FMODSS: A Decision Support System for Solving Multiple Objective Linear Programming Problem with Fuzzy Parameters

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Abstract

A new fuzzy goal approximate algorithm has been developed for solving fuzzy multiple objective linear programming (FMOLP) problems where fuzzy parameters can be in any form of membership function in both objective functions and constraints. Based on the fuzzy goal approximate algorithm, a fuzzy multiple objective decision support system (FMODSS) is developed. This paper focuses on the development and use of FMODSS in detail. An example is presented for demonstrating how to solve a FMOLP problem by using the FMODSS when the fuzzy goals for objective functions are set up.

Keywords

Decision support system; Multiple objective linear programming; Fuzzy multiple objective linear programming; Fuzzy goal

1. INTRODUCTION

Much organizational decision-making involves multiple, noncommensurable, and conflicting objectives that should be considered simultaneously and which are subject to constraints. Such decision problems can hardly be considered through the examination of a single objective or point of view that will lead to the 'optimum' decision (Zopounidis and Doumpos 2000)

Multiple objective linear programming (MOLP) is one of the popular methods to deal with such complex and ill-structured decision-making. In the MOLP problem formulation process, the objective functions and constraints involve many parameters whose possible values may be assigned by experts. In most practical situations, however, it is natural to consider that experts often imprecisely or ambiguously know the possible values of these parameters (Sakawa and Yano 1991). In this case, it may be more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by fuzzy numbers. The fuzzy multiple objective linear programming (FMOLP) problem involving fuzzy parameters would be viewed as a more realistic version than the conventional one (Lai and Hwang 1994; Sakawa and Yano 1991).

Various kinds of FMOLP models, methods and approaches have been proposed to deal with different decision-making situations which involve fuzzy values in objective function parameters, constraints parameters, or goals. Zhang et al. (2002; 2003) proposed two methods: one is to solve a fuzzy linear programming (FLP) problem by transforming it into a corresponding four-objective constrained optimisation problems, and another is to formulate linear programming problems with fuzzy equality and inequality constraints. Based on that, Wu et al. (2003) extends the results and develops three methods and algorithms to satisfy different decision-making situations in solving the proposed FMOLP problem with fuzzy parameters in objective function and constraints. One of them is for solving FMOLP problem when the fuzzy goals of objective functions need to be achieved.

A fuzzy multiple objective decision support system (FMODSS). which is an implementation of these methods and algorithms proposed by Wu et al (2003)., is developed. In this paper, we focus on one situation in FMODSS which is used to solve a FMOLP problem when the fuzzy goals for objective functions are set up.

This paper is organized as follows. A review of related definitions and theorems for solving FMOLP problems with fuzzy parameters in objective functions and constraints is described in Section 2. The fuzzy goal approximate algorithm proposed for solving FMOLP problem is described in Section 3. After the description of use for FMODSS is supplied in detail in Section 4, an example is presented for demonstrating how to solve a FMOLP problem by the FMODSS in Section 5. Finally, Section 6 concludes the paper.

2. FUZZY MULTIPLE OBJECTIVE LINEAR PROGRAMMING (FMOLP)

In this paper, we consider the FMOLP problem in which all coefficients of the objective functions and constraints are real fuzzy numbers. Before supplying the FMOLP model and proposing the fuzzy goal approximate algorithm for solving it, some definitions and theorems are reviewed for further discussion.

Definition 2.1 Let R^1 be the set of all real numbers. Then a real fuzzy number a is defined by its membership function $\mu_{\tilde{a}}(x)$ which satisfies:

a) A continuous mapping from R^1 to the closed internal [0,1],

b) For $\mu_{\tilde{a}}(x)=0$ all $x \in (-\infty, c]$,

c) Strictly increasing and continuous on $[c,a]$,

d) $\mu_{\tilde{a}}(x)=1$ for all $x \in [a,b]$,

e) Strictly decreasing and continuous on $[b,d]$,

f) $\mu_{\tilde{a}}(x)=0$ for all $x \in [d,+\infty)$ (Dubois and Prade 1978).

Let $F(R)$ be the set of all fuzzy numbers *on* R^1. According to the decomposition theorem of fuzzy set, we have

$$\tilde{\alpha} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha_\lambda^L, \alpha_\lambda^R] \quad (1)$$

$$\tilde{\alpha} = \bigcup_{\lambda \in R_0} \lambda [\alpha_\lambda^L, \alpha_\lambda^R] \quad (2)$$

for all $\tilde{\alpha} \in F(R)$, where R_0 is all rational numbers in (0, 1] (Zhang et al. 2003).

Definition 2.2 Let $\tilde{\alpha}, \tilde{\beta} \in F(R)$ with the membership functions $\mu_{\tilde{\alpha}}(x)$ and $\mu_{\tilde{\beta}}(x)$ respectively, and $0 \le \lambda \in R$. The addition of two fuzzy number $\tilde{\alpha} + \tilde{\beta}$ and the scalar product of λ and $\tilde{\alpha}$ are defined by the membership functions

$$\mu_{\tilde{\alpha}+\tilde{\beta}}(z) = \sup_{z=x+y} \min\{\mu_{\tilde{\alpha}}(x), \mu_{\tilde{\beta}}(y)\}, \quad (3)$$

$$\mu_{\lambda\tilde{\alpha}}(z) = \sup_{z=\lambda x} \mu_{\tilde{\alpha}}(x). \quad (4)$$

Form the decomposition theorem of fuzzy set and the Definition 2.2, for any two fuzzy number $\tilde{\alpha}$ and $\tilde{\beta}$, and $0 \le \alpha \in R$, $0 \le \beta \in R$, we have

$$\tilde{\alpha} + \tilde{\beta} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha_\lambda^L + \beta_\lambda^L, \alpha_\lambda^R + \beta_\lambda^R] \quad (5)$$

$$\alpha\tilde{\alpha} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha\alpha_\lambda^L, \alpha\alpha_\lambda^R] \quad (6)$$

$$\alpha\tilde{\alpha} + \beta\tilde{\beta} = \bigcup_{\lambda \in [0,1]} \lambda [\alpha\alpha_\lambda^L + \beta\beta_\lambda^L, \alpha\alpha_\lambda^R + \beta\beta_\lambda^R] \quad (7)$$

Definition 2.3 Let $F(R^n)$ be the set of all n-dimensional real fuzzy numbers on R^1. For any two n-dimensional fuzzy numbers $\tilde{\alpha}, \tilde{\beta} \in F(R^n)$, we define

1. $\tilde{\alpha} \succeq \tilde{\beta}$ iff $\alpha_{i\lambda}^L \geq \beta_{i\lambda}^L$ and $\alpha_{i\lambda}^R \geq \beta_{i\lambda}^R$, $i = 1, \ldots, n$, $\forall \lambda \in [0,1]$,

2. $\tilde{\alpha} \succeq \tilde{\beta}$ iff $\alpha_{i\lambda}^{L} \geq \beta_{i\lambda}^{L}$ and $\alpha_{i\lambda}^{R} \geq \beta_{i\lambda}^{R}$, $i = 1,\ldots,n$, $\forall \lambda \in [0,1]$,
3. $\tilde{\alpha} \succ \tilde{\beta}$ iff $\alpha_{i\lambda}^{L} > \beta_{i\lambda}^{L}$ and $\alpha_{i\lambda}^{R} > \beta_{i\lambda}^{R}$, $i = 1,\ldots,n$, $\forall \lambda \in [0,1]$ (Zhang et al. 2003).

Based on the above definitions, the FMOLP problems can be formulated as follows:

$$\text{(FMLOP)} \quad \begin{cases} \text{Maximize} & \tilde{f}(x) = \tilde{C}x \\ \text{s.t.} & x \in X = \{x \in R^n \mid \tilde{A}x \preceq \tilde{b}, x \geq 0\} \end{cases} \tag{8}$$

where $\tilde{C}$ is an $k \times n$ matrix, each element of which $\tilde{c}_{ij}$ is real fuzzy number represented by membership function $\mu_{\tilde{c}_{ij}}(x)$, $\tilde{A}$ is an $m \times n$ matrix coefficients of the constraints, $\tilde{b}$ is an m-vector of the rhs, each element of matrixes is fuzzy number represented by membership function $\mu_{\tilde{b}_i}(x)$, and x is an n-vector of decision variables, $x \in R^n$.

In the proposed FMOLP models, for each $x \in X$, the values of objective function $\tilde{f}(x)$ are fuzzy numbers. According to Definition 2.3, we have following definitions about FMOLP problems.

Definition 2.4 x^* is said to be a complete optimal solution, if and only if there exists $x^* \in X$ such that $\tilde{f}_i(x^*) \succeq \tilde{f}_i(x)$, $i = 1,\ldots,k$, for all $x \in X$.

Definition 2.5 x^* is said to be a Pareto optimal solution, if and only if there does not exists another $x \in X$ such that $\tilde{f}_i(x) \succeq \tilde{f}_i(x^*)$, for all i.

Definition 2.6 x^* is said to be a weak Pareto optimal solution, if and only if there does not exists another $x \in X$ such that $\tilde{f}_i(x) \succ \tilde{f}_i(x^*)$, for all i.

Associated with the FMOLP problems, let's consider the following multiple objective linear programming ($MOLP_\lambda$) problems:

$$(\text{MOLP}_\lambda) \quad \begin{cases} \text{Maximize} & \left(\langle C_\lambda^L, x\rangle, \langle C_\lambda^R, x\rangle\right)^T, \forall \lambda \in [0,1] \\ \text{s.t.} & x \in X = \{x \in R^n \mid A_\lambda^L x \leq b_\lambda^L, A_\lambda^R x \leq b_\lambda^R, x \geq 0, \forall \lambda \in [0,1]\} \end{cases} \tag{9}$$

where

$$C_\lambda^L = \begin{bmatrix} c_{11\lambda}^L & c_{12\lambda}^L & \cdots & c_{1n\lambda}^L \\ c_{21\lambda}^L & c_{22\lambda}^L & \cdots & c_{2n\lambda}^L \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1\lambda}^L & c_{k2\lambda}^L & \cdots & c_{kn\lambda}^L \end{bmatrix}, C_\lambda^R = \begin{bmatrix} c_{11\lambda}^R & c_{12\lambda}^R & \cdots & c_{1n\lambda}^R \\ c_{21\lambda}^R & c_{22\lambda}^R & \cdots & c_{2n\lambda}^R \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1\lambda}^R & c_{k2\lambda}^R & \cdots & c_{kn\lambda}^R \end{bmatrix}$$

$$A_\lambda^L = \begin{bmatrix} a_{11\lambda}^L & a_{12\lambda}^L & \cdots & a_{1n\lambda}^L \\ a_{21\lambda}^L & a_{22\lambda}^L & \cdots & a_{2n\lambda}^L \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1\lambda}^L & a_{m2\lambda}^L & \cdots & a_{mn\lambda}^L \end{bmatrix}, A_\lambda^R = \begin{bmatrix} a_{11\lambda}^R & a_{12\lambda}^R & \cdots & a_{1n\lambda}^R \\ a_{21\lambda}^R & a_{22\lambda}^R & \cdots & a_{2n\lambda}^R \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1\lambda}^R & a_{m2\lambda}^R & \cdots & a_{mn\lambda}^R \end{bmatrix},$$

$$b_\lambda^L = \left[b_{1\lambda}^L, b_{2\lambda}^L, \cdots, b_{m\lambda}^L\right]^T, \quad b_\lambda^R = \left[b_{1\lambda}^R, b_{2\lambda}^R, \cdots, b_{m\lambda}^R\right]^T$$

Theorem 2.1 Let x^* X be the solution to the MOLPλ problem. Then x^* *is* also a solution to the FMOLP problem.

Proof. The proof is obvious from Definition 2.3.

In the next section, we will describe a fuzzy goal approximate algorithm to solve the MOLPλ problem and the FMOLP problem when the DM sets up some fuzzy goals of objective functions, those fuzzy goals are represented by fuzzy numbers.

3. A FUZZY GOAL APPROXIMATE ALGORITHM FOR SOLVING FMOLP PROBLEM

Considering the FMOLP problem, for each of the fuzzy multiple objective functions $f(x) = (f_1(x), f_2(x), \dots, f_k(x))$, assume that the DM can specify some fuzzy goals g = (gl, g....gk.)^r which reflects the desired values of the objective functions of the DM. Based on the definition of FMOLP problem and MOLPλ problem and Theorem 2.1, we can make the conclusion that the solution of MOLPλ problem is equally the solution of FMOLP problem. From the definition of MOLPλ problem, when the DM sets up some fuzzy goals $g =$ (g1,g2...,gk)^r, the corresponding Pareto optimal solution, which is, in the minimax sense, the nearest to the fuzzy goals or better than that if the fuzzy goals is attainable, is obtained by solving the following minimax problem:

$$(\text{MOLP}_{\lambda m}) \quad \begin{cases} \text{Minimize} \;\; \max \left(\langle C_\lambda^L, x \rangle - g_\lambda^L, \langle C_\lambda^R, x \rangle - g_\lambda^R \right)^T, \forall \lambda \in [0,1] \\ \text{s.t.} \quad x \in X = \{ x \in R^n \mid A_\lambda^L x \le b_\lambda^L, A_\lambda^R x \le b_\lambda^R, x \ge 0, \forall \lambda \in [0,1] \} \end{cases} \tag{10}$$

where $g_\lambda^L = [g_{1\lambda}^L, g_{2\lambda}^L, \cdots, g_{k\lambda}^L]^T$, $g_\lambda^R = [g_{1\lambda}^R, g_{2\lambda}^R, \cdots, g_{k\lambda}^R]^T$.

For the simplicity in presentation, we define

$$X_\lambda = \{ x \in R^n \mid A_\lambda^L x \le b_\lambda^L, A_\lambda^R x \le b_\lambda^R, x \ge 0 \} \;\; \lambda \in [0,1]$$

The main steps of the fuzzy goal approximate algorithm are described as follows:

Let the interval [0,1] be decomposed into l mean sub-intervals with $(l+1)$ nodes $\lambda_i (i = 0, \cdots, l)$ which are arranged in the order of $0 = \lambda_0 < \lambda_1 < \cdots < \lambda_l = 1$, then define $X^l = \bigcap_i^l X_{\lambda_i}$, and denote

$$(\text{MOLP}_{\lambda m})_l \quad \begin{cases} \text{Minimize} \;\; \max \left(\langle C_{\lambda_i}^L, x \rangle - g_{\lambda_i}^L, \langle C_{\lambda_i}^R, x \rangle - g_{\lambda_i}^R \right)^T, 0 = \lambda_0 < \ldots < \lambda_l = 1 \\ \text{s.t.} \quad x \in X^l \end{cases} \tag{11}$$

Step 1: Set $l = 1$, then solve $(\text{MOLP}_{\lambda m})_l$ with $(x)_l$, where $(x)_l = (x_1, x_2, \dots, x_n)_l$, and the solution is obtained subject to constraint $x \in X^l$.

Step 2: Solve $(\text{MOLP}_\lambda)_{2l}$ with $(x)_{2l}$.

Step 3: If $\|(x)_{2l} - (x)_l\| < \varepsilon$, the solution x^* of $\text{MOLP}_{\lambda m}$ problem is $(x)_{2l}$. Otherwise, update l to $2l$ and go to Step 2.

The detail data flow diagram is listed in Figure 1.

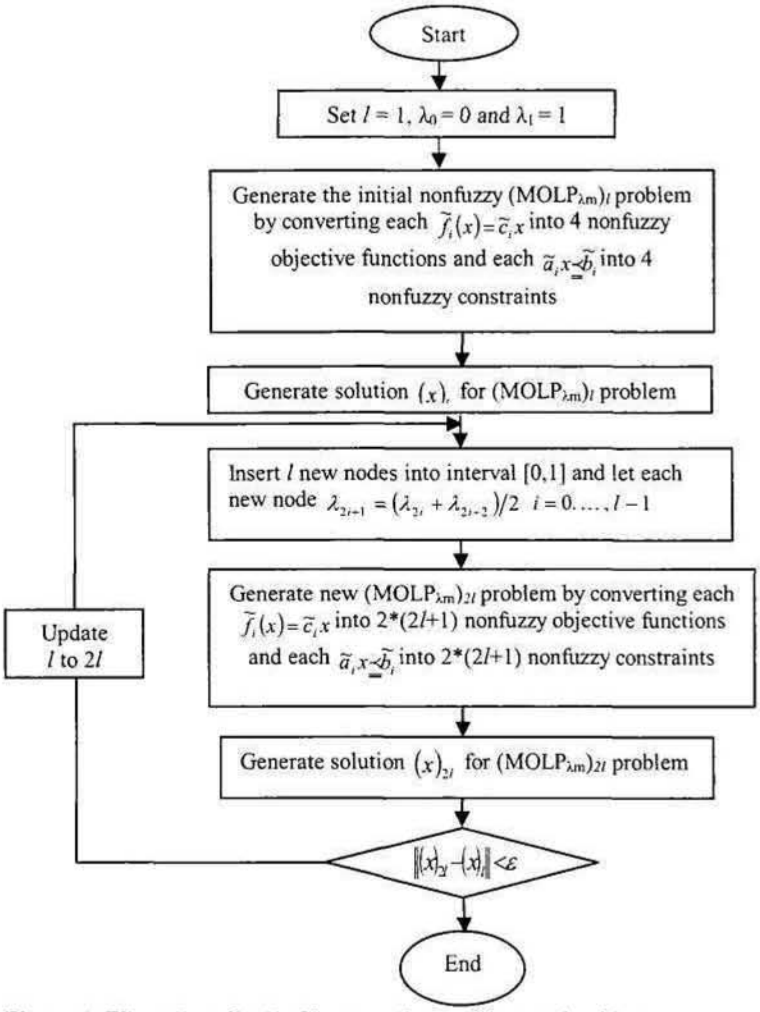

Figure 1: Flow chart for the fuzzy goal approximate algorithm

4. IMPLEMENTATION OF FMODSS

The FMODSS is developed using Microsoft Visual Basic 6.0 (Professional Edition) that provides a modular way for software development. The modular development of FMODSS provides flexibility to the system in order to incorporate new developed methods into it, and to enhance its user interface. The user interface of the FMODSS has the typical form of window-based software. It takes full advantage of the graphical capabilities of Windows environment enabling the user (DM or decision analyst) to exploit the capabilities of the system.

The FMODSS implements four methods and algorithms so far to deal with different decision-making situations. In this paper, we focus on the situation in which the DM sets up the fuzzy goals of the objective functions which are represented by fuzzy numbers, and let the system supply the solution which satisfies those fuzzy goals.

According to the FMOLP model in Section 2, the following common data need to be input from the user for setting up model and other initial data for system.

- The number of decision variables, the number of objective functions, and the number of constraints, as showed in Figure 2,
- The information about objective functions includes the coefficients of objective functions, the max/min for individual objective function, as showed in Figure 3,
- The information about constraints includes the coefficients of constraints, the coefficients of rhs, the relation sign of individual constraint, as showed in Figure 3,

- Especially, as described in FMOLP model in Section 2, the coefficients of objective functions and constraints are represented by fuzzy numbers. A special Dialog Box as showed in Figure 4 is generated in system for entering the fuzzy numbers. The forms of left continuous increasing function and right continuous decreasing function of fuzzy numbers can be selected as linear, quadratic, cubic, exponential, logarithmic, other piecewise form, and four end-points of left continuous increasing function and right continuous decreasing function of fuzzy numbers defined in Definition 2.1 are entered simultaneously.

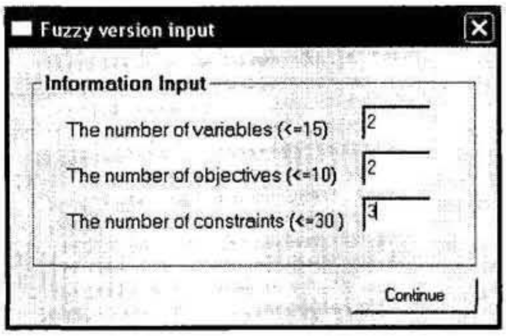

Figure 2: Input window 1 for FMODSS

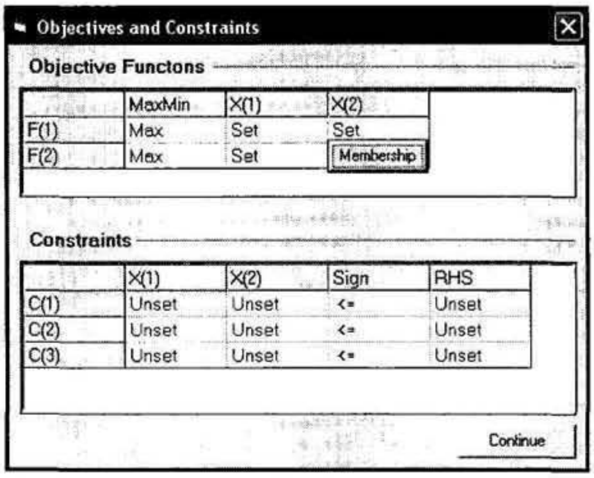

Figure 3: Input window 2 for FMODSS

Refer to Window "Fuzzy Multiple Objective Linear Goal Programming" as shown in Figure 5, in order to input the fuzzy goals which are represented by fuzzy numbers, click the grid in FlexGrid "Fuzzy Goal Input", another Window "Fuzzy Goal Input" as shown in Figure 6 is pop up. In Figure 6, a fuzzy goal is entered. The left membership function of the fuzzy number is linear; the right membership function is also linear. The four end points for left and right membership functions are 6, 7.5, 6.5, and 12 respectively. The diagram in Figure 6 shows the shape of the membership function of the fuzzy goal as well.

In Window "Fuzzy Multiple Objective Linear Goal Programming", we can also set the degree of all of the membership function of the fuzzy numbers. When the degree is set to 1, the original fuzzy problem will be changed to crisp problem, and the values of objective function $\tilde{f}(x)$ will be nonfuzzy numbers.

After the fuzzy goals inputting and the degree setting, press Button "Run", the final solution will be supplied and displayed. In Figure 5, the decision variables are displayed in FlexGrid "Output". In order to show the fuzzy objective functions of the running results, click the grid in FlexGrid "Output" for objective functions, another Window "Fuzzy Objective Function Output" as shown in Figure 7 is pop up. In Figure 7. a fuzzy objective function of the running result is displayed. The four end points for left and right membership functions are 5.6364, 7.8182, 8.9091 and 13.8182 respectively. The diagram in Figure 7 shows also the shape of the membership function of the fuzzy objective function.

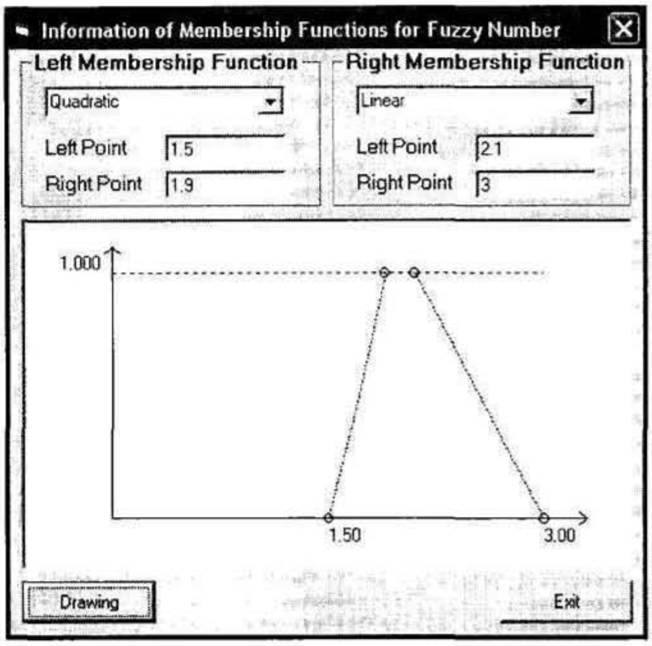

Figure 4: Input window for *fuzzy* number's membership function

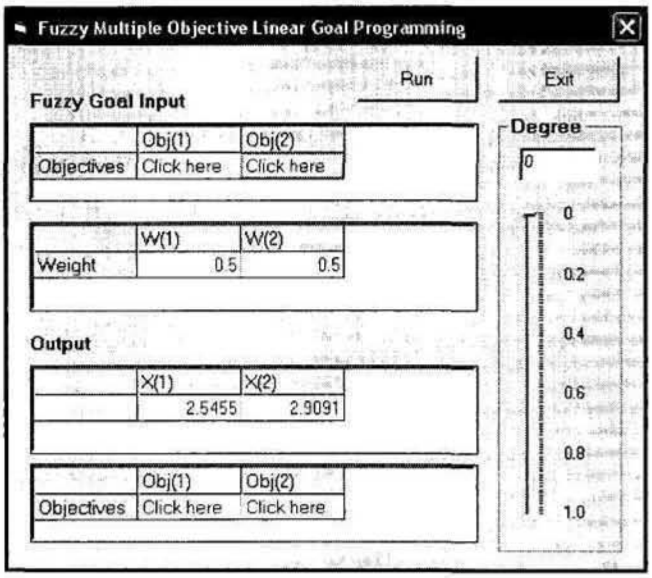

Figure 5: The window for solving Fuzzy Multiple Objective Linear Goal Programming problem

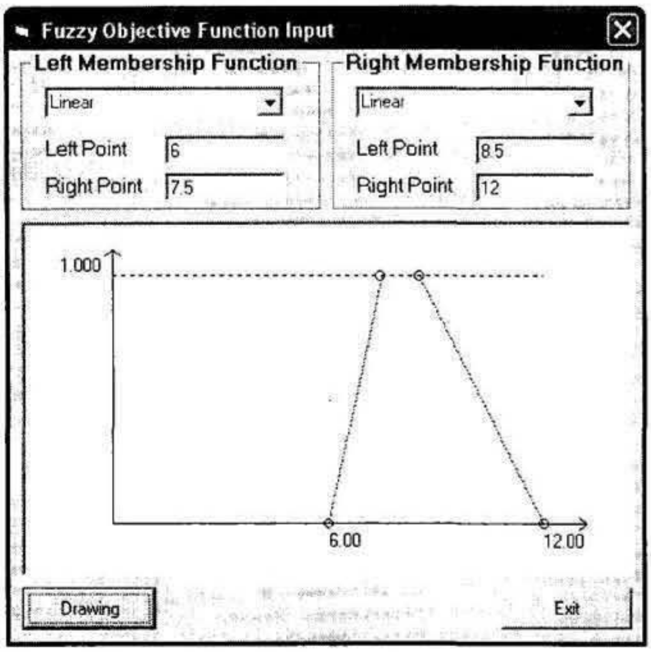

Figure 6: Input window for fuzzy goal

5. APPLICATION

5.1 The description of an example of FMOLP

A company produces two products *P1* and *P2* utilizing three different materials M1, M2, and M3 • The company knows that to produce about 1 ton of *P1* requires around 2 tons of M1, around 8 tons of M2 , and around 3 tons of M_3 while to produce about 1 ton of P2 requires around 6 tons, 6 tons, and 1 ton of M1, M2 and M3, respectively. The total amounts of available materials are limited to around 27 tons, 45 tons, and 15 tons for M1, M2 and M3, respectively. It also knows that *P1* yields a profit of around 1 million dollars per ton, while P2 yields around 2 million dollars. And unfortunately, P1 yields about 3 units of pollution per ton, and P2 yields about 3 units of pollution per ton. In such typical production planning problem, the manager in the company has two objectives: to maximize the total profit and minimize the amount of pollution.

For the production planning problem described above, the FMOLP problem with two objective functions and three constraints can be modelled as follows:

$$\max \tilde{f}(x) = \max\begin{pmatrix} \tilde{f}_1(x) \\ \tilde{f}_2(x) \end{pmatrix} = \max\begin{pmatrix} \tilde{c}_{11}x_1 + \tilde{c}_{12}x_2 \\ \tilde{c}_{21}x_1 + \tilde{c}_{22}x_2 \end{pmatrix} = \max\begin{pmatrix} \tilde{1}x_1 + \tilde{2}x_2 \\ \tilde{3}x_1 + \tilde{2}x_2 \end{pmatrix} \qquad (12)$$

$$\text{s.t.} \quad \begin{cases} \tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 = \tilde{2}x_1 + \tilde{6}x_2 \le \tilde{b}_1 = \tilde{27} \\ \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 = \tilde{8}x_1 + \tilde{6}x_2 \le \tilde{b}_2 = \tilde{45} \\ \tilde{a}_{31}x_1 + \tilde{a}_{32}x_2 = \tilde{3}x_1 + \tilde{1}x_2 \le \tilde{b}_3 = \tilde{15} \\ x_1 \le 0; \quad x_2 \le 0 \end{cases} \qquad (13)$$

We suppose the membership functions of the objective functions and constraints' coefficients are set up as following:

$$\mu_{\tilde{c}_{11}}(x) = \begin{cases} 0 & x < 0.5 \text{ or } 2 < x \\ (x^2 - 0.25)/0.56 & 0.5 \le x < 0.9 \\ 1 & 0.9 \le x \le 1.1 \\ (4 - x^2)/2.79 & 1.1 < x \le 2 \end{cases} \qquad \mu_{\tilde{c}_{12}}(x) = \begin{cases} 0 & x < 1.5 \text{ or } 3 < x \\ (x^2 - 2.25)/1.36 & 1.5 \le x < 1.9 \\ 1 & 1.9 \le x \le 2.1 \\ (9 - x^2)/4.59 & 2.1 < x \le 3 \end{cases}$$

$$\mu_{\tilde{c}_{21}}(x)=\begin{cases}0 & x<2.5 \text{ or } 4<x\\ (x^2-6.25)/2.16 & 2.5\le x<2.9\\ 1 & 2.9\le x\le 3.1\\ (4-x)/0.9 & 3.1<x\le 4\end{cases}\qquad \mu_{\tilde{c}_{22}}(x)=\begin{cases}0 & x<1.5 \text{ or } 3<x\\ (x^2-2.25)/1.36 & 1.5\le x<1.9\\ 1 & 1.9\le x\le 2.1\\ (3-x)/0.9 & 2.1<x\le 3\end{cases}$$

$$\mu_{\tilde{a}_{11}}(x)=\begin{cases}0 & x<1.5 \text{ or } 3<x\\ (x-1.5)/0.4 & 1.5\le x<1.9\\ 1 & 1.9\le x\le 2.1\\ (x^2-4.41)/4.59 & 2.1<x\le 3\end{cases}\qquad \mu_{\tilde{a}_{12}}(x)=\begin{cases}0 & x<5.5 \text{ or } 7<x\\ (x-5.5)/0.4 & 5.5\le x<5.9\\ 1 & 5.9\le x\le 6.1\\ (e^7-e^x)/(e^7-e^{6.1}) & 6.1<x\le 7\end{cases}$$

$$\mu_{\tilde{a}_{21}}(x)=\begin{cases}0 & x<7.5 \text{ or } 9<x\\ (x-7.5)/0.4 & 7.5\le x<7.9\\ 1 & 7.9\le x\le 8.1\\ (e^9-e^x)/(e^9-e^{8.1}) & 8.1<x\le 9\end{cases}\qquad \mu_{\tilde{a}_{22}}(x)=\begin{cases}0 & x<5.5 \text{ or } 7<x\\ (e^x-e^{5.5})/(e^{5.9}-e^{5.5}) & 5.5\le x<5.9\\ 1 & 5.9\le x\le 6.1\\ (e^7-e^x)/(e^7-e^{6.1}) & 6.1<x\le 7\end{cases}$$

$$\mu_{\tilde{a}_{31}}(x)=\begin{cases}0 & x<2.5 \text{ or } 4<x\\ (e^x-e^{2.5})/(e^{2.9}-e^{2.5}) & 2.5\le x<2.9\\ 1 & 2.9\le x\le 3.1\\ (4-x)/0.9 & 3.1<x\le 4\end{cases}\qquad \mu_{\tilde{a}_{32}}(x)=\begin{cases}0 & x<0.5 \text{ or } 2<x\\ x^2-0.25/0.56 & 0.5\le x<0.9\\ 1 & 0.9\le x\le 1.1\\ (2-x)/0.9 & 1.1<x\le 2\end{cases}$$

$$\mu_{\tilde{b}_1}(x)=\begin{cases}0 & x<26.5 \text{ or } 28<x\\ (e^x-e^{26.5})/(e^{26.9}-e^{26.5}) & 26.5\le x<26.9\\ 1 & 26.9\le x\le 27.1\\ (21952-x^3)/2049.289 & 27.1<x\le 28\end{cases}\qquad \mu_{\tilde{b}_2}(x)=\begin{cases}0 & x<44.5 \text{ or } 46<x\\ (e^x-e^{44.5})/(e^{44.9}-e^{44.5}) & 44.5\le x<44.9\\ 1 & 44.9\le x\le 45.1\\ (46-x)/0.9 & 45.1<x\le 46\end{cases}$$

$$\mu_{\tilde{b}_3}(x)=\begin{cases}0 & x<14.5 \text{ or } 16<x\\ (x-14.5)/0.4 & 14.5\le x<14.9\\ 1 & 14.9\le x\le 15.1\\ (e^{16}-e^x)/(e^{16}-e^{15.1}) & 15.1<x\le 16\end{cases}$$

5.2 **Solving MOLP2mproblem by the fuzzy goal approximate algorithm**

From the description of the problem in Subsection 5.1, the number of decision variables, objective functions, and constraints are set to 2, 2, and 3 respectively as showed in Figure 2.

The interface for entering the information of the objective functions and constraints is showed in Figure3. The information includes the max/min of individual objective functions that are all set to max, and the relationship involved in constraints that are all set as "less than".

Also in Figure 3, in order to enter fuzzy coefficients of the objective functions and constraints, the user can click on the corresponding grids in the table, and then another Dialog Box will pop up as showed in Figure 4. For example, the membership function ^ ($_v$) for coefficient ?,,, which is $\tilde{2}$, needs to be input, then the forms of left continuous increasing function and right continuous decreasing function of the membership function //- *(x)* are selected as quadratic and linear respectively, and four end-points are set to 1.5, 1.9, 2.1, and 3 respectively.

After finishing entering FMOLP model, the DM will switch to the window as shown in Figure 5 to solve the problem. Suppose the manager in the company wants to generate about 8 millions dollars profit and about 13 units of pollution. Then the corresponding membership functions of the fuzzy goals are set up as follows:

$$g_1(x)=\begin{cases}0 & x<6 \text{ or } 12<x\\ (x-6)/1.5 & 6\le x<7.5\\ 1 & 7.5\le x\le 8.5\\ (12-x)/3.5 & 8.5<x\le 12\end{cases}\qquad (14)$$

$$g_2(x)=\begin{cases}0 & x<11 \text{ or } 17<x\\ (x-11)/1.5 & 11\le x<12.5\\ 1 & 12.5\le x\le 13.5\\ (17-x)/3.5 & 13.5<x\le 17\end{cases}\qquad (15)$$

And the DM can enter the fuzzy goals of the objective functions by Window "Fuzzy Goal Input" as shown in Figure 6.

Back to the window as shown in Figure 5, press Button "Run". Finally, the running results is that the decision variables are $x_1^* = 2.5455$ and $x_2^* = 2.9091$, which means that 2.5455 tons of P_1 and 2.9091 tons of P_2 should be produced if two fuzzy goals which are set by the manager need to be achieved. Two fuzzy objective functions will be $\tilde{f}_1^*(x_1^*, x_2^*) = \tilde{f}_1^*(2.5455, 2.9091) = 2.5455\tilde{c}_{11} + 2.9091\tilde{c}_{12}$ $\tilde{f}_2^*(x_1^*, x_2^*) = \tilde{f}_2^*(2.5455, 2.9091) = 2.5455\tilde{c}_{21} + 2.9091\tilde{c}_{22}$, which are shown as in Figure 7.

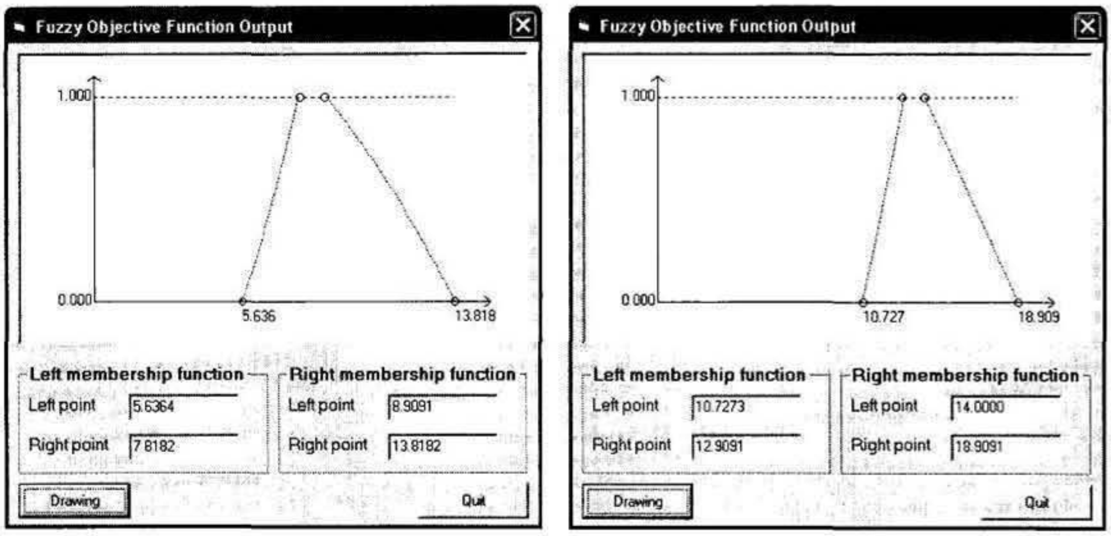

Figure 7: The window for fuzzy objection functions output

In Figure 7, the left diagram represents the form of membership function of fuzzy profit, four end-points of which are 5.6364, 7.8182, 8.9091 and 13.8182. Compared with the membership function in (14), the running result of fuzzy profit is similar to the manager's setting of the first fuzzy goal. Also the right diagram in Figure 7 represents the form of membership function of fuzzy units of pollution, four end-points of which are 10.7273, 12.9091, 14 and 18.9091. Compared with the membership function in (15), the running result of fuzzy units of pollution is similar to the manager's setting of the second fuzzy goal as well.

6. CONCLUSIONS

Decision-making in complex and ill-structured situations is normally affected by uncertainty, which is essentially due to the insufficient and imprecise nature of data evaluated by decision makers. We have developed several methods and algorithms for solving FMOLP problems with fuzzy parameters in any form of membership function in both objective functions and constraints. We have also developed FMODSS a decision support system for fuzzy multiple objective linear programming with fuzzy parameters based on these methods and algorithms. Especially, the detail description is focused on the situation in which some fuzzy goals need to be achieved for FMOLP problem. This system has been initially tested by a number of examples and results are very positive. This system will be put online in order to attract more applications to use the system.

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