Symmetry of modes in coupled photonic crystal waveguides

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Coupled waveguide (CW) geometries are important in a range of optical devices including directional couplers whose key element comprises two waveguides spaced sufficiently closely to facilitate an exchange of energy. CWs have been studied in both conventional guided wave structures[1] and photonic crystals[2-4]—the latter receiving recent attention with the claim that short coupling lengths (the length over which energy couples completely between the guides) can be achieved, heralding the development of compact devices.

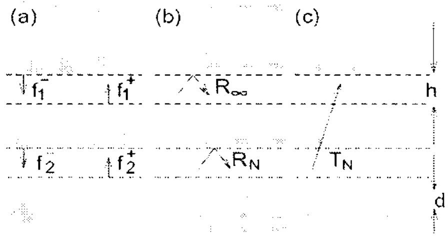

Fig. 1. Schematics of the geometries considered: (a) planar waveguide, (b) one-dimensional layered structure, and (c) two dimensional photonic crystal with square lattice. For each, the electric field is orthogonal to the plane, and the mode propagates in the plane. Dashed lines indicate the waveguide edges.

In recent literature, there has been some debate about the symmetry of the bound modes of photonic crystal waveguides (PCW). While the situation for conventional waveguides—which exhibit an even fundamental mode and an odd second mode[1]—is well understood, the situation for PCWs is less clear. Recently Boscolo *et al*[3] proposed that the fundamental coupled waveguide mode (CWM) is always even, as in planar structures. In this paper, however, we show this is not true in general, and that for such structures the fundamental CWM can be either even or odd depending upon the spacing of the guides.

Our derivation of this result uses a common theoretical framework in which we consider the three geometries of Fig. 1: (a) a conventional planar CW, (b) a CW in a layered Bragg structure (of period d) and (c) CWs in a 2D square symmetric photonic crystal lattice of period d. The analysis of these three structures proceeds in a similar manner. In each of the guides, designated as 1, 2 in Fig. 1, we represent the fields by plane wave expansions

$$E_j(\vec{r}) = \sum_{p=-\infty}^{\infty} \chi_p^{-\frac{1}{2}} \left[f_j^- e^{-i\chi_p(y-y_j^-)} + f_{jp}^+ e^{+i\chi_p(y-y_j^+)} \right] e^{i\beta_p x} \quad (1)$$

In (1), the upward and downward fields in each the guides (j) are denoted by vectors of plane wave coefficients $f_j^\pm = \left[f_{jp}^\pm \right]$, with the phase origins of the fields set at $y = y_j^\pm$ denoted by the dashed lines in Fig. 1. Note that for the structures in Figs 1(a) and 1(b), we use only the specular term ($p = 0$) of (1), while the full series is needed for the 2D crystal of Fig. 1(c). In general, we treat the structure as a stack of diffraction gratings and accordingly, in (1), the direction sines and cosines are $\beta_p = \beta_0 + 2\pi p/d$ and $\chi_p = \sqrt{k^2 - \beta_p^2}$, where $k = 2\pi/\lambda$ is the free space wave number. In a practical implementation of the theory, the field series are truncated and we characterise the action of the individual elements by scattering matrices. For example, R_N and T_N, the reflection and transmission scattering matrices that characterize the properties of the barrier between the guides comprising N layers, contain elements such as $R_{N:pq}$ which denotes the amplitude reflected into order p due to unit amplitude incidence in order q (associated with the direction sine β_q). Similarly, R_∞ is the matrix that characterises the reflection off the semi-infinite cladding of the guides[5,6]. In the case of planar structures in Figs. 1(a) and 1(b), all scattering matrices reduce to a scalar since the plane wave expansions contain only the specular order ($p=0$).

The fields in Fig. 1 are related by

$$f_1^- = R_\infty P f_1^+ , \quad f_1^+ = R_N P f_1^- + T_N P f_2^+ , \quad f_2^+ = R_\infty P f_2^- , \quad f_2^- = T_N P f_1^- + R_N P f_2^+ \tag{2}$$

where $P = \text{diag}[e^{i\chi_p h}]$ is the (diagonal) scattering matrix that propagates the field between the dashed lines in Figs. 1 over the guide of width h. Then solving Eqs. (2), we show that

$$f_1^- = U f_2^+, \quad f_2^+ = U f_1^-, \quad \text{where } U = (I - R_\infty P R_N P)^{-1} R_\infty P T_N P \tag{3}$$

where I is the identity matrix. It then follows from (3) that $(I-U^2)f_1^- = (I-U^2)f_2^+ = 0$, or that

$$(I-\sigma U)f_1^- = 0 \text{ where } I-\sigma U = (I - R_\infty P R_N P)^{-1}\left[I - R_\infty P(R_N + \sigma T_N)P\right] \tag{4}$$

with $\sigma = \pm 1$ and $f_2^+ = \sigma f_1^-$. Thus, for a symmetric structure, the modes must be either even ($\sigma = +1$) or odd ($\sigma = -1$). Now, from the results of Botten *et al*[5,6], it is known that

$$R_N = (R_\infty - Q^N R_\infty Q^N)(I - R_\infty Q^N R_\infty Q^N)^{-1}, \quad T_N = (I - R_\infty^2) Q^N (I - R_\infty Q^N R_\infty Q^N)^{-1}, \tag{5}$$

where the matrix Q expresses the propagation of the Bloch functions of the periodic structure through a layer of the PC. Since the Bloch functions are the eigenfunctions of the structure, Q is a function of the eigenvalues $\{\mu_i\}$ of the translation operator. For propagating Bloch functions, the eigenvalue μ_i lies on the unit circle, whereas for evanescent modes, $|\mu|<1$. Thus, for a mode of the CW to exist, we require that $(I-\sigma U)$ is singular, i.e.,

$$\det\left[I - R_\infty P(R_\infty + \sigma Q^N)(I + \sigma R_\infty Q^N)^{-1} P\right] = 0. \tag{6}$$

From this point forwards, the details of the exact treatments for the three geometries of Fig. 1 differ as the actual form of the fields become significant. However, for long wavelengths, there is an approximation that enables all three structures to be handled in an entirely equivalent manner. In this regime, the physics is dominated by the specular diffracted order $(p=0)$, with all evanescent plane waves being dropped from the calculation. This approximation, which is valid for frequencies in most of the first band gap of the 2D crystal, enables the matrix formulation (6) to be reduced to scalar form

$$R_\infty^2 P^2 \frac{1+\sigma\mu^N / R_\infty}{1+\sigma\mu^N R_\infty} = 1 \tag{7}$$

Here, μ, which is can be shown to be real, represents the decay of the field with propagation into the crystal over a lattice period. If $\mu < 0$ the field changes sign after a period, similar to the behaviour of the field at the edge of the Brillouin zone, while if $\mu > 0$, the sign of the field over a period is unchanged, similar to the behaviour at the Brillouin zone centre. The planar structure in Fig. 1(a) is of arbitrary periodicity and is associated with $\mu > 0$, since the evanescent field, due to total internal reflection (TIR), does not change sign. Noting that that $|R_\infty| = 1$, as occurs for TIR for the conventional waveguide in Fig. 1(a), or for band gap guiding in the PC structures of Figs. 1(b,c), it follows that the dispersion equation (7) can be recast in the simple form

$$\chi_0 h + \arg(R_\infty + \sigma\mu^N) = m\pi, \tag{8}$$

where m is an integer and χ_0 is the direction cosine of the sole propagating order. The term $\sigma\mu^N$ characterises the effect due to the barrier between the guides. Note that when the guides are widely spaced (i.e., $N \to \infty$), the term $\sigma\mu^N \to 0$ and the dispersion equation for a single guide materialises.

As is evident from (8), the key to understanding the symmetry of the modes lies in the sign of the quantity $\sigma\mu^N$. When $\mu > 0$, the sign of $\sigma\mu^N$ is determined by that of σ, while for $\mu < 0$, the sign depends also on the width N of the barrier. To grasp the significance of this, consider the planar geometry in Fig. 1(a). Since this relies on TIR, $\mu > 0$ and $-\pi < \arg(R_\infty) < 0$. Thus for the symmetric mode ($\sigma > 0$), $\arg(R_\infty + \sigma\mu^N)$ increases from its value for a single guide with the consequence that χ_0 must decrease and β_0 must increase. In contrast, for the odd mode ($\sigma < 0$), β_0 must decrease. Since the fundamental mode is that having the largest propagation constant for a given frequency, it follows that it is even while the second mode is odd.

For the 1D PC of Fig. 1(b), the sign of μ varies: in the fundamental gap, and all odd number band gaps, $\mu < 0$ while in all even number band gaps, $\mu > 0$. Thus, if $-\pi < \arg(R_x) < 0$, it follows that in the even gaps the fundamental CWM is always even. However, in odd numbered gaps, the fundamental mode can be either odd or even according to whether N is odd or even.

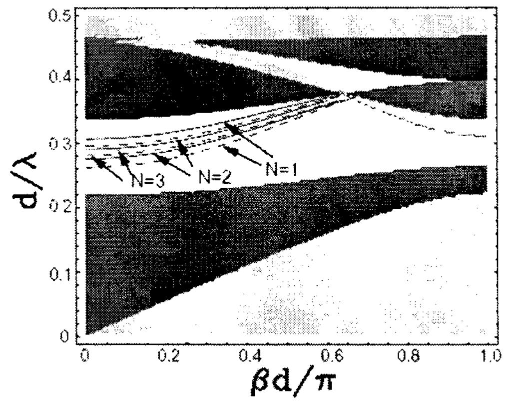

Fig. 2 Projected band structure for a 2D bulk PC with a square lattice (of lattice constant d) of inclusions of radius $a = 0.3d$ and index $n = 3$ in a background of $n = 1$. Dark shaded regions indicate bands, white regions indicate gaps with $\mu < 0$ and light-shaded regions indicate gaps with $\mu > 0$. CWM dispersion relations are also given for even (solid) and odd modes (dashed), for twin guides of width $h = d$ for barriers of width N = 1, 2, 3.

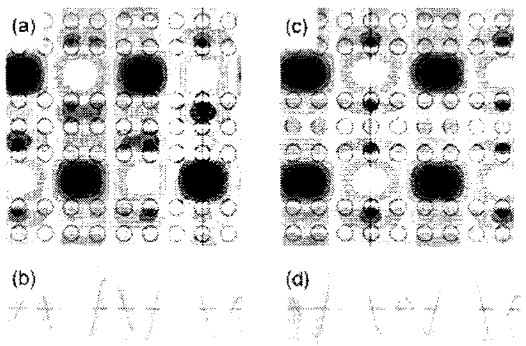

Fig. 3. Electric field in a 2D PC with two coupled waveguides. (a), (b) odd CWM; (c), (d) even CWM. (a), (c) electric field contours—light and dark regions correspond to positive and negative phase; (b) and (d) field profiles through vertical lines in (a) and (c), respectively. The dark circles indicate the cylinders.

The situation is similar for the 2D PC structure of Fig. 1(c) since in much of the lowest gap, where the scalar approximation is valid, $\mu < 0$ as illustrated in Fig. 2. This figure, computed numerically without approximations[5], shows the projected band structure of a 2D PC and also the CWM dispersion relations for the odd and even modes.

Figs. 3 show the electric field for the structure of Fig. 2 with $N = 3$ for $\lambda = 3.05d$. Figs. 3(a,b) refer to the odd CWM with $\mu = -0.465$ and propagation constant β_{odd}=1.353, while Figs. 3(c,d) refer to the even CWM, for which $\mu = -0.487$ and β_{even}=1.239.

In conclusion, the fundamental CWM in the lowest gap of a photonic crystal may be either even or odd, depending on the phase change on ref lection off the bulk photonic crystal, the sign of the dominant evanescent eigenvalue and the separation of the guides. We add that this does not affect the operation of directional couplers which depends only on the beat length $2\pi / |\beta_{odd} - \beta_{even}|$ of the CWMs. This theoretical framework also allows us to interpret interesting phenomena such as the crossing of CWMs in a higher band gap, attributable to a transition between regions of positive and negative μ. Although the results here refer to a specific polarization, we have found qualitatively similar results for modes with other polarizations.

This work was produced with the assistance of the Australian Research Council under the ARC Centres of Excellence program.

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