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DYNAMIC ANALYSIS OF 'SMART' PIN-FRAME SYSTEM USING TIME-FREQUENCY REPRESENTATION OF EARTHQUAKES

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ABSTRACT: This paper presents the dynamic analysis of a 'smart' pin-frame system employing instantaneous frequency (IF) extracted using Time-Frequency Representations (TFR) of earthquake records. The study is, therefore, aimed to demonstrate the applicability of real-time modal frequency shift technique for the non-linear 'smart' pin-frame model, but it is also used to numerically assess the performance of the proposed frequency shift strategy. The results showed the potential use of this approach for vibration mitigation of structures.

KEYWORDS: Smart pin, Earthquakes, Time-Frequency representation, Magneto-rheological fluids.

1. INTRODUCTION

Nowadays, the use of 'smart' or electronically controllable devices for vibration mitigation is commonly found in building structures. The use of TFR of earthquake records, which constitute multi-component signals, is proposed for vibration mitigation of frequency adjustable systems such as 'smart' pin-frame model. The TFR technique [1,2,6] has been used for other applications, such as: signal synthesis (music, speech); image processing (radar, X-rays); signal processing (fault detection, seismic study [3,4]). Among several available TFR's, the classic Spectrogram [1] and Wigner-Ville distribution (WVD) [2,5,6] are commonly used and employed to determine IF of earthquake records, which is required for real-time frequency shift technique in dynamic analysis of the model, particularly in parallel with the use of computer and electronically coded algorithm. For example, the use of 'smart' pin, a prototype Magneto-rheological fluid (MRF) pin connector for beam-column connections, with the ability to instantaneously increase or decrease the connection rotational stiffness, will cause changes to system frequency, leading to a non-linear dynamic system [7] depending on the supplied Direct Current (DC) level to the pin by a power supply unit through a power amplifier. The generated magnetic field inside the pin polarizes the MRF into either liquid or semi-solid state, changing the fluid rheological property, which can be electronically real-time regulated using the algorithm in the computer. It is unlike the common control systems with control force feedback. The non-linear 'smart' pin-frame system feedback a DC current level to shift system modal frequency away from the resonant frequency, leading to smaller dynamic responses, which can be attained by setting the maximum ratio of instantaneous frequency to real-time adjustable system frequency. Unfortunately, this approach is not yet ready for real earthquakes with unpredicted instantaneous frequency.

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2. STRUCTURAL MODEL

'Smart' pin-frame systems, having non-linear stiffness characteristics and Coulomb (zero-field) friction f_d and subjected to earthquake excitations, can mathematically be represented by the Equation of motion in modal coordinate system as,

$$\ddot{y}_i + 2\xi_i\omega_{si}(t)\,\dot{y}_i + \left(\omega_{si}(t)\right)^2 y_i = \gamma_i\,\ddot{x}_g + \beta_i f_d\,\mathrm{sign}(\dot{x})\,\delta(0\mathrm{A}) \tag{1a}$$

$$\gamma_i = -\left(\varphi_i^T \boldsymbol{MI}\right)/\left(\varphi_i^T \boldsymbol{M}\varphi_i\right); \qquad \beta_i = -(\varphi_i^T \boldsymbol{B})/\left(\varphi_i^T \boldsymbol{M}\varphi_i\right) \tag{1b-c}$$

Equation (1) holds for i^th^ mode, where $\boldsymbol{M}$, $\boldsymbol{I}$, $\boldsymbol{B}$, $\delta(*)$, y_i, $\ddot{x}_g$, ϕ_i, ξ_i and ω_{si} are, respectively, the system mass matrix; the unit column vector; the ith modal displacement; the location vector; Kronecker delta function; the ith modal response; the seismic induced ground acceleration; the ith mode shape vector; the ith modal damping ratio; and the ith modal system circular frequency. Using Fourier Transform, Equation (1) can be rewritten as

$$y_i(t) = H_i(t,\omega)\left(\gamma_i\,\ddot{x}_g(t) + \beta_i f_d\,\mathrm{sign}(\dot{x})\,\delta(0\mathrm{A})\right) \tag{2a}$$

$$H_i(t,\omega) = 1/\left(\left(-\left(\omega/\omega_{si}(t)\right)^2 + 2j\xi_i\left(\omega/\omega_{si}(t)\right) + 1\right)\left(\omega_{si}(t)\right)^2\right) \tag{2b}$$

In Equations (2), j, H_i, t and ω are the imaginary number; the transfer function; temporal variable and circular frequency, respectively. Minimization of the transfer function, which reflects the concept of frequency shift technique, will correspondingly minimize the modal responses.

3. TIME-FREQUENCY REPRESENTATION OF EARTHQUAKES

The linear or quadratic TFR can be employed to estimate the IF of earthquakes for each particular time t. For example, the classical method, STFT, which is commonly used to obtain TFR, can be implemented for discrete-time signals of earthquake records using the expression below,

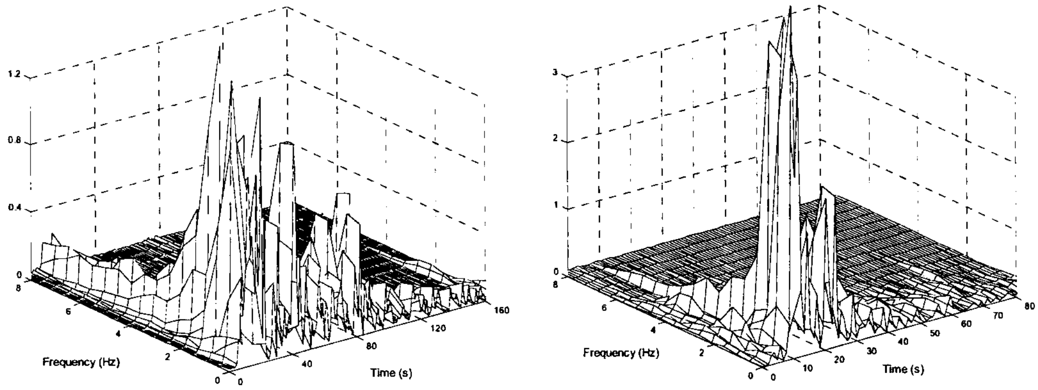
$$\mathrm{STFT}(k,n) = \sum_{m=0}^{L-1} \ddot{x}_g(m)\,\mathrm{h}(m-k)\exp(-j2\pi\, m\, n/L) \tag{3}$$

$$t = k\,\Delta t \qquad k = 0,1,2,3,\ldots\ldots\ldots\ldots N$$
$$\omega = 2\pi n/(L\,\Delta t) \qquad n = 0,1,2,3,\ldots\ldots\ldots\ldots N \tag{4a-b}$$

In Equation (3), h(*), Δt, N and L are the L-point window function; the sampling period (interval); number of discrete signal and the window width of number of points, respectively. More than a dozen window functions, such as: rectangular; Bartlett (triangular); Hanning; Hamming; Blackman; Chebyshev; Kaiser; etc., have been developed with their own merits in resolution for time and frequency [2,3,6]. While, Wigner-Ville Distribution [1,5], a simple and powerful quadratic time frequency distribution, to examine the relationship between IF; particularly for mono-component signal, is examined for its applicability to earthquake records. In order to cope with the problem of half signal interval in formulation, the signal sampling rate is doubled using a low-pass interpolation filter (FIR) with an up-sampling factor of 2 in order to obtain an aliasing-free discrete-time WVD. To do so, the real signals need to be transformed into analytical signals using Hilbert transform to suppress the cross term interference/attenuation [6]. The WVD can be expressed as,

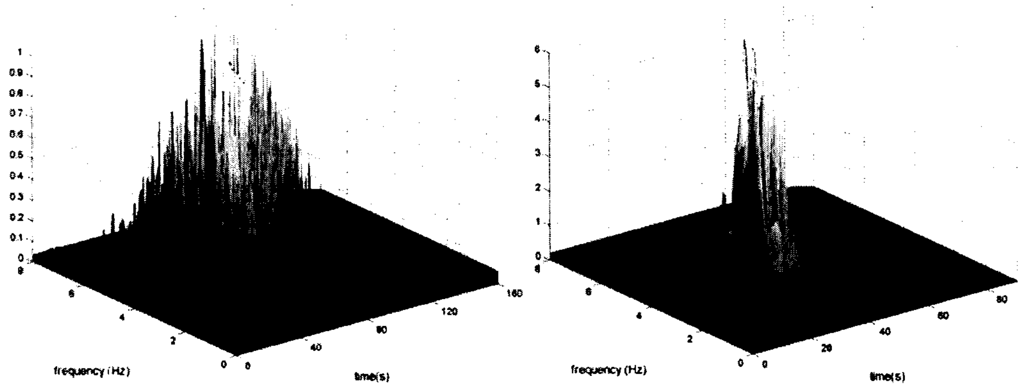
$$W(k,n) = 2\sum_{m=1,3}^{2N-1} (\ddot{x}_g)(k+m)(\ddot{x}_g)^*(k-m)\exp(j4\pi\, n\, m/N);\ k,n = 0,2,4,\ldots\ldots\ldots\ldots\ldots 2N-1 \tag{5}$$

In Equation (5), the superscript * indicates conjugate of earthquake acceleration. In order to impose large vibration responses, two time-scaled earthquakes with a scaling factor of 3, El Centro NS 1940 and Northridge NS 1994, were, respectively, selected to represent far- and near-field earthquake excitations. The joint-time-frequency distributions for these two earthquake records as shown in Figures 1 and 2 were generated using STFT method with Hanning window length (L) of 32 points or discrete signals.



(a) El Centro earthquake (b) Northridge earthquake

Figure 1 Three dimensional STFT time-frequency representations of time-scaled earthquake records



(a) El Centro earthquake (b) Northridge

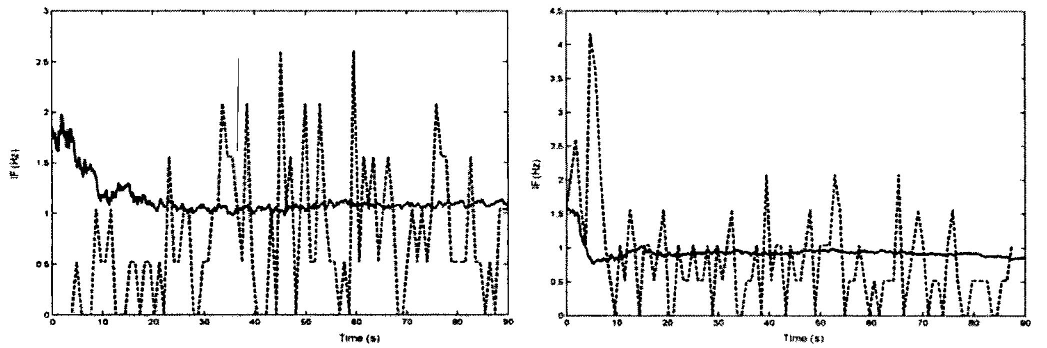
Figure 2 Three dimensional Wigner-Ville distributions (WVD) of time-scaled earthquake records

Three dimensional time-frequency distributions (Figure 1) show the same patterns for both methods, but less dense distributions for WVD (Figure 2) due to the use of window function in STFT method. One can see that at an instance of time, the time-scaled El Centro and Northridge earthquake signals contain a broad band of frequencies for the time range of 10 to 80 seconds and 10 to 40 seconds, respectively. The use of small window length, although, increases the time resolution, but decreases the frequency resolution. The IF f_i in function of discrete time k, which is an 'average' of the frequencies at a given time [1] can be estimated using various methods. The IF can be estimated using peak detection of squared STFT (Equation (3)) with the assumption of stationary IF over the local analysis time [1] and the first moment of time-frequency distribution of WVD [1, 6], respectively, as

$$f_i(k)=\sum_{n=0}^{N-1} f(n)\ \delta(\mathrm{STFT}(k,n)-\max(\mathrm{STFT}(k,n))) \qquad \text{(6a-c)}$$

$$f_i(k)=\sum_{n=0}^{N-1} f(n)W(k,n)/\sum_{n=0}^{N-1} W(k,n) \quad ; \qquad f(n)=n/N$$

For multi-component signals like earthquake records, some instantaneous frequencies can be identified from the instantaneous Fourier frequency spectrum at a particular time *t* or *k*.



(a) El Centro earthquake

(b) Northridge earthquake

Figure 3 Instantaneous Frequencies versus time of time-scaled earthquake records (dash line – STFT)

Interestingly, Figure 3, which shows different IF resulting from STFT and WVD, respectively, due to the use of two different approaches: peak detection and averaging, leads to ambiguity of the 'real' instantaneous frequency of earthquakes. Nevertheless, the Figure provides useful information for system frequency shifting strategy.

4. APPLICATION TO 'SMART' MR PIN-FRAME SYSTEM

With the knowledge of the IF of time-frequency-based earthquake excitations, the structural frequency will electronically be adjusted using the 'smart' MR pin, which is connected to a computer system with frequency shift algorithm. The algorithm, will real-time set the system frequency to attain the maximum frequency ratio, and hence the minimum transfer function. Only two settings of system frequencies: minimum and maximum system frequency (Table 1), which are associated with 'pin-pin' (0 Ampere case) and 'pin-rigid' (1.80 Ampere case) beam-column connections of the 'smart' MR pin-frame model (Figure 4) with two additional mass Δm cases (0 and 31.5 kg) were used for dynamic analysis. The model and 'smart' MR pin were fabricated in the Structures Laboratory of the Faculty of Engineering, University of Technology Sydney (UTS).

In view of Eq. (2b), the system modal transfer function can be rewritten in terms of frequency ratio f_r as

$$\begin{aligned} H_i(k) &= 1/\left(((1-(f_r(k))^2)^2+(2\xi(k)f_r(k))^2))^{1/2}(2\pi f_{si})^2\right) \\ f_r(k) &= \max\left(1-(f_i(k)/(f_{si})_{0\mathrm{A}})^2, 1-(f_i(k)/(f_{si})_{1.8\mathrm{A}})^2\right) \end{aligned} \qquad \text{(7a-b)}$$

where $f_i = \omega/2\pi$ and $f_{si} = \omega_{si}/2\pi$. The use of large system modal frequency ratio associated with 'pin-pin' case is applicable if the following condition for the denominator holds,

$$\frac{\left(((1-(f_r(k))^2)^2+(2\xi(k)f_r(k))^2))^{1/2}(2\pi f_{si})\right)_{0\mathrm{A}}}{\left(((1-(f_r(k))^2)^2+(2\xi(k)f_r(k))^2))^{1/2}(2\pi f_{si})\right)_{1.8\mathrm{A}}} > 1 \qquad (8)$$

The earthquake signals with the sampling frequency of 50 Hz, resulting from sudden release tests (7), were used to identify the system frequencies, damping ratio and zero-field friction, as tabulated in Table 1. The identified system frequency range, which can be considered as the first mode frequency range for moderate high rise buildings, gives two benchmark frequencies for the system frequency shift strategy (Equations (7-8)) in studying the vibration mitigation of 'smart' MR pin-frame model. Results presented with time-history displacement responses are computed using discrete-time state-space method with the system frequency shift strategy, which was developed based on the IF to invoke alternate DC current level (0 and 1.8 Ampere) and constant 1.8 Ampere or 'pin-rigid' beam end conditions for the 'smart' MR pin.

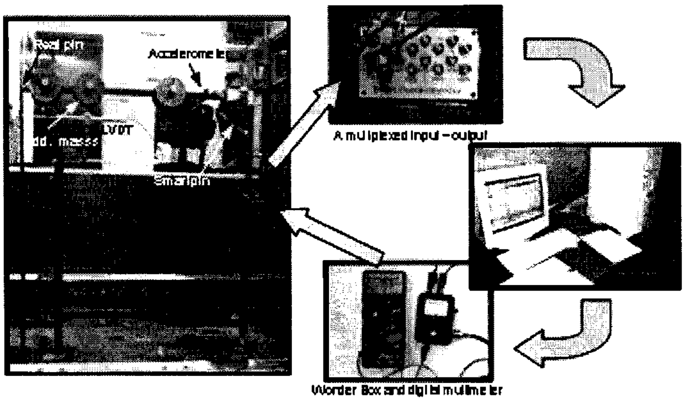
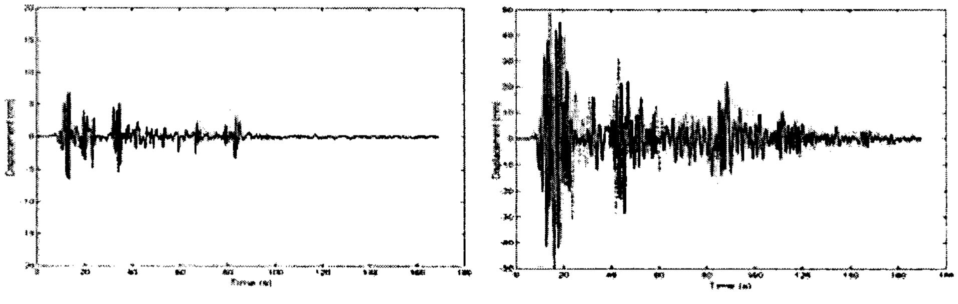


Figure 4 'Smart' MR pin-frame model on the UTS shake table

In order to asses the performance of IF, three DC current levels, namely, 0, 1.8 and 'flip-flop' or alternate 0 and 1.8 Ampere with two additional mass cases (0 and 31.5 kg), are numerically studied as shown in Figures 5 and 6. Both Figures 5 and 6 indicate the same trend, that is, the 'flip-flop' DC current level of either 0 or 1.80 Ampere, based on the instantaneous frequency, is superior to 0 Ampere for no additional mass case and similarly to 1.80 Ampere for 31.5 kg additional mass case.

Table 1 System frequency f_s (Hz), damping ratio (%) and zero-field friction (N) of 'Smart' MR pin-frame model

Δm (kg)	0 A			1.80 A	
	f_s (Hz)	ξ (%)	f_d (N)	f_s (Hz)	ξ (%)
0	0.78	4.78	0.50	1.17	6.33
8.75	0.61	9.12	0.45	0.96	1.43
21.875	0.44	14.28	0.30	0.68	1.43
31.5	0.36	13.71	1.00	0.68	1.43



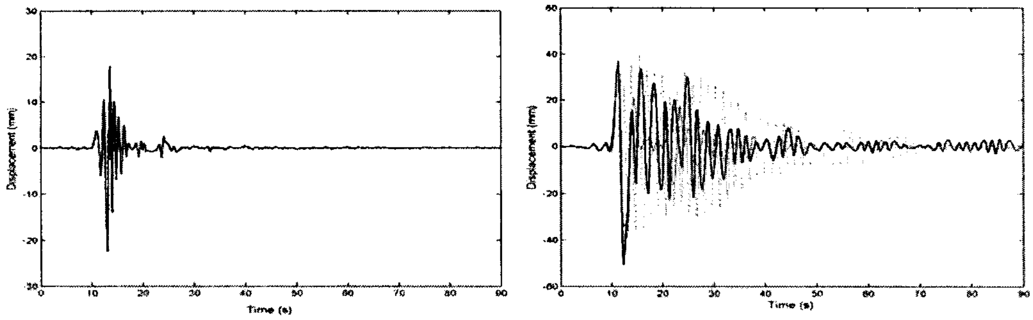
(a) 0 kg additional

(b) 31.5 kg additional

Figure 5 Displacement responses for 0 Ampere (grey); 1.80 Ampere (dashed) and 'flip-flop' (0/1.80) Ampere cases due to El Centro earthquake with intensity factor of 8%.

5. DISCUSSION AND CONCLUSION

The use of IF of known earthquake records for vibration mitigation of structures by employing the classic technique of frequency shift, which can nowadays be implemented in real time applications using electronically controllable fluid devices like 'smart' MR pin is becoming more common in practice. The only shortcoming is our understanding and powerful methods to extract not only the 'real' instantaneous frequency of available earthquake records, but also the capability to predict the frequency of real earthquakes during such hazards. The STFT method has limitation of either time or frequency resolution. Similarly, WVD uses averaged frequency concept. The available methods, which are developed for mono-component signals, require further development for multi-component signals such as earthquakes. Nevertheless, this preliminary study shows the potential to use of instantaneous frequency for real-time frequency shift controller for vibration mitigation of 'smart' MR pin-frame system.



(a) 0 kg additional

(b) 31.5 kg additional

Figure 6 Displacement responses for 0 Ampere (grey); 1.80 Ampere (dashed) and 'flip-flop' (0/1.80) Ampere cases due to Northridge earthquake with intensity factor of 3%.

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