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IMPACT OF DIFFERENT NUMERICAL TECHNIQUES ON DAMAGE IDENTIFICATION IN STRUCTURES

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ABSTRACT: Damage identification techniques have been widely investigated and used for structural damage evaluation. Many researchers have shown good results in detecting, locating and quantifying damage in structures using various damage identification algorithms and methods. One of the popular and promising damage identification methods is modified damage index (MDI) which utilises modal strain energy (MSE) and a statistical approach. However, when using this damage identification method, numerical techniques used in realising the damage detection algorithm plays an important role for the final outcome. The use of different techniques in detection of damage has not been widely investigated. In this paper, a finite element (FE) model of a timber beam was developed as a test structure. Modal responses of the test structure were generated using a FE software package. The damage index algorithm, utilising modal strain energy as its damage indicator, was computed. In the computation process, different numerical techniques at different stages were utilised to process the data. Since in practice, the number of modal data is usually limited, it is recommended that the mode shape data to be expanded using mode shape reconstruction technique. Thus, the raw data was reconstructed using two different mode shape reconstruction techniques, namely Shannon's sampling theorem and cubic spline. The computation of MDI is enabled by numerical integration method. In this paper, two numerical integration methods were performed viz trapezoidal and rectangular rules. The manipulated data is subsequently transformed into standard normal space. The mode shape was mass normalised and the mode shape curvature was normalised with respect to the maximum value of each considered mode. For practicality purposes, the first two flexural mode shapes were used in the algorithms computation. Among the two proposed numerical integration methods, the rectangular rule has shown greater potential. The cubic spline mode shape reconstruction technique shows better results compared to the Shannon's sampling theorem.

KEYWORDS: damage, timber, modal strain energy, mode shape reconstruction, finite element methods.

1 INTRODUCTION

For the past few decades, non-destructive evaluation (NDE) for damage detection has widely been investigated. Many research studies have been performed utilising NDE by using vibration response to detect, locate and characterise damage in structural and mechanical systems [1]. Studies have found that vibration based methods are capable to detect, locate and quantify damage [2-6]. In experimental and field applications, modal parameters, especially mode shape, which are essential in many damage detection algorithms, are usually acquired with limited number of sensors. This limitation can be overcome with the help of numerical techniques, whereby more data can be generated between the

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original acquired data. Such techniques are crucial in determining the success of a damage detection algorithm, but not many detailed works have been reported in comparing different numerical techniques in the area of damage detection of civil engineering structures. Park and Stubbs [6] and Peterson et al. [3] have utilised Shannon's sampling theorem to reconstruct mode shape in damage detection and the results appeared to agree well between the predicted and actual damage sites. Others used cubic spline or cubic polynomial for their intended purpose in damage identification of structural systems [2, 4-5]. Another aspect in the computation of damage detection algorithms using modal strain energy is the impact of different numerical integration techniques on the damage detection results.

The objective of this paper is to investigate the effect of reconstructing mode shapes of structural systems with limited number of sensors using Shannon's sampling theorem and cubic spline as well as the implication of two different numerical integration methods on the outcomes of damage detection results. In order to achieve these goals, the following tasks are carried out: first, the numerical integration methods are compared using a modified damage index (MDI) algorithm proposed by the authors [2] with multiple damage scenarios simulated in a finite element (FE) model of a timber beam. A fine mesh was used for this purpose. With the knowledge gained from the first task, a better numerical integration method was adopted for the second task. The Shannon's sampling theorem and Spline were used to reconstruct mode shapes from a coarser mesh FE model. The generated mode shapes from both techniques were then applied to the MDI [2] for comparison.

2 FINITE ELEMENT MODEL

2.1 UNDAMAGED MODEL

An analytical model was developed using a commercially available finite element (FE) analysis package, namely ANSYS. The FE model was constructed for sawn timber beams, which are widely available in Australia. The set up of the model of the beam is illustrated in Figure 1. The specimen's breath and depth were 45mm and 90mm, respectively, with a span length of 4,500mm. The model beam was of radiata pine timber with modulus of elasticity of 12,196N/mm^2, obtained from four point bending test. The solid elements (SOLID45) were utilised to model the beam, for which different damage scenarios can be easily created. For the finer mesh model, there are 41 nodes and 40 elements used in the model, which are denoted by numbers with and without italics, respectively, as shown in Figure 1. The coarser mesh model utilises the finer mesh results with intervals of 1/8 of the span length. Basically, there are 41 and 9 data points for fine and coarse meshes, respectively.

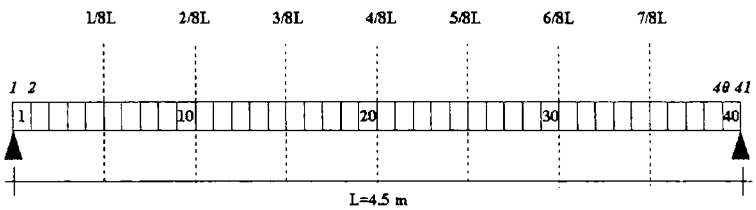


Figure 1. Nodes and elements for FE model of the timber beam

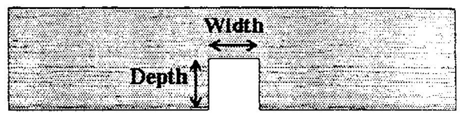


Figure 2 Configuration of damage cases

2.2 DAMAGED MODEL

Different damage scenarios for multiple damage were used to complete the two tasks mentioned earlier. Damage cases 4S5M6S and 2S4S5S6M as tabulated in Table 1 were used to demonstrate the differences between the two numerical integration methods applied to a damage detection algorithm. The former consisted of a rectangular opening from the soffit of the beam located at mid-span, 5/8 and 6/8 of the span length to simulate rotten wood, which usually starts from the surface in timber girders. The latter has an extra damage located at 2/8 of the span length. Another two damaged models (4S6L and 4S6S) were utilised to study the changes in the MDI due to different mode shape reconstruction techniques. The dimensions of the opening, simulating damage for various cases, are given in Table 1. In this paper and the discussions that follow, L, M and S denote light, medium and severe damage, respectively. The length of all inflicted damage is 1% of the total span length (45mm) with depths equal to 10%, 30% and 50% of the beam depth, designated as damage cases L, M and S, respectively. The configuration of the damage cases is shown in Figure 2.

Table 1 Dimensions of inflicted damage

Damage Case	Location per 8th of span length	Width (mm)	Depth (mm)	% reduction of 'I'
4S6L	4, 6	45	45, 9	87.5, 27.1
4S6S	4, 6	45	45, 45	87.5, 87.5
4S5M6S	4, 5, 6	45	45, 18, 45	87.5, 65.7, 87.5
2S4S5S6M	2, 4, 5, 6	45	45, 45, 45, 18	87.5, 87.5, 87.5, 65.7

3 DAMAGE DETECTION ALGORITHM

3.1 MODIFIED DAMAGE INDEX (MDI)

In Equation 1, the terms $\bar{\phi}_i^{//*}$ or $\bar{\phi}_i^{//}$ are normalized mode shape curvature coordinates, which are normalized with respect to the maximum value of the corresponding mode, corresponding to mode i for a beam structure. The asterisk denotes the damage cases. The derivation of this equation is discussed in more details in Li et al. [2].

$$\overline{\beta_{ij}} = \frac{\left[\int_j \left\{\bar{\phi}_i^{//*}(x)\right\}^2 dx + \int_0^L \left\{\bar{\phi}_i^{//*}(x)\right\}^2 dx\right] \int_0^L \left\{\bar{\phi}_i^{//}(x)\right\}^2 dx}{\left[\int_j \left\{\bar{\phi}_i^{//}(x)\right\}^2 dx + \int_0^L \left\{\bar{\phi}_i^{//}(x)\right\}^2 dx\right] \int_0^L \left\{\bar{\phi}_i^{//*}(x)\right\}^2 dx} \tag{1}$$

The modified damage index method introduced above, has been developed to account for all available mode shapes, which are normalized with respect to mass, through the summation of the combination of mode shape curvatures. Although mode shape vectors have been normalized with respect to the Euclidean norm of the matrix, the mode shape curvatures used for the damage index calculation are not normalized. Values of mode shape curvature are dependant on the shapes of each individual mode shape. Instead of reflecting the changes in the curvature due to damage, the summation of non-normalized mode shape curvatures will distort the damage index in favour of higher modes, which results in false damage identifications. Transforming the damage indicator values into the standard normal space, normalized damage index Z_j is obtained as follows:

$$Z_j = \frac{\overline{\beta}_j - \mu_{\overline{\beta}_j}}{\sigma_{\overline{\beta}_j}} \tag{2}$$

where $\mu_{\overline{\beta}_j}$ = mean of $\overline{\beta}_j$ values for all j elements and $\sigma_{\overline{\beta}_j}$ = standard deviation of $\overline{\beta}_j$ for all j elements.

4 NUMERICAL INTEGRATION METHODS

To calculate damage index $\overline{\beta}_{ij}$ in Equation 1, the mode shape and its derivatives are required. The central difference scheme for the second derivative as shown in Equation 3, based on three data points, is adopted for computing the mode shape curvature:

$$f''[x] = \frac{f[x-h] - 2f[x] + f[x+h]}{h^2} + HOT \tag{3}$$

To perform integration of Equation 1, two commonly used methods are adopted, namely the rectangular rule and the trapezoidal rule. Figures 3 and 4 illustrate application of the two methods in performing the numerical integration of Equation 1. The rectangular rule is producing 'n' output data and the trapezoidal rule with only 'n-1' from an n-point data set. It is important to note that the outcomes of damage detection using Equation 1 will heavily rely on the accuracy of derivatives estimation, which will require fine increment of data points.

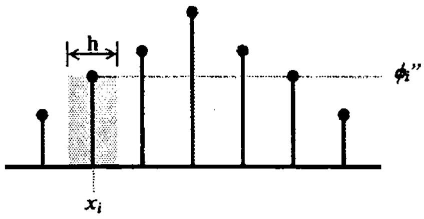


Figure 3. Numerical integration using the rectangular rule [8]

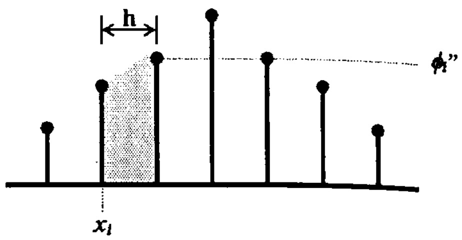


Figure 4. Numerical integration using the trapezoidal rule [8]

5 MODE SHAPE RECONSTRUCTION ALGORITHMS

5.1 SHANNON'S SAMPLING THEOREM

Shannon's sampling theorem defines correct interpolated signal values at points equally spaced between the samples. In here, the variables of interest are the position of the sensors on the beam. The interpolated mode shape should represent the exact reconstruction of the mode shape of interest when the summation is for all 'n' data points between negative and positive infinity, i.e., for an infinite number of spatially repeated experimental mode shape vectors. Acceptable results can be obtained using several repeats of the experimental mode shape coordinates, i.e., for a finite range of values of 'n'. The governing equation is shown in Equation 4. A more detailed explanation of how to apply Shannon's sampling theorem to reconstruct mode shapes can be found in [7].

$$S(x) = \sum_{n=-\infty}^{\infty} S(nT) \frac{\sin \pi(\frac{x}{T} - n)}{\pi(\frac{x}{T} - n)} \tag{4}$$

S(x) and *S(nT)* represent the values which are interpolated along *x* (the span length) and the sampling points from sensors, respectively. *T* is the spacing of the sensors.

5.2 CUBIC SPLINE

Matlab provides easy access to the cubic spline data interpolation using Spline function. A tri-diagonal linear system (with, possibly, several right sides) is being solved for the information needed to describe the coefficients of the various cubic polynomials which make up the interpolating spline.

6 RESULTS AND DISCUSSIONS

In the following results, the statistically normalized damage indicator values (Zj) for each of the damage cases are plotted against the beam span length. The damage locations are shown with dashed line in all the figures.

6.1 NUMERICAL INTEGRATION METHODS

Figures 5 and 6 show the comparison between the rectangular and the trapezoidal rules, for damage cases 4S5M6S and 2S4S5S6M, respectively, used for the computation of the damage algorithm. It is apparent that the rectangular rule is superior to the trapezoidal rule in damage detection application. In Figure 5, the medium damage can be detected when using the rectangular rule for integration, but failed to do so when the trapezoidal rule was applied. It is also observed that the indication of damage is more pronounced when rectangular rule is applied. This is particularly important when the damage index is used for the experimental results, which are usually contaminated with noise. The rectangular rule takes advantage of every mode shape curvature at each node, instead of the average between two nodes taken into account in the trapezoidal rule one, thus making it better. Hence, the rectangular rule was chosen for all the subsequent results and discussions.

(a) Rectangular rule **(b) Trapezoidal rule**

Figure 5. Comparison of numerical integration methods for 4S5M6S

(a) Rectangular rule **(b) Trapezoidal rule**

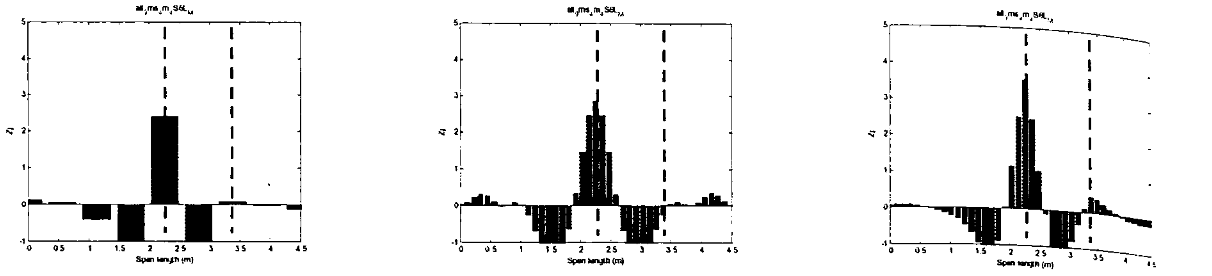
Figure 6. Comparison of numerical integration methods for 2S4S5S6M

6.2 MODE SHAPE RECONSTRUCTION

It is clear from Figure 7(a) that without sufficient data points the damage detection algorithm fails to identify all damage locations. Even with the Shannon's sampling theorem to reconstruct the mode shape, the light damage at ¾ of the span length was not detected as shown in Figure 7(b). In Figure 7(c), when Spline was used to reconstruct mode shape for calculating ϕ_i", both damage regions were detected and it also well predicted the severity of damage through the estimation of probability of damage. The advantage of Spline is confirmed with respect to the damage case 4S6S as depicted in Figure 8.

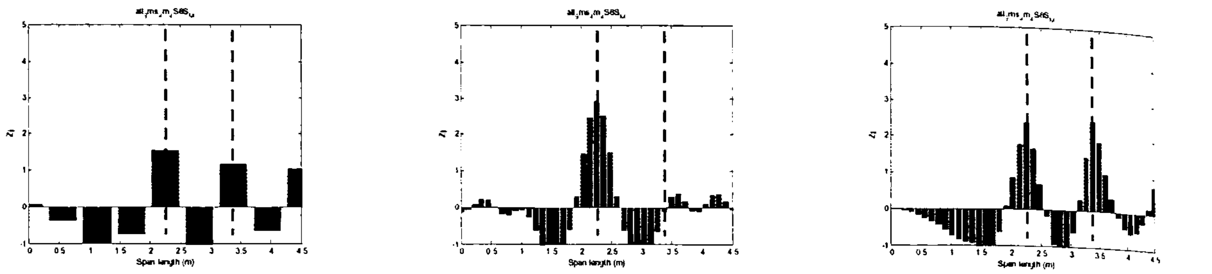
7 CONCLUSIONS

It is clear that numerical techniques used in a damage detection process can significantly affect the outcomes of the detection results. For example, despite its popularity, the Shannon's sampling theorem is not as effective as the cubic spline for mode shape reconstruction, especially for multiple damage cases. When mode shape derivatives need to be calculated, using the rectangular rule for the numerical integration yields better results than the trapezoidal rule. With these findings, the corresponding experimental work will be conducted in the structures laboratory as the next stage of this investigation.



(a) As is (b) Shannon's sampling theorem (c) Spline

Figure 7. Comparison of mode shape reconstruction techniques for 4S6L



(a) As is (b) Shannon's sampling theorem (c) Spline

Figure 8. Comparison of mode shape reconstruction techniques for 4S6S

8 ACKNOWLEDGEMENT

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