Virtual Spiral Architecture

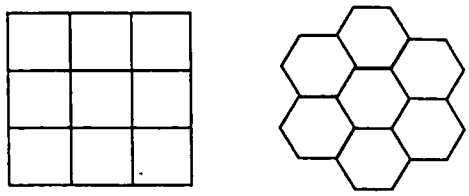
Qiang Wu, Xiangjian He, Tom Hintz
Department of Computer Systems
Faculty of Information Technology
University of Technology, Sydney
{wuq,sean,hintz}@it.uts.edu.au

Abstract

Spiral Architecture is a relative new and powerful approach to machine vision system. The geometrical arrangement of pixels on Spiral architecture can be described in terms of a hexagonal grid. However, all the existing hardware for capturing image and for displaying image are produced based on rectangular architecture. It has become a serious problem affecting the advanced research on Spiral Architecture. In this paper, a novel mimic Spiral Architecture called virtual Spiral Architecture is presented. Using virtual Spiral Architecture, images on rectangular architecture can be smoothly transferred to hexagonal architecture. Moreover, such mimicking operation does not introduce distortion and reduce image resolution, which is an advantage over other mimicking methods. There is no doubt that virtual architecture will be helpful to research on Spiral Architecture and other hexagonal architectures.

1. Introduction

The possibility of using a hexagonal grid to represent digit images has been studied for more than thirty years. Hexagonal grids have higher degrees of symmetry than the rectangular grids. This symmetry results in a considerable saving of both storage and computation time [1, 2]. On hexagonal architecture, each grid has only six neighboring grids which have the same distance to the centre hexagon of the seven-hexagon unit. It is a different grid arrangement scheme from rectangular architecture which uses a set of 3×3 vision unit (see Figure 1.). Because of the uniformly connected and close-packed form, greater angular resolution, higher efficiency, and better performance are achieved in many hexagonal systems [1-3].



(a) Rectangular Architecture (b) Hexagonal Architecture

Figure 1. Vision unit in two different image architectures. In the Spiral Architecture, unlike the rectangular architecture, each pixel has only six neighboring pixels which have the same distance to the centre hexagon of the seven-hexagon unit of vision.

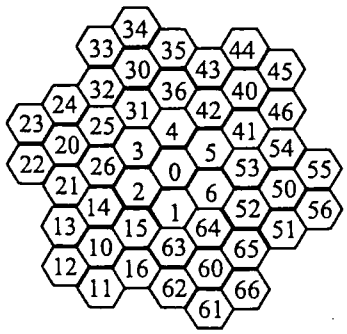


Figure 2. Cluster of size 49 including spiral addresses

In order to widely utilize hexagonal grid system on image processing area, many mathematic foundations have been specially developed for hexagonal grid. Her [3] proposed a symmetrical hexagonal coordinate frame to identify each grid on hexagonal architecture. According to symmetrical hexagonal coordinate frame, each grid point uses integer coordinates with three components, which makes hexagonal grid system

compatible to digital computation. However, Her still borrowed mathematics from the rectangular system to apply his hexagonal coordinates frame on image transformation, so it is inevitable to perform floating metric computation. Moreover, because one more component was introduced in his coordinate system than traditional rectangular architecture with two components, it increased computational complexities in some degree. Sheridan [4] proposed another single component hexagonal addressing system called spiral addressing. Normally, hexagonal grid system with spiral addressing scheme is called Spiral Architecture. On Spiral Architecture, each pixel is identified by a designated positive number. The numbered hexagons form the cluster of size 7^n. The hexagons tile the plane in a recursive modular manner along the spiral direction. An example of a cluster with size of 49 and the corresponding addresses are shown in Figure 2. Moreover, Sheridan [4] also developed two algebraic operations on Spiral Architecture: Spiral Addition and Spiral Multiplication. The neighboring relation among the pixels on Spiral Architecture can be expressed uniquely by these two operations. These two operations also define two transformations on spiral address space respectively, which are image translation and image rotation. Spiral Addition and Spiral Multiplication are pure spiral address counting operations and do not include floating computation, so spiral addressing is a higher efficient and more convenient coordinate frame than symmetrical hexagonal coordinate frame [3]. The work presented in this paper is part of advanced research based on Spiral Architecture.

In spite of the many advantages of Spiral Architecture, it has not been used widely in image processing area. The main reason is that there are no mature devices for capturing image and for displaying image based on hexagonal architecture. In order to further research on Spiral Architecture, it is inevitable to use mimic Spiral Architecture based on the existing rectangular image architecture. Mimic Spiral Architecture plays an important role in image processing applications on Spiral Architecture. It forwards the image data between image processing algorithms on Spiral Architecture and rectangular image architecture for display purpose (see Figure 3.). Such mimic Spiral Architecture' must retain the symmetrical properties of hexagonal grid system. In addition, mimic Spiral Architecture cannot reduce the resolution of the original image. The purpose of this paper is to examine the current mimicking methods and find out the existing problems which affect the research work on Spiral Architecture. Then, a novel mimic

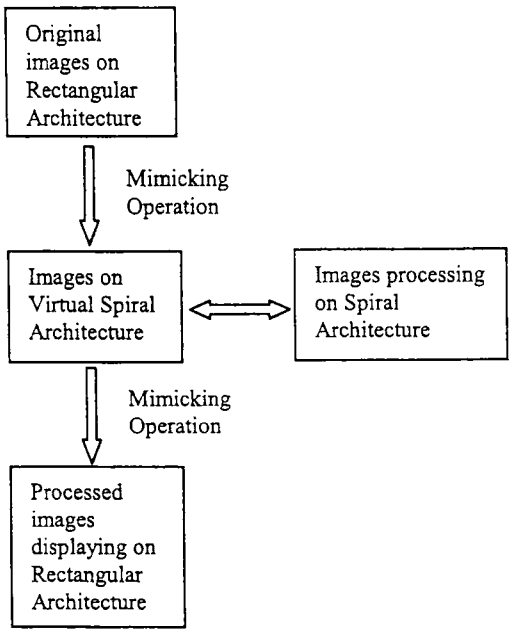


Figure 3. Image processing on virtual Spiral Architecture

Spiral Architecture called virtual Spiral Architecture is developed. Images mimicked on Virtual Spiral Architecture have the similar resolution as the original images without distortion.

The organization of this paper is as follows. Existing mimicking methods are reviewed in Section 2. In Section 3, novel virtual Spiral Architecture is presented followed by the experimental results in Section 4. We conclude in Section 5.

2. A Review of the existing mimicking methods

In order to implement the theoretical results on Spiral Architecture, it is necessary to mimic Spiral Architecture from the existing image structure, Rectangle Architecture. The method used in current research on Spiral Architecture [5-7] is to use a set of four rectangular pixels which are adjacent to each other to mimic a hexagonal pixel. Figure 4. shows the arrangement of seven hexagonal pixels on mimic spiral architecture, which are identified by the different filling pattern, with spiral addresses 0, 1, 2, 3, 4, 5 and 6. Each of them consists of four rectangular pixels.

However, because the data of mimic hexagonal pixel, for example a grey value for a grey-level image, is the average of the four corresponding rectangle pixels, it introduces loss of resolution.

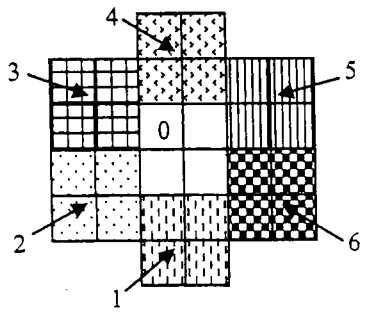


Figure 4. Seven adjacent mimic hexagonal pixels; each of them consists of four rectangular pixels.

In addition, according to Spiral Architecture theory the distance between the six surrounding pixels and the central pixel is the same. However, this property is lost in the mimic Spiral Architecture (See Figure 4.). This effect weakens the advantages of Spiral Architecture on image processing particularly on edge detection [8] and object recognition. Moreover, these problems will affect some particular image processing applications. For example, distortion will damage the results of image rotation and will also affect the performance of edge detection algorithms based on Spiral Architecture. Decreasing resolution will affect comparison of the results between image compression algorithm on Spiral Architecture [6] and common compression algorithms on rectangular architecture.

In the next section, a new mimicking method is presented, which resolves the problems mentioned in the previous paragraph.

3. A novel virtual Spiral Architecture

Based on the review of current mimicking methods, a new mimic Spiral Architecture called virtual Spiral Architecture is presented. Such virtual Spiral Architecture only exists during the procedure of image processing. It builds up a virtual hexagonal grid system on memory space in computer. Then, different algorithms can be implemented on such virtual spiral space. Finally, output data can be mapped back to rectangular architecture for display (see Figure 3).

In order to make such mimicking methods reliable, virtual Spiral Architecture retains the resolution of the image on virtual spiral space. That is, the resolution does not change during mimicking procedure. In addition, virtual Spiral Architecture retains the advantages of real hexagonal grid system like higher degrees of symmetry, uniformly connected and closed-packed form (see Figure 2), so mimicking procedure will not introduce shape distortion.

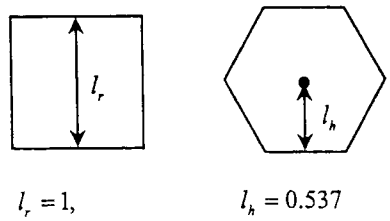


Figure 5. A square grid and a hexagonal grid which have the same size of area.

3.1 Retaining image resolution

For a given picture represented on rectangular architecture, if it is re-represented on Spiral Architecture on which each hexagonal grid has the same area size as square grid on rectangular architecture, the image resolution is retained.

In order to work out the size of hexagonal gird, the length of the side in a square grid is defined as 1 unit length. Namely, the area of a square grid is 1 unit area. Then, for a hexagonal grid which has the same area size as square grid, the distance from the centre to the side in a hexagonal grid is 0.537 (see Figure 5).

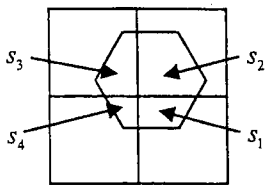
3.2 Rounding on hexagonal grids

In Section 3.1, the size of the hexagonal grid has been calculated. Using such hexagonal grid system to build up virtual Spiral Architecture, the resolution of the image will not be affected during mimicking procedure.

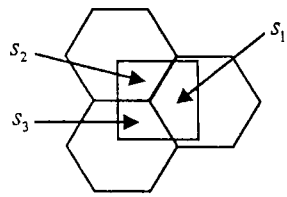
Besides the size of grid, the other important information of grid is the RGB values of the grid for color image and grey values for grey image. In order to simplify the explanation, only the grey images are considered in this paper. But the methods can be applied on color images without difficulties.

In order to work out the grey value of a hexagonal gird, the relations between the hexagonal grid and its connected square grid must be investigated. The purpose is to find out the different contribution of each connected square grid's grey value to the referenced hexagonal grid (see Figure 6).

Let N denote the number of square grids which are connected to a particular hexagonal grid. s_i denotes the size of overlap area between square grid i, one of connected square grid, and the hexagonal grid. Because the size of grid is 1 unit area (see Section 3.1), the percentage of overlap area in a referenced hexagonal



(a) One virtual hexagonal grid and its four connected square grids



(b) One square grid and its three connected virtual hexagonal grids

Figure 6. Relation between virtual hexagonal grid and the connected square grid. s_i is the size of overlap area.

grid is,

$$p_i = s_i / 1 \times 100\% = s_i. \quad (1)$$

Let gh denote the grey value of hexagonal grid, and gs denote the grey value of square grid. Thus, the grey value of hexagonal grid is calculated as the weighted average of the grey values of the connected square grids as,

$$gh = \sum_{i=1}^{N} p_i \cdot gs_i. \quad (2)$$

On the other hand, the reverse operation must be considered in order to map the images from virtual Spiral Architecture to rectangular architecture after image processing on Spiral Architecture (see Figure 3). After image processing on Spiral Architecture, the grey values of virtual hexagonal grids have been changed. Thus, the aim is to calculate the grey values of square grid from the connected hexagonal grids (see Figure 6.b). The same way as Equation (2) is used to calculate the grey value of square grid. However, p_i stands for the percentage of overlap area in a referenced square grid (see Figure 6.b). Supposing there are M virtual hexagonal grids connected to a particular square grid, the square grid's grey value is,

$$gs = \sum_{i=1}^{M} p_i \cdot gh_i. \quad (3)$$

Using Equation (2) and (3), the grey values of grids can be calculated easily as long as p_i can be calculated.

However, for digital images, it is impossible to calculate the size of a pixel, so approximate methods must be used.

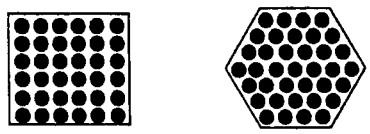


Figure 7. One grid is composed of many small points

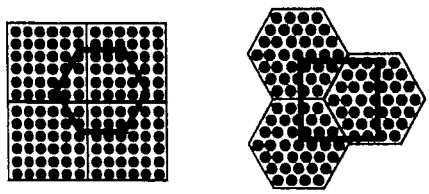


Figure 8. The size of overlapped area is represented by the number of small points

In virtual Spiral Architecture and rectangular architecture, each grid is considered as a set which is composed of many small points (see Figure 7). Then, alternatively, the size of grid and the size of overlap area between hexagonal grid and square grid (see Figure 8) can be approximated by the number of small points. The accuracy of approximation will be improved when the number of small points in a grid is increased.

Moreover, the rectangular architecture and the Spiral Architecture are mapped onto a two-dimension Cartesian coordinate system. The centers of these two architectures are overlapped on the origin of Cartesian coordinate system. On such a Cartesian coordinate system, each hexagonal grid is determined by six lines, $l_{ik}(a_{ik}, b_{ik})$, where $i = 1, 2, \ldots, W$ and $k = 1, 2, \ldots, 6$. W is the number of hexagonal grids on virtual Spiral Architecture (see Figure 9). a_{ik} is the intercept of k-th line on grid i. b_{ik} is the slope of k-th line on grid i.

The central hexagonal grid whose spiral address is 0 is identified as the first grid with six lines below,

$$\begin{aligned} line1&: l_{11}(-0.537, 0) \\ line2&: l_{12}(-1.075, -1.732) \\ line3&: l_{13}(1.075, 1.732) \\ line4&: l_{14}(0.537, 0) \\ line5&: l_{15}(1.075, -1.732) \\ line6&: l_{16}(-1.075, 1.732). \end{aligned} \quad (4)$$

The lines of other grids can be obtained according to coordinate translation principles. Any method can be used here to go through the hexagonal grids. In our work, Spiral Addition [4] is used to go through the

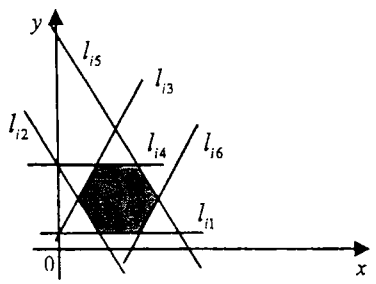


Figure 9. A hexagonal grid on Cartesian coordinate system

grids, which is a fast method, because Spiral Addition is special mathematical operation designed on hexagonal grid system.

After the hexagonal grids are located by six lines as above, the size of overlap area between hexagonal grid and square grid can be calculated by counting the number of small points in the corresponding regions.

Since the center of rectangular architecture is on the origin of Cartesian coordinate, the coordinate of each small point in each square grid (see Figure 8) will be known. Thus, small point (x, y) is located in hexagonal grid i, if,

$$\begin{aligned} &a_{i1} < y < a_{i4} \\ &a_{i2} + b_{i2} \times x < y < a_{i5} + b_{i5} \times x \\ &a_{i6} + b_{i6} \times x < y < a_{i3} + b_{i3} \times x. \end{aligned} \tag{5}$$

It will be easier to find out the dependence relationship between the small point and the square grid. Because the central grid of rectangular architecture is on the origin of Cartesian coordinate, its coordinate is (0,0). The distance between the centers of two adjacent square grids is 1 unit length, so every square grid has integer coordinate. Let $sign(r)$ denote the sign of number r. Namely,

$$sign(r) = \begin{cases} 1 & if\ r \geq 0 \\ -1 & if\ r < 0. \end{cases} \tag{6}$$

Then, small point (x, y) is located in square grid (m,n) if,

$$\begin{aligned} &[x + sign(x) \times 0.5] = m \\ &[y + sign(y) \times 0.5] = n. \end{aligned} \tag{7}$$

Using Equation (5) and (7), the dependence relationship between the small points (see Figure 8) and the corresponding grids including hexagonal grids and square grids can be worked out. At the same time, the connecting relationship between the hexagonal grid and the square grid is determined by the shared small points. That is, a square grid is connected with a hexagonal grid if one or more small points in the square grid are also included in the hexagonal grid. The size of overlap area is calculated by counting the number of points located in the overlap area.

Supposing the number of small points included in a hexagonal grid is num_h and the number of small points in the overlap area between this hexagonal grid and one of its connected square grids is sn_i, percentage p_i in Equation (2) is,

$$p_i = {sn_i}/{num_h} \times 100\%. \tag{8}$$

where $i = 1, 2, \ldots, N$ and N is number of square grids which are connected to the referenced hexagonal grid. Using the same way p_i is Equation (3) also can be calculated.

Thus, the input images on rectangular architecture can be mapped onto virtual Spiral Architecture using Equation (2). After processing, the images on virtual Spiral Architecture can be mapped back onto rectangular architecture using Equation (3).

4. Experimental Results

In the experiments, an image with 300×300 square pixels is used for testing (see Figure 10). On rectangular architecture, each square grid is assumed to be composed of 10×10 small points. In order to test the performance of mimicking algorithm, image rotating algorithm using Spiral Multiplication [7] is implemented on virtual Spiral Architecture to rotate the image for 60°. The experimental results are shown in Figure 11.

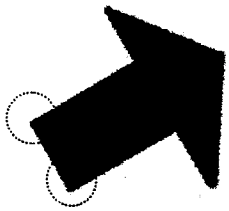
As shown in Figure 11, mimic Spiral Architecture using four square pixels to mimic one hexagonal pixel produces distortion on object shape (see circled position). Moreover, this kind of mimicking method also reduces image resolution (see the blurred object boundary). Because four square pixels are used to mimic one hexagonal pixel, the grey value of the hexagonal pixel is the average of grey values of the four corresponding square pixels.

Compared with common mimic Spiral Architecture, performance of virtual Spiral Architecture is better. It retained the object shape as well as image resolution during mimicking procedure (see Figure 11.b).

5. Conclusions

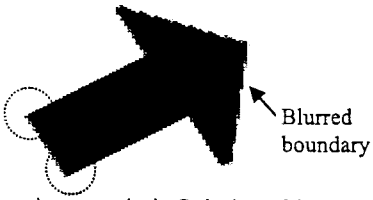
This paper presents a novel virtual Spiral Architecture. It is significant for image processing research on Spiral Architecture and other hexagonal

Figure 10. An image with 300×300 square pixels on rectangular architecture



(a) Image rotation on rectangular architecture

(b) Image rotation on virtual Spiral Architecture



(c) Image rotation on mimic Spiral Architecture (four square grids mimicking one hexagon grid)

Figure 11. Image rotating on different image architectures

grid systems, because currently there is no mature hardware based on hexagonal architecture.

The proposed virtual Spiral Architecture maps the original image onto a virtual hexagonal grid system in computer memory space. Then, applications on Spiral Architecture are implemented on such a virtual hexagonal grid system. At last, the processing results are mapped back to normal rectangular architecture for display.

During mimicking procedure, virtual Spiral Architecture keeps the features of real hexagonal grid system which include higher degree of symmetry than the rectangular grids, uniformly connected and close-packed form, and greater angular resolution. Images mimicked on Virtual Spiral Architecture have the similar resolution as the original images without distortion which is an advantage of virtual Spiral Architecture over other mimicking methods.

In order to obtain the grey values of virtual hexagonal grids, the size of overlapped area between virtual hexagonal grid and its connected square grids must be calculated. In this paper, each grid is considered as a set of small points. Then, the size of overlapped part is approximated by number of small points in the overlapped area. The approximate accuracy will be improved if the number of small points in each grid is increased. But this will increase computational complexity. It is a trade-off which must be considered in the practical applications.

6. References

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