads the base learner BL at the next learning step to acquire a new classifier $c_{t+1}$, for which it is more possible to correctly classify the examples that are misidentified by the acquired classifier $c_t$. At the end of learning, the AdaBoost technique outputs the acquired classifier set S and the weight set $\overline{W}$ . The classifier ensemble is represented as the weighted average of the acquired classifier outputs $c_t(X(\phi))$. The AdaBoost technique, as a meta-type method, provides a flexible way to boost supervised learning models.

In the AdaBoost technique, the candidate model (e.g. classifier) set can contain finite or infinite models. Moreover, it has been proven for two-class classification that the classification error rate of the ensemble of classifiers acquired by the AdaBoost technique decreases exponentially fast as long as in the learning process, the base learner BL can consistently acquire the classifier that achieves slightly higher classification accuracy on the training example set than randomly guessing. When the confidence $Cnf(\phi)$ is greater (or less) than 0, it means that the classifier $c(X(\phi))$ correctly (or incorrectly) classifies the example $\phi$. Greater positive (or negative) $Cnf(\phi)$ represents higher confidence, with which the classifier $c(X(\phi))$ correctly (or incorrectly) classifies the example $\phi$. So the confidence $Cnf(\phi)$ is expected to be as great as possible. The minimum confidence $\min_{\phi\in\Phi}(Cnf(\phi))$ is named the classifier $c(X(\phi))$'s margin, which can be regarded as one of a classifier's risk measures. A theoretical conclusion about the AdaBoost technique is that the margin increases as the learning steps increase. While these advantages of the

AdaBoost technique are just what to be expected for diversification across strategies, the AdaBoost technique cannot be applied directly to strategy diversification because it is discussed mostly in the supervised learning field while the strategy defined in this thesis essentially differs from a supervised learning model.

In supervised learning, each training example is marked with a label for identification. The AdaBoost technique for two-class classification updates the probability distribution at each learning step based on the acquired classifier's edge, its output and the training example label. But in training a strategy, each training example cannot be marked with a label before a candidate strategy set is given. An investment example is that investors make decision about whether to buy or sell a stock at each time point over a period of time. Following the methodology of supervised learning, a training example set needs to be given for determining a strategy and each training example comprises a multi-attribute input variable and a label. Suppose that the multi-attribute input variable of a training example be information at each time point such as real-time stock price and trading volume and its label be the decision on buying or selling. In this case, an exact label actually does not exist for each training example because all decisions during the period of time interact with each other and each individual decision depends on its previous and next decisions. Suppose that the multi-attribute input variable of a training example be all information in the whole investment process and its label be the investment return. In this case, the investment return actually does not exist before a strategy is given since its existence depends on a specific strategy. The research of reinforcement learning has given a deep insight into this kind of difference (Sutton and Barto 1998).

However, this thesis points out that the key idea of adaptively updating the probability distribution in the AdaBoost technique is not just applicable to supervised learning if it is illustrated in a general manner as below. Higher probability is assigned to the "worse" training examples according to the acquired model's "performance" at each learning step. The word "worse" means that the acquired model does not identify the training example well. Assigning higher probability to the "worse" examples aims to lead the base learner to acquire a new model, which is more possible to identify the "worse" examples well at the next learning step. This update mechanism can acquire various models, which identify different training examples well. In the AdaBoost technique, the "worse" training examples are referred to as those misclassified by the acquired classifier. The DAB technique extends the meaning of "worse" from misidentification in supervised learning to lower reward in training a strategy. Thus, the DAB technique assigns higher probability to the training examples, on which the acquired strategy achieves lower reward. This leads the base learner to acquire a new strategy, which is more possible to achieve high reward on the "worse" training examples at the next learning step. Diversification across strategies is then represented as allocating resources to the acquired strategies in proportion with their weights.

To discuss the DAB technique, several symbols are introduced as below. Let $\phi$ denote a time series, which is also called an example. The information at each time point of $\phi$ is represented as a multi-attribute variable. A strategy s makes decision based on each multi-dimensional vector in $\phi$. Let $\rho$ denote resources and w ($0 < w \leq 1$) denote weight. The strategy s achieves the reward $r(s,w,\phi)$ when it is executed on $\phi$ with the weight resources ($w \times \rho$). For example, when the strategy (s) makes profit of \$10,000 by utilizing 10% (w) of a total capital of \$1 million ($\rho$) over a period of time ($\phi$), the reward $r(s,w,\phi)$ equals to 1% for the total capital ($\rho$). Let $\Omega$ denote a candidate strategy set, which contains finite or infinite strategies. This thesis assumes that given a candidate strategy set $\Omega$ and an example $\phi$, the reward $r(s,1,\phi)$ has a lower bound $r_{INF}(\phi)$ and a upper bound $r_{SUP}(\phi)$ as below:

$$r_{INF}(\phi) = INF\{r(s,1,\phi) \mid s \in \Omega\}$$

$$r_{SUP}(\phi) = SUP\{r(s,1,\phi) \mid s \in \Omega\}$$

When the candidate strategy set $\Omega$ contains infinite strategies, the lower bound $r_{INF}(\phi)$ and the upper bound $r_{SUP}(\phi)$ can be estimated by randomly sampling the strategies from $\Omega$. In addition to the above assumption about bounds, there is no other limitation for the reward in the DAB technique. Given an example set $\Phi$ that contains finite examples $\phi$, the normalized reward $\overline{r}(s_t,1,\phi)$ is calculated as below:

$$\bar{r}(s,1,\phi) = \frac{r(s,w,\phi) - w \times r_{INF}(\phi)}{\Delta},$$

where $SUP\{r_{SUP}(\phi) \mid \phi \in \Phi\} - INF\{r_{INF}(\phi) \mid \phi \in \Phi\}$.

The DAB technique inherits the basic idea of the AdaBoost technique, which adaptively updates the probability distribution on the example set at each learning step. The AdaBoost technique acquires a model (e.g. classifier) at each learning step according to the probability distribution. Higher probability is set for the examples misidentified by the acquired model when the AdaBoost technique updates the probability distribution. The AdaBoost technique can make judgment on whether an example is misidentified since each example in the training set is marked with a label in supervised learning. But the label of each example in the training set is unknown in training a strategy. So a normalized target reward $\bar{r}_\theta$ ($0 \le \bar{r}_\theta \le 1$) is set in the DAB technique as the benchmark to evaluate the reward and update the probability distribution at each learning step. Given the candidate strategy set $\Omega$ and the example set $\Phi$, $\bar{r}_\theta = 1$ means that the reward $r(s,1,\phi)$ is expected to reach the upper bound $SUP\{r_{SUP}(\phi) \mid \phi \in \Phi\}$. Table 4.2 shows the pseudo codes to describe the DAB technique for strategy diversification.

**Table 4.2 The DAB technique for strategy diversification**

LEARNING INPUT OF THE DAB TECHNIQUE:

1. Resources $\rho$;
2. An example set $\Phi$;
3. A probability distribution $P_0$ on $\Phi$;
4. A candidate strategy set $\Omega$;
5. A normalized target reward $\bar{r}_\theta$;
6. A base learner BL;
7. Maximum number of learning steps $t_M$.

LEARNING INITIALIZATION OF THE DAB TECHNIQUE:

1. $P_1 \leftarrow P_0$;

2. $T \leftarrow \varnothing$;
3. $S \leftarrow \varnothing$;
4. $W \leftarrow \varnothing$.

LEARNING PROCESS OF THE DAB TECHNIQUE: for $t = 1 \ldots t_M$ do

1. Call BL to acquire a strategy $s_t$ from $\Omega$ ($s_t$ achieves the reward $r(s_t,1,\phi)$ and the normalized reward $\overline{r}(s_t,1,\phi)$ with the resources $\rho$ on the example $\phi$.);
2. Calculate the edge $e_t \leftarrow \sum_{\phi\in\Phi} p_{t,\phi} \times \overline{r}(s_t,1,\phi)$;
3. If $e_t < \overline{r}_\theta$, then break;
4. Calculate the weight $w_t \leftarrow ln\left(\frac{1}{\overline{r}_\theta}-1\right)-ln\left(\frac{1}{e_t}-1\right)$;
5. Update the probability distribution
$$P_{t+1} = \left\{ p_{t+1,\phi} \,\middle|\, p_{t+1,\phi} = \frac{p_{t,\phi} \times exp\left(-w_t \times \overline{r}(s_t,1,\phi)\right)}{p_t}, \phi \in \Phi \right\},$$
where $p_t = \sum_{\phi\in\Phi} p_{t,\phi} \times exp\left(-w_t \times \overline{r}(s_t,1,\phi)\right)$;
6. Let $T \leftarrow T \cup \{t\}$, $S \leftarrow S \cup \{s_t\}$ and $W \leftarrow W \cup \{w_t\}$.

LEARNING OUTPUT OF THE DAB TECHNIQUE:

1. An index set T;
2. A set of strategies $S = \{s_t \mid t \in T\}$;
3. A weight set $\overline{W} = \left\{ \overline{w}_t \,\middle|\, \overline{w}_t = \frac{w_t}{w}, w = \sum_{t\in T} w_t, w_t \in W, t \in T \right\}$;
4. A normalized reward $\overline{r}(S,\overline{W},\phi) = \sum_{t\in T} \overline{w}_t \times \overline{r}(s_t,1,\phi)$.

EXECUTION:

1. Allocates to the resources $\rho$ to each strategy $s_t$ ($s_t \in S$) in proportion with its weight $\overline{w}_t$ ($\overline{w}_t \in \overline{W}$);
2. Execute all strategies $s_t$ ($s_t \in S$) in parallel with their allocated resources $\overline{w}_t \times \rho$;

3. Achieve a reward $r_E(S,\overline{W},\phi) = \sum_{t\in T} r(s_t,\overline{w}_t,\phi)$ and a normalized reward $\overline{r}_E(S,\overline{W},\phi) = \sum_{t\in T} \overline{r}(s_t,\overline{w}_t,\phi)$.

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The DAB technique comprises two parts of learning and execution (see Table 4.2). The DAB technique initially assigns a probability for each example in the example set. Given the probability distribution, the edge in Table 4.2 represents the average reward, which is achieved by a strategy on the training example set. The base learner acquires a strategy, which achieves the edge (i.e. the average reward) as great as possible, from the candidate strategy set. This means that the acquired strategy achieves higher reward on the examples that are assigned with higher probability. The DAB technique then updates the probability distribution on the example set based on the normalized target reward, the reward on each example and the edge. In updating the probability distribution, higher probability is assigned for the examples, on which the acquired strategy achieves lower reward. At the end of learning, the DAB technique outputs the normalized reward $\overline{r}(S,\overline{W},\phi)$, which is the weighted average of rewards that are achieved by the acquired strategy in S. Before parallel execution, the DAB technique allocates the resources $\rho$ to each acquired strategy in S in proportion with its weight in $\overline{W}$. All strategies in S are then executed with their allocated resources in parallel in the execution part. At the end of parallel execution, the reward $r_E(S,\overline{W},\phi)$ and the normalized reward $\overline{r}_E(S,\overline{W},\phi)$ are achieved on the example $\phi$ ($\phi\in\Phi$). The DAB technique, as a meta-type method, is a flexible way for implementing diversification across strategies.

A technique named "Hedge" has been proposed to solve the dynamic allocation problem (Freund and Schapire 1997). The DAB technique has an advantage over the Hedge technique. That is, it can be proven that the k-LPM $r_{\sigma(k)}$ of rewards (one of downside risk measures) drops exponentially fast as the learning steps increase. The next section will theoretically prove the DAB technique's statistical properties.

## 4.2 Theoretical Analysis

Given the resources $\rho$, the example set $\Phi$, the probability distribution $P_0$, the candidate strategy set $\Omega$, the base learner BL, the normalized target reward $\overline{r}_\theta$ and the maximum number $t_M$ of learning steps, the DAB technique inherits and extends the key idea of the AdaBoost technique to acquire the strategy set S, determine the weight set $\overline{W}$ in the learning part and output the normalized reward $\overline{r}(S,\overline{W},\phi)$ on the example $\phi$ ($\phi \in \Phi$) at the end of learning. The DAB technique aims to lower the candidate strategies' risk, which can be measured by different standards, and limit the decrease in reward. From the perspective of candidate strategies' efficient frontier, the DAB technique moves it toward a favorable direction. This section will theoretically analyze several statistical properties about the DAB technique and then explain its reasonability and effectiveness.

When the DAB technique updates the probability distribution at each learning step, higher probability is assigned for the examples, on which the acquired strategy $s_t$ ($s_t \in S$) achieves lower reward. Roughly speaking, this makes it harder for the base learner BL to acquire a strategy with a larger edge $e_t$ at the next learning step. The similar property also exists in the AdaBoost technique (Freund and Schapire 1997). Here, Hypothesis 4.1 is given according to the above property and it is strongly supported by the experiments in the next chapter.

**Hypothesis 4.1 (edge $e_t$)**: The edge $e_t$, which is achieved by the acquired strategy $s_t$ at each learning step, decreases as the learning steps increase.

**Theorem 4.1 ($\overline{r}(S,\overline{W},\phi)$)**:

$$\overline{r}(S,\overline{W},\phi) \geq \frac{ln(p_{0,\phi})}{w} + \frac{\overline{r}_\theta}{w} \times \sum_{t\in T}(1 - exp(-w_t)).$$

**Proof**: $p_{0,\phi} \times \prod_{t\in T} exp(-w_t \times \overline{r}(s_t,1,\phi))$

$= p_{T+1,\phi} \times \prod_{t\in T} p_t$ ($p_{t,\phi}$'s definition)

$$\le \sum_{\phi\in\Phi} p_{T+1,\phi} \times \prod_{t\in T} p_t \qquad (p_{T+1,\phi} \text{ and } p_t \text{ are nonnegative})$$

$$= \prod_{t\in T} p_t \qquad (\sum_{\phi\in\Phi} p_{T+1,\phi} = 1)$$

$$= \prod_{t\in T}\left(\sum_{\phi\in\Phi} p_{t,\phi} \times exp\left(-w_t \times \bar{r}(s_t,1,\phi)\right)\right) \qquad (p_t\text{'s definition})$$

$$\le \prod_{t\in T}\left(\sum_{\phi\in\Phi} p_{t,\phi} \times \left(1-\left(1-exp\left(-w_t\right)\right)\times \bar{r}(s_t,1,\phi)\right)\right) \qquad (a^b \le 1-(1-a)\times b \text{ for } 0\le a,b\le 1)$$

$$= \prod_{t\in T}\left(1-\left(1-exp\left(-w_t\right)\right)\times e_t\right) \qquad (e_t\text{'s definition, } \sum_{\phi\in\Phi} p_{t,\phi} = 1)$$

$$ln\left(p_{0,\phi}\right) - \sum_{t\in T} w_t \times \bar{r}(s_t,1,\phi)$$

$$\le \sum_{t\in T} ln\left(1-\left(1-exp\left(-w_t\right)\right)\times e_t\right) \qquad (\text{according to the above inequality})$$

$$\le \sum_{t\in T}\left(-\left(1-exp\left(-w_t\right)\right)\times e_t\right) \qquad (ln(1\text{-}a) \le \text{-}a \text{ for } 0\le a<1)$$

Thus, $\bar{r}\left(S,\bar{W},\phi\right)$

$$= \frac{1}{w} \times \sum_{t\in T} w_t \times \bar{r}(s_t,1,\phi) \qquad (\bar{r}\left(S,\bar{W},\phi\right)\text{'s definition})$$

$$\ge \frac{ln\left(p_{0,\phi}\right)}{w} + \frac{1}{w} \times \sum_{t\in T}\left(1-exp\left(-w_t\right)\right)\times e_t \qquad (\text{divided by the negative number } -w)$$

$$\ge \frac{ln\left(p_{0,\phi}\right)}{w} + \frac{\bar{r}_\theta}{w} \times \sum_{t\in T}\left(1-exp\left(-w_t\right)\right) \qquad (e_t \ge \bar{r}_\theta) \ \blacksquare$$

**Corollary 4.1** ($\bar{r}\left(S,\bar{W},\phi\right)$): When an equal probability $1/|\Phi|$ is set for each example in $\Phi$ at the beginning of learning, i.e. $P_0$ is a uniform distribution,

$$\bar{r}\left(S,\bar{W},\phi\right) \ge -\frac{ln|\Phi|}{w} + \frac{\bar{r}_\theta}{w} \times \sum_{t\in T}\left(1-exp\left(-w_t\right)\right).$$

**Theorem 4.2 (average of $\bar{r}\left(S,\bar{W},\phi\right)$):**

$$\sum_{\phi\in\Phi} p_{0,\phi} \times \bar{r}\left(S,\bar{W},\phi\right) \ge \frac{\bar{r}_\theta}{w} \times \sum_{t\in T}\left(1-exp\left(-w_t\right)\right).$$

**Proof:** $\prod_{\phi\in\Phi} exp\left(p_{0,\phi} \times ln\left(\prod_{t\in T} exp\left(-w_t \times \bar{r}(s_t,1,\phi)\right)\right)\right)$

$\leq \sum_{\phi\in\Phi} p_{0,\phi} \times \prod_{t\in T} exp\left(-w_t \times \bar{r}(s_t,1,\phi)\right)$ (the finite form of Jensen's inequality, Cover and Thomas 1991)

$= \sum_{\phi\in\Phi} p_{T+1,\phi} \times \prod_{t\in T} p_t$ ($p_{T+1,\phi}$ and $p_t$ are nonnegative)

$= \prod_{t\in T} p_t$ ($\sum_{\phi\in\Phi} p_{T+1,\phi} = 1$)

$= \prod_{t\in T}\left(\sum_{\phi\in\Phi} p_{t,\phi} \times exp\left(-w_t \times \bar{r}(s_t,1,\phi)\right)\right)$ ($p_t$'s definition)

$\leq \prod_{t\in T}\left(\sum_{\phi\in\Phi} p_{t,\phi} \times \left(1-\left(1-exp(-w_t)\right) \times \bar{r}(s_t,1,\phi)\right)\right)$ ($a^b \leq 1-(1-a)\times b$ for $0\leq a,b\leq 1$)

$= \prod_{t\in T}\left(1-\left(1-exp(-w_t)\right)\times e_t\right)$ ($e_t$'s definition, $\sum_{\phi\in\Phi} p_{t,\phi} = 1$)

$-\sum_{\varphi\in\Phi} p_{0,\phi} \times \sum_{t\in T} w_t \times \bar{r}(s_t,1,\phi)$

$\leq \sum_{t\in T} ln\left(1-\left(1-exp(-w_t)\right)\times e_t\right)$ (according to the above inequality)

$\leq \sum_{t\in T}\left(-\left(1-exp(w_t)\right)\times e_t\right)$ ($ln(1\text{-}a) \leq \text{-}a$ for $0\leq a<1$)

Thus $\sum_{\phi\in\Phi} p_{0,\phi} \times r\left(S, \bar{W}, \phi\right)$

$= \sum_{\phi\in\Phi} p_{0,\phi} \times \frac{1}{W} \times \sum_{t\in T} w_t \times \bar{r}(s_t,1,\phi)$ ($\bar{r}\left(S, \bar{W}, \phi\right)$'s definition)

$\geq \frac{1}{W} \times \sum_{t\in T}\left(1-exp(-w_t)\right)\times e_t$ (divided by the negative number $-w$)

$\geq \frac{\bar{r}_\theta}{W} \times \sum_{t\in T}\left(1-exp(-w_t)\right)$ ($e_t \geq \bar{r}_\theta$) ■

**Proposition 4.1** Suppose $w_{t+1} \leq w_t$, where $w_t$ and $w_{t+1}$ are any two consecutive weights in W in Table 4.2. The following function is increasing as |T| increases:

$$f(|T|) = \frac{1}{\sum_{t\in T} w_t} \times \left( a + b \times \sum_{t\in T} \left(1 - exp(-w_t)\right) \right),$$

where a ($a \le 0$), b ($b \ge 0$) are two constants, T is the index set in Table 4.2.

**Proof**: Let $g(x) = \frac{1-exp(-x)}{x}$, where $x \ge 0$.

The function g(x)'s first-order derivative is $\frac{\partial}{\partial x} g(x) = \frac{x \times exp(-x) - (1 - exp(-x))}{x^2}$.

The first-order derivative $\frac{\partial}{\partial x} g(x) \le 0$ for any x ($x \ge 0$) because

$$\frac{\partial}{\partial x}\left(x \times exp(-x) - \left(1 - exp(-x)\right)\right) = -x \times exp(-x) \le 0 \text{ for any x } (x \ge 0),$$

$$\text{and } x \times exp(-x) - \left(1 - exp(-x)\right) = 0 \text{ for x } (x = 0).$$

The function g(x) ($x \ge 0$) is decreasing according to the fact that $\frac{\partial}{\partial x} g(x) \le 0$ for any x ($x \ge 0$).

Thus for any t ($t \in T$), $\frac{1-exp(-w_t)}{w_t} \le \frac{1-exp\left(-w_{|T|+1}\right)}{w_{|T|+1}}$ $\quad (w_{|T|+1} \le w_t)$

That is, $w_t \times \left(1 - exp\left(-w_{|T|+1}\right)\right) - w_{|T|+1} \times \left(1 - exp(-w_t)\right) \ge 0$.

Thus, $\left(1 - exp\left(-w_{|T|+1}\right)\right) \times \sum_{t\in T} w_t - w_{|T|+1} \times \sum_{t\in T}\left(1 - exp(-w_t)\right) \ge 0$.

That is, $\frac{\sum_{t\in T}\left(1-exp(-w_t)\right) + \left(1 - exp\left(-w_{|T|+1}\right)\right)}{\sum_{t\in T} w_t + w_{|T|+1}} - \frac{\sum_{t\in T}\left(1-exp(-w_t)\right)}{\sum_{t\in T} w_t} \ge 0$.

Moreover, $\frac{a}{\sum_{t\in T} w_t + w_{|T|+1}} - \frac{a}{\sum_{t\in T} w_t} \ge 0$ since $a \le 0$ and $w_t \ge 0$.

Thus, $f(|T|+1) - f(|T|)$

$$= \left( \frac{a}{\sum_{t\in T} w_t + w_{|T|+1}} - \frac{a}{\sum_{t\in T} w_t} \right)$$

$$+ \ b\times\left(\frac{\sum_{t\in T}\left(1-exp\left(-w_t\right)\right)+\left(1-exp\left(-w_{|T|+1}\right)\right)}{\sum_{t\in T} w_t + w_{|T|+1}} - \frac{\sum_{t\in T}\left(1-exp\left(-w_t\right)\right)}{\sum_{t\in T} w_t}\right)$$

$\geq 0$

So the function $f(|T|)$ is increasing as $|T|$ increases. ■

Theorem 4.1, Corollary 4.1 and Theorem 4.2 show that the normalized reward $\overline{r}\left(S,\overline{W},\phi\right)$ and its average $\sum_{\phi\in\Phi} p_{0,\phi}\times\overline{r}\left(S,\overline{W},\phi\right)$ are guaranteed by the lower bounds as the DAB technique acquires more strategies with weights in the learning part. Hypothesis 4.1 states that the edge $e_t$ decreases as the learning steps increase. The weight $w_t$ decreases as the learning steps increase according to the relationship between $e_t$ and $w_t$ (see Table 4.2). Proposition 4.1 shows that in Theorem 4.1, Corollary 4.1 and Theorem 4.2, the lower bounds of $\overline{r}\left(S,\overline{W},\phi\right)$ and its average $\sum_{\phi\in\Phi} p_{0,\phi}\times\overline{r}\left(S,\overline{W},\phi\right)$ increase as the learning steps increase. The lower bound involved in Corollary 4.1 is reasonable even for a large example set because it only logarithmically depends on the example set size $|\Phi|$.

The concept of Kullback-Leibler (KL) divergence (Kullback and Leibler 1951) will be used in the theorem and corollary about the k-LPM of $\overline{r}\left(S,\overline{W},\phi\right)$. The KL divergence and its property are introduced as below.

**Definition 4.1 (KL divergence):** Let $D_{KL}(p||q)$ denote the KL divergence of p and q ($0 \leq p, q \leq 1$). Thus

$$D_{KL}\left(p\,||\,q\right) = p\times ln\left(\frac{p}{q}\right)+\left(1-p\right)\times ln\left(\frac{1-p}{1-q}\right).$$

In the information theory, the KL divergence is represented as the difference between the cross entropy and the entropy.

**Proposition 4.2 (KL divergence):** $D_{KL}(p||q) \geq 0$, where $0 \leq p, q \leq 1$.

**Proof:** According to Definition 4.1,

$$D_{KL}\left(p\,||\,q\right) = p\times ln\left(\frac{p}{q}\right)+\left(1-p\right)\times ln\left(\frac{1-p}{1-q}\right) = -p\times ln\left(\frac{q}{p}\right)-\left(1-p\right)\times ln\left(\frac{1-q}{1-p}\right).$$

Since the logarithm function is concave,

$$D_{KL}(p \| q) \geq -ln\left(p \times \frac{q}{p} + (1-p) \times \frac{1-q}{1-p}\right) = 0. \qquad \blacksquare$$

**Theorem 4.3 (k-LPM of $\bar{r}(S, \bar{W}, \phi)$):**

$$\sum_{\phi \in \Phi} \left( max\left(0, \left(\bar{r}_\theta - \bar{r}(S, \bar{W}, \phi)\right)\right)\right)^k \leq \bar{r}_\theta^k \times exp\left(-\sum_{t \in T} D_{KL}(\bar{r}_\theta \| e_t)\right),$$

where $D_{KL}(\bar{r}_\theta \| e_t)$ is the KL divergence of $\bar{r}_\theta$ and $e_t$.

**Proof**: Let $\Phi 0 = \left\{\phi \middle| \phi \in \Phi \wedge \bar{r}(S, \bar{W}, \phi) \leq \bar{r}_\theta\right\}$.

For any $\phi$ ($\phi \in \Phi 0$), $\bar{r}(S, \bar{W}, \phi) = \sum_{t \in T} \bar{w}_t \times \bar{r}(s_t, 1, \phi) \leq \bar{r}_\theta$.

So $-\sum_{t \in T} w_t \times \bar{r}_\theta \leq -\sum_{t \in T} w_t \times \bar{r}(s_t, 1, \phi)$.

Thus $\sum_{\phi \in \Phi 0} p_{0,\phi} \times \prod_{t \in T} \exp(-w_t \times \bar{r}_\theta)$

$$\leq \sum_{\phi \in \Phi 0} p_{0,\phi} \times \prod_{t \in T} \exp(-w_t \times \bar{r}(s_t, 1, \phi))$$

$$\leq \sum_{\phi \in \Phi} p_{0,\phi} \times \prod_{t \in T} \exp(-w_t \times \bar{r}(s_t, 1, \phi)) \qquad (\Phi 0 \subseteq \Phi)$$

$$= \sum_{\phi \in \Phi} p_{T+1,\phi} \times \prod_{t \in T} p_t \qquad (p_{T+1,\phi} \text{ and } p_t \text{ are nonnegative})$$

$$= \prod_{t \in T} p_t \qquad (\sum_{\phi \in \Phi} p_{T+1,\phi} = 1)$$

$$= \prod_{t \in T} \left( \sum_{\phi \in \Phi} p_{t,\phi} \times exp(-w_t \times \bar{r}(s_t, 1, \phi)) \right) \qquad (p_t\text{'s definition})$$

$$\leq \prod_{t \in T} \left( \sum_{\phi \in \Phi} p_{t,\phi} \times \left(1 - (1 - exp(-w_t)) \times \bar{r}(s_t, 1, \phi)\right) \right) \qquad (a^b \leq 1-(1-a) \times b \text{ for } 0 \leq a,b \leq 1)$$

$$= \prod_{t \in T} \left(1 - (1 - exp(-w_t)) \times e_t\right) \qquad (e_t\text{'s definition}, \sum_{\phi \in \Phi} p_{t,\phi} = 1)$$

Thus $\sum_{\phi \in \Phi 0} p_{0,\phi} \leq \prod_{t \in T} f(w_t)$, where $f(w_t) = \frac{1-(1-exp(-w_t)) \times e_t}{exp(-w_t \times \bar{r}_\theta)}$. $\prod_{t \in T} f(w_t)$ can be minimized by minimizing each $f(w_t)$ since $f(w_t)$ is positive for each t ($t \in T$). So $w_t$

$= ln\left(\frac{1}{\overline{r}_\theta}-1\right)-ln\left(\frac{1}{e_t}-1\right)$ by setting the first-order derivative of $f(w_t)$ to be zero, i.e.

$\frac{\partial}{\partial w_t} f(w_t) = 0$. Plug $w_t = ln\left(\frac{1}{\overline{r}_\theta}-1\right)-ln\left(\frac{1}{e_t}-1\right)$ into $\prod_{t\in T} f(w_t)$.

Thus $\sum_{\phi\in\Phi}\left(max\left(0,\left(\overline{r}_\theta-\overline{r}\left(S,\overline{W},\phi\right)\right)\right)\right)^k$

$$= \sum_{\phi\in\Phi 0} p_{0,\phi}\times\left(\overline{r}_\theta-\overline{r}\left(S,\overline{W},\phi\right)\right)^k$$

$$\le \overline{r}_\theta^{\,k}\times\sum_{\phi\in\Phi 0} p_{0,\phi} \qquad \text{-}\ \left(\overline{r}\left(S,\overline{W},\phi\right)\ge 0,\ \text{integral number } k\ge 0\right)$$

$$\le \overline{r}_\theta^{\,k}\times\prod_{t\in T} exp\left(\overline{r}_\theta\times ln\left(\frac{e_t}{\overline{r}_\theta}\right)+\left(1-\overline{r}_\theta\right)\times ln\left(\frac{1-e_t}{1-\overline{r}_\theta}\right)\right)$$

$$= \overline{r}_\theta^{\,k}\times exp\left(-\sum_{t\in T}\left(\overline{r}_\theta\times ln\left(\frac{\overline{r}_\theta}{e_t}\right)+\left(1-\overline{r}_\theta\right)\times ln\left(\frac{1-\overline{r}_\theta}{1-e_t}\right)\right)\right)$$

$$= \overline{r}_\theta^{\,k}\times exp\left(-\sum_{t\in T} D_{KL}\left(\overline{r}_\theta\,||\,e_t\right)\right) \qquad \blacksquare$$

**Corollary 4.2 (k-LPM of $\overline{r}\left(S,\overline{W},\phi\right)$):**

$$\sum_{\phi\in\Phi}\left(max\left(0,\left(\overline{r}_\theta-\overline{r}\left(S,\overline{W},\phi\right)\right)\right)\right)^k \le \overline{r}_\theta^{\,k}\times exp\left(-|T|\times\min_{t\in T} D_{KL}\left(\overline{r}_\theta\,||\,e_t\right)\right),$$

where $D_{KL}(\overline{r}_\theta||e_t)$ is the KL divergence of $\overline{r}_\theta$ and $e_t$.

Proposition 4.2 states that $D_{KL}(\overline{r}_\theta||e_t)$ is nonnegative. Thus, Theorem 4.3 and Corollary 4.2 show that the k-LPM of $\overline{r}\left(S,\overline{W},\phi\right)$ can decrease exponentially fast as long as DAB consistently acquires the strategy $s_t$, which achieves the edge $e_t$ that is slightly higher than the normalized target reward $\overline{r}_\theta$ in the learning process. There are similarities between Theorem 4.3 (and Corollary 4.2) in this chapter and Theorem 3 in (Rätsch et al. 2001) since the normalized target reward $\overline{r}_\theta$ and the exponential decreasing bound are involved in all of them. However, they are different in that Theorem 3 in (Rätsch et al. 2001) is proven in the research of the

AdaBoost technique for supervised learning (e.g. two-class classification) while Theorem 4.3 in this chapter is applicable to the analysis of downside risk for diversification across strategies, i.e. it provides the exponentially decreasing bound for the downside risk of diversification across strategies. Essentially, Theorem 3 in (Rätsch et al. 2001) can be regarded as the discussion on the bound of 0-order LPM (the probability of loss), while Theorem 4.3 in this chapter extends the conclusion to the bound of k-order LPM (k: nonnegative integral number). This extension ($k \geq 0$) is important for evaluating the downside risk of a strategy because the values of k represent the degree of risk aversion or tolerance (Bawa 1975).

For a finite strategy set $\Omega$ and a finite example set $\Phi$, a reward matrix is represented as below:

$$\begin{pmatrix} r(\omega_1,1,\phi_1) & r(\omega_1,1,\phi_2) & \cdots & r\left(\omega_1,1,\phi_{|\Phi|}\right) \\ r(\omega_2,1,\phi_1) & r(\omega_2,1,\phi_2) & \cdots & r\left(\omega_2,1,\phi_{|\Phi|}\right) \\ \vdots & \vdots & \cdots & \vdots \\ r\left(\omega_{|\Omega|},1,\phi_1\right) & r\left(\omega_{|\Omega|},1,\phi_2\right) & \cdots & r\left(\omega_{|\Omega|},1,\phi_{|\Phi|}\right) \end{pmatrix}.$$

The probability vector $P(\Phi)$ on the finite example set $\Phi$ is represented as below:

$$P(\Phi)=\begin{pmatrix} p(\phi_1) \\ p(\phi_2) \\ \vdots \\ p\left(\phi_{|\Phi|}\right) \end{pmatrix}.$$

The probability vector $Q(\Omega)$ on the finite strategy set $\Omega$ is represented as below:

$$Q(\Omega)=\begin{pmatrix} q(\omega_1) \\ q(\omega_2) \\ \vdots \\ q\left(\omega_{|\Omega|}\right) \end{pmatrix}^T.$$

**Definition 4.2 (edge)** Given a probability vector $P(\Phi)$ on the finite example set $\Phi$, the edge $e(P(\Phi))$ is represented as below:

$$e\left(P(\Phi)\right) = \max_{\omega\in\Omega} \sum_{\phi\in\Phi} p(\phi)\, r(\omega,1,\phi).$$

**Definition 4.3 (margin)** Given a probability vector $Q(\Omega)$ on the finite strategy set $\Omega$, the margin $m(Q(\Omega))$ is represented as below:

$$m\left(Q(\Omega)\right) = \min_{\phi\in\Phi} \sum_{\omega\in\Omega} q(\omega) r(\omega,1,\phi).$$

**Theorem 4.4 (minimax theorem, von Neumann 1928)**

$$\max_{Q(\Omega)\in Pr^{|\Omega|}} m\left(Q(\Omega)\right) = \min_{P(\Phi)\in Pr^{|\Phi|}} e\left(P(\Phi)\right)$$

where $Pr^{|\Omega|}$ and $Pr^{|\Phi|}$ are respectively the $|\Omega|$-dimensional and $|\Phi|$-dimensional probability measures.

Given the example set $\Phi$ and the probability vector $P(\Phi)$ on it, the edge $e(P(\Phi))$ can be regarded as the maximum average reward, which is achieved by one strategy in the strategy set $\Omega$. The edge is used in the DAB technique to adaptively update the probability distribution $P_t$ and adjust the weight $w_t$. Hypothesis 4.1 states that the edge decreases as the learning steps increase. Given the strategy set $\Omega$ and the probability vector $Q(\Omega)$, the margin $m(Q(\Omega))$ can be regarded as the minimum reward, which is achieved by diversifying the strategy set $\Omega$ in the finite example set $\Phi$. In the supervised learning field, the margin was proposed to measure the generalization ability of a model (Vapnik 1999, Freund and Schapire 1998). The margin was adopted as one of measures for risk in the investment field (Balzer 1994). Theorem 4.4 connects the margin and the edge, i.e. maximizing the margin is equivalent to minimizing the edge. Thus, the margin (i.e. the minimum reward) is expected to increase as the edge decreases in the DAB technqiue.

The DAB technique allocates the resources $\rho$ to each acquired strategy in S in proportion with its weight in $\overline{W}$ before parallel execution and executes all strategies in S with their allocated resources in parallel in the execution part. The DAB technique achieves the reward $r_E\left(S,\overline{W},\phi\right)$ and the normalized reward $\overline{r}_E\left(S,\overline{W},\phi\right)$ on the example $\phi$ ($\phi\in\Phi$) at the end of parallel execution. Let $r_\mu$, $r_\sigma$, $r_{\sigma(2)}$ and $r_M$ respectively denote $r_E\left(S,\overline{W},\phi\right)$'s average, standard deviation, k-order LPM and minimum. They are calculated as below:

$$r_\mu = \sum_{\phi\in\Phi} p_{0,\phi} \times r_E\left(S,\overline{W},\phi\right),$$

$$r_\sigma = \sqrt{\frac{1}{|\Phi|-1} \times \sum_{\phi\in\Phi}\left(r_E\left(S,\overline{W},\phi\right)-r_\mu\right)^2},$$

$$r_{\sigma(k)} = \sum_{\phi\in\Phi}\left(max\left(0,\left(r_{INF}(\phi)+\overline{r}\times\Delta - r_E\left(S,\overline{W},\phi\right)\right)\right)\right)^k,$$

$$r_M = \min_{\phi\in\Phi}\left(r_E\left(S,\overline{W},\phi\right)\right).$$

This section has proven several statistical properties about $\overline{r}\left(S,\overline{W},\phi\right)$, its average, k-LPM and minimum. Based on these statistical properties, two hypotheses are given as below.

**Hypothesis 4.2 (k-LPM of rewards $r_{\sigma(k)}$ and average of rewards $r_\mu$)**: The k-LPM $r_{\sigma(k)}$ of rewards decreases as the learning steps increase and meanwhile the reduction in the average reward $r_\mu$ is limited. From the perspective of candidate strategies' efficient frontier, the DAB technique moves it toward the favorable direction. The efficient frontier's horizontal and vertical coordinates respectively represent $r_{\sigma(k)}$ and $r_\mu$.

**Hypothesis 4.3 (minimum of rewards $r_M$ and average of rewards $r_\mu$)**: The minimum $r_M$ of rewards increases as the learning steps increase and meanwhile the reduction in the average reward $r_\mu$ is limited. From the perspective of candidate strategies' efficient frontier, the DAB technique moves it toward the favorable direction. The efficient frontier's horizontal and vertical coordinates respectively represent $r_M$ and $r_\mu$.

Since $\overline{r}\left(S,\overline{W},\phi\right)$ is not exactly the same as $\overline{r}_E\left(S,\overline{W},\phi\right)$ in the hypotheses, the analysis in this section is rather theoretical explanation for Hypotheses 4.2 and 4.3 than mathematical proof. The hypotheses will be verified by the experiments on real-life data in the next chapter.

# 4.3 Summary

This chapter has proposed the DAB technique, which reflects the idea of diversification across strategies, for risk management in various practices. The DAB technique extends the key idea of the AdaBoost technique to acquire strategies from a candidate strategy set and determine their weights. The resources are then allocated to each acquired strategy in proportion with its weight before parallel execution and all acquired strategies are executed in parallel with their allocated resources. Theoretical analysis shows several advantages of the DAB technique: 1) it allows the candidate strategy set to contain finite or infinite strategies; 2) as the learning steps increase, it lowers risk that can be measured by different standards (e.g. the k-LPM of rewards and the minimum of rewards) and limits the decrease in average reward. The DAB technique will be applied to implement diversification across the DF strategies for trade execution and will be verified by the experiments on real-life data in the next chapter. In future, the theoretical research of the DAB technique can be further extended to analysis of its generalization ability, i.e. its statistical properties in out-of-sample testing.

# Chapter 5

# Boosted Dynamic Focus Strategy

In this chapter, the DAB technique, which reflects the idea of diversification across strategies, is applied to acquire DF strategies from a candidate DF strategy set and determine their weights. The entire order is allocated to each acquired DF strategy in proportion with its weights and all acquired DF strategies are executed in parallel to fill their allocated order. This parallel execution is called the BONUS strategy. Theoretical analysis shows that as the learning steps increase, the BONUS strategy moves the candidate DF strategies' efficient frontier toward the favorable direction. The efficient frontier's horizontal and vertical coordinates respectively represent the risk measured by different standards and the average shortfall. In-sample test based on 80 datasets strongly supports the theoretical analysis on the BONUS strategy. Out-of-sample test results on most datasets show that the DAB technique and the BONUS strategy outperform the optimal DF strategy and two simply diversification techniques. This chapter is composed of 3 sections. The 1st section describes and analyzes the BONUS strategy. The 2nd section empirically verifies the theoretical analysis on the BONUS strategy and evaluates its effectiveness. The 3rd section summarizes the BONUS strategy.

# 5.1 BONUS Strategy and Analysis

A trade execution strategy is designed to fill a buy or sell order of $\rho$ shares over a period of time (i.e. the example $\phi$) with low transaction cost and risk. A set of DF strategies, which reflects the idea of diversification across time, was proposed in Chapter 3 to improve the limit order strategy for trade execution. The DF strategies dynamically adjust the volume of each small market order over the execution period according to real-time state variables to reduce the trade execution cost and risk, which are brought by the limit order strategy. The DAB technique, which reflects the idea of diversification across strategies, was proposed in Chapter 4 to move a candidate strategy set's efficient frontier toward a favorable direction. It is applied to acquire DF strategies from a candidate DF strategy set and determine their weights. The entire order of $\rho$ shares is allocated to each acquired DF strategy in proportion with its weight w and all acquired DF strategies are executed in parallel to fill their allocated order of $w \times \rho$ shares. This parallel execution is regarded as the BONUS strategy.

Let $\$(s,w,\phi)$ denote the dollar value of the DF strategy s buying (or selling) the order of $w \times \rho$ shares on the example $\phi$. Let $c(s,w,\phi)$ denote the shortfall, which is brought by the DF strategy s when it is executed on the example $\phi$ to fill the order of $w \times \rho$ shares. The shortfall $c(s,w,\phi)$ is calculated as bellow:

$$c(s,w,\phi) = sgn \times 10000 \times \frac{\frac{\$(s,w,\phi)}{\rho} - w \times p_D(\phi)}{p_D(\phi)},$$

where sgn = 1 (or −1) for buying (or selling), $p_D(\phi)$ is the midpoint price between the best bid price and the best ask price at the beginning time of $\phi$, the unit of the shortfall $c(s,w,\phi)$ is BPS.

Given a candidate DF strategy set $\Omega$ and an example $\phi$, any DF strategy s ($s \in \Omega$) brings the shortfall $c(s,1,\phi)$ when it is executed to fill the order of $\rho$ shares on the example $\phi$. It is assumed in this chapter that the shortfall $c(s,1,\phi)$ has a lower bound $c_{INF}(\phi)$ and an upper bound $c_{SUP}(\phi)$ as below:

$$c_{INF}(\phi) = INF\{c(s,1,\phi) \mid s \in \Omega\}$$

$$c_{SUP}(\phi) = SUP\{c(s,1,\phi) \mid s \in \Omega\}.$$

If the candidate DF strategy set $\Omega$ contains infinite DF strategies, the lower bound $c_{INF}(\phi)$ and the upper bound $c_{SUP}(\phi)$ can be estimated by randomly sampling some DF strategies from $\Omega$. Given an example set $\Phi$ that contains finite examples $\phi$, the normalized shortfall $\overline{c}(s,w,\phi)$, which is brought by the DF strategy s when s is executed on the example $\phi$ to fill the order of $w\times\rho$ shares, is calculated as below:

$$\overline{c}(s,w,\phi) = \frac{c(s,w,\phi) - w \times c_{INF}(\phi)}{\Delta},$$

where $\Delta = SUP\{c_{SUP}(\phi) \mid \phi \in \Phi\} - INF\{c_{INF}(\phi) \mid \phi \in \Phi\}$. Table 5.1 shows the pseudo codes to describe the DAB technique and the BONUS strategy.

**Table 5.1 The DAB technique and the BONUS strategy for trade execution**

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LEARNING INPUT:

1. A buy or sell order of $\rho$ shares;
2. An example set $\Phi$;
3. A probability distribution $P_0$ on $\Phi$;
4. A candidate DF strategy set $\Omega$;
5. A normalized target shortfall $\overline{c}_0$;
6. A base learner BL;
7. Maximum number $t_M$ of learning steps.

LEARNING INITIALIZATION:

1. $P_1 \leftarrow P_0$;
2. $T \leftarrow \varnothing$;
3. $S \leftarrow \varnothing$;
4. $W \leftarrow \varnothing$.

LEARNING PROCESS: for $t = 1 \ldots t_M$ do

1. Call BL to acquire a DF strategy $s_t$ from $\Omega$ and $s_t$ brings the shortfall $c(s_t,1,\phi)$ and the normalized shortfall $\bar{c}(s_t,1,\phi)$;
2. Calculate the edge $e_t \leftarrow \sum_{\phi\in\Phi} p_{t,\phi}\times\left(1-\bar{c}(s_t,1,\phi)\right)$;
3. If $e_t < 1-\bar{c}_\theta$, then break;
4. Calculate the weight $w_t \leftarrow \ln\left(\frac{1}{1-\bar{c}_\theta}-1\right)-\ln\left(\frac{1}{e_t}-1\right)$;
5. Update the probability distribution

$$P_{t+1}=\left\{p_{t+1,\phi}\;\middle|\;p_{t+1,\phi}=\frac{p_{t,\phi}\times\exp\left(-w_t\times\left(1-\bar{c}(s_t,1,\phi)\right)\right)}{p_t},\phi\in\Phi\right\},$$

where $p_t=\sum_{\phi\in\Phi} p_{t,\phi}\times\exp\left(-w_t\times\left(1-\bar{c}(s_t,1,\phi)\right)\right)$;

6. Let $T\leftarrow T\cup\{t\}$, $S\leftarrow S\cup\{s_t\}$ and $W\leftarrow W\cup\{w_t\}$.

LEARNING OUTPUT:

1. An index set T;
2. A DF strategy set $S=\{s_t \mid t\in T\}$;
3. A weight set $\bar{W}=\left\{\bar{w}_t\;\middle|\;\bar{w}_t=\frac{w_t}{w},w=\sum_{t\in T} w_t, w_t\in W, t\in T\right\}$;
4. A normalized reward $\bar{c}(S,\bar{W},\phi)=\sum_{t\in T}\bar{w}_t\times\bar{c}(s_t,1,\phi)$.

EXECUTION (THE BONUS STRATEGY):

1. Allocates the entire order of $\rho$ shares to each acquired DF strategy $s_t$ ($s_t\in S$) in proportion with its weight $\bar{w}_t$ ($\bar{w}_t\in\bar{W}$);
2. Execute all acquired DF strategies $s_t$ ($s_t\in S$) in parallel with their allocated order of $\bar{w}_t\times\rho$ shares;
3. Output a shortfall $c_E(S,\bar{W},\phi)=\sum_{t\in T} c(s_t,\bar{w}_t,\phi)$ and a normalized shortfall $\bar{c}_E(S,\bar{W},\phi)=\sum_{t\in T}\bar{c}(s_t,\bar{w}_t,\phi)$.

Table 5.1 shows that given the buy or sell order of $\rho$ shares, the example set $\Phi$, the probability distribution $P_0$, the candidate DF strategy set $\Omega$, the target shortfall $\overline{c}_\theta$, the base learner BL and the maximum number $t_M$ of learning steps, a set of DF strategies with weights is acquired by using the DAB technique in the part of learning, the entire order of $\rho$ shares is allocated to each acquired DF strategy $s_t$ in proportion with its weight $\overline{w}_t$ and all acquired DF strategies are executed in parallel to fill their allocated order of $\overline{w}_t \times \rho$ shares in the part of execution. The BONUS strategy is referred to as the parallel execution and it boost the candidate DF strategy set's performance. In this chapter, an equal probability $1/|\Phi|$ is initially set for each example $\phi$ in the example set $\Phi$, i.e. the probability distribution $P_0$ is a uniform distribution.

The edge $e_t$ in Table 5.1 is reversely related to the average shortfall, which is brought by a DF strategy on the training example set. The base learner acquires a DF strategy, which achieves the edge as high as possible (i.e. achieves the average shortfall as low as possible), from the candidate DF strategy set. The DAB technique in Table 5.1 calculates the edge $e_t$ and updates the probability distribution $P_t$ at each learning step based on the formula $1-\overline{c}(s_t,1,\phi)$ instead of the normalized shortfall $\overline{c}(s_t,1,\phi)$ while the DAB technique in Table 4.2 calculates the edge $e_t$ and updates the probability distribution $P_t$ at each learning step based on the normalized reward $\overline{r}(s_t,1,\phi)$. Intuitively, the expectation on the normalized reward $\overline{r}(s_t,1,\phi)$ and on the normalized shortfall $\overline{c}(s_t,1,\phi)$ is reverse. This means that the essence of the DAB technique in Table 5.1 is the same as that of the DAB technique in Table 4.2. The theoretical analysis in Chapter 4 can be extended to explain the effectiveness of the DAB technique in Table 5.1 if $1-\overline{c}(s_t,1,\phi)$ is regarded as the normalized reward $\overline{r}(s_t,1,\phi)$. Suppose that $\overline{r}(s_t,1,\phi)=1-\overline{c}(s_t,1,\phi)$ and $\overline{r}_\theta=1-\overline{c}_\theta$. Thus, $\overline{r}(S,\overline{W},\phi)=1-\overline{c}(S,\overline{W},\phi)$ since $\sum_{t\in T}\overline{w}_t=1$. Based on the above relationship, the theoretical analysis in Chapter 4 can be illustrated as below in the context of trade execution.

**Hypothesis 5.1 (edge $e_t$)**: The edge $e_t$, which is achieved by the acquired strategy $s_t$ at each learning step, decreases as the learning steps increase.

**Theorem 5.1 ($\bar{c}(S,\bar{W},\phi)$)**:

$$\bar{c}(S,\bar{W},\phi) \leq 1-\frac{ln(p_{0,\phi})}{W}-\frac{1-\bar{c}_\theta}{W}\times\sum_{t\in T}(1-exp(-w_t)).$$

**Corollary 5.1 ($\bar{c}(S,\bar{W},\phi)$)**:

$$\bar{c}(S,\bar{W},\phi) \leq 1+\frac{ln|\Phi|}{W}-\frac{1-\bar{c}_\theta}{W}\times\sum_{t\in T}(1-exp(-w_t)).$$

**Theorem 5.2 (average of $\bar{c}(S,\bar{W},\phi)$)**:

$$\sum_{\phi\in\Phi}p_{0,\phi}\times\bar{c}(S,\bar{W},\phi) \leq 1-\frac{1-\bar{c}_\theta}{W}\times\sum_{t\in T}(1-exp(-w_t)).$$

**Proposition 5.1** Suppose $w_{t+1} \leq w_t$, where $w_t$ and $w_{t+1}$ are any two consecutive weights in W in Table 5.1. The following function is decreasing as $|T|$ increases:

$$f(|T|) = 1-\frac{1}{\sum_{t\in T}w_t}\times\left(a+b\times\sum_{t\in T}(1-exp(-w_t))\right),$$

where a ($a \leq 0$), b ($b \geq 0$) are two constants, T is the index set in Table 5.1.

Theorem 5.1, Corollary 5.1 and Theorem 5.2 show that the normalized shortfall $\bar{c}(S,\bar{W},\phi)$ and its average $\sum_{\phi\in\Phi}p_{0,\phi}\times\bar{c}(S,\bar{W},\phi)$ are limited by the upper bounds as the DAB technique acquires more strategies with weights in the learning part. Hypothesis 5.1 states that the edge $e_t$ decreases as the learning steps increase. So the weight $w_t$ decreases as the learning steps increase according to the relationship between $e_t$ and $w_t$ (see Table 5.1). According to Proposition 5.1, the upper bounds in Theorem 5.1, Corollary 5.1 and Theorem 5.2 decrease as the learning steps increase. The upper bound in Corollary 5.1 is reasonable even for a large example set because it only logarithmically depends on the example set size $|\Phi|$.

**Theorem 5.3 (k-UPM of $\overline{c}(S,\overline{W},\phi)$):**

$$\sum_{\phi\in\Phi}\left(max\left(0,\left(\overline{c}\left(S,\overline{W},\phi\right)-\overline{c}_\theta\right)\right)\right)^k \le \left(1-\overline{c}_\theta\right)^k \times exp\left(-\sum_{t\in T} D_{KL}\left(\left(1-\overline{c}_\theta\right)\| e_t\right)\right).$$

**Corollary 5.2 (k-UPM of $\overline{c}(S,\overline{W},\phi)$):**

$$\sum_{\phi\in\Phi}\left(max\left(0,\left(\overline{c}\left(S,\overline{W},\phi\right)-\overline{c}_\theta\right)\right)\right)^k \le \left(1-\overline{c}_\theta\right)^k \times exp\left(-|T|\times \min_{t\in T} D_{KL}\left(\left(1-\overline{c}_\theta\right)\| e_t\right)\right).$$

$D_{KL}\left(\left(1-\overline{c}_\theta\right)\| e_t\right)$ is nonnegative according to the property of the KL divergence. Theorem 4.3 and Corollary 4.2 show that the k-order UPM of $\overline{c}\left(S,\overline{W},\phi\right)$ can decrease exponentially fast as long as in the learning process, the DAB technique consistently acquires the strategy $s_t$, which achieves the edge $e_t$ that is slightly higher than $1-\overline{c}_\theta$. The 2-order UPM is tested in the following empirical study and more values of k will be testified in future work.

For a finite strategy set $\Omega$ and a finite example set $\Phi$, a shortfall matrix is represented as below:

$$\begin{pmatrix} c(\omega_1,1,\phi_1) & c(\omega_1,1,\phi_2) & \cdots & c\left(\omega_1,1,\phi_{|\Phi|}\right) \\ c(\omega_2,1,\phi_1) & c(\omega_2,1,\phi_2) & \cdots & c\left(\omega_2,1,\phi_{|\Phi|}\right) \\ \vdots & \vdots & \cdots & \vdots \\ c\left(\omega_{|\Omega|},1,\phi_1\right) & c\left(\omega_{|\Omega|},1,\phi_2\right) & \cdots & c\left(\omega_{|\Omega|},1,\phi_{|\Phi|}\right) \end{pmatrix}.$$

The probability vector P(Φ) on the finite example set Φ and the probability vector Q(Ω) on the finite strategy set Ω are represented respectively as below:

$$P(\Phi)=\begin{pmatrix} p(\phi_1) \\ p(\phi_2) \\ \vdots \\ p\left(\phi_{|\Phi|}\right) \end{pmatrix} \text{ and } Q(\Omega)=\begin{pmatrix} q(\omega_1) \\ q(\omega_2) \\ \vdots \\ q\left(\omega_{|\Omega|}\right) \end{pmatrix}^T.$$

**Definition 5.1 (edge)** Given a probability vector P(Φ) on the finite example set Φ, the edge e(P(Φ)) is represented as below:

$$e(P(\Phi)) = \max_{\omega\in\Omega}\sum_{\phi\in\Phi} p(\phi)\times(1-c(\omega,1,\phi)) = 1-\min_{\omega\in\Omega}\sum_{\omega\in\Omega} p(\phi)\times c(\omega,1,\phi).$$

**Definition 5.2 (margin)** Given a probability vector Q($\Omega$) on the finite strategy set $\Omega$, the margin m(Q($\Omega$)) is represented as below:

$$m(Q(\Omega)) = \min_{\phi\in\Phi}\sum_{\omega\in\Omega} q(\omega)\times(1-c(\omega,1,\phi)) = 1-\max_{\phi\in\Phi}\sum_{\omega\in\Omega} q(\omega)\times c(\omega,1,\phi).$$

**Theorem 5.4 (minimax theorem, von Neumann 1928)**

$$\min_{P(\Phi)\in Pr^{|\Phi|}} e(P(\Phi)) = \max_{Q(\Omega)\in Pr^{|\Omega|}} m(Q(\Omega)),$$

where $Pr^{|\Omega|}$ and $Pr^{|\Phi|}$ are respectively the $|\Omega|$-dimensional and $|\Phi|$-dimensional probability measures.

Definition 5.1 shows that given an example set $\Phi$ and a probability vector P($\Phi$) on $\Phi$, the edge e(P($\Phi$)) is reversely proportional to the minimum shortfall average $\min_{\omega\in\Omega}\sum_{\omega\in\Omega} p(\phi)\times c(\omega,1,\phi)$. The edge is used in the DAB technique to adaptively update the probability distribution $P_t$ and adjust the weight $w_t$ at each learning step. Hypothesis 5.1 states that the edge decreases as the learning steps increase. Definition 5.2 shows that given a strategy set $\Omega$ and a probability vector Q($\Omega$) on $\Omega$, the margin m(Q($\Omega$)) is reversely proportional to the maximum shortfall $\max_{\phi\in\Phi}\sum_{\omega\in\Omega} q(\omega)\times c(\omega,1,\phi)$. Theorem 5.4 connects the margin and the edge, i.e. maximizing the margin is equivalent to minimizing the edge. Thus, the margin is expected to increase (i.e. the maximum shortfall is expected to decrease) as the edge decreases in the DAB technique.

After the entire order of $\rho$ shares is allocated to each acquired DF strategy in S in proportion with its weight in $\overline{W}$, all acquired DF strategies are executed in parallel to fill their allocated order of $\overline{w}_t\times\rho$ shares (i.e. the BONUS strategy). Let $c_E(S,\overline{W},\phi)$ denote the shortfall, which is brought by the BONUS strategy on the example $\phi$. Let $c_\mu$, $c_\sigma$, $c_{\sigma(2)}$ and $c_M$ respectively denote $c_E(S,\overline{W},\phi)$'s average, standard deviation, k-order UPM and maximum. They are calculated as below:

$$c_\mu = \sum_{\phi\in\Phi} p_{0,\phi} \times c_E\left(S, \overline{W}, \phi\right).$$

$$c_\sigma = \sqrt{\frac{1}{|\Phi|-1} \times \sum_{\phi\in\Phi}\left(c_E\left(S, \overline{W}, \phi\right) - c_\mu\right)^2},$$

$$c_{\sigma(k)} = \sum_{\phi\in\Phi} p_{0,\phi} \times \left(max\left(0, \left(c_E\left(S, \overline{W}, \phi\right) - c_{INF}(\phi) - \overline{c}_\theta \times \Delta\right)\right)\right)^k,$$

$$c_M = \max_{\phi\in\Phi}\left(c_E\left(S, \overline{W}, \phi\right)\right).$$

$c_\sigma$, $c_{\sigma(k)}$ and $c_M$ are used as different standards to measure risk.

The theoretical analysis in this section illustrates the statistical properties on the learning output $\overline{c}\left(S, \overline{W}, \phi\right)$, its average, k-UPM and maximum. The following two hypotheses are given according the theoretical analysis. Since $\overline{c}\left(S, \overline{W}, \phi\right)$ is not exactly the same as $\overline{c}_E\left(S, \overline{W}, \phi\right)$ in the hypotheses, the analysis in this section is rather theoretical explanation for the following hypotheses than mathematical proof. The following hypotheses will be verified through the experiments on real-life data in the next section.

**Hypothesis 5.2 (k-UPM of shortfalls $c_{\sigma(k)}$ and average of shortfalls $c_\mu$)**: The k-UPM $c_{\sigma(k)}$ of shortfalls decreases as the learning steps increase and meanwhile the increase in the average shortfall $c_\mu$ is limited. From the perspective of candidate strategies' efficient frontier, the BONUS strategy moves it toward the favorable direction. The efficient frontier's horizontal and vertical coordinates respectively represent $c_{\sigma(k)}$ and $c_\mu$.

**Hypothesis 5.3 (maximum of shortfalls $c_M$ and average of shortfalls $c_\mu$)**: The maximum $c_M$ of shortfalls increases as the learning steps increase and meanwhile the increase in the average shortfall $c_\mu$ is limited. From the perspective of candidate strategies' efficient frontier, the BONUS strategy moves it toward the favorable direction. The efficient frontier's horizontal and vertical coordinates respectively represent $c_M$ and $c_\mu$.

## 5.2 Empirical Evaluation

This section evaluates the DAB technique and the BONUS strategy through the detailed in-sample results of the stock "WFT" in four test periods and the statistical summary of in-sample results on all 80 datasets. Figures 5.1, 5.2, 5.3 and 5.4 shows the in-sample results of the stock "WFT" in four test periods: January, February, March and April 2002. The three charts in the left column of each figure respectively show how the edge $e_t$, the maximum of shortfalls $c_M$ and the 2-UPM of shortfalls $c_{\sigma(2)}$ (see the vertical coordinates) change as the learning steps (see the horizontal coordinates) increase. The study of efficient frontier is shown in three charts on the right column of each figure, in which the vertical coordinates represent the average of shortfalls $c_\mu$ and the horizontal coordinates represent respectively the standard deviation of shortfalls $c_\sigma$, the maximum of shortfalls $c_M$ and the 2-UPM of shortfalls $c_{\sigma(2)}$. In the three charts in the right column of each figure, the dot curves are the DF strategies' efficient frontiers and the solid curves are the BONUS strategy's in-sample test results.

The DF strategies and their weights need to be acquired by using the DAB technique before the BONUS strategy is implemented. As an input parameter of the DAB technique, the normalized target shortfall $\overline{c}_\theta$ needs to be determined at the start of learning. Here, $\overline{c}_\theta$ is set in the following way. Different values between 0 and 1 (starting from 0 in an increasing order) are given to the normalized target shortfall $\overline{c}_\theta$ to see whether the in-sample test results can verify Hypotheses 5.1, 5.2 and 5.3 with the given value of $\overline{c}_\theta$. When a value of $\overline{c}_\theta$ is found, with which the in-sample test results verify Hypothesis 5.2, the acquired DF strategies and their weights are applied to out-of-sample test for implementing the BONUS strategy.

It is clear in the left column charts of each figure that $e_t$, $c_M$ and $c_{\sigma(2)}$ decrease as the learning steps increase, though some oscillation happens in the learning process. The right column charts of each figure show that as the learning steps increase, the BONUS strategy based on the DAB technique lowers risk that can be measured by different standards (the standard deviation of shortfalls $c_\sigma$, the maximum of shortfalls $c_M$ and the 2-UPM of shortfalls $c_{\sigma(2)}$) and meanwhile limits the increase in the average of shortfalls $c_\mu$. From the perspective of candidate DF strategies' efficient frontier, the BONUS strategy moves it toward the favorable (left-bottom) direction as the learning steps increase. This supports Hypotheses 5.1, 5.2 and 5.3 on the DAB technique and the BONUS strategy.

The optimization objective is represented as the tradeoff between the average of shortfalls and the risk, i.e. $c_\mu + \gamma \times$ risk, where $\gamma$ ($\gamma \geq 0$) is a tradeoff factor and the variable "risk" is measured by different standards ($c_\sigma$, $c_{\sigma(2)}$ or $c_M$). The tradeoff factor $\gamma$ represents the degree of risk aversion or tolerance in trade execution. When $\gamma$ is assigned with a greater (or smaller) value, it represents higher risk aversion (or tolerance) in trade execution. When $\gamma = 0$, it means that only the average of shortfalls $c_\mu$ is considered in the optimization objective but the variable "risk" is ignored. When $\gamma \to \infty$, it means that only the variable "risk" is considered in the optimization objective but the average of shortfalls $c_\mu$ is ignored. Let $-\gamma_0$ the slope of the straight line in each chart on the left column of each figure. When the tradeoff factor $\gamma$ is set to be less than $\gamma_0$, the optimization objective $c_\mu + \gamma \times$ risk is minimized by the BONUS strategy rather than a strategy in the candidate DF strategy set. Moreover, the in-sample test on all 80 datasets shows that Hypotheses 5.1, 5.2 and 5.3 are strongly supported by in-sample test respectively on 80, 77 and 72 datasets.

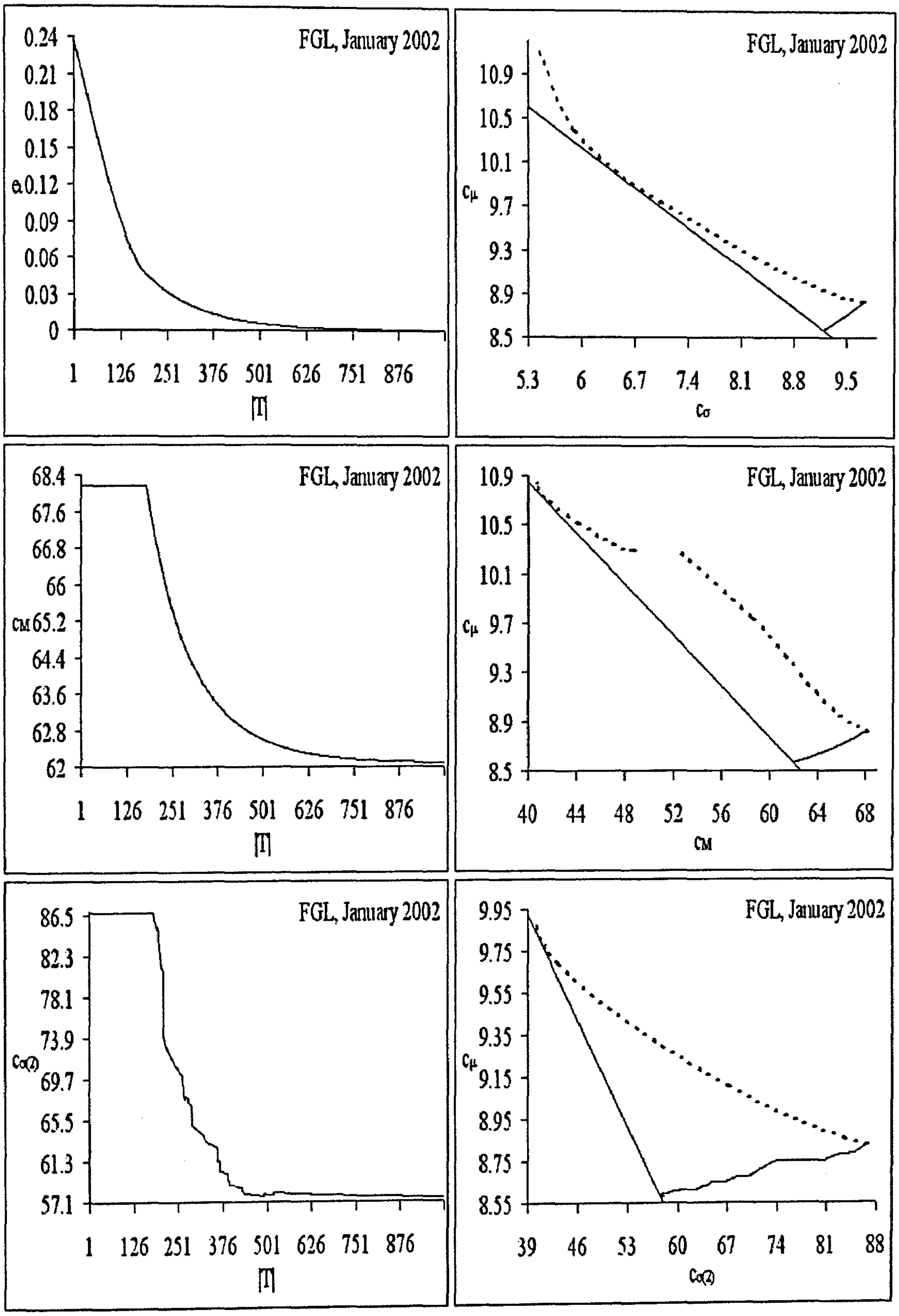


**Figure 5.1 In-sample test on the BONUS strategy (January 2002)**

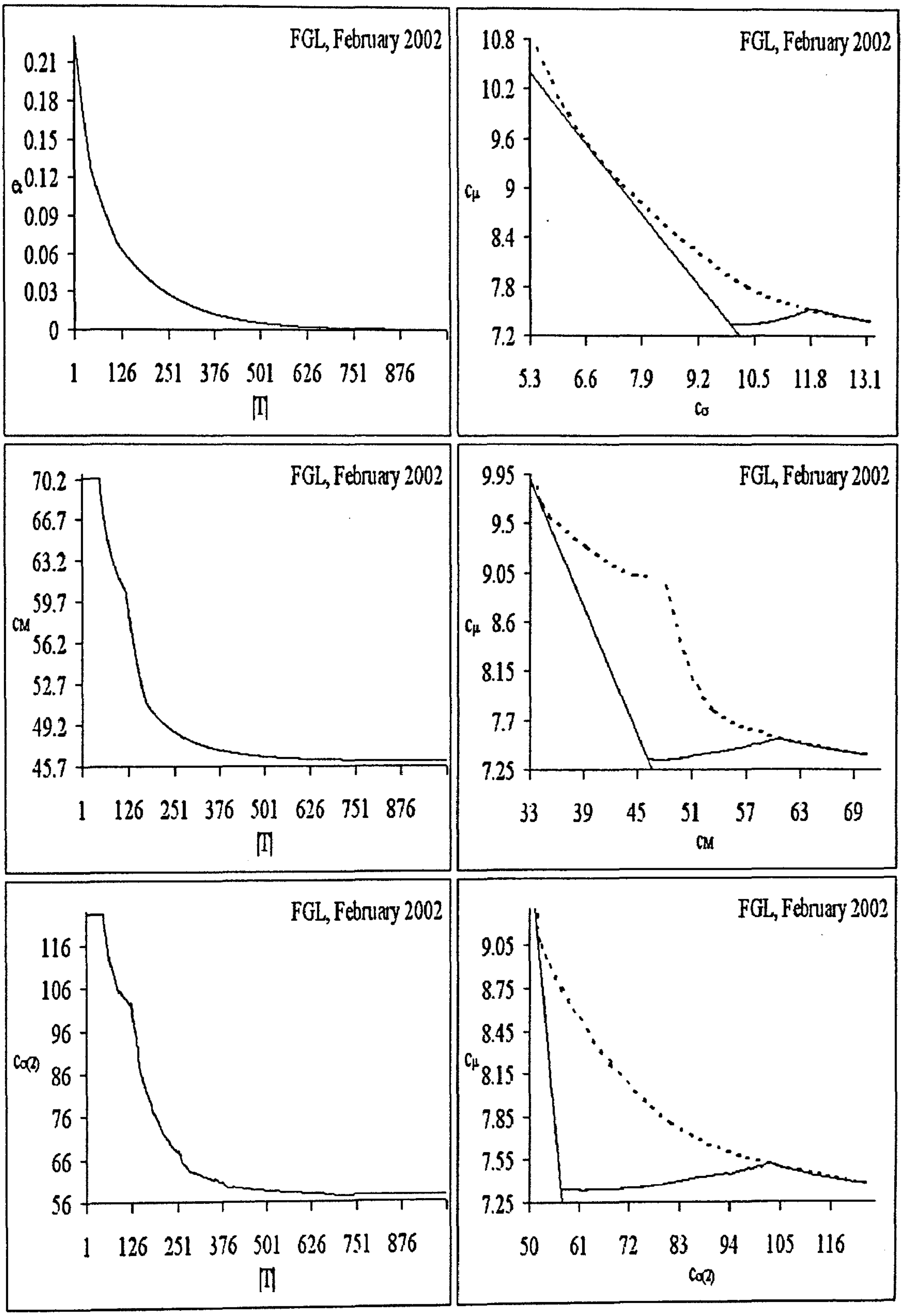


**Figure 5.2 In-sample test on the BONUS strategy (February 2002)**

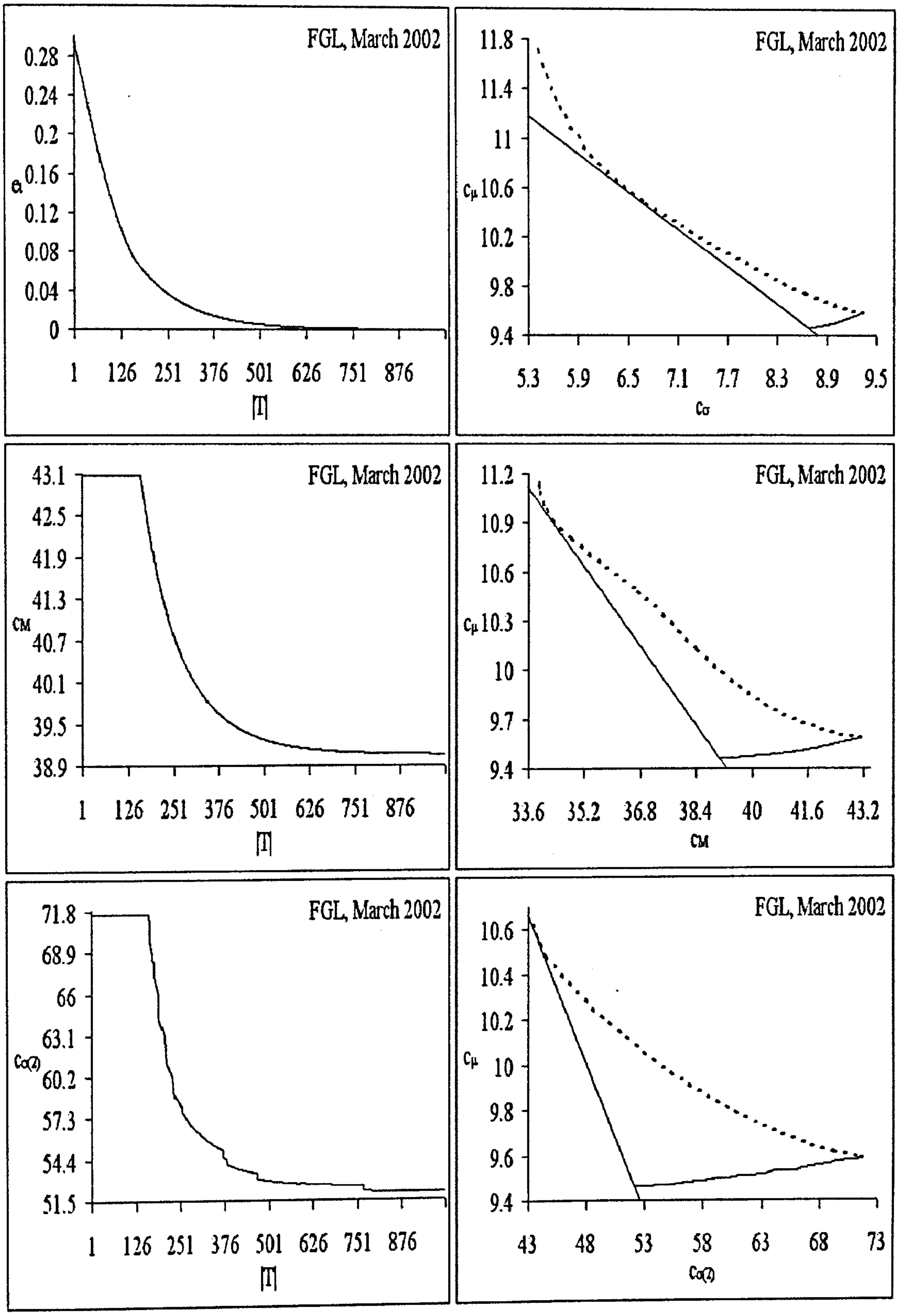


**Figure 5.3 In-sample test on the BONUS strategy (March 2002)**

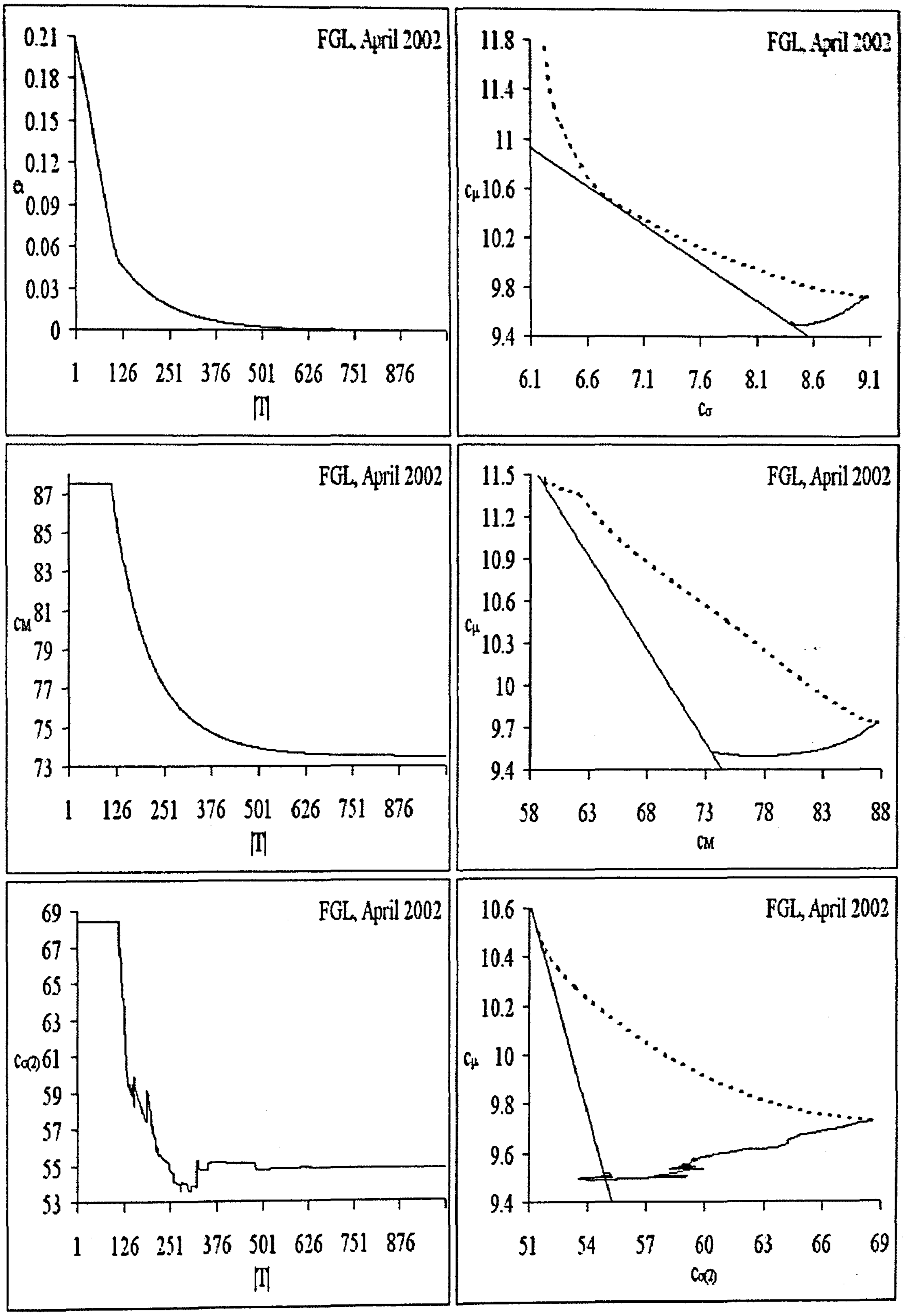


**Figure 5.4 In-sample test on the BONUS strategy (April 2002)**

After the DF strategies and their weights are determined, the BONUS strategy allocates the entire order to each acquired DF strategy in proportion with its weight. All acquired DF strategies are executed in parallel with their allocated order. This section reports the out-of-sample test results of parallel execution on 80 datasets and compared the BONUS strategy to the optimized DF strategy and two simple diversification techniques.

Let "DF0" and "DF1" denote the DF strategy optimized respectively by setting $\gamma = 0$ and 1. Let BONUS denote the BONUS strategy. Let "Average" and "Ranking" denote respectively the 1st and 2nd diversification techniques. The 1st diversification technique "Average" equally allocates the entire order to several DF strategies that bring lower shortfall than other DF strategies in the candidate DF strategy set do. The 2nd diversification technique "Ranking" allocates the entire order to the DF strategies, which bring lower shortfall than other DF strategies in the candidate DF strategy set do, in proportion with the weight $w_t$ calculated as below:

$$w_t = \left( c_t \times \sum_{t \in T} \frac{1}{c_t} \right)^{-1},$$

where T is the index set for the DF strategies, $c_t$ is the shortfall that is brought by the $t^{th}$ strategy.

Tables 5.2, 5.3, 5.4 and 5.5 report the out-of-sample test results of "DF0", "DF1", "BONUS", "Average" and "Ranking" in terms of the average of shortfalls $c_\mu$ and the standard deviation of shortfalls $c_\sigma$. The out-of-sample test results on 80 datasets show that "BONUS" brings lower average of shortfalls $c_\mu$ than "DF0" and "DF1" does respectively on 74 and 79 datasets. The out-of-sample test results also show that in terms of the average of shortfalls $c_\mu$, two simple diversification techniques "Average" and "Ranking" cannot outperform the BONUS strategy (i.e. the DAB technique) on all 80 datasets except the dataset "WFT in April 2002". Moreover, the out-of-sample test on 80 datasets show that the lowest standard deviation of shortfalls $c_\sigma$ is achieved by "BONUS" on 12 datasets while by "DF1" (i.e. $\gamma = 1$) on 68 datasets. This supports the analysis about the effect of $\gamma$ on the optimization objective.

**Table 5.2 Out-of-sample test on BONUS strategies (AMP – CBA)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $C_\sigma$ | $c_\mu$ | $c_\sigma$ |
| AMP | DF1 | 7.70 | 8.21 | 8.65 | **7.51** | 8.97 | **7.70** | 8.36 | 9.31 |
| | DF0 | 7.70 | 8.21 | 8.49 | 9.83 | 8.74 | 9.90 | 7.23 | 11.66 |
| | BONUS | **6.73** | **8.20** | **7.44** | 8.04 | **7.84** | 7.72 | **6.85** | **9.27** |
| | Average | 10.42 | 10.67 | 11.94 | 9.90 | 12.52 | 10.93 | 11.06 | 13.07 |
| | Ranking | 9.77 | 9.96 | 10.59 | 8.94 | 11.36 | 9.77 | 10.07 | 11.87 |
| ANZ | DF1 | 6.10 | **6.06** | 5.94 | **6.98** | 4.70 | **5.93** | 5.55 | **5.80** |
| | DF0 | 5.32 | 8.23 | 5.03 | 8.58 | 4.04 | 7.83 | 4.66 | 7.54 |
| | BONUS | **5.11** | 7.26 | **4.88** | 7.55 | **3.96** | 7.05 | **4.56** | 7.02 |
| | Average | 7.79 | 7.56 | 7.63 | 9.15 | 5.71 | 6.98 | 6.83 | 6.76 |
| | Ranking | 7.02 | 6.99 | 6.76 | 8.26 | 5.12 | 6.58 | 6.11 | 6.27 |
| BHP | DF1 | 3.66 | **5.62** | 4.51 | **5.70** | 4.87 | **6.49** | 5.04 | **4.48** |
| | DF0 | **1.98** | 8.80 | 3.23 | 9.02 | 4.18 | 8.25 | **3.13** | 6.79 |
| | BONUS | 2.13 | 7.91 | **3.16** | 8.67 | **4.16** | 8.10 | 3.16 | 6.70 |
| | Average | 4.17 | 6.13 | 5.13 | 6.36 | 5.12 | 6.57 | 4.75 | 5.55 |
| | Ranking | 3.60 | 6.27 | 4.02 | 6.51 | 4.65 | 6.79 | 4.32 | 5.64 |
| BIL | DF1 | 6.58 | **7.30** | 7.19 | **9.25** | 7.91 | **7.29** | 7.28 | **7.99** |
| | DF0 | 5.98 | 10.09 | 6.13 | 12.06 | 7.49 | 9.90 | 6.41 | 10.03 |
| | BONUS | **5.89** | 9.16 | **6.05** | 11.21 | **7.12** | 8.64 | **6.30** | 9.42 |
| | Average | 7.27 | 8.85 | 8.52 | 10.82 | 9.82 | 9.45 | 8.68 | 9.87 |
| | Ranking | 6.92 | 8.39 | 8.02 | 10.28 | 9.05 | 8.91 | 8.17 | 9.16 |
| CBA | DF1 | 5.53 | **6.60** | 6.17 | **6.89** | 5.55 | **5.49** | 6.54 | **6.76** |
| | DF0 | 4.32 | 8.54 | 4.81 | 8.59 | 4.88 | 7.56 | 5.96 | 8.51 |
| | BONUS | **4.23** | 7.55 | **4.69** | 7.87 | **4.63** | 6.51 | **5.39** | 7.01 |
| | Average | 7.44 | 8.89 | 8.28 | 9.15 | 7.46 | 7.06 | 8.85 | 9.07 |
| | Ranking | 6.83 | 8.16 | 6.84 | 7.85 | 6.29 | 6.28 | 7.65 | 8.12 |

**Table 5.3 Out-of-sample test on BONUS strategies (CML – NCPDP)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| CML | DF1 | 13.29 | **12.11** | 10.92 | **11.52** | 11.51 | **10.17** | 9.93 | **9.87** |
| | DF0 | 13.29 | **12.11** | 10.92 | **11.52** | 9.68 | 12.23 | 8.42 | 12.45 |
| | BONUS | **12.12** | 12.65 | **10.02** | 11.91 | **9.46** | 11.55 | **8.35** | 11.78 |
| | Average | 17.40 | 15.35 | 13.31 | 13.75 | 14.83 | 12.63 | 11.37 | 11.47 |
| | Ranking | 16.72 | 14.65 | 12.66 | 13.09 | 13.78 | 11.89 | 10.67 | 10.91 |
| FGL | DF1 | 9.89 | **6.22** | 11.09 | **5.82** | 11.11 | **6.37** | 10.55 | **7.20** |
| | DF0 | 7.80 | 10.37 | 9.74 | 11.71 | 9.73 | 9.06 | 9.74 | 9.90 |
| | BONUS | **7.61** | 9.52 | **9.22** | 9.11 | **9.56** | 8.53 | **9.46** | 8.87 |
| | Average | 8.58 | 7.99 | 10.14 | 7.43 | 10.35 | 7.69 | 10.38 | 7.91 |
| | Ranking | 8.47 | 8.10 | 9.94 | 7.65 | 10.26 | 7.74 | 10.28 | 7.96 |
| NAB | DF1 | 5.87 | **6.92** | 5.33 | **6.42** | 5.56 | **6.71** | 6.03 | **6.23** |
| | DF0 | 4.71 | 9.23 | 4.16 | 8.39 | 4.76 | 8.71 | 5.06 | 8.06 |
| | BONUS | **4.66** | 8.31 | **4.03** | 7.66 | **4.66** | 7.94 | **4.89** | 7.19 |
| | Average | 8.22 | 8.53 | 7.17 | 8.54 | 6.99 | 8.06 | 7.72 | 7.78 |
| | Ranking | 7.17 | 7.89 | 5.93 | 7.51 | 5.95 | 7.27 | 6.70 | 7.00 |
| NCP | DF1 | 6.08 | **9.08** | 5.54 | **7.64** | 6.76 | **8.18** | 6.25 | **8.43** |
| | DF0 | **4.71** | 11.63 | 4.40 | 10.06 | **4.95** | 10.54 | **4.65** | 10.92 |
| | BONUS | 4.72 | 11.49 | **4.39** | 9.69 | 4.96 | 10.44 | **4.65** | 10.86 |
| | Average | 7.17 | 10.30 | 6.89 | 8.64 | 8.18 | 9.10 | 7.62 | 9.34 |
| | Ranking | 6.44 | 10.08 | 5.81 | 8.47 | 7.04 | 8.95 | 6.42 | 9.26 |
| NCPDP | DF1 | 10.51 | **14.03** | 8.47 | **11.29** | 25.41 | **91.80** | 11.02 | **13.51** |
| | DF0 | 9.04 | 19.18 | 7.64 | 15.10 | 24.58 | 96.03 | 9.73 | 18.36 |
| | BONUS | **8.92** | 16.86 | **7.36** | 13.21 | **23.92** | 93.30 | **9.29** | 15.97 |
| | Average | 12.73 | 17.60 | 10.92 | 14.73 | 29.26 | 92.53 | 14.87 | 17.78 |
| | Ranking | 11.72 | 15.91 | 9.54 | 13.68 | 27.44 | 92.18 | 13.66 | 16.64 |

**Table 5.4 Out-of-sample test on BONUS strategies (PBL – WBC)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $C_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| PBL | DF1 | 15.79 | **17.70** | 21.06 | **18.65** | **16.19** | **14.84** | 18.58 | **15.97** |
| | DF0 | 15.79 | **17.70** | 21.06 | **18.65** | **16.19** | **14.84** | 18.58 | **15.97** |
| | BONUS | **15.15** | 17.86 | **20.82** | 18.70 | 16.28 | 14.96 | **18.12** | 15.98 |
| | Average | 21.27 | 23.17 | 27.08 | 22.99 | 22.20 | 24.05 | 24.44 | 20.05 |
| | Ranking | 20.35 | 21.88 | 25.58 | 21.74 | 20.87 | 22.24 | 22.55 | 18.49 |
| RIO | DF1 | 11.92 | 12.50 | 8.47 | 11.00 | 9.21 | 10.18 | 9.16 | 9.17 |
| | DF0 | 11.92 | 12.50 | 8.47 | 11.00 | 9.21 | 10.18 | 9.16 | 9.17 |
| | BONUS | **10.36** | **11.95** | **7.46** | **10.07** | **8.56** | **9.47** | **8.31** | **9.03** |
| | Average | 16.78 | 18.02 | 11.48 | 15.56 | 12.56 | 15.69 | 12.40 | 13.27 |
| | Ranking | 15.60 | 16.41 | 10.26 | 13.61 | 11.17 | 13.26 | 11.34 | 11.79 |
| SGB | DF1 | 20.11 | **15.48** | 22.00 | 20.22 | 29.01 | **33.33** | 18.19 | **14.71** |
| | DF0 | 20.11 | **15.48** | 22.00 | 20.22 | 29.01 | **33.33** | 18.19 | **14.71** |
| | BONUS | **19.20** | 15.50 | **21.36** | **19.81** | **28.39** | 33.92 | **17.57** | 15.13 |
| | Average | 26.32 | 20.51 | 28.89 | 25.64 | 35.98 | 38.50 | 24.78 | 20.02 |
| | Ranking | 24.65 | 19.57 | 27.51 | 24.73 | 34.67 | 37.47 | 23.73 | 19.34 |
| TLS | DF1 | 8.71 | **4.64** | 8.44 | **5.27** | 9.51 | **4.77** | 9.62 | **4.95** |
| | DF0 | 4.29 | 11.38 | 4.68 | 11.98 | 6.21 | 11.15 | **6.51** | 11.88 |
| | BONUS | **4.21** | 10.48 | **4.67** | 11.01 | **6.16** | 10.51 | 6.55 | 10.20 |
| | Average | 6.65 | 6.78 | 6.61 | 7.30 | 8.02 | 6.58 | 8.11 | 7.52 |
| | Ranking | 6.31 | 7.11 | 6.13 | 7.87 | 7.65 | 6.96 | 7.85 | 7.90 |
| WBC | DF1 | 8.01 | **9.25** | 6.18 | **7.42** | 6.33 | **12.11** | 5.24 | **6.09** |
| | DF0 | 7.00 | 12.09 | 5.21 | 9.47 | 5.49 | 13.39 | 4.41 | 7.84 |
| | BONUS | **6.73** | 10.61 | **5.11** | 8.66 | **5.45** | 13.08 | **4.32** | 7.33 |
| | Average | 9.92 | 10.99 | 7.58 | 9.40 | 7.59 | 12.78 | 6.28 | 7.68 |
| | Ranking | 9.23 | 10.41 | 6.86 | 8.55 | 6.83 | 12.42 | 5.65 | 7.11 |

**Table 5.5 Out-of-sample test on BONUS strategies (WES – WPL)**

| Shortfall (BPS) | | February 2002 | | March 2002 | | April 2002 | | May 2002 | |
|---|---|---|---|---|---|---|---|---|---|
| | | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ | $c_\mu$ | $c_\sigma$ |
| WES | DF1 | 33.88 | 26.22 | 33.47 | 25.54 | 44.90 | 64.44 | 28.51 | 24.23 |
| | DF0 | 33.88 | 26.22 | 33.47 | 25.54 | 44.90 | 64.44 | 28.51 | 24.23 |
| | BONUS | **31.89** | **25.38** | **30.98** | **25.19** | **42.30** | **64.00** | **24.96** | **23.15** |
| | Average | 44.53 | 31.87 | 44.79 | 31.74 | 57.67 | 67.87 | 40.50 | 32.16 |
| | Ranking | 42.99 | 31.33 | 42.38 | 30.84 | 55.82 | 67.65 | 38.16 | 31.03 |
| WFT | DF1 | 16.45 | **6.28** | 16.12 | **6.57** | 15.95 | **6.30** | 16.13 | **5.66** |
| | DF0 | 16.33 | 10.06 | 15.86 | 7.50 | 15.69 | 7.08 | 15.40 | 9.94 |
| | BONUS | **15.64** | 8.29 | **15.60** | 7.76 | 15.60 | 7.17 | **15.32** | 8.84 |
| | Average | 15.98 | 7.89 | 15.69 | 8.06 | **15.52** | 7.38 | 15.66 | 8.45 |
| | Ranking | 15.98 | 7.91 | 15.70 | 8.07 | 15.53 | 7.39 | 15.65 | 8.45 |
| WMC | DF1 | 9.66 | **10.53** | 8.45 | **9.52** | 7.32 | **7.03** | 8.42 | **8.17** |
| | DF0 | 9.66 | **10.53** | 8.13 | 12.66 | 6.98 | 9.22 | 8.35 | 11.15 |
| | BONUS | **8.91** | 11.01 | **7.67** | 11.19 | **6.66** | 7.64 | **7.81** | 9.07 |
| | Average | 12.00 | 13.52 | 10.27 | 11.18 | 9.16 | 9.02 | 10.30 | 10.17 |
| | Ranking | 11.40 | 12.69 | 9.66 | 10.47 | 8.57 | 8.37 | 9.55 | 9.39 |
| WOW | DF1 | 8.55 | **9.16** | 9.78 | **12.58** | 10.17 | **9.90** | 8.13 | **9.27** |
| | DF0 | 8.55 | **9.16** | 9.62 | 15.99 | 10.06 | 12.36 | 7.85 | 12.87 |
| | BONUS | **7.79** | 9.44 | **8.78** | 13.76 | **9.28** | 10.44 | **7.12** | 10.11 |
| | Average | 10.63 | 10.92 | 12.44 | 15.09 | 12.23 | 11.90 | 10.38 | 12.03 |
| | Ranking | 10.06 | 10.33 | 11.48 | 14.11 | 11.50 | 11.25 | 9.79 | 11.26 |
| WPL | DF1 | 9.94 | **10.11** | 10.99 | **10.59** | 11.80 | **12.25** | 12.26 | 11.96 |
| | DF0 | 9.94 | **10.11** | 10.72 | 13.28 | 12.70 | 17.16 | 12.26 | 11.96 |
| | BONUS | **8.97** | 10.52 | **9.73** | 11.58 | **10.53** | 13.47 | **11.20** | **11.63** |
| | Average | 12.26 | 12.50 | 13.79 | 12.69 | 16.20 | 17.24 | 17.62 | 18.31 |
| | Ranking | 11.61 | 11.74 | 12.96 | 12.07 | 14.97 | 15.79 | 15.66 | 15.98 |

# 5.3 Summary

This chapter has applied the DAB technique to trade execution and proposed the BONUS strategy. The DAB technique acquires DF strategies from a candidate DF strategy set and determines their weights. The BONUS strategy allocates the entire order to each acquired DF strategy in proportion with its weight. The acquired DF strategies are then executed in parallel to fill their allocated order. Theoretical analysis shows that the BONUS strategy moves the candidate DF strategies' efficient frontier toward the favorable direction. In other words, as the learning steps increase, the BONUS strategy based on the DAB technique lowers the candidate DF strategies' risk that can be measured by different standards (e.g. the k-UPM of shortfalls and the maximum of shortfalls) and limits the increase in average shortfall. In-sample test on 80 datasets strongly supports the theoretical analysis on the BONUS strategy. Out-of-sample test shows that the DAB technique and the BONUS strategy achieve lower average shortfall the DF strategy and two simple diversification techniques do. In further, the DAB technique, as a meta-type method, can be applied to implementing diversification across strategies in more practices.

# Chapter 6

# Conclusions and Future Work

This thesis has verified the rationale and effectiveness of diversification in the framework of developing trade execution strategies from the theoretical and empirical perspectives. The philosophy of diversification connects all research parts together in this thesis. This thesis is contributed to both finance and computer science fields. Three major contributions compose three research parts of this thesis: the DF strategy, the DAB technique and the BONUS strategy.

In the first part, this thesis proposes the DF strategy that reflects the idea of diversification across time. The DF strategy incorporates a series of small market order with different volume into the limit order strategy and dynamically adjusts each market order volume based on two real-time state variables "inventory" and "order book imbalance". The sigmoid function is adopted to map the state variable to the market order volume. The empirical results on a lot of real-life data show that the DF strategy achieves lower cost and risk brought by the limit order strategy does.

In the second part, this thesis proposes the DAB technique that reflects the idea of diversification across strategies. The DAB technique extends the key idea of adaptively updating the probability distribution in the AdaBoost technique so that it is not just applicable to supervised learning. The DAB technique acquires strategies from a candidate strategy set and determines their weights. The resources are allocated to each acquired strategy in proportion with its weight and all acquired strategies are then executed in parallel with their allocated resources. The DAB technique allows the candidate strategy set to contain infinite strategies. The theoretical analysis shows that as the learning steps increase, the DAB technique lowers the candidate strategy set's risk that can be measured by different standards and limits the decrease in its average reward. From the perspective of the candidate strategy set's efficient frontier, the DAB technique moves it toward the favorable direction.

In the third part, the DAB technique is applied to acquire DF strategies from a candidate DF strategy set and determine their weights. The entire order is allocated to each acquired DF strategy in proportion with its weights and all acquired DF strategies are then executed in parallel to fill their allocated order. The parallel execution is called the BONUS strategy. The empirical results on real-life data strongly support the theoretical conclusion on the DAB technique and the BONUS strategy, i.e. the BONUS strategy moves the candidate DF strategy set's efficient frontier toward the favorable direction as the learning steps increase. The empirical results also show that the BONUS strategy based on the DAB technique achieves lower cost and risk than the optimal DF strategy and two simple diversification techniques do.

The thesis can be further extended from the following several aspects. First, while the DF strategy dynamically adjusts the volume of each market order, it also can be extended to dynamically adjust the limit order volume. Due to the uncertainty of price movement, a limit order may incur adverse price selection while it is waiting for favorable price movement in future. Moreover, a large limit order placed in the market will disclose trading intention. Dynamic volume adjustment of the limit order may be helpful for controlling its adverse price selection and limiting information disclosure.

Second, dynamic price adjustment is suggested to combine with dynamic volume adjustment in trade execution. As two most important attributes of an order in trade execution, price and volume are complementary to each other and both of them should be considered in practice. This combination may be reasonable and advisable for trade execution.

Third, a problem called data-snooping has been pointed out in the finance field (Lo and MacKinlay 1990, Sullivan et al. 1999). This problem is about whether a parameter optimized on in-sample test can be well generalized in out-of-sample test. It is also called the problem of overfitting in the statistical learning field (Vapnik 1999). The problem of overfitting in the DAB technique should be further discussed from the theoretical perspective as some parameters need to be determined in this technique.

Fourth, the target reward (or shortfall) should be related to benchmarks in practice such as historical average of investment returns (or implementation shortfalls). So the DAB technique's effectiveness should be further verified through setting these practical benchmarks for the target reward (or shortfall).

Fifth, the value of k in k-LPM (k-UPM) represents the degree of risk aversion or tolerance in practice. Except the value "2" used in this thesis, more values should be testified to verify the DAB technique's effectiveness.

Sixth, the AdaBoost technique's performance is affected by the candidate learning model set (Meir and Rätsch 2003). In the two-dimensional XOR problem, the classification error rate decreases to zero after four learning steps. However, it cannot decrease to zero if the candidate classifier set is only composed of vertical lines. As a meta-type method, the DAB technique's performance is also related to the candidate strategy set. So it will be helpful to exploiting the effect of the candidate strategy set on the DAB technique.

Seventh, this thesis applies the DAB technique to boost the DF strategies for trade execution and the boosted DF strategy is named the BONUS strategy. In future work, the DAB technique is expected to boost strategies in more applications such as financial investment and inventory management.

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