THE BIVARIABLE FRACTAL ALGORITHM IN DISTRIBUTED NAVIGATIONAL SIMULATION SYSTEM

Dan Liu [1], Xiangjian He [2], Yicheng Jin [1]

1 Institute of Nautical Science&Technology
Dalian Maritime University
Dalian,Liaoning,P.R.China

2 Department of Computer Systems
Falculty of Information Technology,
University of Technology,Sydney
116026PO Box 123,Broadway 2007,Australia

Abstract—*The scene simulation is one of the kernel methods in developing the navigation simulator, and terrain is one of the important components of the scene simulation. According to the basic request on terrain protraction in the visual system of navigation simulatorand the software and hardware platform of the existing navigation simulator. Many useful results of bivariable functions have been obtained by studying the bivariable fractal space. A bivariable fractal algorithm considered with the geometrical precision, the reality and the complicated degree in different scenes has been proposed. The reliable result is available.*

Keywords: *distributed navigational simulation system, scene simulation, bivariable fractal function.*

1 Introduction

The ship handling simulator should provide essential and realistic navigation conditions to the users according to the design motive. The scene here is the seen ambience when the ship is navigating. It is generated in real-time by the computers from the preformed ship navigation condition models. In the near years ,terrain protraction methods are perfect along with the development of GIS and VR etc.

GIS requires more Geometrical precision than real-time display of the graph ,and the VR system is on the opposite. So some of the ready terrain protraction methods can only be used for reference, and some are unsuitable for the scene simulation at all. Exploring and searching for the effective terrain protraction methods in the navigation system are demanded [1].

Fractals is the new branch of nonlinear mathematics, it can simulate almost all the natural phenomena. In the later part, some useful results of bivariable functions have been obtained by studying the bivariable fractal space.

2 Biavariable fractal function

Let X be the complete metric space and let $W_i : X \to X$, $i=1,2,\cdots,n$ be continuous mappings. Then the collection $\{X; W_i : i=1,2,\cdots,n\}$ is called an iterated function system, abbreviated IFS.A set $A \subseteq X$ is said to be an attractor of the IFS if A is nonempty, compact and with $A=\bigcup_{i=1}^{n} W_i(A)$.We now introduce some notations which we will put to use throughout the rest of this paper [2].

Let $\lambda \in (-1,1)$ be a constant ,m,n ≥ 12 be integers. Denote $M=\{1, \cdots,m\}$ and $N=\{1, \cdots,n\}$.

Let $-\infty \le x_0 < x_1 < \cdots < x_m < \infty$ and $-\infty < y_0 < y_1 < \cdots < y_n < \infty$ Denote D=[x_0, x_m] × [y_0, y_n] and $D_{i,j} = [x_{i-1}, x_i] \times [y_{j-1}, y_j]$ for all i∈M and j∈N.

Let C(D) denote the collection of real-valued continuous functions defined on D. The partition of D given by D=$\bigcup_{i,j} D_{i,j}$ will be denoted by Δ. For $i \in M$ and $j \in N$ define maps $A_i : [x_0, x_m] \to [x_{i-1}, x_i]$ and $B_j : [y_0, y_n] \to [y_{j-1}, y_j]$ by

$$A_i(x) = \frac{x_i - x_{i-1}}{x_m - x_0}(x - x_0) + x_{i-1}, x \in [x_0, x_m]$$

And

$$B_j(y) = \frac{y_j - y_{j-1}}{y_n - y_0}(y - y_0) + y_{j-1}, y \in [y_0, y_n]$$

Respectively.

Definition 1.Let $W_{i,j} : D \times R \to D \times R, (i,j) \in M \times N,$ be continuous mappings. The IFS $\{D \times R; W_{i,j} : (i,j) \in M \times N\}$ is said to be iterated generating system, called IGS[3] for short, if there is a function $f \in C(D)$ such that {(x, y, f (x, y)):(x, y)∈ D}, the graph of f, is the unique attractor of the IFS, in which case we also write.

3 Spaces of bivariable fractal functions

Theorem 1[4] .Let $\phi_{i,j} \in C(D), (i,j) \in M \times N$ be Lipschitz. Define $W_{i,j} : D \times R \to D \times R$ by

$$W_{i,j}(x, y, z) = (A_i(x), B_j(y), \lambda z + \phi_{i,j}(x, y))$$

For (x, y, z) $\in D \times R$ and (i, j) $\in M \times N$. then $\{D \times R; W_{i,j} : (i,j) \in M \times N\}$ is an IGS if and only if there exist $u \in C[x_0, x_m]$ and $v \in C[y_0, y_n]$ such that the following equations hold:

$$\lambda u(x) = \phi_{i,j}(x, y_n) - \phi_{i,j+1}(x, y_0)$$
$$x \in [x_0, x_m], i \in M, j = 1, \cdots, n-1$$
$$\lambda v(x) = \phi_{i,j}(x_m, y) - \phi_{i+1,j}(x_0, y)$$
$$y \in [y_0, y_n], j \in N, j = 1, \cdots, m-1$$
$$u(A_i(x)) = \lambda u(x) + \phi_{i,1}(x, y_0) - \phi_{i,n}(x, y_n)$$
$$x \in [x_0, x_m], i \in M$$
$$v(B_j(y)) = \lambda v(y) + \phi_{1,j}(x_0, y) - \phi_{m,j}(x_m, y)$$
$$y \in [y_0, y_n], j \in N$$

Definition 2 Let H be a linear subspace of C (D) then the set

$F(\Delta, \lambda, H) =$

{ $f \in C(D)$: there exist $\phi_{i,j} \in H, (i,j) \in M \times N,$ such that { $D \times R; W_{i,j} : (i,j) \in M \times N\}$ ⇔ f , where $W_{i,j}: D \times R \to D \times R$ is defined by $W_{i,j}$ (x, y, z)=($A_i(x), B_j(y), \lambda z + \phi_{i,j}(x, y, z))$}

is called the set of fractal functions with respect to the partition Δ,the contractive factor λ and the bottom space H.

Theorem 2.Denote K={$\phi \in C(D)$:there exist a, b, c and d in R such that ϕ(x, y)= axy+ bx + cy + d for all (x, y) $\in D$}

Then $F(\Delta, \lambda, K)$ is a linear subspace of C (D). Furthermore, we have

$$\dim F(\Delta,\lambda,K)=\begin{cases} mn+3 & if\lambda \neq 0 \\ (m+1)(n+1) & if\lambda = 0 \end{cases}$$

Let us denote

$$I(\Delta)=\{(i,j): i=1,\cdots,m-1; j=1,\cdots,n-1\}$$

Called the interior indices, and

$$B(\Delta)=\{(i,j): i=0,m; j=0,1,\cdots,n\}\bigcup\{(i,j): i=0,1,\cdots,m; j=0,n\}$$

Called the boundary indices.

4 Conclusion

Theorem 3. Suppose $\lambda \neq 0$, Let J be a subset of $\{(i,j):\ i=0,1,\cdots,m\ ;\ j=0,1,\cdots,n\ \}$.Denote $J_I = J\bigcup I(\Delta)$ and $J_B = J\bigcup B(\Delta)$ .Let $\{Z_{i,j}:(i,j)\in J\}$ be a set of real numbers. Then we have (1). There exists an $f\in F(\Delta,\lambda,K)$ such that

$$f(x_i,y_i)=z_{i,j},(i,j)\in J \tag{1}$$

If and only if the following equations

$$(x_m-x_{i-1})(z_{0,0}-z_{0,n})-(x_m-x_0)(z_{i-1,0}-z_{i-1,n}) + (x_{i-1}-x_0)(z_{m,0}-z_{m,n})=0, i=2,\cdots,m \tag{2}$$

$$(y_n-y_{j-1})(z_{0,0}-z_{m,0})-(y_n-y_0)(z_{0,j-1}- z_{m,j-1})+(y_{j-1}-y_0)(z_{0,n}-z_{m,n})=0 \tag{3}$$
$$j=2,\cdots,n$$

In the unknowns $z_{i,j}(i,j)\in B(\Delta)\setminus J_B$,are linearly solvable.

There exists one and only $f\in F(\Delta,\lambda,K)$ such that (1) holds if and only if $J_I = I(\Delta)$ and the equation (2) and (3) in the unknowns $z_{i,j}(i,j)\in B(\Delta)\setminus J_B$ (see [2]), are linearly solvable.

Corollary. Suppose $\lambda \neq 0$ and let

$$J=\{(i,j): i=0,1,\cdots,m;\ j=0,1,\cdots,n\}\bigcup\{(0,n)(m,0),(m,n)\}$$

Then for any set of mn+3 real numbers $\{\ z_{i,j}:(i,j)\in J\ \}$, there exists a unique $f\in F(\Delta,\lambda,K)$ such that (1)holds. Furthermore, for each (p, q) $\in J$, let $g_{p,q}\in F(\Delta,\lambda,K)$ be the unique function with

$$g_{p,q}(x_i,y_j)=\begin{cases}1 & if(i,j)=(p,q) \\ 0 & if(i,j)\in J\setminus\{(p,q)\}\end{cases}$$

Then $\{g_{i,j}:(i,j)\in J\}$ forms a basis of the linear space $F(\Delta,\lambda,K)$.

According to Theorem 3 and the Corollary, we can outline mountain in the screen plane of computer. Figure 1 is the experimental result of this method.

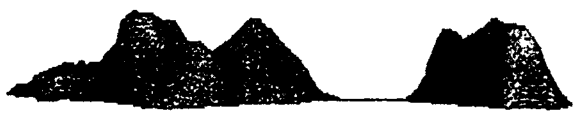

Figure 1. the 3D model of three hill island , Dalian, China

In fact, common ship often navigates along the recommended sea-route when it navigates along the seashore. The outline of a mountain can be seen to be composed of the points of intersection between the normal of the center of the sea-route and the contour line with the most height and the points with the nearest distance to the sea-route center when the measurer observes the mountain in the positive horizontal direction. Therefore, the means to pick up the outline points of the mountain is: firstly, separate the center line of the sea-route with equal intervals; secondly, fine the intervals. The unequal interval partition can be used according to such factors as the ridge lines, the changes of height value and the curvature of the sea-route center line. Rearrange the sampling points in a certain order, then put them into the computer. The following figure is the mountain outline of

Huangbaizui, Dalian.

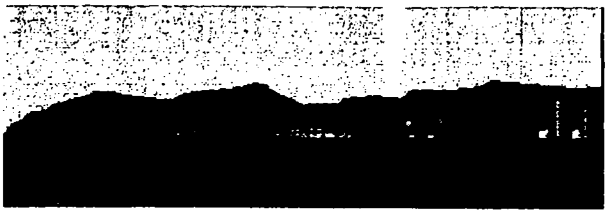

Figure 2. the mountain outline of Huangbaizui, Dalian, China

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