Group decision making method with fuzzy preference of decision makers

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Abstract: A decision is often made in a group and decision makers often utilize fuzzy judgments in attempting to reach an optimal solution. In order to deal with the fuzziness of decision makers' judgments in solutions, this paper proposes a fuzzy group decision-making method for multi-objective decision problems. The method allows group members to express their fuzzy preferences for decision objectives, fuzzy judgements for solution selection rules and weights for group members. A final decision can be made based on a group selection matrix. This group decision-making method aggregates all group members' fuzzy opinions and judgments into the final group decision in a most acceptable degree.

Keywords: Group decision-making, Multi-objective decision-making, Fuzzy decision-making, Fuzzy preference.

1. Introduction

Real-world decision-making problems often involve multiple, non-commensurable, and conflicting objectives that should be considered simultaneously and subjected to constraints. Such decision-making problems are called multi-objective decision-making (MODM) problems. One of the major systems-analytic approaches to multi-objective decision-making is multi-objective optimisation, known as MODM methods (Hwang & Masud, 1979). The multiple objectives in a MODM problem cannot be combined into a single objective. Moreover, the objectives usually conflict with each other and any improvement of one objective can be achieved only at the expense of another. With this observation, decisions with optimality are not uniquely determined. The final decision must be selected from among the set of optimal solutions. Consequently, the aim in solving MODM problems is to derive a satisfaction solution for decision makers based on subjective value judgments.

Since an organisation is frequently required to make decisions in a cooperative group, group decision-making involving MODM has been paid attention and called as multi-objective group decision-making (MOGDM) (Bui 1989). Among multi-person decision-making setting, there is a set of solutions and a set of individuals who provide their preferences over the set of solutions and extend to find a solution that is most acceptable by the group of individuals as a whole (Bui 1989; Korhonen & Wallenius1990; Lu & Quaddus 2000).

The decision problem here is no longer only the design of a satisfactory solution according to one individual's goals. The analysis must be extended to account for the conflict and aggregation among different group members who have different goals for objectives and judgments for satisfactory solutions. The preferences of the group members are expected to vary from one of another. Consequently, determining a 'best' satisfactory solution to a multi-objective problem in a group requires the aggregation of individual opinions and judgments. This is especially true for an interactive procedure that requires group feedback to generate a 'most' acceptable solution. There is no rule for combining individual preferences into a group preference unless interpersonal comparison of utilities is allowed. Therefore most utility group aggregation methods require explicit interpersonal comparisons of utility and follow a normative approach assuming that a group decision rule can be constructed by aggregating the utility functions of group members (Iz & Jelassi 1990).

Unfortunately, a real-world situation is often not so deterministic. The decision objectives are frequently fuzzy and decision makers often utilise fuzzy judgments in attempting to reach optimal solutions. The precise mathematical models are not enough to tackle all practical problems. To deal with the fuzziness of decision makers' judgments in solutions, fuzzy group decision-making approaches were preposed. A relatively practical introduction of fuzzy set theory (Zadeh 1965) into conventional MODM models was first presented by Zimmermann (1978, 1987). Following this, Nishizaki (1994) reported an interactive fuzzy trade-off evaluation method in a group decision-making. Lee (1996) presented a method for group decision-making using fuzzy set theory for evaluating the rate of aggregative risk in software development. Hsu and Chen (1996) provided a similarity aggregation method for aggregating individual fuzzy opinions into a group fuzzy consensus opinion.

This paper proposes a group decision-making method for multi-objective decision problems. The method addresses few issues together: decision makers have different weights, decision makers can express different preferences for alternative solutions, and decision makers can given fuzzy judgement for solution selection criteria. Under the proposed method, group members are allowed to provide their fuzzy weights for each objective and indicate their fuzzy preference for alternative solutions that may be obtained by different members. The final solution is a 'best' satisfactory solution that is most acceptable by the group of individuals as a whole.

Following the introduction, Section 2 gives all preliminaries used in the research. Section 3 presents the group decision-making method. An example of the method's application is shown in Section 4.

2. Preliminaries

In the following, we briefly review some basic definitions and properties of fuzzy sets from Zhang (1998), Zhang et al. (2002a-c) and Sakawa (1993). These basic definitions and notations will be used throughout the paper until otherwise stated.

Definition 2.1 A fuzzy set $\widetilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\widetilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. The function value $\mu_{\widetilde{A}}(x)$ is termed the grade of membership of x in $\widetilde{A}$.

Definition 2.2 A fuzzy set $\widetilde{A}$ in a universe of discourse X is convex if and only if for any $x_1, x_2 \in X$,

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)), \tag{1}$$

where $\lambda \in [0, 1]$.

Definition 2.3 A fuzzy set $\widetilde{A}$ in a universe of discourse X is called a normal fuzzy set implying that there exists $x_0 \in X$ such that $\mu_{\widetilde{A}}(x_0) = 1$.

Definition 2.4 A fuzzy number $\widetilde{a}$ is a fuzzy subset on the space of real number R that is both convex and normal.

Definition 2.5 The λ-cut of fuzzy number $\widetilde{a}$ is defined

$$a_\lambda = \{x;\ \mu_{\widetilde{a}}(x) \geq \lambda,\ x \in R\} \tag{2}$$

a_λ is a non-empty bounded closed interval contained in X and it can be denoted by $a_\lambda = [a_\lambda^-, a_\lambda^+]$, a_λ^- and a_λ^+ are the lower and upper bounds of the closed interval, respectively.

Let $F(R)$ be the set of all fuzzy numbers. By the decomposition theorem of fuzzy set, we have

$$\widetilde{a} = \bigcup_{\lambda \in (0,1]} \lambda [a_\lambda^L, a_\lambda^R], \tag{3}$$

for every $\widetilde{a} \in F(R)$.

For any real number $\lambda \in R$, we define $\mu_\lambda(x)$ by

$$\mu_\lambda(x) = \begin{cases} 1 & \text{iff} \quad x = \lambda \\ 0 & \text{iff} \quad x \neq \lambda. \end{cases}$$

Then $\lambda \in F(R)$.

Let $F^*(R)$ be the set of all finite fuzzy numbers on R.

Theorem 2.1 *Let $\widetilde{a}$ be a fuzzy set on R, then $\widetilde{a} \in F(R)$ if and only if $\mu_{\widetilde{a}}$ satisfies*

$$\mu_{\widetilde{a}}(x) = \begin{cases} 1 & x \in [m, n] \\ L(x) & x < m \\ R(x) & x > n \end{cases},$$

where $L(x)$ is the right continuous monotone increasing function, $0 \leq L(x) < 1$ and $\lim_{x \to -\infty} L(x) = 0$, $R(x)$ is the left continuous monotone decreasing function, $0 \leq R(x) < 1$ and $\lim_{x \to \infty} R(x) = 0$.

Corollary 2.1 *If $\widetilde{a} \in F^*(R)$, then there exist $|x_i| < \infty, i = 1,2$ such that $L(x_1) = R(x_2) = 0$, i.e., the support of $\widetilde{a}$ is a bounded set.*

Corollary 2.2 *For every* $\widetilde{a} \in F(R)$ *and* $\lambda_1, \lambda_2 \in (0, 1]$, *if* $\lambda_1 \leq \lambda_2$, *then* $a_{\lambda_2} \subset a_{\lambda_1}$.

Definition 2.6 A triangular fuzzy number $\widetilde{a}$ can be defined by a triplet $\left(a_0^-, a, a_0^+\right)$ and the membership function $\mu_{\widetilde{a}}(x)$ is defined as:

$$\mu_{\widetilde{a}}(x) = \begin{cases} 0 & x < a_0^- \\ \dfrac{x - a_0^-}{a - a_0^-} & a_0^- \leq x \leq a \\ \dfrac{a_0^+ - x}{a_0^+ - a} & a \leq x \leq a_0^+ \\ 0 & a_0^+ < x \end{cases}, \tag{4}$$

where $a = a_1^- = a_1^+$.

Definition 2.7 If $\widetilde{a}$ is a fuzzy number and $a_\lambda^- > 0$ for any $\lambda \in (0, 1]$, then $\widetilde{a}$ is called a positive fuzzy number. Let $F_+^*(R)$ be the set of all finite positive fuzzy numbers on R.

Definition 2.8 For any $\widetilde{a}, \widetilde{b} \in F_+^*(R)$ and $0 < \lambda \in R$, the sum, difference, product and quotient of two fuzzy numbers $\widetilde{a} + \widetilde{b}$ and the scalar product and scalar quotient of λ and $\widetilde{a}$ are defined by the membership functions

$$\mu_{\widetilde{a}+\widetilde{b}}(t) = \sup_{t=u+v} \min\{\mu_{\widetilde{a}}(u), \mu_{\widetilde{b}}(v)\}, \tag{5}$$

$$\mu_{\lambda\widetilde{a}}(t) = \max\{0, \sup_{t=\lambda u} \mu_{\widetilde{a}}(u)\}, \tag{6}$$

where we set $\sup\{\phi\} = -\infty$.

Theorem 2.2 For any $\widetilde{a}, \widetilde{b} \in F_+^*(R)$ and $0 < \alpha \in R$,

$$\widetilde{a} + \widetilde{b} = \bigcup_{\lambda \in (0,1]} \lambda[a_\lambda^- + b_\lambda^-, a_\lambda^+ + b_\lambda^+],$$

$$\alpha\widetilde{a} = \bigcup_{\lambda \in (0,1]} \lambda[\alpha a_\lambda^-, \alpha a_\lambda^+].$$

Definition 2.9 Let $\widetilde{a} = (a_0^-, a, a_0^+)$ and $\widetilde{b} = (b_0^-, b, b_0^+)$ be two triangular fuzzy numbers. Then $\widetilde{a} = \widetilde{b}$ if $a_0^- = b_0^-$, $a = b$ and $a_0^+ = b_0^+$.

Definition 2.10 If $\widetilde{a}$ is a triangular fuzzy number and $0 < a_\lambda^- \leq a \leq a_\lambda^+ \leq 1$, fro any $\lambda \in (0, 1]$, then $\widetilde{a}$ is called a normalized positive triangular fuzzy number (Yoon, K.P. & Hwang, C.L., 1995). Let $F_T^*(R)$ be the set of all normalized positive triangular fuzzy numbers on R.

Definition 2.11 A linguistic variable is a variable whose values are linguistic terms (Sakawa, 1993).

The concept of linguistic variable is very useful in dealing with situations that are too complex or ill-defined to be reasonably described in conventional quantitative expressions (Yoon, K.P. & Hwang, C.L. 1995). For example, "length" is a linguistic variable, its values are very short, short, medium, long, very long, etc. These linguistic values can also be represented by normalized positive triangular fuzzy numbers.

Definition 2.12 Let $\tilde{a}, \tilde{b} \in F_T^*(R)$, then the vertex method is defined to calculate the distance between them as

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_0^- - b_0^-)^2 + (a - b)^2 + (a_0^+ - b_0^+)^2]} \tag{7}$$

Definition 2.13 Let $\tilde{a}, \tilde{b} \in F_T^*(R)$, then fuzzy number $\tilde{a}$ is closer to fuzzy number $\tilde{b}$ as $d(\tilde{a}, \tilde{b})$ approaches 0.

Many distance measurement functions are proposed, but here the vertex method is an effective and simple method to calculate the distance between two triangular fuzzy numbers. Some important properties of the vertex method are described as follows:

Proposition 2.1 *If both* $\tilde{a}$ and $\tilde{b}$ *are real numbers, then the distance measurement* $d(\tilde{a}, \tilde{b})$ *is identical to the Euclidean distance.*

Proposition 2.2 *Let* $\tilde{a}, \tilde{b} \in F_T^*(R)$. *Then they are identical if and only if* $d(\tilde{a}, \tilde{b}) = 0$.

Proposition 2.3 *Let* $\tilde{a}, \tilde{b}, \tilde{c} \in F_T^*(R)$.*Then* $\tilde{b}$ *is closer to* $\tilde{a}$ *than* $\tilde{c}$ *if and only if* $d(\tilde{b}, \tilde{a}) < d(\tilde{c}, \tilde{a})$.

Proposition 2.4 *Let* $\tilde{a}, \tilde{b} \in F_T^*(R)$. *If* $d(\tilde{a}, 0) < d(\tilde{b}, 0)$, *then* $\tilde{a}$ *is closer to* 0 *than* $\tilde{b}$.

3. A group decision-making method with member's fuzzy preference for MODM

In a multi-objective group decision-making situation, the goals of a MODM problem can be as selection criteria (rules), and group members are allowed to give a weight for each goal. The alternative solutions are conducted through using MODM methods by the group members, and members can indicate their preferences for each solution. The final decision will be made based on the alternative solutions and group members' preferences (weights). The purpose of this method is to enhance group consensus on the group decision outcome. The method consists of seven steps with in two levels:

Level 1. *Individual Preference Generation*:

Let $\boldsymbol{S} = \{\boldsymbol{S}_1, \boldsymbol{S}_2, \ldots, \boldsymbol{S}_m\}$, $m \geq 3$, be a given finite set of solutions of MODM; $\boldsymbol{C} = \{\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_t\}$ be a given finite set of selection rules; $\boldsymbol{P} = \{\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots, \boldsymbol{P}_n\}$, $n \geq 2$, be a given finite set of decision makers. The steps of generating individual preference are shown below:

Step 1: Decision maker P_k (k = 1, 2, ..., n) determines the weights (importances) of selection rules C (goals of a MODM problem) by using Analytic Hierarchy Process (AHP) method.

By pairwise comparison of the relative importance of selection rules, the pairwise comparison matrix E = $[e_{ij}^k]_{t \times t}$ is established, where e_{ij}^k represents the quantified judgments on pairs of selection rules C_i and C_j. The comparison scale ranges from 1 to 9, each representing the concepts of: 1 - equally important; 3 - weakly more important; 5 - strongly more important; 7 - demonstratively more important; 9 - absolutely more important, 2, 4, 6 and 8 are intermediate values between adjacent judgments. For example, e_{ij} = 5 means C_i is strongly more important than C_j.

The consistent weights for every selection rule can be determined by calculating the geometric mean of each row of the matrix, and then the result numbers are normalized. The weights are denoted as $w_1^k, w_2^k, \cdots, w_t^k$, where $w_i^k \in$ [0, 1], $i = 1, 2, \cdots, t$, $k = 1, 2, \cdots, n$ and $\sum_{i=1}^{t} w_i^k = 1$, $k = 1, 2, \cdots, n$.

Step 2: Against every selection rule C_j (j = 1, 2, ..., t), assign '1' to preferred (choose) solutions and '0' to unwanted (reject) solutions. As the choice is a decision maker's subjective judgments, decision makers may often have the situation where it is difficult to choose or reject a solution. Thus the yes/no method needs to be improved. A belief level can be introduced to express the possibility of selecting a solution (i) under rule (j) for a decision maker (k). The belief level b_{ij}^k ($i = 1, 2, \cdots, t$, $j = 1, 2, \cdots, m$, k = 1, 2, ..., n) belongs to a set of linguistic terms that contain various degrees of preference required by the decision maker P_k (k = 1, 2, ..., n). Linguistic terms are words in natural. For example, "very low", "low", "medium", "high", "very high" are linguistic terms. Linguistic terms are ill-defined and can be hardly described by single numerical values. The linguistic terms used in the paper are shown in Table 1.

Table 1. Linguistic variables for the belief levels of selection rules for selection solutions

Linguistic variables	Triangular fuzzy numbers
Very low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1)
Very high (VH)	(0.9, 1, 1)

Step 3: The belief matrix $\left(b_{ij}^k\right)$ $(k = 1, 2, \cdots, n)$ is aggregated to belief vector $\left(\bar{b}_j^k\right)$ (j = 1, 2, ..., m, k = 1, 2, ..., n).

$$\bar{b}_j^k = w_1^k * b_{1j}^k + w_2^k * b_{2j}^k + \cdots + w_t^k * b_{tj}^k \tag{8}$$

The aggregation method is similar to that of Baas and Kwakernaak (1997). Based on the belief vector $\left(\bar{b}_j^k\right)$ (j = 1, 2, ..., m, k = 1, 2, ..., n), the decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) makes an overall judgment on the solutions. The results are called the individual selection vector. All individual selection vectors can be composed by a group selection matrix $\left(\bar{b}_j^k\right)_{n\times m}$.

Level 2. *Group Aggregation*:

Step 4: As group members play different roles in an organization, the relative importance of each decision maker may not equal. Some are more important than the others. Therefore, the relative importance weight of each decision maker should be considered. The most important person(s) $\boldsymbol{P}_k$ among the group is assigned a weight '1', i.e., $v_k = 1$. We compare $\boldsymbol{P}_k$ with the ith decision maker $\boldsymbol{P}_i$, (i = 1, 2, ..., n) to determine $0 \le v_i \le 1$ (i = 1, 2, ..., n; $i \neq k$). The normalized weight of a decision maker $\boldsymbol{P}_k$ (k = 1, 2, ..., n) is denoted as

$$v_k^* = \frac{v_k}{\sum_{i=1}^{n} v_i}, \quad \text{for } k = 1, 2, \cdots, n.$$

Step 5: Considering the weights of all decision makers, we can construct the weighted normalized fuzzy decision vector

$$(\tilde{r}_1, \tilde{r}_2, \cdots, \tilde{r}_m) = (v_1^*, v_2^*, \cdots, v_n^*) \begin{pmatrix} \bar{b}_1^1 & \bar{b}_2^1 & \cdots & \bar{b}_m^1 \\ \bar{b}_1^2 & \bar{b}_2^2 & \cdots & \bar{b}_m^2 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{b}_1^n & \bar{b}_2^n & \cdots & \bar{b}_m^n \end{pmatrix},$$

where $\tilde{r}_j = \sum_{k=1}^{n} v_k^* \bar{b}_j^k$.

Step 6: In the weighted normalized fuzzy decision vector, the elements $\tilde{v}_j$, $j = 1, 2, \cdots, m$, are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. We can then define the fuzzy positive-ideal solution (FPIS, r^*) and fuzzy negative-ideal solution (FNIS, r^-) as:

$$r^* = (1, 1, 1) \quad \text{and} \quad r^- = (0, 0, 0).$$

The distance between each $\tilde{r}_j$ and r^*, $\tilde{r}_j$ and r^- can be currently calculated as:

$$d_j^* = d(\tilde{r}_j, r^*) \quad \text{and} \quad d_j^- = d(\tilde{r}_j, r^-), \quad j = 1, 2, \cdots, m,$$

where $d(.,.)$ is the distance measurement between two fuzzy numbers.

Step 7: A closeness coefficient is defined to determine the ranking order of all solutions once the d_j^* and d_j^- of each decision solution $\boldsymbol{S}_j$ (j = 1, 2, ..., m) are obtained. The closeness coefficient of each solution is calculated as:

$$CC_j = \frac{1}{2}\left(d_j^* + (1 - d_j^-)\right), \quad j = 1, 2, \cdots, m.$$

The solution S_j that corresponds to the largest CC_j is the best satisfactory solution of the decision group.

4. A numerical example

Suppose there are three decision makers, say P_1, P_2 and P_3, in a group. They have obtained three alternative solutions, $S = \{S_1, S_2, S_3\}$, for a given MODM problem. They will use three selection rules (criteria) $C = \{C_1, C_2, C_3\}$ to select a most satisfied solution as the group decision.

Step 1: Assesses:

$$E^1 = \begin{pmatrix} 1 & 1 & 1/3 \\ 1 & 1 & 1/3 \\ 3 & 3 & 1 \end{pmatrix}, \quad E^2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, E^3 = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 1/2 \\ 3 & 2 & 1 \end{pmatrix}.$$

Through computing the geometric mean of each row of the matrix, the normalized resulting numbers are obtained.

$$\begin{pmatrix} (1\times1\times1/3)^{1/3} = 0.6934 \\ (1\times1\times1/3)^{1/3} = 0.6934 \\ (3\times3\times1)^{1/3} = 2.0801 \end{pmatrix} = \begin{pmatrix} 0.2000 \\ 0.2000 \\ 0.6000 \end{pmatrix} = \begin{pmatrix} w_1^1 \\ w_2^1 \\ w_3^1 \end{pmatrix}$$

$$\text{sum} = 3.4669 \qquad 1.0000$$

$$\begin{pmatrix} (1\times1\times1)^{1/3} = 1 \\ (1\times1\times1)^{1/3} = 1 \\ (1\times1\times1)^{1/3} = 1 \end{pmatrix} = \begin{pmatrix} 0.3333 \\ 0.3333 \\ 0.3333 \end{pmatrix} = \begin{pmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \end{pmatrix}$$

$$\text{sum} = 3.0000 \qquad 1.0000$$

$$\begin{pmatrix} (1\times1/2\times1/3)^{1/3} = 0.5503 \\ (2\times1\times1/2)^{1/3} = 1 \\ (3\times2\times1)^{1/3} = 1.817 \end{pmatrix} = \begin{pmatrix} 0.1634 \\ 0.2970 \\ 0.5396 \end{pmatrix} = \begin{pmatrix} w_1^3 \\ w_2^3 \\ w_3^3 \end{pmatrix}$$

$$\text{sum} = 3.3673 \qquad 1.0000$$

Step 2: Assesses:

$$\begin{pmatrix} b_{11}^1 & b_{12}^1 & b_{13}^1 \\ b_{21}^1 & b_{22}^1 & b_{23}^1 \\ b_{31}^1 & b_{32}^1 & b_{33}^1 \end{pmatrix} = \begin{pmatrix} VL & VH & L \\ M & H & VL \\ H & M & M \end{pmatrix}$$

$$\begin{pmatrix} b_{11}^2 & b_{12}^2 & b_{13}^2 \\ b_{21}^2 & b_{22}^2 & b_{23}^2 \\ b_{31}^2 & b_{32}^2 & b_{33}^2 \end{pmatrix} = \begin{pmatrix} VL & H & VL \\ M & H & VL \\ H & M & M \end{pmatrix}$$

$$\begin{pmatrix} b_{11}^3 & b_{12}^3 & b_{13}^3 \\ b_{21}^3 & b_{22}^3 & b_{23}^3 \\ b_{31}^3 & b_{32}^3 & b_{33}^3 \end{pmatrix} = \begin{pmatrix} L & VH & L \\ M & H & VL \\ VH & M & L \end{pmatrix}.$$

Step 3: We have

$$\begin{aligned}
\overline{b}_1^1 &= w_1^1 b_{11}^1 + w_2^1 b_{21}^1 + w_3^1 b_{31}^1 \\
&= 0.2(0, 0, 0.1) + 0.2(0.3, 0.5, 0.7) + 0.6(0.7, 0.9, 1) = (0.48, 0.64, 0.76) \\
\overline{b}_2^1 &= w_1^1 b_{12}^1 + w_2^1 b_{22}^1 + w_3^1 b_{32}^1 \\
&= 0.2(0.9, 1, 1) + 0.2(0.7, 0.9, 1) + 0.6(0.3, 0.5, 0.7) = (0.5, 0.68, 0.82) \\
\overline{b}_3^1 &= w_1^1 b_{13}^1 + w_2^1 b_{23}^1 + w_3^1 b_{33}^1 \\
&= 0.2(0, 0.1, 0.3) + 0.2(0, 0, 0.1) + 0.6(0.3, 0.5, 0.7) = (0.18, 0.32, 0.5) \\
\overline{b}_1^2 &= w_1^2 b_{11}^2 + w_2^2 b_{21}^2 + w_3^2 b_{31}^2 \\
&= 0.33(0, 0, 0.1) + 0.33(0.3, 0.5, 0.7) + 0.33(0.7, 0.9, 1) = (0.33, 0.47, 0.57) \\
\overline{b}_2^2 &= w_1^2 b_{12}^2 + w_2^2 b_{22}^2 + w_3^2 b_{32}^2 \\
&= 0.33(0.7, 0.9, 1) + 0.33(0.7, 0.9, 1) + 0.33(0.3, 0.5, 0.7) = (0.57, 0.77, 0.9) \\
\overline{b}_3^2 &= w_1^2 b_{13}^2 + w_2^2 b_{23}^2 + w_3^2 b_{33}^2 \\
&= 0.33(0, 0, 0.1) + 0.33(0, 0, 0.1) + 0.33(0.3, 0.5, 0.7) = (0.1, 0.17, 0.3) \\
\overline{b}_1^3 &= w_1^3 b_{11}^3 + w_2^3 b_{21}^3 + w_3^3 b_{31}^3 \\
&= 0.16(0, 0.1, 0.3) + 0.3(0.3, 0.5, 0.7) + 0.54(0.9, 1, 1) = (0.57, 0.7, 0.8) \\
\overline{b}_2^3 &= w_1^3 b_{12}^3 + w_2^3 b_{22}^3 + w_3^3 b_{32}^3 \\
&= 0.16(0.9, 1, 1) + 0.3(0.7, 0.9, 1) + 0.54(0.3, 0.5, 0.7) = (0.52, 0.7, 0.84) \\
\overline{b}_3^3 &= w_1^3 b_{13}^3 + w_2^3 b_{23}^3 + w_3^3 b_{33}^3 \\
&= 0.16(0, 0.1, 0.3) + 0.3(0, 0, 0.1) + 0.54(0, 0.1, 0.3) = (0, 0.07, 0.24).
\end{aligned}$$

Step 4: Assesses: $v_1^* = v_2^* = v_3^* = 1/3$.

Step 5: We have

$$
\begin{aligned}
\widetilde{r}_1 &= v_1^*\overline{b}_1^1 + v_2^*\overline{b}_1^2 + v_3^*\overline{b}_1^3 \\
&= 0.333[(0.48, 0.64, 0.76) + (0.33, 0.47, 0.57) + (0.57, 0.7, 0.8)] \\
&= 0.333(1.38, 1.81, 2.13) = (0.5, 0.6, 0.71) \\
\widetilde{r}_2 &= v_1^*\overline{b}_2^1 + v_2^*\overline{b}_2^2 + v_3^*\overline{b}_2^3 \\
&= 0.333[(0.5, 0.68, 0.82) + (0.57, 0.77, 0.9) + (0.52, 0.7, 0.84)] \\
&= 0.333(1.59, 2.15, 2.56) = (0.53, 0.72, 0.85) \\
\widetilde{r}_3 &= v_1^*\overline{b}_3^1 + v_2^*\overline{b}_3^2 + v_3^*\overline{b}_3^3 \\
&= 0.333[(0.18, 0.32, 0.5) + (0.1, 0.17, 0.3) + (0, 0.07, 0.24)] \\
&= 0.333(0.28, 0.56, 1.04) = (0.09, 0.19, 0.35).
\end{aligned}
$$

Step 6: We get

$$d_1^* = d(\widetilde{r}_1, r^*) = \sqrt{\frac{1}{3}[(1-0.5)^2 + (1-0.6)^2 + (1-0.71)^2]} = 0.41$$

$$d_2^* = d(\widetilde{r}_2, r^*) = \sqrt{\frac{1}{3}[(1-0.53)^2 + (1-0.72)^2 + (1-0.85)^2]} = 0.33$$

$$d_3^* = d(\widetilde{r}_3, r^*) = \sqrt{\frac{1}{3}[(1-0.09)^2 + (1-0.19)^2 + (1-0.35)^2]} = 0.8$$

$$d_1^- = d(\widetilde{r}_1, r-) = \sqrt{\frac{1}{3}[0.5^2 + 0.6^2 + 0.71^2]} = 0.61$$

$$d_2^- = d(\widetilde{r}_2, r-) = \sqrt{\frac{1}{3}[0.53^2 + 0.72^2 + 0.85^2]} = 0.71$$

$$d_3^- = d(\widetilde{r}_3, r-) = \sqrt{\frac{1}{3}[0.09^2 + 0.19^2 + 0.35^2]} = 0.24$$

Step 7: Finally, we have

$$
\begin{aligned}
CC_1 &= \frac{1}{2}(d_1^- + (1-d_1^*)) = \frac{1}{2}(0.61 + (1-0.41)) = 0.6 \\
CC_2 &= \frac{1}{2}(d_2^- + (1-d_2^*)) = \frac{1}{2}(0.71 + (1-0.33)) = 0.69 \\
CC_3 &= \frac{1}{2}(d_3^- + (1-d_3^*)) = \frac{1}{2}(0.24 + (1-0.8)) = 0.22.
\end{aligned}
$$

Since CC_2 is higher than both CC_1 and CC_3, the solution $\boldsymbol{S}_2$ is selected as a most satisfied solution for the decision group. The solution aggregates maximally all group members' judgments and preferences for a MODM solution in whole.

References

Bass, M.S. and Kwakernaak, H. 1977, 'rating and ranking of multiple aspect alternatives using fuzzy sets', *Outomatica,* Vol.13, 47-58.

Bui, T. 1989, *Co-oP: A group decision support system for cooperative multiple criteria group decision-making,* Springer-Verlag, Berlin.

Chen, C. T. 2000, 'Extensions of the TOPSIS for group decision-making under fuzzy environment', *Fuzzy Sets and Systems,* Vol. 114, pp: 1-9.

Chen, S. M. 2000, 'Aggregation fuzzy opinions in the group decision-making environment', *Cybernetics and Systems.* Vol. 29, pp363-376.

Dubois, D. and Prade, H., 1980, *'Fuzzy sets and systems: Theory and applications',* Academic Press, New York.

Eom, S. B. 1996, 'Mapping the intellectual structure of research in decision support systems through author cocitation analysis (1971-1993)', *Decision Support Systems,* Vol. 16, no. 4, pp. 315-338.

Eom, S. B. 1998, 'Relationships between the decision support system subspecialties and reference disciplines: An empirical investigation', *European Journal of Operational Research,* Vol. 104, no. 1, pp. 31-45.

Hsu, H. M. and Chen, C. T. 1996, 'Aggregation of fuzzy opinions under group decision-making', *Fuzzy Sets & Systems.* Vol. 55, pp 241-253.

Hwang, C. L. and Masud, A. S. M. 1979, *'Multiple Objective Decision-making - Methods and Applications: A State of the Art Survey',* Springer, New York.

Lai, Y. J. and Hwang, C. L., 1996, *'Fuzzy multiple objective decision-making methods and application',* Springer-Verlag Berlin Heidelberg.

Iz, P. H. and Jelassi, M. T. 1990, 'An Interactive Group Decision Aid for Multi-objective Problems: An Empirical Assessment', *International Journal of Management Science,* Vol. 18, no. 6, pp. 245-253.

Kacprzyk, J., Fedrizzi, M. and Nurmi, H. 1992 'Group decision making and consensus under fuzzy preference and fuzzy majority,' *Fuzzy Sets and Systems* ,Vol. 49, pp: 21-31.

Karacapilidis, N.and Pappis, C. 2000, 'Computer-supported collaborative argumentation and fuzzy similarity measures in multiple criteria decision-making', *Computers & Operations Research*; New York; Vol. 27, no 7,8, pp: 653-671.

Korhonen, P. and Wallenius, J. 1990, 'A Multiple Objective Linear Programming Decision Support System', *Decision Support Systems,* Vol. 6, pp. 243 - 251.

Kuo, R. J., Chi, S. C. and Kao, S. S. 1999, 'A decision support system for locating convenience store through fuzzy AHP', *Computers & Industrial Engineering,* Vol. 37, No 1,2, pp. 232-236.

Lee, H. M. 1996, 'Group decision-making using fuzzy set theory for evaluating the rate of aggregative risk in software development', *Fuzzy Sets & Systems.* Vol. 80, pp 261-271.

Lu, J. and Quaddus, M.A. 2000, 'An intelligence-based multi-objective group decision support system and its Application, Information Technology for Business Management', *16th World Computer Congress (WCC2000),* Publishing House of Electronics Industry, China.

Lu, J. 2000, *'A framework and prototype of intelligent multiple objective group decision support systems',* PhD thesis, Curtin University library.

Nishizaki, I. and Seo, F. 1994, 'Interactive support for fuzzy trade-off evaluation in group decision-making', *Fuzzy Sets and Systems* ,Vol. 68, pp: 309-325.

Nunamaker, J. F., JR., M. C. and Titus, P. D. M. 1990-1991, 'Systems development in information systems research', *Journal of management information systems,* Vol. 7, no. 3, pp. 88-106.

Parra, M A., Terol, A B. and Rodriguez, M V. 1999, 'Uria Solution of a possibilistic multi-objective linear programming problem', *European Journal of Operational Research*, Vol. 119, no: 2, pp 338-344.

Sakawa, M.1993, *'Fuzzy sets and interactive multiobjective optimization'*, Plenum Press, New York.

Sakawa, M., Yano, H. and Yumine T. 1987 'An interactive fuzzy satisficing method for multiobjective linear programming problems and its application', *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-17, pp. 654-661

Sprague, R. H. & Watson, H. J. 1989, *'Decision Support Systems: Putting Theory into Practice'*, 2nd edition, Prentice-Hall, London.

Tanaka, H. and Asai, K. 1984a, 'Fuzzy solution in fuzzy linear programming problems', *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-14, pp. 325-328.

Tanaka, H. and Asai, K. 1984b, 'Fuzzy linear programming problems with fuzzy numbers', *Fuzzy Sets and Systems*, Vol.13, no. 1, pp. 1-10.

Yoon, K.P. and Hwang, C.L. 1995, *'Multiple attribute decision making – An introduction'*, SAGE Publications, Inc., Thousand Oaks.

Zadeh, L. A. 1965, 'Fuzzy sets', *Inform & Control,* Vol. 8, pp. 338-353

Zhang, G.Q. 1998, *'Fuzzy number-valued measure theory'*, Tsinghua University Press, Beijing.

Zhang, G.Q., Wu, Y.H., Remia, M. and Lu, J. 2002a, 'Formulation of fuzzy linear programming problems as four-objective constrained problems', *Applied Mathematics and Computation*, in press.

Zhang, G.Q., Wu, Y.H., Remia, M. and Lu, J. 2002b, 'Formulation of linear programming problems with fuzzy equality and inequality constrains', Submitted to *IEEE Transactions on Systems, Man and Cybernetics*: Part B: *Cybernetics.*

Zhang, G.Q., Wu, Y.H., Remia, M. and Lu, J. 2002c, 'An α-fuzzy max order and solution of linear constrained fuzzy optimization problems', *Proceedings of the International Conference on Computational Mathematics and Modeling*, Bangkok, Thailand, May 22-24, 2002, and *East-West Journal of Mathematics*, Special Vol. May 2002, pp84.

Zimmermann, H.J. 1978, 'Fuzzy programming and linear programming with several objective functions', *Fuzzy Sets and Systems,* Vol. 1, pp. 45-55

Zimmermann, H.J. 1987, *'Fuzzy sets, decision making and expert systems'*, Kluwer-Nijhoff Publishing, Dordecht.