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STUDY ON MODEL REDUCTION OF LARGE STRUCTURAL SYSTEMS FOR ACTIVE VIBRATION CONTROL

J. Boffa, N. Zhang and B. Samali

Mechatronics and Mechanical Systems, Faculty of Engineering, University of Technology, Sydney

ABSTRACT

This paper studies the applicability of the Dynamic Model Reduction Method that is used for direct plant order reduction in active vibration control of large and flexible structures. By using a twenty-storey building model as the plant, the accuracy of the frequency responses of the reduced 3 degrees of freedom models was firstly evaluated with sinusoidal applied forces. For the closed-loop dynamic analysis, the tall building model with an active mass damper at the top storey and recorded earthquake excitation was used. The results of the simulations have indicated that the dynamic model reduction method can be used as an alternative approach for model reduction of structural systems for the purpose of active vibration control. Compared with the Guyan model reduction method, the dynamic model reduction method has many advantages, especially in terms of its accuracy at the high frequency range. The mode-displacement method produces reduced models that are good for dynamic analysis of open-loop systems but it is inconvenient for the use in active vibration control.

1. INTRODUCTION

For the active vibration control of complicated mechanical or structural systems a reduced dynamic model of the system with a very limited number of degrees of freedom and yet sufficient accuracy is often required. One of the typical applications is the active vibration control of high rise and flexible building structures subjected to earthquake excitations and wind loads that have mainly low frequency components. In this case, the dynamic responses of the structural systems contain mainly the contributions made by a few of the lowest vibration modes of the structural system. So the vibrations can be effectively controlled based on a reduced-order plant model that contains only a few of the lowest modes of the structure [1].

The use of a reduced plant model within the controller can minimise the computation time for determining the feed-back gains required by the actuators and therefore improve the overall performance of the combined plant-controller system [2,3]. Seto et al. [1,2] pointed out the importance of having reduced plant models in terms of meaningful physical parameters such as mass, damping, stiffness parameters and presented a few successful applications of active vibration of flexible structures based on reduced physical low-order plant models. Ma and Hagiwara[4] developed the mode-displacement method for obtaining the reduced model of a large structural system. The resulted models often perform well in structural analysis. Zhang[5] presented a dynamic model reduction method that produces reduced models of systems with a large number degrees of freedom for dynamic analysis. The reduced models are formulated from condensed mass, damping and stiffness coefficient matrices and retain a small number of lowest modes of the original system. Care needs to be taken in choosing the reference frequency for taking into account the dynamic effect of high modes, and in choosing the master coordinates that are retained in the reduced models.

This paper begins with a description of the three different model reduction techniques that were used in this investigation, and then presents both simulation results of dynamic responses of the open-loop and closed-loop systems under earthquake inputs. The three different plant model reduction techniques are: Guyan, Mode-Displacement and the Dynamic Model Reduction Method (DMRM). From the results, it is clear that the DMRM is superior particularly when applied to closed-loop active vibration control of large and flexible structures.

2. DESCRIPTION OF THREE DIFFERENT MODEL REDUCTION METHODS

2.1 The Guyan Model Reduction Method

The Guyan model reduction method is the most common procedure for reducing the size of mass and stiffness matrices that form the reduced model of a plant. While the reduced stiffness matrix preserves its accuracy, the reduced mass matrix produced by this method does not. The reason for this inaccuracy is that the Guyan method uses a static transformation between the eliminated and retained coordinates for obtaining the reduced mass matrix. This static transformation ignores the dynamic effect of the applied loads and creates an increasing error as the frequency of excitation is increased. For this reason, the Guyan model reduction method is only accurate in the low frequency range, and this will be demonstrated later in the simulation results.

2.2 The Mode-Displacement Model Reduction Method

The mode-displacement method, (which is a special case of the Ma-Hagiwara mode superposition method[4], when the reference frequency is set to infinity) produces a low-order plant model in terms of retained modal coefficients, natural frequencies and a few of modal coordinates. This method is simple and convenient as far as the open-loop dynamic analysis of the plants is concerned, when the applied loads are known.

2.3 The Dynamic Model Reduction Method (DMRM)

In order for DMRM [5] condensed models to best approximate the original one, the condensed model retains n_c number of natural frequencies and the corresponding modes at the chosen master coordinates of interest from the original model. For the same unique harmonic forces applied at the master coordinates, the response matrix X_c determined from the condensed model must also be the same as that determined from the original model. To achieve the first requirement, the system matrix $M_c^{-1}K_c$ is determined as

$$B_c = M_c^{-1}K_c = \Phi\Lambda\Phi^{-1}, \tag{1}$$

where Λ is the eigenvalue matrix and Φ is the corresponding modal matrix from the full-size model, (all damping is ignored here for simplicity purposes). To meet the second requirement, the response matrix X_c must be determined from the original structural system which has a large number of degrees of freedom, or alternatively from vibration testing.

The mass matrix of the reduced model can then be determined as,

$$M_c = X_c^{-1}(-\omega^2 I + M_c^{-1}K_c)^{-1} \tag{2}$$

Consequently, the stiffness matrix is determined as,

$$K_c = M_c B_c. \tag{3}$$

After the condensed model is obtained, the responses at the master coordinates due to the applied forces can then be computed, and hence the dynamic responses at those eliminated coordinates can also be obtained in terms of the computed responses at the master coordinates.

As damping always exists in actual structural systems and is difficult to be modelled accurately, modal damping is therefore used for the reduced models. The level of the modal damping is determined by experience or by experimental modal testing on the systems.

Unlike the Guyan method, the DMRM includes the dynamic effect of the applied loads when formulating the reduced mass matrix, and has therefore much greater accuracy at the high frequency range of excitation. Both the Guyan and DMRM use real coordinates, and this is their greatest advantage over the Mode-Displacement Method, particularly for closed-loop control applications.

3. FREQUENCY RESPONSE ANALYSIS (OPEN-LOOP SYSTEMS)

The frequency responses were determined by applying harmonic excitation forces to the open-loop reduced plant models with three degrees of freedom. The peaks in the graph represent the three lowest natural frequencies of the plant models (2.3, 14.1, and 39.3Hz), and they are compared against the full-scale 20 degree of freedom system (the solid line). Figure 1 shows a comparison between the Guyan (dotted line) and the DMRM (dashed line). Figure 2 shows a comparison between the Mode-Displacement Reduction Method (dotted line) and the DMRM (dashed line). Modal damping was used throughout these simulations, with a damping ratio equal to 0.01.

As can be seen in Figure 1, the reduced model based on the DMRM retains the chosen three natural frequencies and corresponding modal shapes accurately from the original system. Therefore the predicted dynamic responses based on the DMRM have very good accuracy at a particular frequency range that often covers the frequency range of excitation forces applied to the system. In contrast to this, the reduced model obtained by the Guyan method does not retain all of the chosen vibration modes precisely and consequently has poor accuracy in the higher frequency range (third natural frequency). Figure 2 shows that that there are no significant discrepancies between the DMRM and the mode-displacement reduction methods, as both of these models perform closely to the solid line (full-scale model).

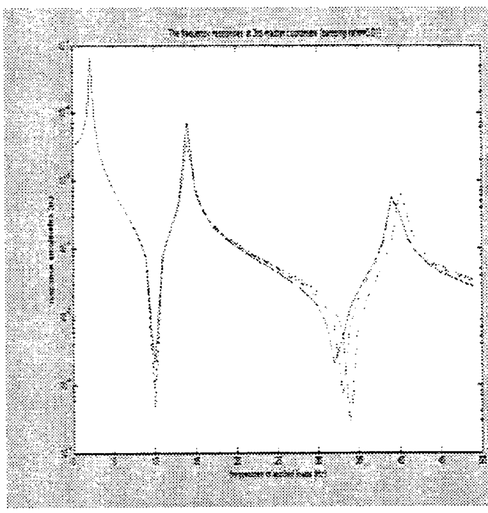

Figure 1. DMRM versus GUYAN (Frequency Response).

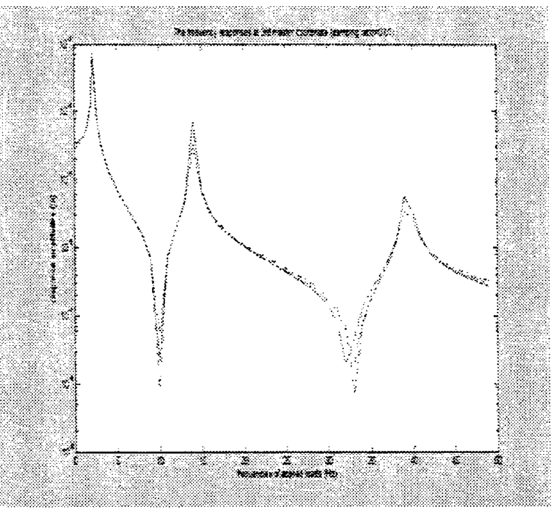

Figure 2. DMRM versus Mode-Displacement (Freq. Response)

4. OPEN-LOOP SYSTEM RESPONSE UNDER EARTHQUAKE EXCITATION

For the simulations of the responses of open-loop system, the recorded El Centro earthquake data was used. The original data had many dominate low frequency components and was sampled at 50Hz. The original sampling frequency was scaled up by a factor of 8 (i.e., 400Hz sampling frequency) in order to shift the dominating frequency components to a higher range. In doing so, the higher modes of the reduced plant models were also excited under the modified earthquake input. The 400Hz sampled earthquake input has major dominating frequencies between 9 and 17Hz, with some minor dominating frequencies occurring between 17 to 47Hz. At this sampling frequency, the total frequency content of the earthquake ranges from zero to approximately 130Hz.

The three degrees of freedom (3dof) reduced models were used as the plant model in each simulation, and the accelerations of the top storey of the building were plotted (below). These three reduced models were compared against a 6dof mode-displacement model, as it was considered to be more representative of the full-scale model due to its inclusion of twice as many modes of vibration. As can be seen in the plots below, the 6dof response has greater acceleration amplitudes because it includes the extra modes of vibration. For this reason, all of the 3dof responses appear to underestimate the true response of the full-structure as estimated by the 6dof model.

Of all the 3dof models, the mode-displacement appears to perform most accurately when this particular modified earthquake input is used, and the Guyan model performs with the least accuracy. When the data sample frequency of the earthquake is increased, the deviation of the 3dof models from the 6dof model increases further, and as the data sample frequency is decreased the deviation decreases as well. When the earthquake input was modified by using a sample frequency of 100Hz, all four models gave almost identical responses. The reason for this is that only the low modes of vibration are excited at this low frequency range, and these modes are all included in the 3dof reduced models.

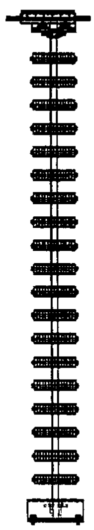

Figure 3. Structural Model

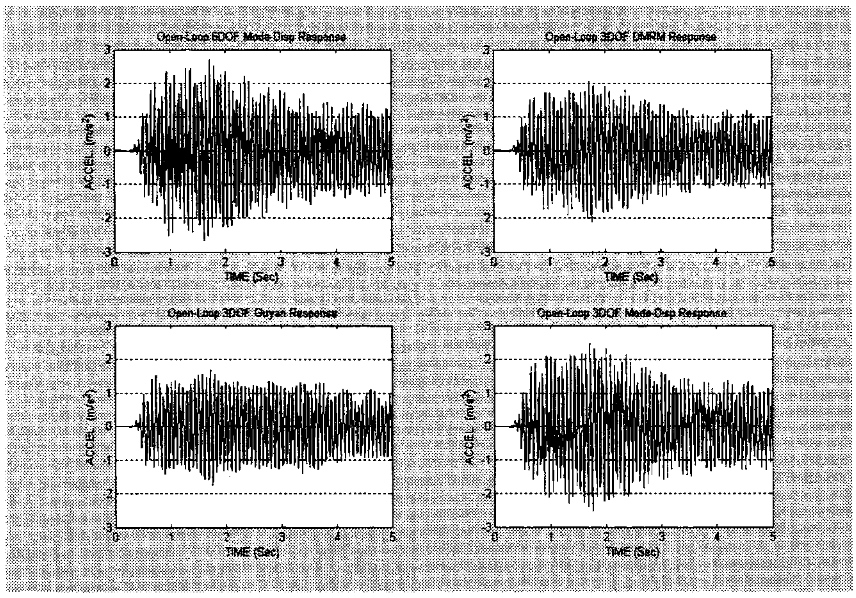

Figure 4. Open-Loop Earthquake Response of Top Storey of Reduced Models (with DSF = 400Hz)

5. CLOSED-LOOP RESPONSES UNDER EARTHQUAKE EXCITATION

Although it is widely acknowledged that the Linear Quadratic Regulator is the most common control method for structural control, the pole placement control technique was used here because of its simplicity. The pole placement control was configured so that parameters such as the desired closed-loop damping ratios (active-damping ratios) and the desired closed-loop natural frequency of the active mass could be adjusted. By increasing these parameters, more control force is produced. It should be noted however, that in a real system, an unlimited control force is not usually available.

In the simulation of closed-loop system responses under earthquake input, we added an active mass damper to the top level of the building model and therefore increased the reduced models by one degree of freedom. The same 6dof (now 7dof) mode-displacement model was used to replace the actual/physical plant in all the closed-loop simulations. The previous 3dof (now 4dof) models of the plant were used within the controller for the purpose of estimating any unmeasurable states[6].

All system parameters were kept unchanged in these simulations, so that the performance of the 4dof observers could be isolated. The simulations below use an earthquake data sample frequency (DSF) of 100Hz because the Guyan model performs best at this frequency. The graphs show that the DMRM performs slightly better then the Guyan method, during closed-loop control. Both the DMRM and Guyan models dampen-out the response effectively as they estimate states in an accurate manner.

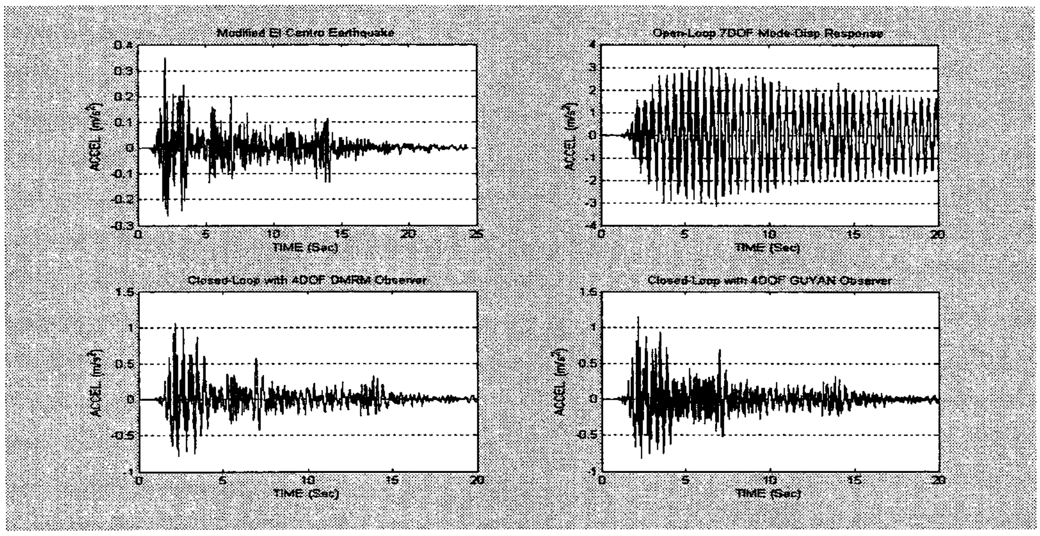

Figure 5. (Upper): Earthquake ground acceleration, and Open-Loop Plant Response of Top Storey (with DSF =100Hz).
Figure 5. (Lower): Closed-Loop Performance Comparison between DMRM and Guyan 4DOF models (with DSF =100Hz).

The data sample frequency of the earthquake was altered again so that the effect of higher frequency contents of excitation could be examined on the closed-loop system performance of the 4dof models. The simulations used an earthquake data sample frequency of 400Hz. Figure 6 clearly shows that the DMRM performs significantly better then the Guyan method, during closed-loop control. The DMRM dampens-out the response more effectively then the Guyan, because it estimates unmeasurable states in a more accurate manner.

We also attempted to use the 4dof reduced observer model obtained by the model-displacement method in the simulation of closed-loop system responses under the earthquake inputs, but unstable responses were obtained when the control input was applied to the 7dof plant model. However, the Mode-Displacement method works exceptionally well when the complete plant-controller system remains in principal co-ordinates, but this is inapplicable to a real/physical plant system/model. Further investigation is currently being conducted to solve this issue.

6. CONCLUDING REMARKS

The applicability of the Dynamic Model Reduction Method to active structural control of large structural systems has been demonstrated from the presented simulations. The reduced plant model obtained by DMRM performs slightly better in the low frequency range and significantly better in high frequency range than the one obtained by the Guyan method. Among the three model reduction methods, the model-displacement method produces a reduced model that performs best in dynamic analysis of open-loop systems over the frequency range covered by the reduced models, however, the mode-displacement method is not conveniently used within the control model of a large system for active vibration control purposes.

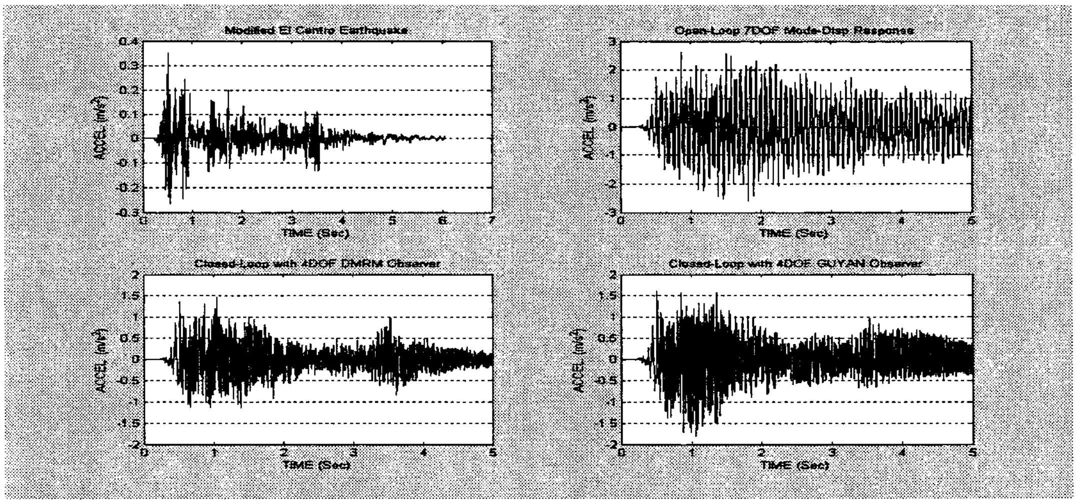

Figure 6. (Upper): Earthquake ground acceleration, and Open-Loop Plant Response of Top Storey (with DSF = 400Hz).
Figure 6. (Lower): Closed-Loop Performance Comparison between DMRM and Guyan 4DOF models (with DSF = 400Hz).

7. ACKNOWLEDGEMENTS

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