Two-Graph Stamps for Linear Controlled Sources

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Abstract – **The simple process of formulation the state equations for RLCEI-g_m networks without CE-loops and LI-cutsets, makes the method very attractive for symbolic analysis. The only serious drawback of this method is its inability to include directly controlled sources other than the voltage-controlled current sources (VCCS). In this paper we introduce two-graph stamps to overcome this difficulty and allow formulation of the state equations of linear and nonlinear networks with CE-loops and LI-cutsets in the same manner as for linear RLCEI networks.**

1 INTRODUCTION

Recently [1], a simple method has been proposed for determining the symbolic state equations for RLCEI-g_m (R,L,C,E,I-g_m stand for resistors, inductors, capacitors, independent voltage and current sources and voltage-controlled current sources) without CE-loops and LI-cutsets. The simplicity of the method is due to the fact the branch conductance matrix **G** and the branch resistance matrix **R** are diagonal, making it a simple matter to apply a modification of Binet-Cauchy Theorem. For networks containing other controlled sources (voltage-controlled voltage sources -VCVS, current-controlled voltage sources-CCVS, current-controlled current sources-CCCS) the matrices **G** and **R** are no longer diagonal.

This paper describes two-graph stamps which allow formulation of the state equations for linear and nonlinear improper networks in the same way as ordinary linear networks without excess elements.

2 DIAGONAL SIGNAL-FLOW GRAPHS FOR LINEAR CONTROLLED SOURCES

There are four types of controlled sources [2], shown in Fig. 1, whose definition and usual acronyms are given below.

(1) voltage-controlled current source (g_m or VCCS or VC type)
$i = g_m v' , i' = 0$

(2) Voltage-controlled voltage sources (μ or VCVS or VV type)
$v = \mu v' , i' = 0$

(3) Current-controlled voltage source (r_m or CCVS or CV type)
$v = r_m i' , v' = 0$

(4) Current-controlled current sources (β or CCCS or CC type)
$i = \beta i' , v' = 0$

Each definition involves two branches : the primed variables are for the controlling branch, and the unprimed for the controlled branch.

Case 1. RLCEI networks without CE-loops and LI-cutsets.

For RLCEI networks, the state equations may be obtained from the primitive signal-flow graph (Fig. 1) [1].

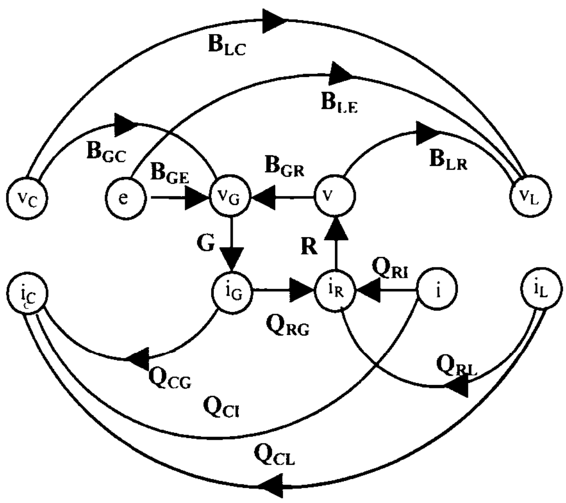

Fig. 1: Primitive signal-flow graph of RLCEI networks without CE-loops and LI-cutsets.

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In this primitive signal-graph $\mathbf{B}_{GC}$, $\mathbf{B}_{GE}$, $\mathbf{B}_{GR}$, $\mathbf{B}_{LC}$, $\mathbf{B}_{LR}$, $\mathbf{B}_{LE}$ are the loop matrices, $\mathbf{Q}_{CG}$, $\mathbf{Q}_{RI}$, $\mathbf{Q}_{RG}$, $\mathbf{Q}_{CL}$, $\mathbf{Q}_{RL}$ $\mathbf{Q}_{CI}$ – the cutest matrices.

For RCLEI networks without CE-loops and LI-cutsets, we have $\mathbf{Q}_{CG} = -\mathbf{B}_{GC}^t$, $\mathbf{Q}_{RG} = -\mathbf{B}_{BR}^t$, $\mathbf{Q}_{LC} = -\mathbf{B}_{LC}^t$, $\mathbf{Q}_{RL} = -\mathbf{B}_{LR}^t$, and **R** and **G** are diagonal matrices.

Case 2. RLCEI-g_m networks without CE-loops and LI-cutsets.

Now we will consider connected, linear, time-invariant networks modeled with RLC elements, independent voltage and current sources and voltage-controlled current sources (g_m) without CE-loops and LI-cutsets.

In this case the matrices **G** and **R** are no longer diagonal. We will see that the matrix **R** and the matrix **G** of networks modeled with controlled sources can also be expressed as diagonal matrices.

Consider first the primitive signal-flow graph for voltage-controlled current source (Fig. 2).

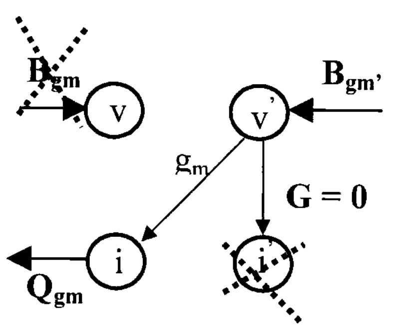


Fig. 2: Primitive signal-flow graph for the voltage-controlled current source

It is easy to convert the original signal-flow graph (Fig. 2) to its diagonal form (Fig. 3).

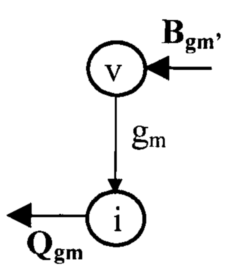


Fig. 3: The diagonal signal-flow graph of the voltage-controlled current source

This diagonal signal-graph can be obtained from the current and voltage graphs shown in Table 1.

Since the voltage-controlled current source has now its diagonal form we can analyze RLCEI-g_m networks without CE-loops and LI-cutset in such way as ordinary RLCEI networks without excess elements.

Case 3. RLCEI-r_m networks without CE-loops and LI-cutsets.

Now we will consider RLCEI networks with current-controlled voltage sources (RLCEI-r_m networks) without excess elements.

In this case the matrices **G** and **R** are also not diagonal (Fig. 4).

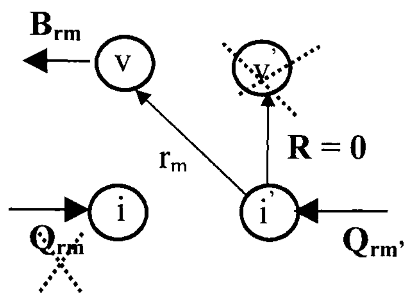


Fig. 4: Primitive signal-flow graph for the current-controlled voltage source

We can also convert this original signal-flow graph (Fig. 4) to its diagonal form (Fig. 5).

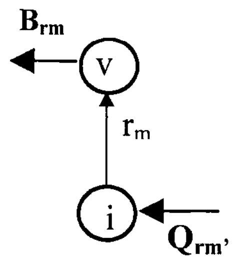


Fig. 5: The diagonal signal-flow graph of the current controlled voltage source

This diagonal signal-flow graph can be obtained from the current and voltage graphs shown in Table 1 and we can analyze such networks as RLCEI-networks without excess elements.

Case 4. RLCEI-μ networks with LI-cutsets and without CE-loops.

We first consider RLCEI networks with voltage-controlled voltage sources (RLCEI-μ networks) without excess elements.

In this case the matrices **G** and **R**, similarly as in cases 1 and 2, are not diagonal (Fig. 6).

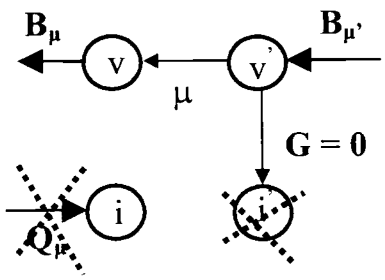


Fig. 6: Primitive signal-flow graph for the voltage-controlled voltage source

In this case it is necessary to increase the complexity of the diagonal signal-flow graph (Fig. 7).

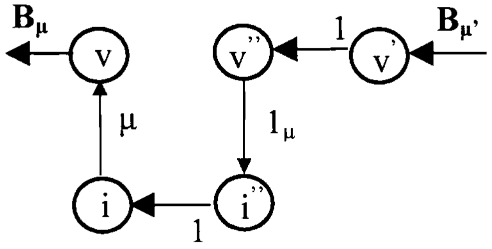


Fig. 7: The diagonal signal-flow graph of the voltage-controlled voltage source

In order to obtain a diagonal pattern of the voltage-controlled voltage source we must insert an additional branch represented by symbol 1_μ, i.e. the branch with the conductance equal to one.

This diagonal signal-flow graph can be obtained from the current and voltage graphs in Table 1.

Because the excess inductors can be represented as a series combination of voltage-controlled voltage sources and independent voltage sources [3] we can now analyze circuits RLCEI-μ with LI-cutset.

Case 5. RLCEI-β networks with CE-loops and LI-cutsets.

Finally, we consider RLCEI networks with current-controlled current sources (RLCEI-β networks) without excess elements.

The matrices **G** and **R** , as in cases 1,2 and 3, are again not diagonal (Fig. 8).

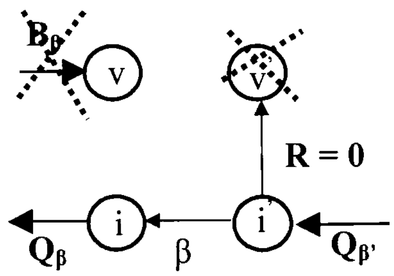


Fig. 8: Primitive signal-flow graph for the current-controlled current source

Also in this case it is necessary to increase the complexity of the diagonal signal-flow graph (Fig. 9).

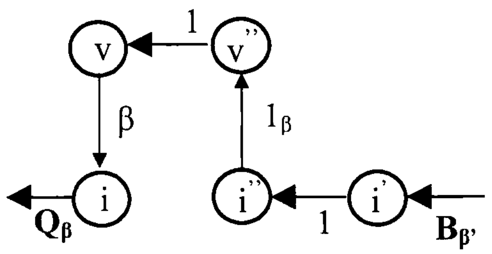


Fig. 9: The diagonal signal-flow graph of the current-controlled current source

In order to obtain a diagonal pattern of the current-controlled current source we must insert an additional branch represented by symbol 1_β, i.e. the branch with the resistance equal to one.

This diagonal signal-flow graph can be obtained from the current and voltage graphs in Table 1.

Because the excess inductors can be represented as a series combination of voltage-controlled voltage sources and independent voltage sources whereas CE-loops as a parallel combination of current-controlled current sources and independent current sources [3] we can now analyze circuits RLCEI-cs with LI-cutsets and CE-loops (where RLCEI-cs stands for resistors, inductors, capacitors, independent voltage and current sources and all types of controlled sources).

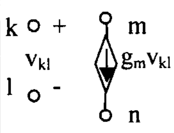
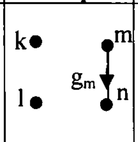
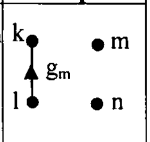
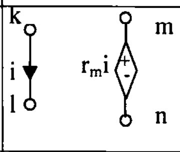
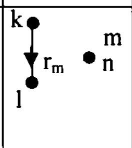
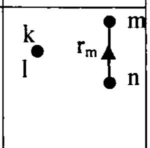
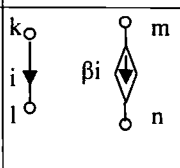
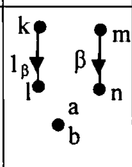
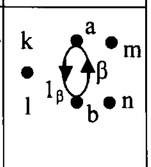
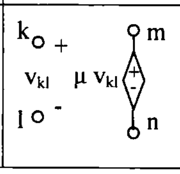
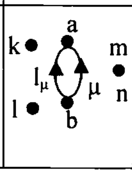
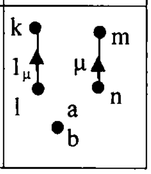
3 TWO-GRAPH STAMPS FOR LINEAR CONTROLLED SOURCES

The corresponding two-graph stamps representing four types of controlled sources may be obtained in the simple way from Fig. 3,5,7 and 9, respectively.

From Fig. 3 it follows that the branch labels g_m must be placed in the current graph between nodes of controlled branch while the nodes of controlling branch must be opened. In the voltage graph this branch must be placed between nodes of controlling branch whilst the nodes of controlled branch must be opened (Table 1).

Similarly, from Fig. 5 we see that branch labels r_m must be placed in the current graph between nodes of controlling branch while the nodes of controlled branch must be shorted. In the voltage graph this branch must be placed between nodes of controlled branch whilst the nodes of controlling branch must be shorted (Table 1).

Table 1: Two-graph stamps for controlled sources

Element	Symbol	Current Graph	Voltage Graph
VVCS			
CCVS			
CCCS			
VCVS			

A more elaborate procedure is needed for the Fig. 7 and 9 but it is very similar to given above – see Table 1.

Using two-graph stamps for linear controlled sources given in Table 1, RLCEI-cs networks can be represented by a linear graph consisting a voltage graph G_V and a current graph G_I and then the matrices **R** and **G** are diagonal.

4 CONCLUSIONS

In the two-graph method of Mayeda and Seshu [4] for a lumped, linear, time-invariant circuit, two graph are constructed: the voltage graph G_V, and the current graph G_I. Both graph have the same number of vertices and edges, but for active circuits (i.e. circuits with controlled sources) have different topology. The only serious drawback of the original method of Mayeda and Seshu is its inability to directly include controlled sources other than the voltage-controlled current sources (designated as g_m or VCCS).

In this paper we introduce the idea of the diagonal signal-flow graphs for four types of controlled sources and the appropriate two-graph stamps for these controlled sources are given. In this case the matrices **G** and **R** are diagonal and it is possible to apply the modification of the Binet-Cauchy theorem to formulation of the state equations.

References

[1] M. Pierzchała, "Symbolic State Equations for Improper Networks", in *Proc. SMACD 2004*, Wrocław, Poland, September, 2004

[2] P.-M. Lin, *Symbolic Network Analysis*. Elsevier. Amsterdam. 1991.

[3] M. Pierzchała, Comments on "Writing State Equations for an Improper Network Without Having to Invert Matrices", *IEEE Trans. Circuits and Systems*, vol. CAS-22, August 1975, pp. 699-702.

[4] W. Mayeda, *Graph Theory*. Willey-Interscience, New York, 1972.