Impedance models of photon conductance in photonic crystal waveguides

N A Nicorovici[1], L C Botten[1], A A Asatryan[1], C M de Sterke[2] and R C McPhedran[2]

[1]Centre for Ultrahigh-bandwidth Devices for Optical Systems
Department of Mathematical Sciences, University of Technology, Sydney,
PO Box 123, Broadway NSW 2007, Australia

[2]Centre for Ultrahigh-bandwidth Devices for Optical Systems
School of Physics, University of Sydney, NSW 2006, Australia

Introduction

There are strong analogies between the behaviour of electrons in solids and the corresponding properties for photons in periodic structures. Indeed, photonic crystals, which exhibit both pass bands and band gaps are intended to perform the same functions for photons as do semiconductors for electrons. In this paper we discuss an extension of the analogy by exploring the photonic analogue of electron conductance first studied by Landauer [1] who showed, in the context of mesoscopic wires, that electron conductance g was proportional to the energy transmittance T of the wave function.

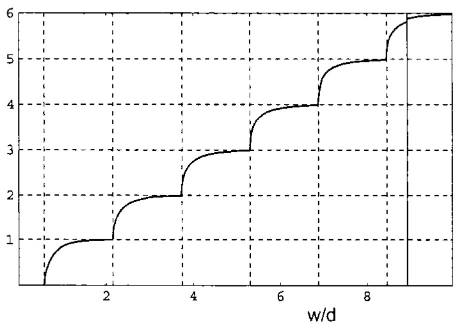

Fig. 1 Conductance of a semi-infinite photonic crystal waveguide as a function of the normalised width of the guide. The bulk PC is a square array of cylinders of normalised radius $a/d = 0.3$ (where d is the lattice constant) and refractive index $n = 3$ for a normalised wavelength $\lambda/d = 3.15$ located in the first band gap.

Here, we apply Landauer's conductance formula extended to multiple channels

$$g = \sum_{m,q} \left| t_{mq}^{12} \right|^2 \qquad (1)$$

where t_{mq}^{12} is the complex transmission coefficient in channel m of medium M_2 due to unit amplitude incidence in channel q of medium M_1. In this context, a channel is a mechanism that allows energy to be carried to infinity, e.g., a propagating plane wave or mode in the optical context.

While the study of electron conductance in nanostructures has been an active area of research for many years, it is only comparatively recently that van Wees [2] discovered the beautifully regular "ladder" or "staircase" structure that is exhibited by the conductance, as a function of gate width for a quantum point contact. Shortly thereafter, the corresponding phenomenon for photons was exhibited by Montie *et al* [3] who showed that the transmission cross-section increased in a step-like fashion as a function of the slit width, as in Fig. 1.

In this paper, we analyse the conductance problem for a semi-infinite photonic crystal waveguide (bounded above by free space) and seek to compute the total transmission into the guide according to (1) using a new scattering matrix method [4]. We then proceed to show the close parallels that exist between the behaviour of the PC waveguide and that of a perfect metal guide and, in turn, we are led to the formulation of an impedance model for propagation in the PC structure.

Photonic crystal waveguide analysis

Our formulation of the problem uses a supercell model in which we have a periodic array of waveguides each of width w encased in a photonic crystal with sufficient separating layers of the PC to ensure isolation of the waveguides. The supercell periodicity defines the directions of the plane wave channels in free space (region M_1) while the modes of the photonic crystal waveguide (region M_2) are computed using our Bloch mode technique. In (1), the indices q and m respectively denote a propagating plane wave and a propagating mode. It is helpful, however, to recast the problem purely in terms of waveguide modes by exploiting symmetry and the reciprocity theorem, i.e.,

$$g=\sum_{m,q}\left|t^{12}_{mq}\right|^2=\sum_{m,q}\left|t^{21}_{qm}\right|^2=\mathrm{Tr}\left(T^{\mathrm{H}}_{21}T_{21}\right), \qquad (2)$$

with the summation taken over the set of all propagating modes m and plane wave orders q, and with the scattering matrix T_{21} truncated appropriately. We next exploit an energy conservation relationship [4] appropriate to reflection and transmission scattering matrices that have been truncated to include only propagating plane waves and modes, i.e.,

$$R^H_{21}R_{21}+T^H_{21}T_{21}=I_2. \qquad (3)$$

In (3), I_2 is an identity matrix whose dimension is the number of propagating modes in the photonic crystal (M_2). This enables the conductance to be expressed exclusively in terms of PC waveguide modes through the introduction in (2) of R_{21}, the scattering matrix that characterises the reflection of modes, propagating in the crystal, from the interface of the PC with free space. Thus, the conductance becomes:

$$\begin{aligned} g&=\mathrm{Tr}\left(\mathrm{I}_2-R^{\mathrm{H}}_{21}R_{21}\right)\\ &=\sum_m\left(1-\sum_l\left|r^{21}_{lm}\right|^2\right)\approx\sum_m\left(1-\left|r^{21}_{mm}\right|^2\right). \end{aligned} \qquad (4)$$

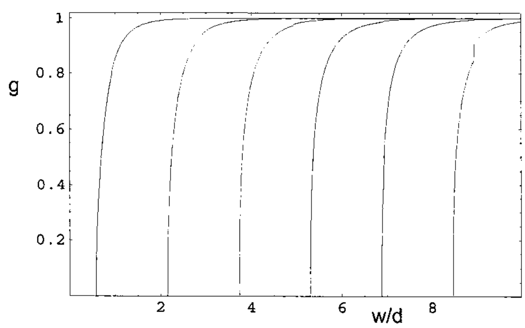

Fig. 2 $1-\left|r^{21}_{mm}\right|^2$ (m=1,2, …, 6) versus normalised width of the PC waveguide. The parameters of the crystal are those of Fig. 1.

As is apparent from Fig. 2, each of the terms $1-\left|r^{21}_{mm}\right|^2$ in (4) represents a step in the conductance ladder of Fig. 1. Once the guide width increases sufficiently to support each additional mode (marked by the dotted lines) for the given wavelength, the conductance of each propagating state appears to rise rapidly until it saturates at unity. Note also that the cut-off guide widths for each mode are equally separated. In fact, in the first PC band gap, for which propagation is determined largely by the specular diffraction order [5], we can show the cut-off widths of each mode are

$$w_n=\frac{n\pi-\arg(R_{12})}{k}, \qquad (5)$$

where $\arg(R_{12})$ is the phase on reflection of the specular plane wave from the semi-infinite crystal. The form of Eq. (5) is identical to that for a perfect metal guide, with the choice of $\arg(R_{12})=-\pi$. This in turn suggests that our study of photon conductance should be advanced by considering the properties of an isolated guide in a perfect metal, for which a semi-analytic treatment is possible.

Waveguides in metallic structures

We next consider an isolated perfect metal guide and expand the free space fields in a Fourier integral of plane waves and the waveguide fields in a linear combination of modes

$$u_n(x)=\sin\left(\nu_n x\right)e^{\pm i\beta_n y},\ \nu_n=\frac{n\pi}{w},\ \beta_n=\sqrt{k^2-\nu_n^2}. \qquad (6)$$

The conductance analysis proceeds in the same manner as above necessitating the calculation of the corresponding reflection matrix R_{21} using a semi-analytic treatment [6]. We thus derive

$$\begin{aligned} R_{21}&=(I+Y)^{-1}(I-Y), Y=\beta^{-1/2}B\beta^{-1/2},\\ B_{nm}&=\int_{-\infty}^{\infty}\chi(\alpha)\bar{J}_n(\alpha)J_m(\alpha)\,d\alpha,\\ J_n(\alpha)&=\frac{1}{\sqrt{2\pi}}\int_{-w/2}^{w/2}e^{-i\alpha x}u_n(x)\,dx. \end{aligned} \qquad (7)$$

Thus, from (7), the reflection matrix R_{21} appears in the form of a generalised Fresnel coefficient involving a relative admittance matrix Y defined in terms of the admittance of free space B and the admittance of the guide β, both expressed in the waveguide modal basis $\{u_n(x)\}$. The actual form of B derives from a change of basis of the free space admittance $\chi(a)=\sqrt{k^2-\alpha^2}$, in the plane wave basis, to the modal basis.

Fig. 3 shows metallic guide analogue of Fig. 1 (for the PC guide) and it is clear that these are structurally identical, differing only with respect to shifted modal cut-off widths. The rapid rise of each "step" in the conductance ladder can be studied using an asymptotic treatment of (7). This reveals that the impedance mismatch between the guide and free space diminishes quickly with guide width once a given mode has begun to propagate, leading quickly to saturation of the conductance. In fact, for a perfect metal guide, the conductance of each mode can be calculated to high accuracy using the scalar approximation

$$g_m = 1 - |\rho_m|^2, \ \rho_m = \frac{1 - Y_{mm}}{1 + Y_{mm}}, \qquad (8)$$

in which only the self-reflection (i.e. diagonal) term is taken from the admittance matrix Y.

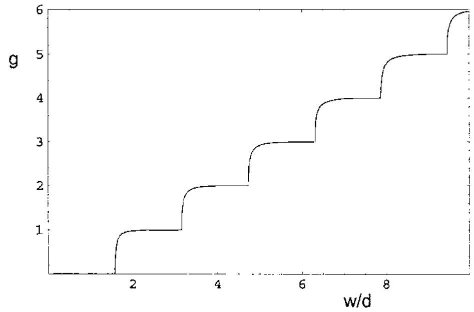

Fig.3 The conductance of an isolated semi-infinite metal waveguide as a function of guide width.

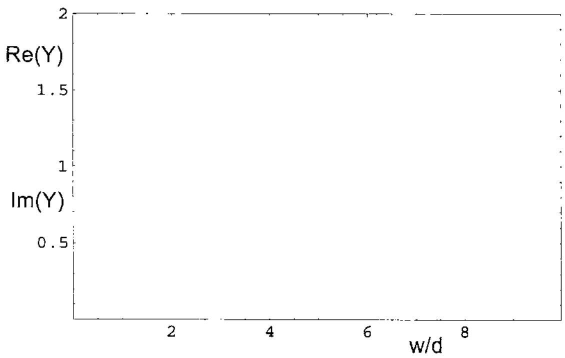

Fig. 4 Real (upper) and imaginary (lower curves) parts of Y_{mm} for an isolated semi-infinite metal guide versus guide width.

Impedance formulation for a PC

The close parallel between the formulation of the ideal metal waveguide and a PC waveguide suggests there must exist a corresponding impedance treatment for the PC waveguide. In fact, a Bloch mode analysis[4,6] shows that a modal admittance Y can be inferred directly from the reflection coefficient R_{21} according to

$$R_{21} = (I + Y)^{-1}(I - Y). \qquad (9)$$

Following the transfer matrix treatment in Ref. [6], it can be show that $R_{21} = -F_-^{-1}F_+$, where the matrices $F_\mp$ comprise the downward and upward components of the Bloch mode eigenvectors. The full matrix treatment and the single mode approximation (8) then allow us to compute the contributions to the conductance ladder for the PC guide.

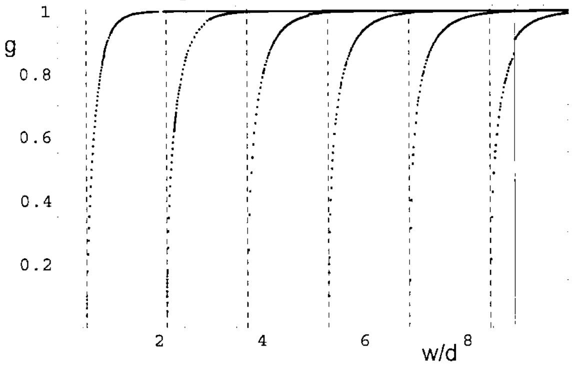

Fig. 5. $g_m = 1 - |\rho_m|^2$ vs. w/d for the PC guide. Solid curve: full matrix calculation. Dotted curve: from (8)

As is evident from Fig. 5, the single mode admittance differs slightly from full treatment, converging to the full matrix result with increasing numbers of modes.

Conclusions

Our study of photon conductance in PC waveguides has demonstrated strong parallels with the corresponding properties of metal guides and has led to the characterisation of the system using impedance matrices. If impedances are able to efficiently and accurately characterise PC devices , it will be an important advance enabling such devices to be modelled using generalisations of thin film optics.

This work was produced with the assistance of the Australian Research Council under the ARC Centres of Excellence program.

References

1. R Landauer, Phil. Mag. **21**, 863 (1970).
2. B J van Wees *et al*, Phys. Rev. Lett. **60**, 848 (1988).
3. E A Montie *et al*, Nature, **350**, 594 (1991).
4. L C Botten et al, Opt. Express **12**, 1592-1604, (2004) http://www.opticsexpress.org.
5. C M de Sterke, L C Botten, A A Asatryan, T P White, R C McPhedran, "Modes of photonic crystal waveguides", Opt. Letters, in press.
6. L C Botten, N A Nicorovici, A A Asatryan, C M de Sterke and R C McPhedran, "Conductance of photonic crystal waveguides", in preparation.