Cascade Sliding Mode-PID Controller for Non-overshoot Time Responses

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***Abstract*— Overshoot is a serious problem in industrial process control. This paper presents a new method for elimination of step response overshoot in a conventionally PID-controlled system by cascading a sliding mode controller in the outer loop. The general idea is to use the cascade control principle to model the under-damped system with PID control with a second-order system and then to make use of the sliding mode control feature to obtain a robust, reduced-order response, and thus, suppressing the control overshoot. The validity of the proposed approach is verified through simulation for an uninhabited ground vehicle's braking system suffering from highly nonlinear uncertainties.**

Keywords-* *sliding mode, cascade control, overshoot, robustness.

I. INTRODUCTION

The classical PID controller is still popular in industry because it is a general-purpose controller and does not require complex design procedures. The most important issue of a PID controller is that its parameters need to be tuned properly. However, tuning a PID is not easy and in fact, many PID controllers in industry are not well-tuned.

There are some methods for tuning PID parameters. Based on knowledge of characterizing effects of each control parameter, engineers can adjust the P, I, and D gains until a desired response is obtained. However, this manual method is time-consuming and not always yields a desired response because changing one parameter may affect the performance designated by other two parameters. Although developed for over half a century, two methods proposed Ziegler and Nichols are still being cited in the context of auto-tuning for PID controllers [1]. In the first method, controller parameters are calculated from an open-loop response of the process to a step input (process reaction curve). In the second one, both I and D parameters are set to zero while P parameter is increased gradually until the system oscillates. The period of the oscillation (called ultimate period) and the P gain (called ultimate gain) are used to calculate the desired controller parameters. The Ziegler-Nichols rules can help the tuning process faster than the trial-error method. However, they are not practical in many situations when experiments with open-loop or instable closed-loop can damage the process. To avoid this problem, many techniques such as relay feedback[2], approximate system identification [3], and cross-correlation [4] have been developed to estimate the ultimate gain and ultimate period in Ziegler-Nichols rules.

It is well-known that the control performance obtained by the Ziegler-Nichols tuning methods is just acceptable and the controller parameters need to be fine-tuned to provide the desired response [5]. While eliminating the steady error and shortening the settling time, the Ziegler-Nichols rules still result in a reasonable overshoot (quarter decay ratio). However, this overshoot can be excessive and not acceptable in many processes such as chemical or mechanical systems. Hang *et al.* proposed a method to reduce the overshoot [6]. Using the set-point weighting, this method can reduce the overshoot to 10% or 20%, depending on applications. This may still appear inadequate for overshoot-sensitive systems.

When single-loop PID control systems can not satisfy the control requirement, cascade PID control systems are often used. In [7], both optimization and auto-tuning methods are used for tuning cascade control systems. The results show that a cascade control system gives better responses with shorter settling time and smaller overshoot compared with its single-loop control option.

In this paper, we propose to use the cascade control principle coupled with a sliding mode controller (SMC) at the outer loop to eliminate the overshoot of a step response of the PID-controlled inner loop. It is expected that not only overshoot is alleviated but also such SMC prominent property as robustness to external disturbance, uncertainties and nonlinearities can be retained [8].

Using this method, the PID controller just needs to be tuned to obtain the desired settling time and steady-state error. while overshoot is not considered in the first stage. Based on the resulting closed-loop transfer function modeled by using the cascade control principle, a sliding mode controller (SMC) is then designed to control the input of the inner loop system in such a way that overshoot is entirely suppressed. Simulation results are provided to show the effectiveness of the proposed controller.

II. CONTROLLER DESIGN

A. *Cascade control*

A cascade control system, quite popular in industrial processes, is a multi-loop control system, which can be represented typically by two loops as shown in Fig. 1. The outer loop controller (K_1) is designed based on the process (G_1) and the equivalent closed-loop transfer function of the inner system comprising the inner loop controller (K_2) andd the process (G_2). In that way, one can close the loops for cascading more controllers.

Cascade control has many advantages compared with single-loop control [9]. For example, disturbances (d) in the inner loop can be corrected before they affect the whole system performance. Furthermore, the inner loop can also correct the influence of parameter variations and reduce the effect of nonlinearity in the process. Therefore, cascade control usually performs better than single-loop control, especially in complex processes.

B. *PID controller*

Fig. 2 shows a basic PID controller in a closed-loop feedback system. Output of the controller is a function of the difference (error, e) between the reference (desired output) and the current output:

$$V = K_P e + K_I \int edt + K_D \frac{de}{dt}. \tag{1}$$

Responses of a PID controller is decided by its parameters. The proportional gain (K_P) has the effect of reducing the rise time and it also reduces, but never eliminates, the steady-state error. The integral gain (K_I) has the effect of eliminating the steady-state error, but it may make the transient response worse. The derivative gain (K_D) has the effect of increasing the stability of the system, reducing the overshoot, and improving the transient performance.

To obtain a desired response. PID parameters need to be tuned properly. By manually tuning or auto-tuning methods, the desired setting time and steady-state error can be obtained. In some systems, no matter how the PID tuning procedures are, overshoot of the step response still exists.

C. *Closed-loop Transfer Function for the PID-Controlled Inner Loop*

In this paper, the PID controller is used in the inner-loop. As its step response exhibits a certain amount of overshoot, the transfer function of the closed-loop system of the inner loop (with PID controller) can be modeled equivalently by a second-order function:

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{2}$$

where ζ is the damping ratio and ω_n is the natural frequency. The percentage of overshoot and peak time are calculated as [5]:

$$M_P = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \tag{3}$$

$$t_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}. \tag{4}$$

From the closed-loop step response of the inner loop, an equivalent transfer function can be obtained as (2), where the damping ratio and the natural frequency can be calculated respectively from (3) and (4):

$$\zeta = -\frac{\ln(M_P)}{\sqrt{\pi^2 + \ln^2(M_P)}}, \tag{5}$$

$$\omega_n = \frac{\pi}{t_P\sqrt{1-\zeta^2}}. \tag{6}$$

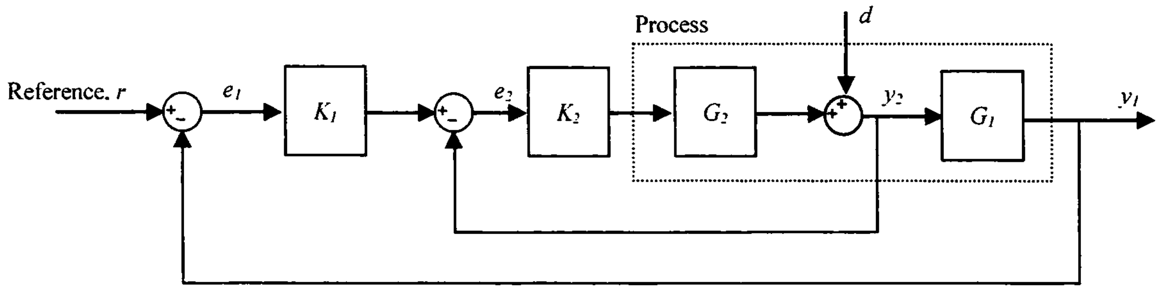

Figure 1. Cascade control system

correction: $\delta = \zeta$ = damping ratio

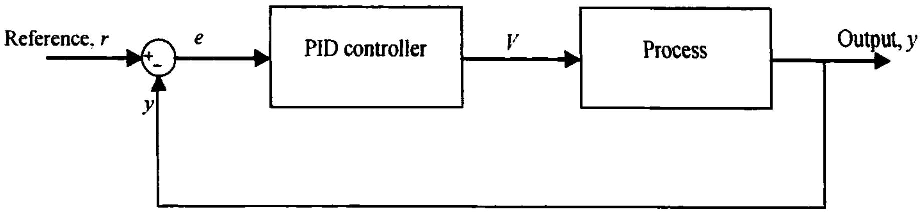

Figure 2. PID-Controlled Inner Loop

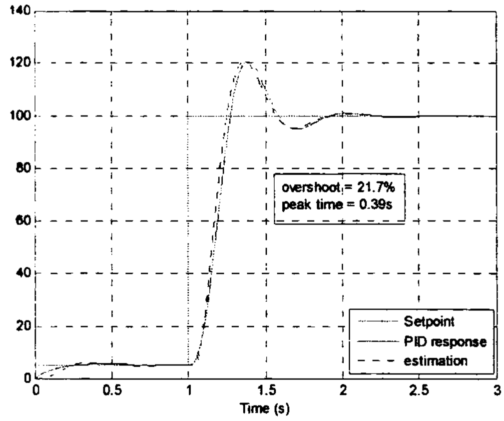

Figure 3. PID-Controlled and Second-Order Step Responses

Fig. 3 shows the difference between the closed-loop step response of a PID controller and the step response of the equivalent second-order transfer function.

D. Sliding Mode Controller development

By considering equivalently the PID closed-loop control as a second-order transfer function, a sliding mode controller is designed to control the whole loop in a cascade control configuration, where the input of the PID controller is regulated by the output of the SMC as shown in Fig. 4. In this figure, v is an unknown input accounting for external disturbance, modeling error and parametric uncertainties.

Let the error be defined as

$$e = y_{ref} - y, \tag{7}$$

where y_{ref} is the desired output (reference).

With the sliding function chosen as $S = \dot{e} + \lambda e$, where λ is a positive scalar to be selected, consider a Lyapunov function $V = \frac{1}{2}S^2$. Taking the first time derivative of V yields $\dot{V} = S\dot{S}$, where

$$\dot{S} = \ddot{e} + \lambda\dot{e} = \left(\ddot{y}_{ref} - \ddot{y}\right) + \lambda\dot{e}. \tag{8}$$

Equation (2) gives

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 u. \tag{9}$$

Substitution from (9) into (8) gives

$$\begin{aligned}\dot{S} &= \ddot{y}_{ref} + 2\delta\omega_n\dot{y} + \omega_n^2 y - \omega_n^2 u + \lambda\dot{e} \\ &= \ddot{y}_{ref} + 2\delta\omega_n\dot{y}_{ref} - 2\delta\omega_n\left(\dot{y}_{ref} - \dot{y}\right) + \omega_n^2 y_{ref} \\ &\quad - \omega_n^2\left(y_{ref} - y\right) - \omega_n^2 u + \lambda\dot{e} \\ &= \ddot{y}_{ref} + 2\delta\omega_n\dot{y}_{ref} + \omega_n^2 y_{ref} - \left(2\delta\omega_n - \lambda\right)\dot{e} \\ &\quad - \omega_n^2 e - \omega_n^2 u \\ &= \omega_n^2\varphi_{ref} - \left(2\delta\omega_n - \lambda\right)\dot{e} - \omega_n^2 e - \omega_n^2 u,\end{aligned} \tag{10}$$

where

$$\varphi_{ref} = \frac{\left(\ddot{y}_{ref} + 2\delta\omega_n\dot{y}_{ref} + \omega_n^2 y_{ref}\right)}{\omega_n^2}.$$

The equivalent control, u_{eq}, is obtained at the nominal regime ($v = 0$) from $\dot{S} = 0$:

$$u_{eq} = \varphi_{ref} - \frac{\left(2\delta\omega_n - \lambda\right)}{\omega_n^2}\dot{e} - e. \tag{11}$$

Now for $v \neq 0$ the control law for SMC has the form of [10]:

$$u = u_{eq} + u_r. \tag{12}$$

Assuming v is upper-bounded, $\|v\| \leq \rho$, one can easily verify that if the robust control, u_r, is chosen as

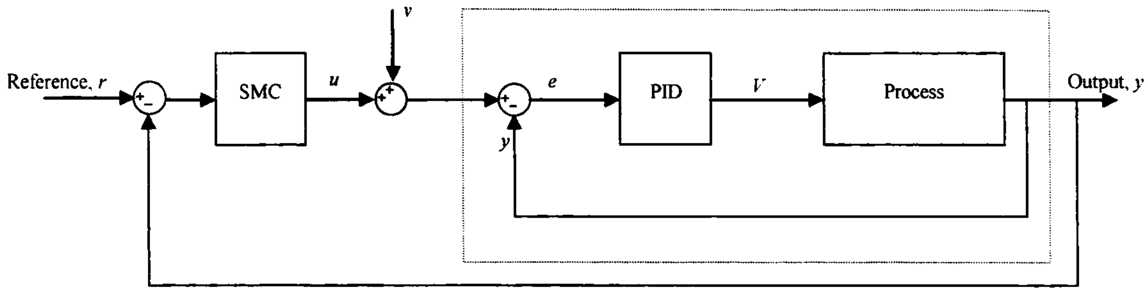

Figure 4. Cascade Sliding Mode - PID controller

$$u_R = \rho sign(S), \tag{13}$$

then the sliding condition $\dot{V} < 0$ is satisfied since

$$\begin{aligned} \dot{V} &= S\dot{S} \\ &= S\left[\omega_n^2 \varphi_{ref} - (2\delta\omega_n - \lambda)\dot{e} - \omega_n^2 e - \omega_n^2 (u_{eq} + u_R + v)\right] \\ &= -S\left[\omega_n^2 (u_R + v)\right]. \end{aligned}$$

The control output of the SMC is then

$$\begin{aligned} u &= u_{eq} + u_R \\ &= \varphi_{ref} - \frac{(2\delta\omega_n - \lambda)}{\omega_n^2}\dot{e} - e + \rho sign(S). \end{aligned} \tag{13}$$

The signum function in (13) creates fast oscillations in the control output, or so-called chattering. A saturation function can be used to reduce this effect [8].

Remark 1: The proposed method may be applied generally for any overshoot-sensitive systems provided that their PID-controlled inner-loop step responses are known.

Remark 2: With robustness of the SMC, the proposed cascade control may tolerate modeling errors, as well as deal with such problems as external disturbance, uncertainties and nonlinearities.

III. SIMULATION RESULTS

The proposed method is tested first with a simple DC motor position control for linear systems. It is then applied for the skid-steering braking system of an autonomous ground vehicle where nonlinear hydraulic drive and other uncertain sources make it difficult to obtain a non-overshoot response.

A. *DC motor position control*

Based on the PID-controlled positioning system using a DC motor provided in [11] as a benchmark, a SMC is designed to control the motor position. In this example, the PID controller is not well-tuned and provides a large overshoot.

The responses with PID (---) and SMC-PID (—) are shown in Fig. 5. When PID is used, a step reference (set-point) at the input (Fig. 5a) creates an oscillated voltage at the input of the motor (Fig. 5b) and results in a large overshoot at the motor shaft (Fig. 5c). In contrast, the SMC forces the PID input (control output of SMC, *u*) (Fig. 5a) and the motor input (Fig. 5b) to eliminate completely the overshoot while it still keeps the desired settling time for the whole system (Fig. 5c).

B. *Hydraulic braking system*

Fig. 6 shows a hydraulic braking system of a skid-steering unmanned ground vehicle (UGV) [12, 13]. It has two components that suffer from nonlinearity, namely the actuator and the hydraulic cylinder. The actuator consists of a DC motor, gears and ball-screw with some dead-zone, as provided by the manufacturer, as shown in Fig. 7. The complicated relationship between output and input of the hydraulic cylinder, estimated from experimental data by using the least square identification method, can be represented in Fig. 8.

A pressure controller is designed to control the system with the assumption that the braking force is proportional to of the pressure inside the hydraulic cylinder. A PID controller is designed first for hydraulic pressure control. It took us a great deal of time to tune the PID controller. However, the best PID response still exhibits a large overshoot which does not satisfy the requirement of skid steering. A cascade SMC has been designed to solve the problem.

The results are shown in Fig. 9 for both the PID and SMC-PID controllers. From step references, it is observed that the PID case possesses a large overshoot at the output (pressure) while the SMC can control the PID input (control output of SMC, *u*) (Fig. 9a) to force the system output to a non-overshoot step response (Fig. 9c).

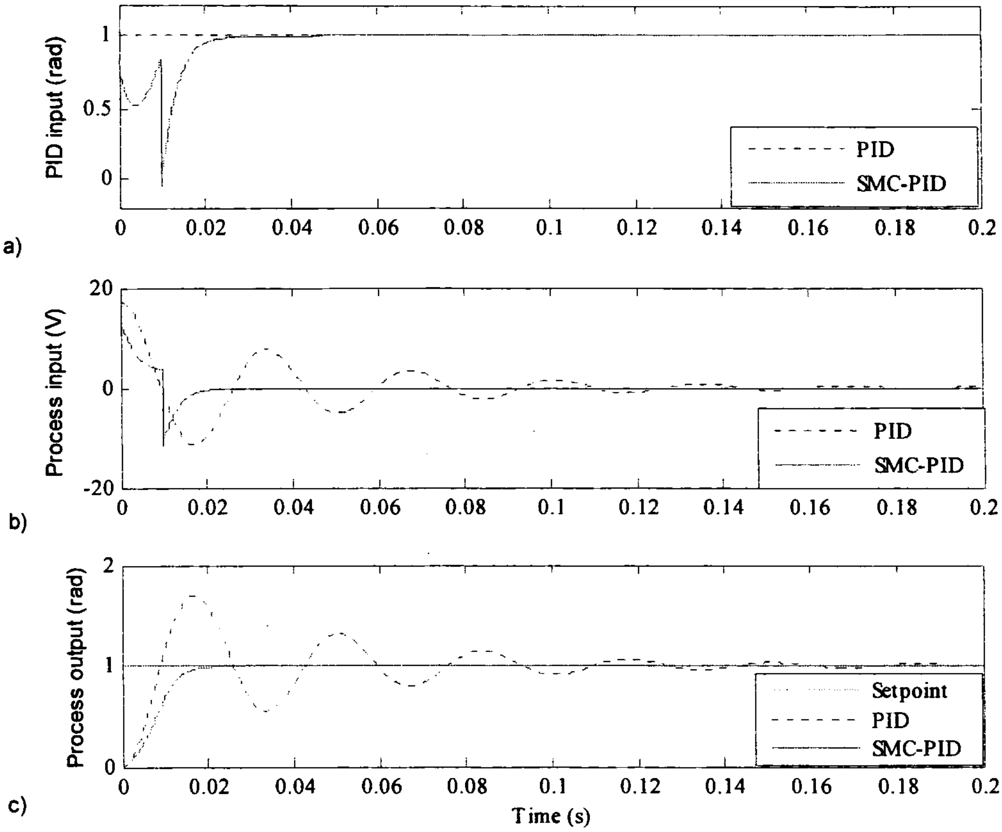

Figure 5. Response of PID controller (---) and SMC-PID (—) for DC motor position

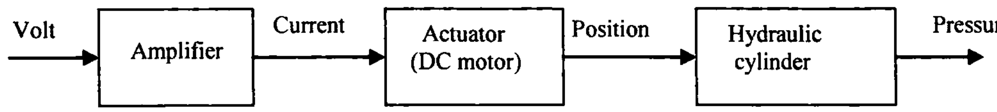

Figure 6. Block diagram of the braking system of an UGV

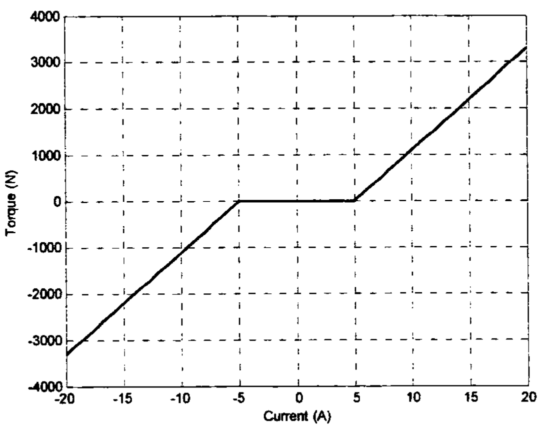

Figure 7. Dead-zone of actuator

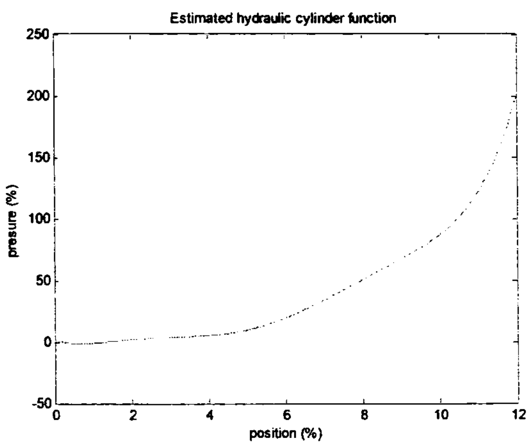

Figure 8. Estimated I/O relationship of hydraulic cylinder

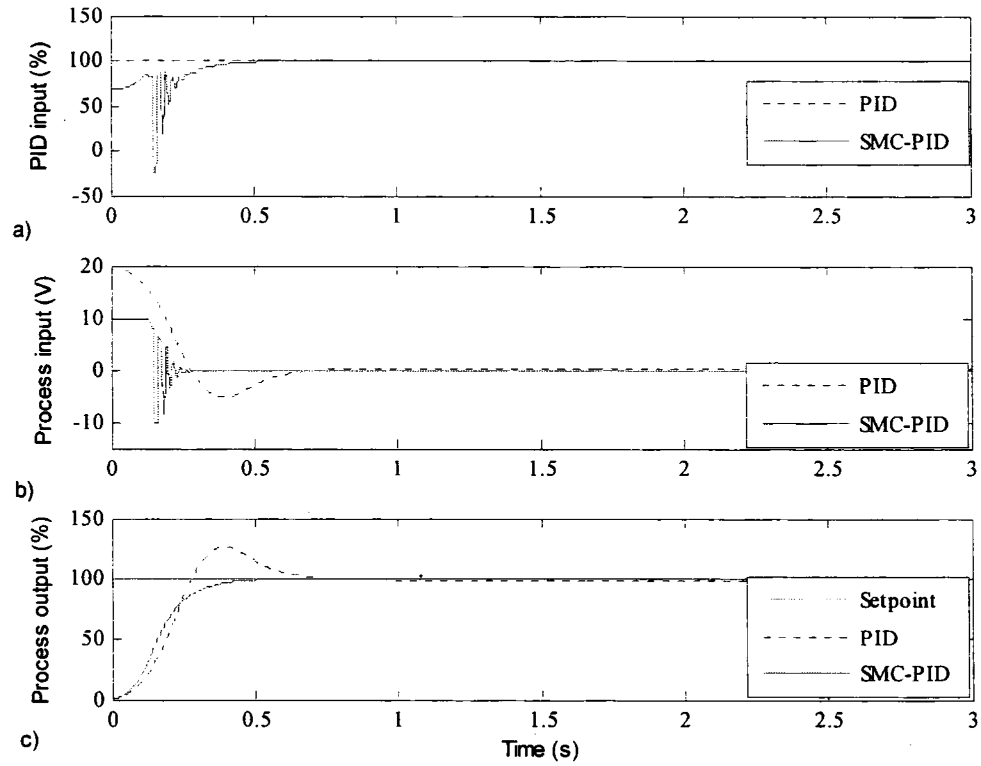

Figure 9. Responses of PID (---) and SMC-PID (—) control for UGV hydraulic braking system

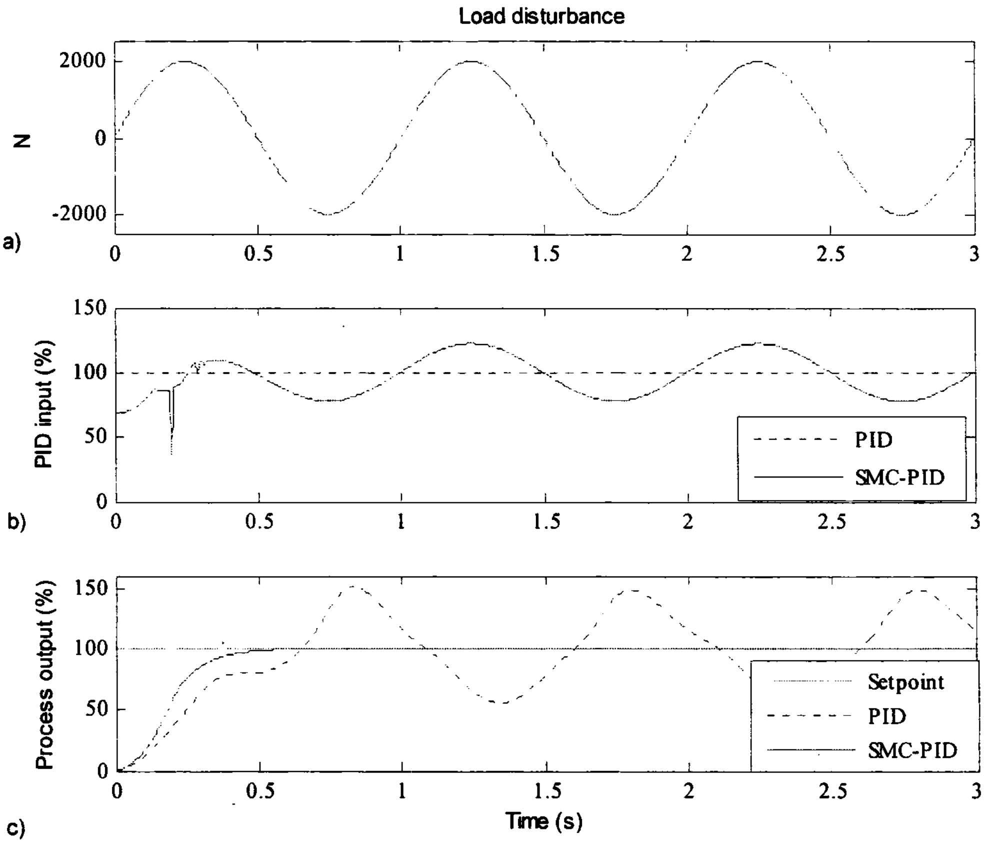

Figure 10. Responses of PID (---) and SMC-PID (—)control with external disturbance

Fig. 10 shows responses of the controllers with a disturbance representing a load change. The amplitude of disturbance is about 60% of maximum torque provided by the actuator (Fig. 10a). The PID controller can not regulate the output braking disc to the desired value while the SMC is able to control the PID input (SMC output, u) (Fig. 10b) in a robust way to compensate for the disturbance. As a result, the error of the SMC-PID is found less than 0.6% compared with 50% of the PID (Fig. 10c). This is explained by the prominent feature of sliding mode control in producing robust, reduced-order time responses, and thus, suppressing successfully the step response control overshoot.

IV. CONCLUSION

We have presented a cascade SMC-PID controller for non-overshoot robust responses. The proposed method can be applied for any PID-controlled system if its closed-loop responses are known. From an equivalent transfer function of the PID inner-loop system, a SMC is designed to force the input of the PID so that overshoot of its step response is completely eliminated. Simulation results for a hydraulic braking system of an unmanned ground vehicle indicate that the proposed method is very effective in suppressing completely control overshoot while retaining the settling time and steady-state error, and also in achieving strong robustness against external disturbance and nonlinearities.

ACKNOWLEDGEMENT

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