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A λ−cut Approximate Approach to Supporting Fuzzy Goal Based Bilevel Decision Making in Risk Management

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Abstract

Bilevel decision making techniques are developed to handle the decentralized problem, in many areas such as risk management, with two-level decision makers, the leader and the follower, who may set goals for their fuzzy objective functions. This paper proposes a λ−cut approximate approach to solving the Fuzzy Goal based Bilevel Decision Making (FGBLDM) problem. Firstly, the λ−cut set based FGBLDM model and related definitions and theorems are given. Then, the programmable algorithm for FGBLDM is presented in detail. Finally, a numerical example is given to illustrate the executing procedure of this algorithm.

Keywords: Bilevel decision making, Goal programming, Fuzzy sets, Optimization, Risk management

1. Introduction

Bilevel decision making techniques are mainly developed for solving decentralized management problems with decision makers in a hierarchical organization, the upper the leader and the lower the follower. Both the leader and the follower try to optimize their own objective functions and the corresponding decisions do not control but affect that of the other level. A bilevel problem in which the parameters, either in objective functions or constrains of leader or follower, are described by fuzzy values is called a fuzzy bilevel decision making [1], which is recognized as an effective technique in finding potential risks and generating warnings.

The goal programming was originally proposed by Charnes and Cooper [2] in 1961 for a linear model. It has been further developed by Lee [3] Ignizio [4, 5], Charnes and Cooper [6]. Recent research can be found from [7–10]. The method requests the decision maker to set a goal for the objective that he/she wishes to attain. A preferred solution is then defined to minimize the deviation from the goal. Therefore goal programming seems to yield a satisfactory solution rather than an optimal one.

In fuzzy bilevel decision making, when both leader and follower set goals for their objectives, the problem of FGBLDM occurs. This study is addressed to this very FGBLDM problem with the adjustable satisfactory degree α, which means, when comparing two fuzzy numbers all values with membership grades smaller than α are neglected.

Bilevel decision making techniques have been applied with remarkable success in different domains. For risk management, which aims to measure or assess risk and develop strategies to manage it [11], FGBLDM plays a significant role as well. The way to allocating resources of supplies, transportation ability, rescue aid and whatever to minimize the effect of threat realization is the key point faced by decision makers located at different levels within the managing network, and thus having inconsistent concerns. For example, when a severe earthquake occurs [12], the roadway systems usually get different degrees of damage, thus reducing the capacity of those roadways, and causing traffic congestion. How to maintain traffic functions reasonably to facilitate saving more lives will be the utmost mission task after quakes. The commander of Emergency-Response Center of government in county and city level (the upper level) aims at allowing traffic to go through the disaster areas as much as possible within the roadway capacity, while the road users (located at the lower level) always choose the shortest route to actualize emer-

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gency rescues. In this situation, bilevel programming should be a suitable techniques to solve this decision making problem.

This paper is organized as: following the introduction, Section 2 presents a λ–cut approximate algorithm together with relevant models and theorems. A numerical example is shown in Section 3 for illustrating the proposed approach. Conclusions and further study are discussed in Section 4.

2. The λ–cut Approximate Algorithm for FGBLDM

Definition 1 *The Fuzzy Linear Bilevel Programming (FLBLP) is defined as [13]:*

For $x \in X \subset R^n$, $y \in Y \subset R^m$, $F : X \times Y \to F^*(R)$, and $f : X \times Y \to F^*(R)$,

$$\min_{x \in X} F(x,y) = \tilde{c}_1 x + \tilde{d}_1 y \tag{1a}$$

$$\text{subject to } \tilde{A}_1 x + \tilde{B}_1 y \precsim \tilde{b}_1 \tag{1b}$$

$$\min_{y \in Y} f(x,y) = \tilde{c}_2 x + \tilde{d}_2 y \tag{1c}$$

$$\text{subject to } \tilde{A}_2 x + \tilde{B}_2 y \precsim \tilde{b}_2 \tag{1d}$$

where $\tilde{c}_1$, $\tilde{c}_2 \in F^*(R^n)$, $\tilde{d}_1$, $\tilde{d}_2 \in F^*(R^m)$, $\tilde{b}_1 \in F^*(R^p)$, $\tilde{b}_2 \in F^*(R^q)$, $\tilde{A}_1 = (\tilde{a}_{ij})_{p\times n}$, $\tilde{a}_{ij} \in F^*(R)$, $\tilde{B}_1 = \left(\tilde{b}_{ij}\right)_{p\times m}$, $\tilde{b}_{ij} \in F^*(R)$, $\tilde{A}_2 = (\tilde{e}_{ij})_{q\times n}$, $\tilde{e}_{ij} \in F^*(R)$, $\tilde{B}_2 = (\tilde{s}_{ij})_{q\times m}$, $\tilde{s}_{ij} \in F^*(R)$, and $F^*(R)$ is the set of all finite fuzzy numbers.

Goals set for the objectives of the leader and the follower in (1) are denoted by fuzzy numbers $\tilde{g}_L$ and $\tilde{g}_F$ with membership functions μ_{g_L} and μ_{g_F} respectively, and our concern is to make the objectives of both the leader and the follower as near to their goals as possible. The differences between $F(x,y)$ and $\tilde{g}_L$, $f(x,y)$ and $\tilde{g}_F$ are usually defined as deviation functions. We use λ-cut set of fuzzy number to format the model of FGBLDM as:

Definition 2 *The λ-cut set based FGBLDM model is defined as:*

$$\begin{aligned} &\min_{x\in X} |c^L_{1\lambda_j} x + d^L_{1\lambda_j} y - g^L_{L\lambda_j}|,\\ &\min_{x\in X} |c^R_{1\lambda_j} x + d^R_{1\lambda_j} y - g^R_{L\lambda_j}|, \end{aligned} \tag{2a}$$

$$\begin{aligned} &\text{subject to } A_{1}{}^L_{\lambda_j} x + B_{1}{}^L_{\lambda_j} y \leqq b_{1}{}^L_{\lambda_j},\\ &A_{1}{}^R_{\lambda_j} x + B_{1}{}^R_{\lambda_j} y \leqq b_{1}{}^R_{\lambda_j}, \end{aligned} \tag{2b}$$

$$\begin{aligned} &\min_{y\in Y} |c^L_{2\lambda_j} x + d^L_{2\lambda_j} y - g^L_{F\lambda_j}|,\\ &\min_{y\in Y} |c^R_{2\lambda_j} x + d^R_{2\lambda_j} y - g^R_{F\lambda_j}|, \end{aligned} \tag{2c}$$

$$\begin{aligned} &\text{subject to } A_{2}{}^L_{\lambda_j} x + B_{2}{}^L_{\lambda_j} y \leqq b_{2}{}^L_{\lambda_j},\\ &A_{2}{}^R_{\lambda_j} x + B_{2}{}^R_{\lambda_j} y \leqq b_{2}{}^R_{\lambda_j} \end{aligned} \tag{2d}$$

where $j = 0, 1, 2, ..., l$, $\tilde{c}_1$, $\tilde{c}_2 \in F^*(R^n)$, $\tilde{d}_1$, $\tilde{d}_2 \in F^*(R^m)$, $\tilde{b}_1 \in F^*(R^p)$, $\tilde{b}_2 \in F^*(R^q)$, $\tilde{A}_1 = (\tilde{a}_{ij})_{p\times n}$, $\tilde{B}_1 = \left(\tilde{b}_{ij}\right)_{p\times m}$, $\tilde{A}_2 = (\tilde{e}_{ij})_{q\times n}$, $\tilde{B}_2 = (\tilde{s}_{ij})_{q\times m}$, $\tilde{a}_{ij}$, $\tilde{b}_{ij}$, $\tilde{e}_{ij}$, $\tilde{s}_{ij} \in F^*(R)$.

For a clear understanding of the idea adopted, define:

$$\begin{aligned}
v^{L-}_{1\lambda_j} &= \frac{1}{2}[|c^L_{1\lambda_j} x + d^L_{1\lambda_j} y - g^L_{L\lambda_j}| \\ &\qquad - (c^L_{1\lambda_j} x + d^L_{1\lambda_j} y - g^L_{L\lambda_j})]\\
v^{L+}_{1\lambda_j} &= \frac{1}{2}[|c^L_{1\lambda_j} x + d^L_{1\lambda_j} y - g^L_{L\lambda_j}| + \\ &\qquad (c^L_{1\lambda_j} x + d^L_{1\lambda_j} y - g^L_{L\lambda_j})]\\
v^{R-}_{1\lambda_j} &= \frac{1}{2}[|c^R_{1\lambda_j} x + d^R_{1\lambda_j} y - g^R_{L\lambda_j}| - \\ &\qquad (c^R_{1\lambda_j} x + d^R_{1\lambda_j} y - g^R_{L\lambda_j})]\\
v^{R+}_{1\lambda_j} &= \frac{1}{2}[|c^R_{1\lambda_j} x + d^R_{1\lambda_j} y - g^R_{L\lambda_j}| + \\ &\qquad (c^R_{1\lambda_j} x + d^R_{1\lambda_j} y - g^R_{L\lambda_j})]\\
v^{L-}_{2\lambda_j} &= \frac{1}{2}[|c^L_{2\lambda_j} x + d^L_{2\lambda_j} y - g^L_{F\lambda_j}| - \\ &\qquad (c^L_{2\lambda_j} x + d^L_{2\lambda_j} y - g^L_{F\lambda_j})]\\
v^{L+}_{2\lambda_j} &= \frac{1}{2}[|c^L_{2\lambda_j} x + d^L_{2\lambda_j} y - g^L_{F\lambda_j}| + \\ &\qquad (c^L_{2\lambda_j} x + d^L_{2\lambda_j} y - g^L_{F\lambda_j})]\\
v^{R-}_{2\lambda_j} &= \frac{1}{2}[|c^R_{2\lambda_j} x + d^R_{2\lambda_j} y - g^R_{F\lambda_j}| - \\ &\qquad (c^R_{2\lambda_j} x + d^R_{2\lambda_j} y - g^R_{F\lambda_j})]\\
v^{R+}_{2\lambda_j} &= \frac{1}{2}[|c^R_{2\lambda_j} x + d^R_{2\lambda_j} y - g^R_{F\lambda_j}| + \\ &\qquad (c^R_{2\lambda_j} x + d^R_{2\lambda_j} y - g^R_{F\lambda_j})]
\end{aligned}$$

Associated with the FBLGP problem (2), we now consider the following bilevel programming problem:

For $(v^{L-}_{1\lambda_j}, v^{L+}_{1\lambda_j}, v^{R-}_{1\lambda_j}, v^{R+}_{1\lambda_j}) \in R^4$, $X' \subseteq X \times R^4$, $(v^{L-}_{2\lambda_j}, v^{L+}_{2\lambda_j}, v^{R-}_{2\lambda_j}, v^{R+}_{2\lambda_j}) \in R^4$, $Y' \subseteq Y \times R^4$, let $x = (x_1, \cdots, x_n) \in X$, $x' = (x_1, \cdots, x_n, v^{L-}_{1\lambda_j}, v^{L+}_{1\lambda_j}, v^{R-}_{1\lambda_j}, v^{R+}_{1\lambda_j}) \in X'$, $y = (y_1, \cdots, y_m) \in Y$, $y' = (y_1, \cdots, y_m, v^{L-}_{2\lambda_j}, v^{L+}_{2\lambda_j}, v^{R-}_{2\lambda_j}, v^{R+}_{2\lambda_j}) \in Y'$, and

$v^L_{1\lambda_j}, v^R_{1\lambda_j}, v^L_{2\lambda_j}, v^R_{2\lambda_j} : X' \times Y' \to F^*(R)$.

$$\min_{x' \in X'} v^L_{1\lambda_j} = v^{L-}_{1\lambda_j} + v^{L+}_{1\lambda_j}$$
$$\min_{x' \in X'} v^R_{1\lambda_j} = v^{R-}_{1\lambda_j} + v^{R+}_{1\lambda_j} \tag{3a}$$

$$\begin{aligned}
\text{subject to } & c^L_{1\lambda_j} x + d^L_{1\lambda_j} y + v^{L-}_{1\lambda_j} - v^{L+}_{1\lambda_j} = g^L_{L\lambda_j}, \\
& c^R_{1\lambda_j} x + d^R_{1\lambda_j} y + v^{R-}_{1\lambda_j} - v^{R+}_{1\lambda_j} = g^R_{L\lambda_j}, \\
& v^{L-}_{1\lambda_j}, v^{L+}_{1\lambda_j}, v^{R-}_{1\lambda_j}, v^{R+}_{1\lambda_j} \geq 0, \\
& v^{L-}_{1\lambda_j} \cdot v^{L+}_{1\lambda_j} = 0, \\
& v^{R-}_{1\lambda_j} \cdot v^{R+}_{1\lambda_j} = 0, \\
& A_1{}^L_{\lambda_j} x + B_1{}^L_{\lambda_j} y \leqq b_1{}^L_{\lambda_j}, \\
& A_1{}^R_{\lambda_j} x + B_1{}^R_{\lambda_j} y \leqq b_1{}^R_{\lambda_j},
\end{aligned} \tag{3b}$$

$$\min_{y' \in Y'} v^L_{2\lambda_j} = v^{L-}_{2\lambda_j} + v^{L+}_{2\lambda_j}$$
$$\min_{y' \in Y'} v^R_{2\lambda_j} = v^{R-}_{2\lambda_j} + v^{R+}_{2\lambda_j} \tag{3c}$$

$$\begin{aligned}
\text{subject to } & \\
& c^L_{2\lambda_j} x + d^L_{2\lambda_j} y + v^{L-}_{2\lambda_j} - v^{L+}_{2\lambda_j} = g^L_{F\lambda_j}, \\
& c^R_{2\lambda_j} x + d^R_{2\lambda_j} y + v^{R-}_{2\lambda_j} - v^{R+}_{2\lambda_j} = g^R_{F\lambda_j}, \\
& v^{L-}_{2\lambda_j}, v^{L+}_{2\lambda_j}, v^{R-}_{2\lambda_j}, v^{R+}_{2\lambda_j} \geq 0, \\
& y v^{L-}_{2\lambda_j} \cdot v^{L+}_{2\lambda_j} = 0, \\
& y v^{R-}_{2\lambda_j} \cdot v^{R+}_{2\lambda_j} = 0, \\
& A_2{}^L_{\lambda_j} x + B_2{}^L_{\lambda_j} y \leqq b_2{}^L_{\lambda_j}, \\
& A_2{}^R_{\lambda_j} x + B_2{}^R_{\lambda_j} y \leqq b_2{}^R_{\lambda_j}
\end{aligned} \tag{3d}$$

Theorem 1 *Let* $(x'^*, y'^*) = (x^*, v^{L-*}_{1\lambda_j}, v^{L+*}_{1\lambda_j}, v^{R-*}_{1\lambda_j}, v^{R+*}_{1\lambda_j}, y^*, v^{L-*}_{2\lambda_j}, v^{L+*}_{2\lambda_j}, v^{R-*}_{2\lambda_j}, v^{R+*}_{2\lambda_j})$ *be the optimal solution to the bilevel problem (3), then* (x^*, y^*) *is the optimal solution to the bilevel problem defined by (2).*

By adopting weighting method [14], (3) can be further transferred into (4):

$$\min_{x' \in X'} v^{L-}_{1\lambda_j} + v^{L+}_{1\lambda_j} + v^{R-}_{1\lambda_j} + v^{R+}_{1\lambda_j} \tag{4a}$$

$$\begin{aligned}
\text{subject to } & \\
& c^L_{1\lambda_j} x + d^L_{1\lambda_j} y + v^{L-}_{1\lambda_j} - v^{L+}_{1\lambda_j} = g^L_{L\lambda_j}, \\
& c^R_{1\lambda_j} x + d^R_{1\lambda_j} y + v^{R-}_{1\lambda_j} - v^{R+}_{1\lambda_j} = g^R_{L\lambda_j}, \\
& v^{L-}_{1\lambda_j}, v^{L+}_{1\lambda_j}, v^{R-}_{1\lambda_j}, v^{R+}_{1\lambda_j} \geq 0, \\
& v^{L-}_{1\lambda_j} \cdot v^{L+}_{1\lambda_j} = 0, \\
& v^{R-}_{1\lambda_j} \cdot v^{R+}_{1\lambda_j} = 0, \\
& A_1{}^L_{\lambda_j} x + B_1{}^L_{\lambda_j} y \leqq b_1{}^L_{\lambda_j}, \\
& A_1{}^R_{\lambda_j} x + B_1{}^R_{\lambda_j} y \leqq b_1{}^R_{\lambda_j},
\end{aligned} \tag{4b}$$

$$\min_{y' \in Y'} v^{L-}_{2\lambda_j} + v^{L+}_{2\lambda_j} + v^{R-}_{2\lambda_j} + v^{R+}_{2\lambda_j} \tag{4c}$$

$$\begin{aligned}
\text{subject to } & \\
& c^L_{2\lambda_j} x + d^L_{2\lambda_j} y + v^{L-}_{2\lambda_j} - v^{L+}_{2\lambda_j} = g^L_{F\lambda_j}, \\
& c^R_{2\lambda_j} x + d^R_{2\lambda_j} y + v^{R-}_{2\lambda_j} - v^{R+}_{2\lambda_j} = g^R_{F\lambda_j}, \\
& v^{L-}_{2\lambda_j}, v^{L+}_{2\lambda_j}, v^{R-}_{2\lambda_j}, v^{R+}_{2\lambda_j} \geq 0, \\
& v^{L-}_{2\lambda_j} \cdot v^{L+}_{2\lambda_j} = 0, \\
& v^{R-}_{2\lambda_j} \cdot v^{R+}_{2\lambda_j} = 0, \\
& A_2{}^L_{\lambda_j} x + B_2{}^L_{\lambda_j} y \leqq b_2{}^L_{\lambda_j}, \\
& A_2{}^R_{\lambda_j} x + B_2{}^R_{\lambda_j} y \leqq b_2{}^R_{\lambda_j}
\end{aligned} \tag{4d}$$

The nonlinear conditions of $v^{L-}_{i\lambda_j} \cdot v^{L+}_{i\lambda_j} = 0$, and $v^{R-}_{i\lambda_j} \cdot v^{R+}_{i\lambda_j} = 0$, $i = 1, 2$ need not be maintained if the Kuhn-Tucker approach [15] together with Simplex algorithm are adopted, since only equivalence at an optimum is wanted. Further explanation can be found from [16]. Thus the problem of (4) is further transformed into:

For $(v^-_{1\lambda_j}, v^+_{1\lambda_j}) \in R^2$, $\bar{X}' \subseteq X \times R^2$, $(v^-_{2\lambda_j}, v^+_{2\lambda_j}) \in R^2$, $\bar{Y}' \subseteq Y \times R^2$, let $x = (x_1, \cdots, x_n) \in X$, $\bar{x}' = (x_1, \cdots, x_n, v^-_{1\lambda_j}, v^+_{1\lambda_j}) \in \bar{X}'$, $y = (y_1, \cdots, y_m) \in Y$, $\bar{y}' = (y_1, \cdots, y_m, v^-_{2\lambda_j}, v^+_{2\lambda_j},) \in \bar{Y}'$, and $v_{1\lambda_j}, v_{2\lambda_j} : \bar{X}' \times \bar{Y}' \to F^*(R)$.

$$\min_{\bar{x}' \in \bar{X}'} v_{1\lambda_j} = v^-_{1\lambda_j} + v^+_{1\lambda_j} \tag{5a}$$

$$\begin{aligned}
\text{subject to } & \\
& (c^L_{1\lambda_j} + c^R_{1\lambda_j}) x + (d^L_{1\lambda_j} + d^R_{1\lambda_j}) y \\
& \quad + v^-_{1\lambda_j} - v^+_{1\lambda_j} = g^L_{L\lambda_j} + g^R_{L\lambda_j}, \\
& A_1{}^L_{\lambda_j} x + B_1{}^L_{\lambda_j} y \leqq b_1{}^L_{\lambda_j}, \\
& A_1{}^R_{\lambda_j} x + B_1{}^R_{\lambda_j} y \leqq b_1{}^R_{\lambda_j},
\end{aligned} \tag{5b}$$

$$\min_{\bar{y}' \in \bar{Y}'} v_{2\lambda_j} = v^-_{2\lambda_j} + v^+_{2\lambda_j} \tag{5c}$$

$$\begin{aligned}
\text{subject to } & \\
& (c^L_{2\lambda_j} + c^R_{2\lambda_j}) x + (d^L_{2\lambda_j} + d^R_{2\lambda_j}) y \\
& \quad + v^-_{2\lambda_j} - v^+_{2\lambda_j} = g^L_{F\lambda_j} + g^R_{F\lambda_j},
\end{aligned} \tag{5d}$$

$$\begin{aligned}
& A_2{}^L_{\lambda_j} x + B_2{}^L_{\lambda_j} y \leqq b_2{}^L_{\lambda_j}, \\
& A_2{}^R_{\lambda_j} x + B_2{}^R_{\lambda_j} y \leqq b_2{}^R_{\lambda_j}
\end{aligned} \tag{5e}$$

where $v^-_{i\lambda_j} = v^{L-}_{i\lambda_j} + v^{R-}_{i\lambda_j}$, $v^+_{i\lambda_j} = v^{L+}_{i\lambda_j} + v^{R+}_{i\lambda_j}$, $i = 1, 2$.

The problem of (5) is a standard linear bilevel programming problem, which can be solved by Kuhn-Tucker approach [15].

Based on discussions above, the λ−cut approximate algorithm is detailed as:

[Step 1](Input)

Get relevant parameters which include: Coefficients of (1); Coefficients of $\tilde{g}_L$ and $\tilde{g}_F$; Satisfactory degree α; and $\varepsilon > 0$.

[Step 2](Initializing)

Let $k = 1$, which is the counter to record current loop.

In (2), where $\lambda_j \in [\alpha, 1]$, let $\lambda_0 = \alpha$ and $\lambda_1 = 1$ respectively, then each objective will be transferred into four non-fuzzy objective functions, and each fuzzy constraint is converted into four non-fuzzy constraints.

[Step 3](Computing)

By introducing auxiliary variables $v_{i\lambda_j}^-$ and $v_{i\lambda_j}^+$, $i = 1, 2$, we get the format of (5).

The solution $(x,\ v_{1\lambda_j}^-,\ v_{1\lambda_j}^+,\ y,\ v_{2\lambda_j}^-,\ v_{2\lambda_j}^+)_2$ of (5) is obtained by Kuhn-Tucker approach.

[Step 4](Comparison)

If $(k = 1)$ **Then**

$(x,\ v_{1\lambda_j}^-,\ v_{1\lambda_j}^+,\ y,\ v_{2\lambda_j}^-,\ v_{2\lambda_j}^+)_1 = (x,\ v_{1\lambda_j}^-,\ v_{1\lambda_j}^+,\ y,\ v_{2\lambda_j}^-,\ v_{2\lambda_j}^+)_2$;

goto [Step 5];

Else If

$$|(x, v_{1\lambda_j}^-, v_{1\lambda_j}^+, y, v_{2\lambda_j}^-, v_{2\lambda_j}^+)_2 - (x, v_{1\lambda_j}^-, v_{1\lambda_j}^+, y, v_{2\lambda_j}^-, v_{2\lambda_j}^+)_1| < \varepsilon$$

Then goto [Step 7] ;

EndIf

[Step 5](Splitting)

Suppose there are $(L + 1)$ nodes $\lambda_j, (j = 0, 2, ..., 2L)$ in the interval $[\alpha, 1]$, insert L new nodes λ_j, $(j = 1, 3..., 2L - 1)$ which satisfy: $\lambda_{2j+1} = (\lambda_{2j} + \lambda_{2j+2})/2, (j = 0, 1, ..., L - 1)$.

[Step 6](Loop)

k=k+1;

goto [Step 3];

[Step 7](Output)

$(x, y)_2$ is obtained as the final solution.

3. Numerical Example

To illustrate the approximate algorithm introduced in Section 2, we consider the following FGBLDM problem.

[Step 1]: (Input relevant parameters)

Suppose the problem is:

$$\text{Leader} : \max_{x \in X} F(x, y) = \tilde{c}_1 x + \tilde{d}_1 y$$

$$\text{subject to } \tilde{A}_1 x + \tilde{B}_1 y \leqslant \tilde{b}_1$$

$$\text{Follower} : \min_{y \in Y} f(x, y) = \tilde{c}_2 x + \tilde{d}_2 y$$

$$\text{subject to } \tilde{A}_2 x + \tilde{B}_2 y \leqslant \tilde{b}_2$$

where $x \in R^1, y \in R^1$, and $X = x \geq 0, Y = y \geq 0$.

The membership functions of the coefficients of the objective functions and the constraints of both the leader and the follower are as follows:

$$\mu_{\tilde{c}_1}(x) = \begin{cases} 0 & x < 5 \\ (x^2 - 25)/11 & 5 \leqq x < 8 \\ 1 & x = 6 \\ (64 - x^2)/28 & 6 < x \leq 8 \\ 0 & x > 8 \end{cases}$$

$$\mu_{\tilde{d}_1}(x) = \begin{cases} 0 & x < 2 \\ (x^2 - 4)/5 & 2 \leqq x < 3 \\ 1 & x = 3 \\ (25 - x^2)/16 & 3 < x \leq 5 \\ 0 & x > 5 \end{cases}$$

$$\mu_{\tilde{c}_2}(x) = \begin{cases} 0 & x < -4 \\ (16 - x^2)/7 & -4 \leqq x < -3 \\ 1 & x = -3 \\ (x^2 - 1)/8 & -3 < x \leq -1 \\ 0 & x > -1 \end{cases}$$

$$\mu_{\tilde{d}_2}(x) = \begin{cases} 0 & x < 5 \\ (x^2 - 25)/11 & 5 \leqq x < 6 \\ 1 & x = 6 \\ (64 - x^2)/28 & 6 < x \leq 8 \\ 0 & x > 8 \end{cases}$$

$$\mu_{\tilde{A}_1}(x) = \begin{cases} 0 & x < -2 \\ (4 - x^2)/3 & -2 \leqq x < -1 \\ 1 & x = -1 \\ (x^2 - 0.25)/0.75 & -1 < x \leq -0.5 \\ 0 & x > -0.5 \end{cases}$$

$$\mu_{\tilde{B}_1}(x) = \begin{cases} 0 & x < 2 \\ (x^2 - 4)/5 & 2 \leqq x < 3 \\ 1 & x = 3 \\ (25 - x^2)/16 & 3 < x \leq 5 \\ 0 & x > 5 \end{cases}$$

$$\mu_{\tilde{A}_2}(x) = \begin{cases} 0 & x < 0.5 \\ (x^2 - 0.25)/0.75 & 0.5 \leqq x < 1 \\ 1 & x = 1 \\ (4 - x^2)/3 & 1 < x \leq 2 \\ 0 & x > 2 \end{cases}$$

$$\mu_{\tilde{B}_2}(x) = \begin{cases} 0 & x < 2 \\ (x^2-4)/5 & 2 \leqq x < 3 \\ 1 & x = 3 \\ (25-x^2)/16 & 3 < x \leq 5 \\ 0 & x > 5 \end{cases}$$

$$\mu_{\tilde{b}_1}(x) = \begin{cases} 0 & x < 19 \\ (x^2-361)/80 & 19 \leqq x < 21 \\ 1 & x = 21 \\ (625-x^2)/184 & 21 < x \leq 25 \\ 0 & x > 25 \end{cases}$$

$$\mu_{\tilde{b}_2}(x) = \begin{cases} 0 & x < 25 \\ (x^2-625)/104 & 25 \leqq x < 27 \\ 1 & x = 27 \\ (961-x^2)/232 & 27 < x \leq 31 \\ 0 & x > 31 \end{cases}$$

The membership functions for the fuzzy goals of $\tilde{g}_L$ and $\tilde{g}_F$ are:

$$\mu_{\tilde{g}_L}(x) = \begin{cases} 0 & x < 15 \\ (x^2-225)/175 & 15 \leqq x < 20 \\ 1 & x = 20 \\ (900-x^2)/500 & 20 < x \leq 30 \\ 0 & x > 30 \end{cases}$$

$$\mu_{\tilde{g}_F}(x) = \begin{cases} 0 & x < 4 \\ (x^2-16)/48 & 4 \leqq x < 8 \\ 1 & x = 8 \\ (225-x^2)/161 & 8 < x \leq 15 \\ 0 & x > 15 \end{cases}$$

Satisfactory degree: $\alpha = 0.2$. $\varepsilon = 0.01$.

[Step 2](Initializing): let k=1. Associated with this example, the corresponding BLGP$_\lambda$ problem is:

$$\min_{x\in X} |\sqrt{11\lambda+25}x + \sqrt{5\lambda+4}y - \sqrt{175\lambda+225}|$$
$$\min_{x\in X} |\sqrt{64-28\lambda}x + \sqrt{25-16\lambda}y - \sqrt{900-500\lambda}|$$
$$\text{subject to } -\sqrt{4-3\lambda}x + \sqrt{5\lambda+4}y \leqslant \sqrt{80\lambda+36}$$
$$-\sqrt{0.75\lambda+0.25}x + \sqrt{25-16\lambda}y \leqslant \sqrt{625-184\lambda}$$
$$\min_{y\in Y} | -\sqrt{16-7\lambda}x + \sqrt{11\lambda+25}y - \sqrt{48\lambda+16}|$$
$$\min_{y\in Y} | -\sqrt{8\lambda+1}x + \sqrt{64-28\lambda}y - \sqrt{225-161\lambda}|$$
$$\text{subject to } \sqrt{0.75\lambda+0.25}x + \sqrt{5\lambda+4}y \leqslant \sqrt{104\lambda+625}$$
$$\sqrt{4-3\lambda}x + \sqrt{25-16\lambda}y \leqslant \sqrt{961-232\lambda}$$

where $\lambda \in [0.2, 1]$.

Referring to the algorithm, only $\lambda_0 = 0.2$ and $\lambda_1 = 1$ are considered initially. Thus 4 non-fuzzy objective functions and 4 non-fuzzy constraints for the leader and follower will be generated respectively.

[Step 3](Computing): By introducing auxiliary variables $v_i^-, v_i^+, i = 1, 2$, we get:

$$\min_{(x,v_1^-,v_1^+)\in \bar{X}'} v_1^- + v_1^+$$
$$\text{subject to } 24.8x + 12.9y + v_1^- - v_1^+ = 84.4,$$
$$-1.8x + 2.2y \leqslant 19.4$$
$$-x + 3y \leqslant 21$$
$$-0.6x + 4.7y \leqslant 24.3$$
$$-x + 3y \leqslant 21$$
$$\min_{(y,v_2^-,v_2^+)\in \bar{Y}'} v_2^- + v_2^+$$
$$\text{subject to } -11.4x + 24.8y + v_2^- - v_2^+ = 35,$$
$$0.6x + 2.2y \leqslant 25.4$$
$$x + 3y \leqslant 7$$
$$1.8x + 4.7y \leqslant 30.2$$
$$x + 3y \leqslant 27$$

Using Branch-and-bound approach [17], current solution is (2.15366,0,0, 2.39243,0,0).

[Step 4](Comparison) : Because $k = 1$, goto **[Step 5]**

[Step 5](Splitting): By inserting a new node $\lambda_1 = (0.2+1)/2 = 0.6$, there are total 3 nodes of $\lambda_0 = 0.2, \lambda_1 = 0.6$ and $\lambda_2 = 1$. Then total 12 non-fuzzy objective functions for the leader and follower together with 12 non-fuzzy constraints for the leader and follower respectively are generated.

[Step 6](Loop): k=1+1=2, goto **[Step 3]**, and current solution of (2.17093,0,0, 2.41756,0,0) is obtained. As $|2.15366 - 2.17093| + |2.39243 - 2.41756| = 0.04 > \varepsilon = 0.01$, the algorithm keep going until the solution of (2.13535,0,0, 2.42797,0,0) is obtained. The computing results are listed at Table 1.

k	x	y	$v_{1\lambda}^+$	$v_{1\lambda}^-$	$v_{2\lambda}^+$	$v_{2\lambda}^-$
1	2.15366	2.39243	0	0	0	0
2	2.17093	2.41756	0	0	0	0
3	2.12393	2.43436	0	0	0	0
4	2.13535	2.42797	0	0	0	0

Table 1: Summary of the running solution

[Step 7](Output): As $|2.12393 - 2.13535| + |2.43436 - 2.42797| = 0.0178 < \varepsilon = 0.02$, $(x^*, y^*) = (2.1354, 2.4280)$ is the final solution

of this FGBLDM probl
tained for the leader anc
(2.1354, 2.4280) are:

$$\begin{cases} F(x^*, y^*) = 2. \\ f(x^*, y^*) = 2. \end{cases}$$

4. Conclusion

Many organizational de
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group [1, 7, 13, 15, 18, 1
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idea of this method.

Future study includ
will handle the situatio
the leader and followe
algorithm for fuzzy go
bilevel decision making

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of this FGBLDM problem. The objectives obtained for the leader and follower under $(x^*, y^*) = (2.1354, 2.4280)$ are:

$$\begin{cases} F(x^*, y^*) = 2.1354\tilde{c}_1 + 2.4280\tilde{d}_1 \\ f(x^*, y^*) = 2.1354\tilde{c}_2 + 2.4280\tilde{d}_2 \end{cases}$$

4. Conclusion

Many organizational decision problems can be formulated by bilevel programming models. Based on the previous researching results from our research group [1, 7, 13, 15, 18, 19], this paper proposes a λ−cut approximate approach to solving the fuzzy goal based bilevel decision making problem, and presents a numerical example to further explain the idea of this method.

Future study includes developing methods that will handle the situation of multiple objectives for the leader and follower. Beside, the interactive algorithm for fuzzy goal based multiple objective bilevel decision making problem will be our focus.

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