Angle-of-Arrival Estimation Using Different Phase Shifts across Subarrays in Localized Hybrid Arrays

J. Andrew Zhang, Wei Ni, Peng Cheng and Yin Lu

***Abstract*—Angle-of-Arrival (AoA) estimation, even for a single arriving signal, in a localized hybrid array is challenging and time-consuming due to a phase ambiguity problem. Subarrays use the same phase shifting values conventionally to exploit cross-correlation between them. This results in the requirement of scanning multiple angles over excessively long periods to resolve the phase ambiguity problem. In this paper, we propose an approach which allows subarrays to use different phase shifts per estimation and can resolve the ambiguity problem by directly estimating the desired AoA parameter. This can potentially speed up the estimation and improve the estimation performance significantly. Simulation results are provided to demonstrate the effectiveness of the proposed approach.**

I. Introduction

Millimeter wave (mm-wave) hybrid array [1] is regarded as a promising low-cost solution for 5G cellular networks. Since line-of-sight (LOS) propagation is dominating in mm-wave channels, angle-of-arrival (AoA) estimation becomes very important and forms the basis for many advanced processing techniques in mm-wave systems.

The AoA estimation in a hybrid array has been studied for multiple incoming signals in, e.g. [2]–[4] and a single signal in, e.g. [5], [6]. For multiple incoming signals, techniques such as Sparse Bayesian Learning [2], adaptive Compressed Sensing [3] and spectrum analysis [4] are proposed. These schemes typically have high computational complexity. For a single signal, the constant phase difference between corresponding elements in two neigbouring subarrays can be exploited to develop algorithms with a much lower complexity. However, in localized hybrid arrays [1] where antennas in one subarray are adjacent to each other, the phase ambiguity problem [5] is a big challenge for estimating a single AoA. The directly estimated parameter is the phase shift between two subarrays. Breaking down this direct estimate to the desired AoA parameter between two antennas faces ambiguity, because of an unknown integer times of 2π up to the number of antennas in one subarray. This ambiguity problem can be solved by, for example, repeatedly scanning more angles and choosing one estimate closest to the scanned angle with maximum energy [5] or using a combination of scanning and tracking [6] over multiple symbol periods. In either case, same phase shifting values need to be used across different subarrays to make the cross-correlation approach work. Although signal energy may be increased, this significantly increases the estimation time, particularly when N is large.

In this paper, we consider the estimation of a single AoA and propose an approach where different phase shifts can be applied across different subarrays. This approach has great potential to speed up the estimation. We also develop three methods to solve the phase ambiguity problem. Two of them enable the direct estimation of the desired AoA parameter between two antennas, by forming an elegant discrete Fourier Transform (DFT) expression. In the noiseless case, these two methods will generate an exact estimate. This paper only focuses on the processing in the first symbol period, leaving the possibly various options of integrating our schemes as open problems.

II. Problem Formulation

Consider a rectangular localized hybrid array which consists of $M_x \times M_y$ rectangular analog subarrays. Each subarray has $N_x \times N_y$ adjacent antennas connected with analogue adjustable phase shifters. The readers are referred to [1] for more details of the array structure.

Assume that each antenna has an omni-directional radiation pattern, i.e., spatial response 1 over all angles. For an incoming signal $\tilde{s}(t)$ with wavelength λ_c at elevation angle θ and azimuth angle ϕ, the received signal at the (m_x, m_y)-th subarray can be represented as [5]

$$\begin{aligned} &s_{m_x,m_y}(t) \\ &= \tilde{s}(t) P_{m_x,m_y}(\theta,\phi) e^{j(m_x N_x u_x + m_y N_y u_y)} + \xi_{m_x,m_y}(t), \quad (1) \end{aligned}$$

where $\xi_{m_x,m_y}(t)$ is the sum of all the additive white Gaussian noise (AWGN) from $N_x N_y$ antenna elements, and

$$u_x = 2\pi d \sin(\theta)\cos(\phi)/\lambda_c, \;\; u_y = 2\pi d \sin(\theta)\sin(\phi)/\lambda_c,$$

$$P_{m_x,m_y}(\theta,\phi) = \sum_{n_x=0}^{N_x-1} \sum_{n_y=0}^{N_y-1} e^{j(n_x u_x + n_y u_y + \alpha_{m_x,m_y}(n_x,n_y))},$$

where d is the distance between adjacent antennas, $\alpha_{m_x,m_y}(n_x,n_y)$ is the phase shift value at the (n_x,n_y)-th antenna in the (m_x,m_y)-th subarray, and $j=\sqrt{-1}$. A typical analog phase shifter can only be set to one of K values and we assume $\alpha_{m_x,m_y}(i_x,i_y) = 2\pi k/K, k=0,\cdots,K-1$.

In [5], [6], $\alpha_{m_x,m_y}(i_x,i_y)$ is chosen as the same for the (i_x,i_y)-th antenna in all subarrays, such that $P_{m_x,m_y}(\theta,\phi)$ becomes independent of m_x and m_y and cross-correlation

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over different subarrays in the same column or row can be calculated to get the estimate for $N_y u_y$ or $N_x u_x$. Note that any phase estimate is constrained to $[-\pi, \pi]$. Hence a phase ambiguity problem arises as $N_x u_x \in [-\pi, \pi]$ can lead to up to N_x possible values for u_x; similarly for u_y. Additional approaches such as trying different phase shifts over multiple symbols need to be applied to solve the phase ambiguity problem, and hence the AoA estimation is slow.

III. Proposed AoA Estimation Scheme

In our proposed scheme, different phase shifting values are applied to different subarrays and u_x or u_y can be estimated directly without phase ambiguity in the absent of noise. This scheme can potentially speed up the estimation process significantly, particularly when SNR is not very low. For clarity, we will present the scheme with reference to a linear array with M $N\times 1$ linear subarrays, and then extend the results to a rectangular array. Thus we will drop the subscript x and y in the symbols hereafter, except for Section III-E.

In a hybrid array, typically $N \gg M$, and $K \geq M$ as at least M beamforming directions are needed to support up to M spatially separated users. For simplicity, we assume $K = N = QM$ where $Q \geq 1$ is an integer.

Now for the first symbol $t = 1$, without any prior knowledge of the AOA, the hybrid array uses the following phase shifting values in the m-th subarray

$$\alpha_m(n) = -2\pi nm/M. \tag{2}$$

Equivalently the array scans M directions at a time.

In this case, $s_{m_x,m_y}(t)$ in (1) becomes

$$s_m(1) = \tilde{s}(1)e^{jmNu} \underbrace{\sum_{n=0}^{N-1} e^{jnu} e^{-j2\pi nm/M}}_{P_m} + \xi_m(1) \tag{3}$$

$$= \tilde{s}(1)e^{jmNu} e^{(j(N-1)\omega_m)} \frac{\sin(N\omega_m)}{\sin(\omega_m)} + \xi_m(1), \tag{4}$$

where $\omega_m = (u - 2\pi m/M)/2$.

By letting $n = n' + qM$, $n' = 0, \cdots, M-1$, the term P_m can be further written as

$$\begin{aligned} P_m &= \sum_{n'=0}^{M-1} \sum_{q=0}^{Q-1} e^{j(n'+qM)u} e^{-j2\pi(n'+qM)m/M} \\ &= \sum_{n'=0}^{M-1} \underbrace{\frac{1 - e^{jQMu}}{1 - e^{jMu}} e^{jn'u}}_{g_{n'}(u)} e^{-j2\pi n'm/M} \\ &= \sum_{n'=0}^{M-1} \underbrace{e^{jn'u} e^{j(Q-1)Mu/2} \frac{\sin(QMu/2)}{\sin(Mu/2)}}_{g_{n'}(u)} e^{-j2\pi n'm/M} \end{aligned} \tag{5}$$

From (5) we can see that $P_m, m = 0, \cdots, M-1$ are actually the DFT coefficients of $g_{n'}(u), n' = 0, \cdots, M-1$. According to the expression of $g_{n'}(u)$, only the term $e^{jn'u}$ is a function of n'. Hence we can compute the phase of the cross-correlation $g_{n'}(u)g^*_{n'+1}(u)$s to get an estimate for u, where $(\cdot)^*$ denotes conjugate operation.

Referring to (4), we can see that $\tilde{s}(1)$ is common for all subarrays and can hence be absorbed to $g_{n'}(u)$. The only unknown is e^{jmNu} for estimating P_m. Computing the cross-correlation between two neighbouring subarrays m and $m+1$ generates

$$\begin{aligned} \rho_m =& s^*_m(1)s_{m+1}(1) \\ =& |\tilde{s}(1)|^2 e^{j((1-N)\pi/M + Nu)} \frac{\sin(N\omega_m)}{\sin(\omega_m)} \cdot \frac{\sin(N\omega_{m+1})}{\sin(\omega_{m+1})} + \\ & \xi^*_m(1)\xi_{m+1}(1). \end{aligned} \tag{6}$$

When suppressing the noise term, the value of e^{jNu} can be determined from the phase of ρ_m up to the difference of ± 1. This is because the sign of the four sine functions can be either -1 or 1.

Now we want to remove the impact of ± 1 from ρ_m so that we can combine the signals $\rho_m, m = 0, \cdots, M-2$ constructively to estimate Nu. The proposed method is as follows:

S1. Compute the phase difference between the one with the maximum magnitude and the others among ρ_ms;
S2. If a phase difference is larger than $\pi/2$, update that ρ_m by multiplying it with a phase term $e^{j\pi}$. Otherwise no update;
S3. Calculate the mean of all these (updated) ρ_ms in S2;
S4. Calculate the angle β of the product between the mean and $e^{j(N-1)\pi/M}$. The estimate of Nu, $\widehat{Nu}$, is

$$\widehat{Nu} = \text{mod}(\beta, 2\pi) - \pi, \quad \text{or} \tag{7}$$

$$\widehat{Nu} = \text{mod}(\beta - \pi, 2\pi) - \pi, \tag{8}$$

where $\text{mod}(\cdot, \cdot)$ is the modular operator.

Then u will be one out of N possible values

$$\hat{u} = \text{mod}((\widehat{Nu} + 2\pi n)/N, 2\pi) - \pi, n = 0, \cdots, N-1 \tag{9}$$

Next we propose three methods for estimating u.

A. Method 1: Peak Alignment

Among all N possible values in (9), we simply choose the one closest to the phase of $s_m(1), m \in [0, M-1]$ with the maximal power. This can be implemented as follows.

S1. Find $s_m(1), m \in [0, M-1]$ with the maximal power. Denote it as $s_{\tilde{m}}(1)$;
S2. Use $\hat{u}$ in (9) to compute $|\hat{u}^* s_{\tilde{m}}(1)|$ for all $n \in [0, N-1]$ and two possible $\widehat{Nu}$ values;
S3. The one $\hat{u}$ leading to the maximal $|\hat{u}^* s_{\tilde{m}}(1)|$ is the estimate of u.

B. Method 2: Rotate Outputs towards Common Directions

Method 2 uses (4) and (5) to estimate u. For every possible $\widehat{Nu}$, the estimation process is as follows.

S1. Multiply $e^{-jm\widehat{Nu}}$ to $s_m(1)$ in (4), and get a noisy version of $P_m\tilde{s}(1)$, $m = 0, \cdots, M-1$;
S2. Compute inverse DFT of $e^{-jm\widehat{Nu}} s_m(1)$, and get estimates of $a_{n'} = \tilde{s}(1)g_{n'}(u)$, $n' = 0, \cdots, M-1$;
S3. Compute cross-correlation between adjacent $a_{n'}$, and average over these $M-2$ correlation values;

S4. Compute the angle of the averaged correlation and get $\hat{u}_2$, which is the estimated value of u.

The two calculated values of $\widehat{Nu}$ are used to generate two estimates of u, and the final estimate is chosen as the one having the minimum Euclidean distance between the estimates and possible values in (9). This is because in the noiseless case, one of the values in (9) should be identical to the one estimated using the correct $\widehat{Nu}$.

C. Method 3: Separate Even and Odd Outputs

Method 3 works when M is an even number. It is based on the observation that a phase term $e^{j\pi}$ only changes the sign of $s_m(1)$ when m is odd. Hence we can separate $s_m(1)$ to even and odd samples, and test a single $\widehat{Nu}$. At the same time, we can separate P_m to even and odd samples and re-express them as $M/2$-point inverse DFTs of two sequences.

For an even sample, $P_{m=2m'}, m' = 0, \cdots, M/2-1$, has a similar expression with (5), with M and Q being replaced by $M/2$ and $2Q$ respectively. For an odd sample, $P_{m=2m'+1}$ becomes

$$P_m = \sum_{n'=0}^{M/2-1} \underbrace{\frac{1-e^{jQMu}}{1-e^{jMu/2}} e^{jn'(u-2\pi/M)}}_{g_{n'}(u)} e^{-j2\pi n'm/(M/2)}.$$

Therefore the process described in Section III-B can be applied to the even and odd samples of $s_m(1)$ separately. Let ϱ_o and ϱ_e denote the two averaged values obtained in S3 in Method 2 for odd and even sequences respectively. The value of u can be estimated by combining these two as

$$\hat{u}_3 = \text{angle}(\varrho_o e^{j2\pi/M} + \varrho_e). \quad (10)$$

D. Subsequent Processing

In the preceding sections, we have discussed how to estimate u in the first symbol period. Although in the noiseless cases, methods 2 and 3 will provide an exact estimate of u. In the presence of noise, particularly when SNR is low, estimation over multiple symbol periods is needed.

There could be many options on how to use the estimates obtained from the first symbol, which are beyond the scope of this paper. Here, we only discuss how one may continue to apply one of the preceding estimation methods in the following $N/M-1$ symbol periods, such that N angles can be scanned to generate a complete set of samples including those from the so-called "mainlobe" of an underlying waveform. Note that similar to that in the first symbol period, the estimate in every following symbol period is also "exact" in the noiseless case. They can be treated independently but can also be combined.

Recall P_m in (4). we actually sampled a periodic signal of the period 2π

$$f(x) = \sum_{n=0}^{N-1} e^{jn(x-u)} = e^{j(N-1)(x-u)/2} \frac{\sin(N(x-u)/2)}{\sin((x-u)/2)}$$

at $x = 2\pi m/M$. Note that the waveform of $f(x)$ has one mainlobe of width $4\pi/N$ and $(N-2)$ sidelobes of width $2\pi/N$, as shown in Fig.1.

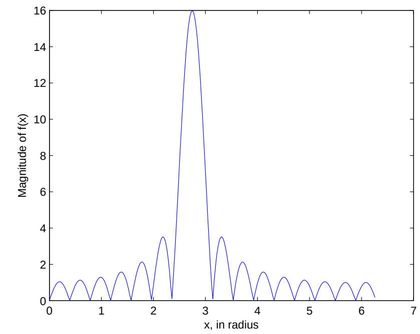

Fig. 1. Magnitude of the function $f(x)$, with $u = 2.746$.

Without any prior knowledge of u, it can only be guaranteed that one sample is from the mainlobe after getting at least $(N-1)$ samples. In the t-th symbol period, we set

$$\alpha_m(n) = -2\pi n(Qm + t - 1)/N, m = 0, \cdots, M-1. \quad (11)$$

In this case, P_m becomes

$$P_m = \sum_{n=0}^{N-1} e^{jn(u-2\pi(t-1)/N)} e^{-j2\pi nm/M}$$
$$= \sum_{n'=0}^{M-1} \underbrace{\frac{1-e^{jQM(u-\varphi_t)}}{1-e^{jM(u-\varphi_t)}} e^{jn'(u-\varphi_t)}}_{g_{n'}(u)} e^{-j2\pi n'm/M}, \quad (12)$$

where $\varphi_t = 2\pi(t-1)/N$. From (12) we can see that u can be estimated similarly to that in the first symbol (where $t = 1$), except that a phase term $e^{j\varphi_t}$ needs to be multiplied to e.g., the averaged cross-correlation output in S3 in Section III-B, before proceeding to S4. Such multiplied outputs can also be averaged over multiple symbols. Note that when noise is dominating, this averaging may not always lead to improved performance.

Since $\exp(-j2\pi nm/N) = \exp(-j2\pi \operatorname{mod}(nm, N)/N)$, $n, m = 0, \cdots, N-1$, only N values are required for each phase shifter in a subarray. This will enable the array to scan N angles uniformly distributed from 0 to 2π, and guarantee to get two samples from the mainlobe of $f(x)$.

E. Extension to 2-D Arrays

The above proposed schemes can be extended to a 2-D localized rectangular array. A brief description of the key points are as follows:

- Use $\alpha_{m_x,m_y}(n_x,n_y) = -2\pi n_x(Q_x m_x + t - 1)/N_x - 2\pi n_y(Q_y m_y + t - 1)/N_y$;
- Apply any of the proposed methods to subarrays in each column and get an estimate for u_y. The SNR can be improved by averaging over different columns. For example, using Method 2, the averaged cross-correlation values in S3 could be further averaged over columns from 0 to $M_x - 1$, before S4 is implemented. Likewise, u_x can be estimated in the per-row direction.
- A complete set of samples need to be obtained after scanning $N_x N_y$ directions over a total of $Q_x Q_y$ symbol periods.

IV. Simulation Results

In this section, we present simulation results for the proposed methods, compared to well known compressive sensing (CS) techniques [2], [3]. We consider a linear array with $d = \lambda/2$. The signal AoA is uniformly distributed on $[0, \pi]$. Every simulation result is obtained by averaging over 40000 trials. SNR in dB is measured per antenna.

For CS, sparse Bayesian learning (SBL) [2] and grid-based formulation are used to solve the problem, with a quantization step of $\pi/32$ for AoAs in the dictionary. Phase shifts are randomly chosen from the dictionary for each subarray. To be consistent with our proposed algorithm, only one shot of M measurements are used for each estimation.

In Fig. 2, we present the root mean squared error (RMSE) of the estimates for different methods. Method 1 achieves lower RMSE at low SNRs, especially when $N = 8$ and $M = 4$. Methods 2 and 3 outperform Method 1 when SNR is high. This is because Methods 2 and 3, which are DFT-based, are more sensitive to SNR. Hence Method 1 is suitable when SNR is low and/or $M < N$ where collected signal power can be low. Method 2 shows slightly better performance than Method 3, attributed to the extra averaging in Method 2. Although SBL is powerful for estimating multiple AoAs simultaneously, it shows inferior performance to all our methods for single AoA estimation when SNR is practically small. Our methods are also much simpler. The average Matlab running speed of the proposed schemes is approximately 94 times of that for SBL.

The above findings can be seen more clearly from Fig. 3, which demonstrates the cumulative distribution function (CDF) of the MSE of the estimates, i.e., the probability of the MSE larger than a given threshold. Methods 2 and 3 show better performance than Method 1 for most threshold values when $M = 8$, and for thresholds up to 0.5 when $M = 4$. For SBL, a large portion of its estimates, 80% and 60% at the left and right subfigures respectively, have high accuracy, but the rest have large estimation errors, partially attributed to the quantization error floor. This leads to large RMSE overall in Fig. 2. This is also the phenomenon observed in our extensive simulations under various SNRs and N, M values.

V. Conclusions

In this paper, we proposed an approach to speeding up the estimation of a single AoA in a hybrid array. Our approach generates cross-correlations between different pairs of adjacent subarrays with ambiguity in a single sign. Three methods were further developed to address the sign-difference. Two of the proposed methods enable the estimation of the desired AoA between two antenna elements by formulating a DFT relationship between the observed signals and the signals leading to the AoA estimate. The proposed methods can be extended over multiple symbols with the extension formula provided. The methods can be implemented over only a limited number of symbols alone, or be integrated as part of complete AoA estimation that involves iterations between estimation and tracking [6], particularly when the SNR is low.

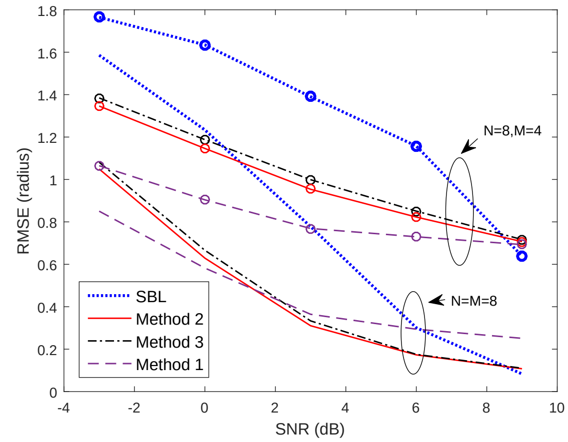

Fig. 2. RMSE of the estimates obtained by different methods versus SNR, for two setup of $N = M = 8$ and $N = 8, M = 4$.

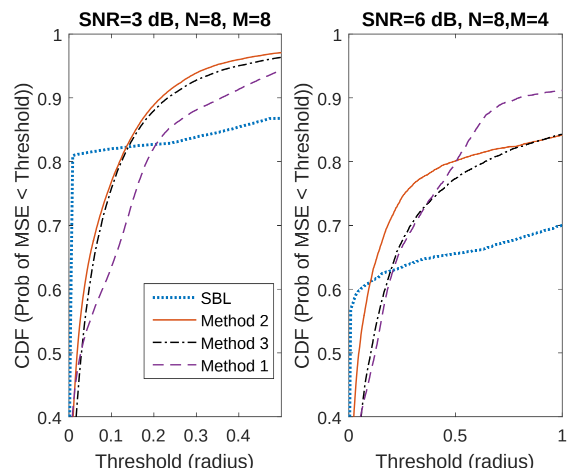

Fig. 3. CDF of MSE for two setups: (Left) $N = M = 8$, SNR =3 dB, and (Right) $N = 8, M = 4$, SNR=6 dB.

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