

A decision support system prototype for fuzzy multiple objective optimization

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Abstract

An interactive fuzzy multiple objective optimization method was proposed for solving fuzzy multiple objective linear programming problems where fuzzy parameters in both objective functions and constraints and fuzzy goals of objectives can be in any form of membership function. Based on the method, in this paper, a fuzzy multiple objective decision support system prototype is developed. A detailed description of the method and system are then supplied.

Keywords: Fuzzy number, Optimization, Fuzzy multiple objective linear programming, Decision support systems.

1 Fuzzy Multiple Objective Linear Programming

Multiple objective linear programming (MOLP) is one of the popular approaches used to deal with some complex and ill-structured decision-making situations. Due to the unavoidable influence of uncertainty, MOLP models must take into account vague information, imprecise requirements, and modifications of the original input data from the modeling phase. During the MOLP problem formulation process, the possible values of the parameters in the objective functions and constraints may be assigned in an experimental, a statistical or a subjective manner through some experts' understanding to the decision problem. With this observation, the values of these parameters are often imprecisely or ambiguously understood to the experts, and are difficult to be determined by precise values rather than fuzzy numbers [1] [5].

In this paper, the Fuzzy MOLP (FMOLP) problem in which all coefficients of the objective functions and constraints are real fuzzy numbers is considered. Then the FMOLP problems can be formulated as follows:

$$\left. \begin{array}{l} \max \tilde{f}(x) = \tilde{C}x \\ \text{s.t. } x \in X = \{x \in R^n \mid \tilde{A}x \preceq \tilde{b}, x \geq 0\} \end{array} \right\} \quad (1)$$

where \tilde{C} is an $k \times n$ matrix, each element of which \tilde{c}_{ij} is fuzzy number with membership function $\mu_{\tilde{c}_{ij}}(x)$, \tilde{A} is an $m \times n$ matrix, each element of which \tilde{a}_{ij} is fuzzy number with membership function $\mu_{\tilde{a}_{ij}}(x)$, \tilde{b} is an m -vector, each element of which \tilde{b}_i is fuzzy number with membership function $\mu_{\tilde{b}_i}(x)$, and x is an n -vector of decision variables, $x \in R^n$.

2 An Interactive Fuzzy Multiple Objective Optimization Method

Many optimization methods and techniques for modeling and solving FMOLP problems have been proposed [2-6]. Wu et al. [7] [8] also developed some algorithms and methods for solving FMOLP problems. Based on these algorithms and methods, an interactive fuzzy multiple objective optimization (IFMOO) method for solving FMOLP problems with fuzzy parameters and fuzzy goals under the different satisfactory degree α . was developed. The main structure of the IFMOO method which includes two stages with six steps is outlines as following:

Stage 1: Initialization

Step 1: Set up FMOLP model, including inputting the membership function of each fuzzy parameter in the model.

Step 2: Ask the decision maker to select satisfactory degree α ($0 \leq \alpha \leq 1$) and the individual weight for

fuzzy objective functions. Under the current degree α and weights, solve the FMOLP problem..

Step 3: Ask the decision maker whether to be satisfied with the initial fuzzy Pareto optimal solution calculated in Step 2. If satisfied, the whole interactive process stops, and the initial solution is to be the final satisfying solution. Otherwise, go to Stage 2.

Stage 2: Iterations

Step 4: Specify new fuzzy goals $\tilde{g} = (\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_k)^T$ for the fuzzy objective functions based on the current fuzzy Pareto optimal solution and new degree α if needed.

Step 5: Calculate the fuzzy Pareto optimal solution based on the current fuzzy goals of objective functions and degree α specified in Step 4.

Step 6: If the decision maker is satisfied with the solution calculated in Step 5, the whole interactive process stops, and the current fuzzy Pareto optimal solution is the final satisfying solution of FMOLP problem. Otherwise, go back to Step 4 for another iteration.

3 Fuzzy Multiple Objective Decision Support System

From the system designer’s point of view, the fuzzy multiple objective decision support system (FMODSS) developed consists of four major software components, which are input-and-display component, model management component, optimization component and data management component. All the flows of data within the system, such as models, criteria and alternative definition and values, intermediate and/or final results, even the data from the external sources, will be store in the database by the data management component.

According to the FMOLP model (1) in Section 1 and the method in Section 2, through the input-and-display component, the following common data need to be input from the user for setting up model and other initial data for system .

- The number of decision variables, the number of fuzzy objective functions, and the number of fuzzy constraints;
- The coefficients of fuzzy objective functions, the max/min for individual fuzzy objective function, as showed in Figure 1;

- The coefficients of fuzzy constraints, the coefficients of right-hand-sides, the relation sign of individual fuzzy constraint, as showed in Figure 1;
- A Dialog Box as showed in Figure 2 is designed and generated specially for entering fuzzy numbers. Referring to the fuzzy number to be entered, the forms of left continuous increasing function and right continuous decreasing function of a fuzzy numbers can be selected as linear, quadratic, cubic, exponential, logarithmic, and other form from the dropdown lists, and four end-points of left and right function of fuzzy numbers are entered in the textboxes simultaneously.

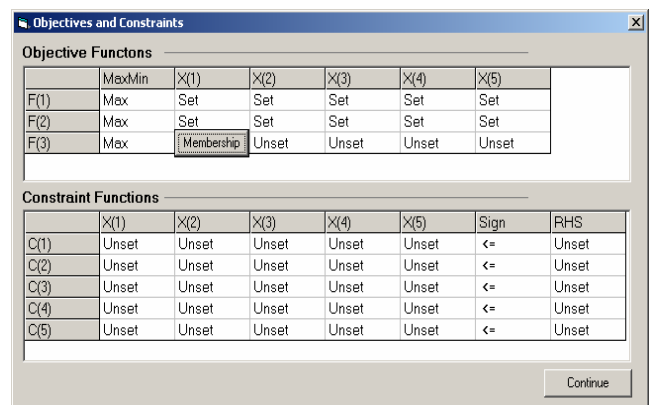


Figure 1: Input window for fuzzy objective functions and fuzzy constraints

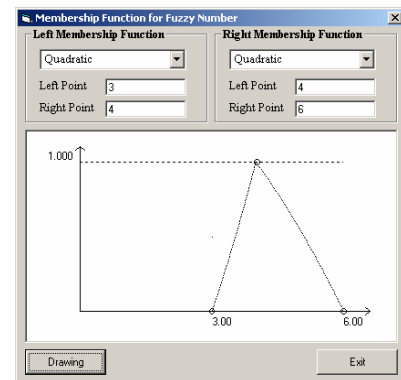


Figure 2: Input window for the membership function of a fuzzy number

An important component of the system is model management. Generally, the model management component is combined with the database management component and provides facilities for the definition, storage, retrieval and execute of a wide range of models. It also gives the opportunity for the user to specify or build entirely new model

by using a model-building facility associated with input-and-display component.

The optimization component of FMODSS presently includes IFMOO method to satisfy the flexibility required of the system. In window as shown in Figure 4, the different weights for fuzzy objective functions can be entered in FlexGrid 1, and the degree of all of the membership functions of the fuzzy numbers can also be set by the slider as well.

Following the IFMOO method described in Section 2, click Button “Initiate”, the initial solution of the problem will be shown as decision variables and fuzzy objective functions in FlexGrid 2 and 3 respectively. To display the membership functions of the fuzzy objective function output, click the corresponding grid in the first row “Objectives” of FlexGrid 3, then another window will be pop up similarly as shown in Figure 3.

When not satisfied with the solution calculated in the previous trial, the decision maker can specify the fuzzy goals to be achieved in the next trial by two ways. One is by increasing or decreasing the previous individual fuzzy objective function solution by percentage through clicking the corresponding grid in the second row “By %” of FlexGrid 3 in Figure 4. The other is by inputting the new fuzzy goals. In order to input the fuzzy goals which are represented by fuzzy numbers, click the corresponding grid in the third row “By value” of FlexGrid 3 in Figure 4, then another Window “Fuzzy Goal Input” similar as shown in Figure 2 will appear. After the new fuzzy goals being created, click on Button “Continue” for getting new solution at the current trial.

The optimal solutions for each trial during the interactive decision-making process are recorded and listed in FlexGrid 4 in Figure 4.

At the time pressing Button “Satisfy”, the final solution of the problem will display and be stored in the database through the data management component. The interactive decision-making process ends at this iteration.

4 An illustrative example

To illustrate the system developed, let us consider the following FMOLP problem with three fuzzy objective functions and five fuzzy constraints:

$$\begin{aligned} \max \begin{pmatrix} \tilde{f}_1(x) \\ \tilde{f}_2(x) \\ \tilde{f}_3(x) \end{pmatrix} &= \max \begin{pmatrix} \tilde{c}_{11}x_1 + \tilde{c}_{12}x_2 + \tilde{c}_{13}x_3 + \tilde{c}_{14}x_4 + \tilde{c}_{15}x_5 \\ \tilde{c}_{21}x_1 + \tilde{c}_{22}x_2 + \tilde{c}_{23}x_3 + \tilde{c}_{24}x_4 + \tilde{c}_{25}x_5 \\ \tilde{c}_{31}x_1 + \tilde{c}_{32}x_2 + \tilde{c}_{33}x_3 + \tilde{c}_{34}x_4 + \tilde{c}_{35}x_5 \end{pmatrix} \quad (2) \\ &= \max \begin{pmatrix} \tilde{1}x_1 + \tilde{9}x_2 + \tilde{10}x_3 + \tilde{1}x_4 + \tilde{3}x_5 \\ \tilde{9}x_1 + \tilde{2}x_2 + \tilde{2}x_3 + \tilde{7}x_4 + \tilde{4}x_5 \\ \tilde{4}x_1 + \tilde{6}x_2 + \tilde{7}x_3 + \tilde{4}x_4 + \tilde{8}x_5 \end{pmatrix} \\ \text{s.t.} \quad &\begin{cases} \tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 + \tilde{a}_{13}x_3 + \tilde{a}_{14}x_4 + \tilde{a}_{15}x_5 \\ \quad = \tilde{3}x_1 + \tilde{9}x_2 + \tilde{9}x_3 + \tilde{5}x_4 + \tilde{3}x_5 \leq \tilde{b}_1 = 1039 \\ \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 + \tilde{a}_{23}x_3 + \tilde{a}_{24}x_4 + \tilde{a}_{25}x_5 \\ \quad = -\tilde{4}x_1 - \tilde{1}x_2 + \tilde{3}x_3 - \tilde{3}x_4 - \tilde{2}x_5 \leq \tilde{b}_2 = 94 \\ \tilde{a}_{31}x_1 + \tilde{a}_{32}x_2 + \tilde{a}_{33}x_3 + \tilde{a}_{34}x_4 + \tilde{a}_{35}x_5 \\ \quad = \tilde{3}x_1 - \tilde{9}x_2 - \tilde{9}x_3 - \tilde{4}x_4 - \tilde{0}x_5 \leq \tilde{b}_3 = 61 \\ \tilde{a}_{41}x_1 + \tilde{a}_{42}x_2 + \tilde{a}_{43}x_3 + \tilde{a}_{44}x_4 + \tilde{a}_{45}x_5 \\ \quad = \tilde{5}x_1 + \tilde{9}x_2 + \tilde{10}x_3 - \tilde{1}x_4 - \tilde{2}x_5 \leq \tilde{b}_4 = 924 \\ \tilde{a}_{51}x_1 + \tilde{a}_{52}x_2 + \tilde{a}_{53}x_3 + \tilde{a}_{54}x_4 + \tilde{a}_{55}x_5 \\ \quad = \tilde{3}x_1 - \tilde{3}x_2 + \tilde{0}x_3 + \tilde{1}x_4 + \tilde{5}x_5 \leq \tilde{b}_5 = 420 \\ x_1 \geq 0; \quad x_2 \geq 0 \end{cases} \end{aligned}$$

In this model, the unified form of all membership functions of the parameters of are assumed to be as following:

$$\mu_{c_{ij}}(x) = \begin{cases} 0 & x < a \text{ or } d < x \\ (x^2 - a^2)/(b^2 - a^2) & a \leq x < b \\ 1 & b \leq x \leq c \\ (d^2 - x^2)/(d^2 - c^2) & c < x \leq d \end{cases} \quad (3)$$

where left and right membership function are all quadratic. For simplicity, we will represent the above form of membership function as quadruple pair (a, b, c, d) in latter on. For the FMOLP model, all membership functions of the fuzzy parameters are to be represented in quadruple pair form and listed in Table 1, 2 and 3.

According to the description of IFMOO method listed in Section 2, the IFMOO method involves two stages with six steps. The interactive process for solving the problem by FMODSS is supposed to be as following:

Stage 1: Initialization

Step 1: Initially, the FMOLP model (2) of the problem will be input into the system. The window for entering the model is showed in Figure 1. The information includes the max/min of individual objective functions that are all set to max, and the relationship involved in constraints that are all set as

“equal or less than”. In order to enter fuzzy coefficients, click on the corresponding grid in the table, and then another Dialog Box will pop up as showed in Figure 2. All fuzzy coefficients as listed in Table 1, 2 and 3 are to be entered through Window as in Figure 1 and Dialog Box as in Figure 2 sequentially.

Table 1: The membership functions of fuzzy objective functions' parameters

\tilde{c}_{ij}	1	2	3	4	5
1	(0.5,1,1,2)	(8,9,9,11)	(9,10,10,12)	(5,1,1,2)	(2,3,3,5)
2	(8,9,9,11)	(1,2,2,4)	(1,2,2,4)	(6,7,7,9)	(3,4,4,6)
3	(3,4,4,6)	(5,6,6,8)	(6,7,7,9)	(3,4,4,6)	(7,8,8,10)

Table 2: The membership functions of fuzzy constraints' parameters

\tilde{a}_{ij}	1	2	3	4	5
1	(2,3,3,5)	(8,9,9,11)	(8,9,9,11)	(4,5,5,7)	(2,3,3,5)
2	(-6,-4,-4,-3)	(-2,-1,-1,-0.5)	(2,3,3,5)	(-5,-3,-3,-2)	(-4,-2,-2,-1)
3	(2,3,3,5)	(-11,-9,-9,-8)	(-11,-9,-9,-8)	(-6,-4,-4,-3)	(0,0,0,0)
4	(4,5,5,7)	(8,9,9,11)	(9,10,10,12)	(0.5,1,1,2)	(-4,-2,-2,-1)

Table 3: The membership functions of right-hand-side's parameters

\tilde{b}_i	1
1	(1038, 1039, 1039, 1041)
2	(93, 94, 94, 96)
3	(60, 61, 61, 63)
4	(923, 924, 924, 926)
5	(419, 420, 420, 422)

Step 2: After finished establishing FMOLP model, the decision maker will switch to the window as shown in Figure 4 to solve the problem. Suppose the satisfactory degree α is set to 0.25. Each weight for three fuzzy objective functions is all equally set to 0.333. Click the Button “Initiate”, the initial compromise solution to the FMOLP model is generated. The decision variables are:

$$x_1^* = 62.39, x_2^* = 53.60, x_3^* = 0, x_4^* = 1.76, x_5^* = 38.92$$

and the fuzzy objective functions are

$$\begin{cases} \tilde{f}_1^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 62.39\tilde{c}_{11} + 53.6\tilde{c}_{12} + 1.76\tilde{c}_{14} + 38.92\tilde{c}_{15} \\ \tilde{f}_2^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 62.39\tilde{c}_{21} + 53.6\tilde{c}_{22} + 1.76\tilde{c}_{24} + 38.92\tilde{c}_{25} \\ \tilde{f}_3^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 62.39\tilde{c}_{31} + 53.6\tilde{c}_{32} + 1.76\tilde{c}_{34} + 38.92\tilde{c}_{35} \end{cases} \quad (4)$$

By clicking the corresponding grids in the first row “Objectives” of FlexGrid 3 in Figure 4 one by one,

the membership functions of the fuzzy objective functions solution $\tilde{f}_1^*, \tilde{f}_2^*$ and \tilde{f}_3^* (4) are shown as in Figure 3, and the initial solution is logged and listed in the first row ‘Trial 1’ of FlexGrid 4.

Step 3: Suppose the decision maker is not satisfied with the initial solution calculated in Step 2, the interactive process will continue to Stage 2.

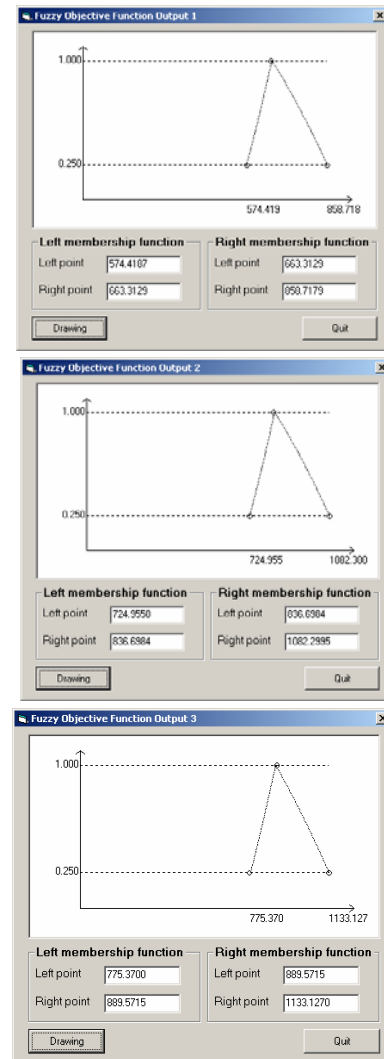


Figure 3: The membership functions of $\tilde{f}_1^*, \tilde{f}_2^*$ and \tilde{f}_3^* at Trial 1

Stage 2: Iterations

Iteration No 1:

Step 4: In this step, the decision maker specifies new fuzzy goals for the fuzzy objective functions to be achieved. Suppose these new fuzzy goals are assigned by decreasing both of the first and third fuzzy objective functions by 5% as the first and third fuzzy goals respectively, and increasing the second fuzzy objective function by 5% as the second fuzzy goals based on the initial solution at Stage 1. That is:

$$\begin{pmatrix} \tilde{g}_1 \\ \tilde{g}_2 \\ \tilde{g}_3 \end{pmatrix} = \begin{pmatrix} 0.95 * \tilde{f}_1^*(x_1^*, x_2^*, x_3^*) \\ 1.05 * \tilde{f}_2^*(x_1^*, x_2^*, x_3^*) \\ 0.95 * \tilde{f}_3^*(x_1^*, x_2^*, x_3^*) \end{pmatrix} \quad (5)$$

By clicking the corresponding grid in the second row ‘‘By %’’ of FlexGrid 3 in Figure 4, one textbox will appear, and the increasing and decreasing numbers are filled in these textboxes one by one.

Step 5: Pressing Button ‘‘Continue’’, the new solution to the FMOLP problem based on achieving the fuzzy goals (5) is generated. Consequently, the decision variables are

$$x_1^* = 64.84, x_2^* = 50.28, x_3^* = 0.0, x_4^* = 8.71, x_5^* = 34.14,$$

and the fuzzy objective functions are

$$\begin{cases} \tilde{f}_1^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 64.84\tilde{c}_{11} + 50.28\tilde{c}_{12} + 8.71\tilde{c}_{14} + 34.14\tilde{c}_{15} \\ \tilde{f}_2^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 64.84\tilde{c}_{21} + 50.28\tilde{c}_{22} + 8.71\tilde{c}_{24} + 34.14\tilde{c}_{25} \\ \tilde{f}_3^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 64.84\tilde{c}_{31} + 50.28\tilde{c}_{32} + 8.71\tilde{c}_{34} + 34.14\tilde{c}_{35} \end{cases} \quad (6)$$

The membership functions of \tilde{f}_1^* , \tilde{f}_2^* and \tilde{f}_3^* (6) are displayed as in Figure 5. The new solution is also logged and listed in the second row ‘‘Trial 2’’ of FlexGrid 4 as in Figure 4.

Step 6: Suppose the decision maker is also not satisfied with the solution calculated in Step 5, the interactive process will carry on to next iteration and go back to Step 4.

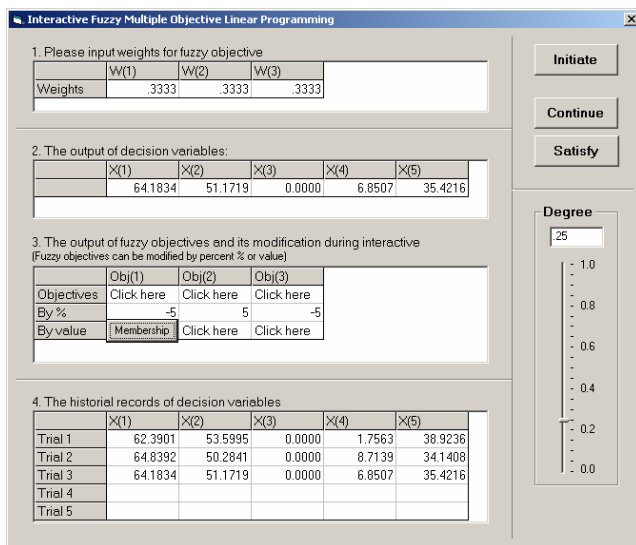


Figure 4: Main window with IFMOLP method for solving FMOLP problem interactively

Iteration No 2:

Step 4: Now, suppose the decision maker creates some new fuzzy goals as followings:

$$(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) = (700, 900, 800), \quad (7)$$

the membership functions of which in quadruple pair format are listed as:

$$\begin{cases} u_{g_1} = (690, 700, 700, 715) \\ u_{g_2} = (890, 900, 900, 915) \\ u_{g_3} = (790, 800, 800, 815) \end{cases} \quad (8)$$

By clicking on the corresponding grids in the third row ‘‘By value’’ of FlexGrid 3 in Figure 4, one Button ‘‘Membership’’ will appear, then click on the Button ‘‘Membership’’, another Dialog Box ‘‘Fuzzy Goal Input’’ similar as shown in Figure 2 will pop up. And the membership functions u_{g_1} , u_{g_2} and u_{g_3} (8) can be input by this Dialog Box sequentially.

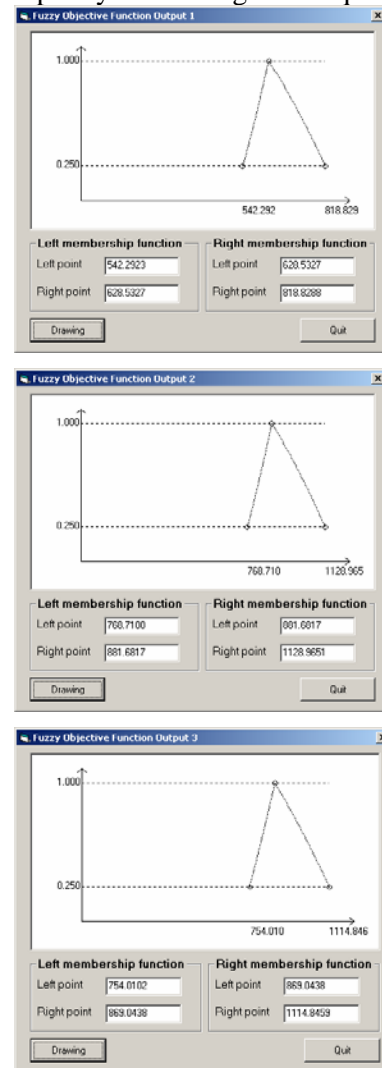


Figure 5: The membership functions of \tilde{f}_1^* , \tilde{f}_2^* and \tilde{f}_3^* at Trial 2

Step 5: As the result of clicking Button ‘‘Continue’’, the solution to the FMOLP problem based on achieving the fuzzy goals (7) (8) is generated. The decision variables are

$$x_1^* = 64.18, x_2^* = 51.17, x_3^* = 0.0, x_4^* = 6.85, x_5^* = 35.42,$$

and the fuzzy objective functions are

$$\begin{cases} \tilde{f}_1^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 64.18\tilde{c}_{11} + 51.17\tilde{c}_{12} + 6.85\tilde{c}_{14} + 35.42\tilde{c}_{15} \\ \tilde{f}_2^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 64.18\tilde{c}_{21} + 51.17\tilde{c}_{22} + 6.85\tilde{c}_{24} + 35.42\tilde{c}_{25} \\ \tilde{f}_3^*(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) = 64.18\tilde{c}_{31} + 51.17\tilde{c}_{32} + 6.85\tilde{c}_{34} + 35.42\tilde{c}_{35} \end{cases} \quad (9)$$

The membership functions of \tilde{f}_1^* , \tilde{f}_2^* and \tilde{f}_3^* (9) are shown as in Figure 6. The solution is also logged and listed in the second row 'Trial 3' of FlexGrid 4 as in Figure 4.

Step 6: Now the decision maker is satisfied with the solution calculated in Step 5, the whole interactive process stops, and the current solution is the final satisfactory solution of the FMOLP problem to the decision maker.

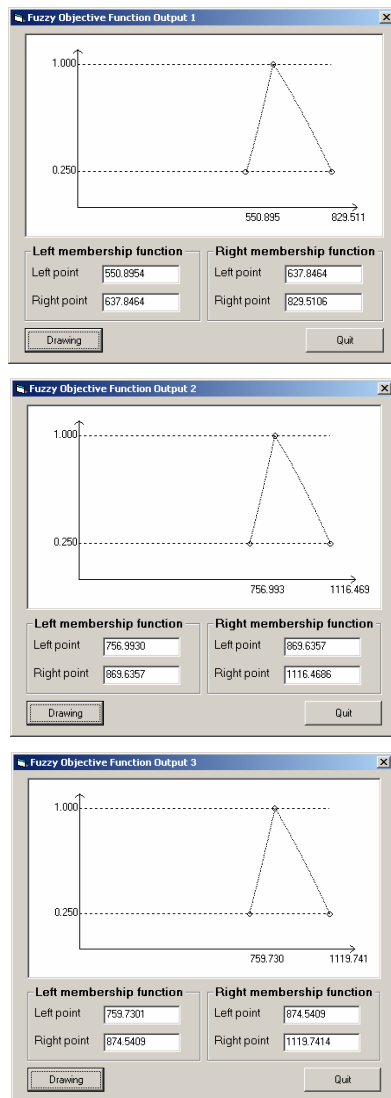


Figure 6: The membership functions of \tilde{f}_1^* , \tilde{f}_2^* and \tilde{f}_3^* in Trial 3

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