A Chattering-Free Variable Structure Controller for Tracking of Robotic Manipulators

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Abstract

This paper describes the design of a novel sliding mode controller for tracking of robotic manipulators. By using a proportional-integral combination of the sliding function in the boundary layer, control chattering is eliminated without deterioration of system robustness. Theoretical developments are verified by simulation results for the tracking problem of a 2 DOF robotic manipulator.

1 Introduction

Variable Structure Systems (VSS) with Sliding Mode Control (SMC) are characterized by control laws that are discontinuous on a certain manifold in the state space, the so-called sliding surface [Itkis, 1976; Utkin, 1987]. A VSS control law is designed such that the representative point's trajectories of the closed-loop system are attracted to the sliding surface and once on the sliding surface they slide towards the origin. By properly designing the sliding surface, VSS attain robustness regardless of parametric uncertainty and external disturbances [Slotine and Li, 1991]. In that context, considerable effort has been made in design of controllers for tracking of robotic manipulators, and variable structure control approaches have inspired a large amount of research work and surveys published in the field [Slotine and Li, 1991; Slotine and Asada, 1986; Arashima *et al.*, 1986; Hung *et al.*, 1993; Young *et al.*, 1999].

Typically, VSS suffer from the chattering phenomenon because of the discontinuous change of control laws across the sliding surface. In practical engineering systems, chattering may cause damage to system components, as well as excite unmodelled and high frequency plant dynamics [Kwatny and Siu, 1987]. There exist several techniques to eliminate chattering. The widely-adopted approach to chattering-free VSS is the so-called boundary layer one, where the discontinuous VSS control law's signum function is approximated using a saturation nonlinearity within a small vicinity around the sliding surface [Slotine and Li, 1991; Hung *et al.*, 1993]. Unfortunately, boundary layer controllers do not guarantee asymptotic stability but rather uniform ultimate boundedness [Corless and Leitmann, 1981; Esfandiari and Khalil, 1991]. As a consequence, there exists a trade-off between the smoothness of control signals and the control accuracy. Some boundary layer width modification techniques to improve tracking precision are discussed in [Slotine and Li, 1991; Arashima *et al.*, 1986; Yu *et al.*, 1994; Chen *et al.*, 2002]. Nevertheless, these proposed methods in practice lead to a computational burden when implemented, or are applicable only to linear systems.

In this paper, a systematic and feasible design of VSS controllers for robust tracking of robot manipulators is proposed. Section 2 provides a brief summary of VSS design for robot tracking. To alleviate chattering in SMC of nonlinear systems, a proportional-integral combination of the sliding function is proposed in the boundary layer in place of the signum function. This continuous controller can force the system states to reach the sliding surface and attain high tracking performance. The main result is presented in Section 3. A rigid two-link manipulator with planned trajectories is simulated to verify the validity of the proposed method as given in Section 4. Finally, a conclusion is reached in Section 5.

2 VSS controller design for tracking of robot manipulators

2.1 Manipulator Model

The dynamic equations of motion for a robot manipulator consisting of n-rigid links are described in joint-space as follows [Spong and Vidyasagar, 1989]:

$$u = H(q)\ddot{q} + h(q,\dot{q}),$$
$$h(q,\dot{q}) = C(q,\dot{q})\dot{q} + g(q), \tag{1}$$

where $u \in R^n$ is the vector of applied generalized forces, $q \in R^n$ is the vector of joint variables, $g \in R^n$ is the vector of the gravitational forces, $C(q,\dot{q})\dot{q} \in R^n$ is the vector of Coriolis and centrifugal forces, and $H=H^T>0$, $H \in R^{nxn}$ is the manipulator inertia matrix.

The dynamics (1) can be cast in the space representation as follows:

$$\ddot{q} = a(q,\dot{q}) + B(q).u + d(t) \ , \tag{2}$$

where $a(q,\dot{q}) = H^{-1}(q)h(q,\dot{q})$, $B(q) = H^{-1}(q)$, (3)

and $d(t) \in R^n$ is the vector of unknown external disturbances.

Note that representing inertia of the robot manipulator, matrix H and its estimate, $\hat{H}$, are positive definite and invertible.

For the SMC design, the following assumption is made:

Assumption 1

The nonlinear dynamics $B(q)$ and $a(q,\dot{q})$ are not exactly known, but can be estimated as $\hat{B}(q)$ and $\hat{a}(q,\dot{q})$. The estimation errors are bounded by known functions:

$$\left\|\hat{H}^{-1}(q).H(q)\right\| = \left\|\hat{B}(q).B^{-1}(q)\right\| \le \beta(q),$$
$$\left\|\hat{a}(q,\dot{q}) - a(q,\dot{q})\right\| \le \Delta a_{\max}(q,\dot{q}) \quad , \tag{4}$$

and the external disturbance is bounded $\|d(t)\| \le D$.

2.2 SMC law

Let $q_d \in R^n$ be the desired trajectory and $e = q_d - q$ and $\dot{e} = \dot{q}_d - \dot{q}$ be the state tracking errors and their time derivatives.

We firstly define the sliding surfaces as follows:

$$S = Ce + \dot{e} = 0 \quad , \tag{5}$$

where $C = diag(C_1,\dots,C_n)$; $C_i \in R$; $C_i > 0$; $i = 1,\dots,n$

If system states remain on the sliding surfaces chosen, tracking errors will tend to zero asymptotically $\forall t \ge 0$. This is because each tracking error e_i is obtained from the function S_i through a first-order low-pass filter with a time constant $\frac{1}{C_i}$ [Slotine and Li, 1991].

The control law u is then designed such that the system state trajectories driven to the sliding surfaces. The key problem is to select a Lyapunov function of the form $V = 0.5S^T.S \ge 0$, and choose a control law such that:

$$\dot{V} = S^T.\dot{S} < 0 \, ; S \ne 0 \text{, or:} \tag{6a}$$

$$S^T\dot{S} \le -\alpha|S| = -\alpha.S^T.\text{sgn}(S) \, , \tag{6b}$$

where α is a positive scalar.

The inequality (6) is often called the sliding condition.

The control law u is now chosen as follows:

$$u = \hat{B}^{-1}[u_{eq} + K.\text{sgn}(S)] \, , \tag{7}$$

where

$$u_{eq} = C.\dot{e} + \ddot{q}_d - \hat{a}(q,\dot{q}) \, , \tag{8}$$

$$\text{sgn}(S) = [\text{sgn}(S_1), \text{sgn}(S_2), \dots, \text{sgn}(S_n)]^T \, , \tag{9a}$$

$$K = diag(K_1,\dots,K_n); \quad K_i > 0; \quad i = 1,\dots,n \ . \tag{9b}$$

Theorem 1

For system (2), if Assumptions 1 and 2 are satisfied, and the control law is chosen as given in (7), with $K_i \ge |\beta - 1|.\|u_{eq}\| + \beta.(\alpha + D + \Delta a_{\max}) \quad i = 1,\dots,n$, then the tracking errors will asymptotically converge to zero.

Proof

Taking the time derivative of (5) gives

$$\dot{S} = C\dot{e} + \ddot{q}_d - \ddot{q} \, .$$

Using (2), (7) and (8), the above equation becomes

$$\begin{aligned} \dot{S} &= C\dot{e} + \ddot{q}_d - a - d - B\hat{B}^{-1}[u_{eq} + K.\text{sgn}(S)] \\ &= (I - B.\hat{B}^{-1})u_{eq} + \hat{a} - a - d - B.\hat{B}^{-1}.K.\text{sgn}(S) \end{aligned} \tag{10}$$

where $I \in R^{nxn}$ is an identity matrix.

Using (10), the sliding condition (6b) becomes

$$\begin{aligned} & S^T\{(I - B.\hat{B}^{-1})u_{eq} + \hat{a} - a - d - B.\hat{B}^{-1}.K.\text{sgn}(S) + \alpha.\text{sgn}(S)\} \le 0 \\ \Leftrightarrow \ & S^T B.\hat{B}^{-1}\{(\hat{B}B^{-1} - I)u_{eq} + \hat{B}B^{-1}(\hat{a} - a) - \hat{B}B^{-1}d - K.\text{sgn}(S) \\ & \qquad + \hat{B}B^{-1}\alpha.\text{sgn}(S)\} \le 0 \end{aligned} \tag{11}$$

Inequality (11) can be brought into:

$$\begin{aligned} & S_i.\{[(\hat{B}.B^{-1} - I)u_{eq} + \hat{B}.B^{-1}.(\hat{a} - a - d + \alpha\,\text{sgn}(S))]_i \\ & \quad - K_i.\text{sgn}(S_i)\} \le 0, \qquad i = 1,\dots,n \end{aligned} \tag{12}$$

where $[V]_i$ denotes the i^{th} component of a vector V.

In view of Assumption 1, condition (12) can be expressed as:

$$K_i \geq |\beta - 1|.\|u_{eq}\| + \beta.(\alpha + D + \Delta a_{\max}),\; i = 1,...,n \quad (13)$$

Therefore, the sliding condition (6b) is satisfied with K_i (i=1,..,n) chosen as in (13), and the system state trajectories are driven onto the sliding surfaces in a finite time. When the system states remain on the sliding surfaces, tracking errors tend asymptotically to zero.

3 Proposed SMC

3.1 Continuous control laws to approximate SMC

Define thin boundary layers neighboring the sliding surfaces:

$$B_i(t) = \{q_i, |S_i(q,t)| \leq \Phi_i\} \quad ,\Phi_i > 0\,; i = 1,...,n\,, \tag{14}$$

where Φ_i is the boundary layer thickness, as illustrated in Figure 3.1 for i=1 in the error phase space.

To remedy the control discontinuity in the boundary layer, the signum function *sgn(S)* in (7) is replaced by a saturation function of the form [Slotine and Sastry, 1983]:

$$sat(S_i) = \begin{cases} \mathrm{sgn}(S_i), & |S_i| \geq \Phi_i \\ \dfrac{S_i}{\Phi_i}, & |S_i| < \Phi_i \end{cases} \quad i = 1,...,n\,, \tag{15}$$

However, the system state applied this control law is uniform ultimate bounded with respect to a small neighborhood of the origin [Gao and Liu, 1995].

Let $\varepsilon_i = C_i^{-1}\Phi_i, i = 1,...,n$ be the boundary layer width. As shown in [Slotine and Li, 1991], the tracking errors exist within a guaranteed precision ε_i. Therefore, the larger the boundary layers the smaller the control chattering and the greater the tracking errors.

3.2 Proposed PI sliding control law

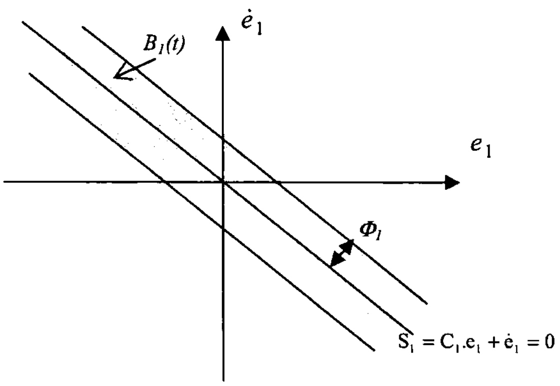

Figure 3.1 The sliding surface and the boundary layer

Let $\sigma_i = \dfrac{S_i}{\Phi_i}; i = 1,...,n$, we first introduce the saturated proportional-integral functions:

$$\rho_i(\sigma_i) = \begin{cases} 1 & if \quad \sigma_i > 1 \\ \sigma_i + K_{I_i} \displaystyle\int_{t_{io}}^{t_i} \sigma_i dt & if \quad -1 \leq \sigma_i \leq 1\,, \\ -1 & if \quad \sigma_i < -1 \end{cases} \tag{16}$$

$i = 1,...,n$

where $K_{I_i} > 0$ is an integral gain, and t_{i0} is the initial time when the system states enter the boundary layer $B_i(t)$ in (14). If $|\sigma_i| \geq 1$, the integration term in (16) will be reset to zero to prepare for the system state entering boundary layer.

The control law u is then chosen as of the form:

$$u = \hat{B}^{-1}[u_{eq} + K.\rho(\sigma)], \tag{17}$$

where

$$\rho(\sigma) = [\rho_1(\sigma_1), \rho_2(\sigma_2), ..., \rho_n(\sigma_n)]^T, \tag{18}$$

and u_{eq}, K are chosen as (8), (9) and (13).

Assumption 2

It is assumed that the chosen integration gains K_{I_i} (i=1,...,n) are sufficiently large such that:

$$\begin{aligned} \dot{\sigma}_i + K_{Ii}\sigma_i > 0 \;\; \text{for all}\, \sigma_i > 0 \\ \dot{\sigma}_i + K_{Ii}\sigma_i < 0 \;\; \text{for all}\, \sigma_i < 0 \end{aligned} \tag{19}$$

Inequalities (19) imply that ρ_i increases for all $\sigma_i > 0$, and ρ_i decreases for all $\sigma_i < 0$ [Salas and Hill, 1990].

Proposition

For system (2), if Assumptions 1 and 2 are satisfied, and the control law is given in (17), with K chosen using (9) and (13), and $\rho(\sigma)$ selected as in (16) and (18), then the tracking errors will tend to zero.

Clearly, outside $B_i(t)$ (i=1, ..., n), we choose control law as (7) (because $\rho_i(\sigma_i) = \mathrm{sgn}(S_i)$), which guarantees that the boundary layers are attractive, and hence all trajectories starting outside of $B_i(t=0)$ are forced to reach $B_i(t)$.

Inside $B_i(t)$ (i=1, ..., n), as from Assumption 2, after a finite time we have with the value K_{I_i} chosen,

$$\begin{aligned} \sigma_i + K_{Ii} \int_{i0}^{t} \sigma_i dt \geq 1 \quad for \quad S_i > 0 \\ \sigma_i + K_{Ii} \int_{i0}^{t} \sigma_i dt \leq -1 \quad for \quad S_i < 0 \end{aligned} \tag{20}$$

Thus,

- For $S_i > 0$:

From (20) and with K_i is chosen in (13), one can obtain:

$$K_i \rho_i(\sigma_i) \geq \left| \left(\hat{B}.B^{-1}\text{-}I\right) u_{eq} + \hat{B}.B^{-1}.(\hat{a} - a - d + \alpha.\text{sgn}(S)) \right|_i \tag{21}$$

- For $S_i < 0$:

Similarly, one can also have:

$$K_i \rho_i(\sigma_i) \leq \left| \left(\hat{B}.B^{-1}\text{-}I\right) u_{eq} + \hat{B}.B^{-1}.(\hat{a} - a - d + \alpha.\text{sgn}(S)) \right|_i \tag{22}$$

Inequalities (21) and (22) imply that sliding condition (12) is satisfied, and tracking errors tend to zero as seen in the proof of Theorem 1.

4 Simulation of a two-link manipulator

Consider a two-link planar arm with two revolute rigid links as shown in Figure 4.1. The control system is designed such that the manipulator's end-effector moves unknown loads along the desired trajectories $q(t) \equiv q_d(t)$ from an initial position (x_0, y_0) to a final position (x_C, y_C) within a finite time t_f. The manipulator payload is varying subject to $m_t = 0 \div m_{t\max}; J_t = 0 \div J_{t\max}$.
The dynamics of the manipulator can be obtained as [Spong and Vidyasagar, 1989]:

$$\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \frac{1}{D_H}\left\{ -\begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}\left(\begin{bmatrix} T.\dot{\theta}_2^2 + 2T.\dot{\theta}_1\dot{\theta}_2 \\ -T.\dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}\right) + \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right\},$$

where

$$\begin{aligned}
h_{11} &= J_1 + m_1 l_{g1}^2 + m_2\left(l_1^2 + l_{g2}^2 + 2 l_1 l_{g2}\cos\theta_2\right) \\
&\quad + J_2 + J_t + m_t\left(l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta_2\right) \\
h_{12} &= h_{21} = m_2\left(l_{g2}^2 + l_1 l_{g2}\cos\theta_2\right) + J_2 + J_t + m_t l_2^2 \\
h_{22} &= m_2 l_{g2}^2 + J_2 + J_t + m_t l_2^2 \\
T &= -\left(m_2 l_1 l_{g2} + m_t l_1 l_2\right)\sin\theta_2 \\
g_1 &= m_1 g.l_{g1}\cos\theta_1 + m_2 g\left(l_1\cos\theta_1 + l_{g2}\cos(\theta_1 + \theta_2)\right) \\
&\quad + m_t g\left(l_1\cos\theta_1 + l_2\cos(\theta_1 + \theta_2)\right) \\
g_2 &= m_2 g.l_{g2}\cos(\theta_1 + \theta_2) + m_t g.l_2\cos(\theta_1 + \theta_2) \\
D_H &= h_{11}.h_{22} - h_{12}^2 ,
\end{aligned}$$

with the following data: m_1=0,3936kg, m_2=0,0946kg, l_1=0,203m, l_2=0,1524m, l_{g1}=0,1554m, l_{g2}=0,0818m, J_1=0,00114kgm^2, J_2 = 0,0003kgm^2, m_{tmax} = 0,294kg, J_{tmax} = 0,001kgm^2, x_0 = 0m, y_0 = 0,3m, x_C = 0,3554m, y_C = 0m, and t_f = 3s.

The nonlinear dynamics estimates $\hat{B}(q)$ and $\hat{a}(q,\dot{q})$ are determined with respect to the case that m_t=0, J_t=0.

The bounds in Assumption 1 can be calculated when $m_t = m_{tmax}$, $J_t = J_{tmax}$ and disturbances $d_1(t) = d_2(t)$=0.5 sin(20t).

The entire system is simulated with an arbitrary payload m_t = 0.147kg, J_t = 0.0005kgm^2.

Fig. 4.2 shows the desired time histories of the position of the two links with a cubic polynomial time law.

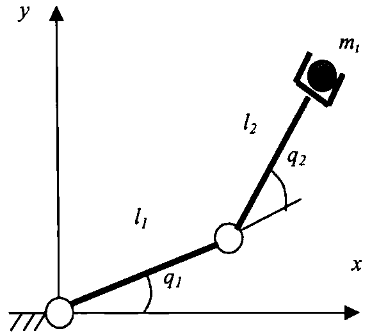

Figure 4.1 : Two-link planar arm

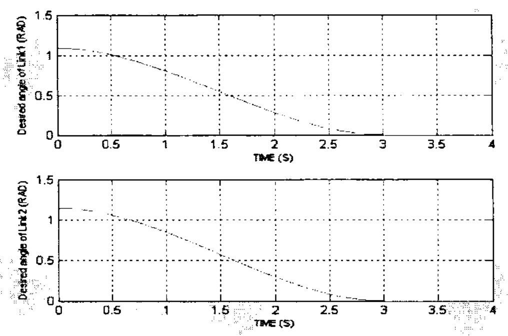

Figure 4.2. The desired time histories of position of two links

Fig. 4.3 shows the tracking errors and control inputs of the two links when the control system uses the conventional VSS controller presented in Section 2. Although the tracking errors tend to zero, the control inputs suffer chattering when the state trajectories are on the sliding surfaces.

In this VSS controller, if the signum functions are replaced by saturation functions as in the well-adopted boundary layer method, system tracking errors and control inputs are displayed in Figure 4.4.

It can be observed from Fig. 4.4 that the control inputs are now smooth but the tracking errors increase.

As shown in Figure 4.5 when the signum functions are replaced by the proposed saturated proportional-integral functions (16), with $K_{I1} = K_{I2} = 75$, perfect tracking performance can still be obtained while chattering of the control input is eliminated completely.

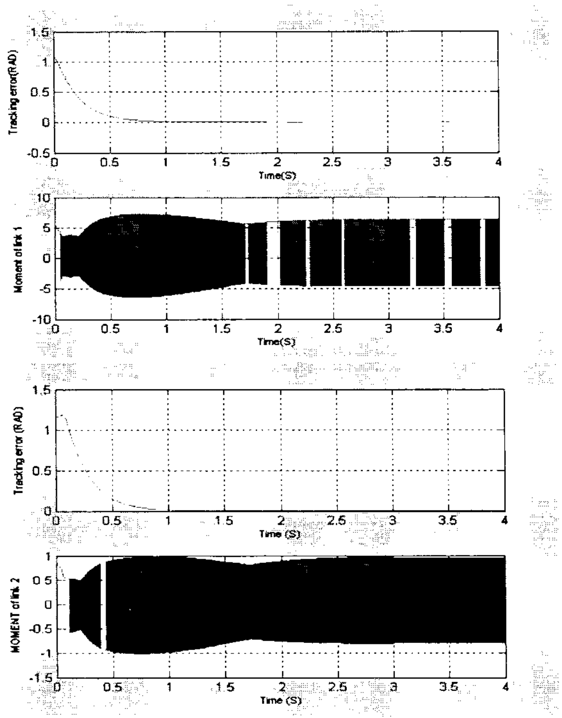

Fig 4.3: The tracking errors and control inputs of the two links under SMC with parameters $\alpha = 0.1$, and $C_1 = C_2 = 5$. Note the errors as e_1=0.00003rad and e_2=0.00048rad at t=2s

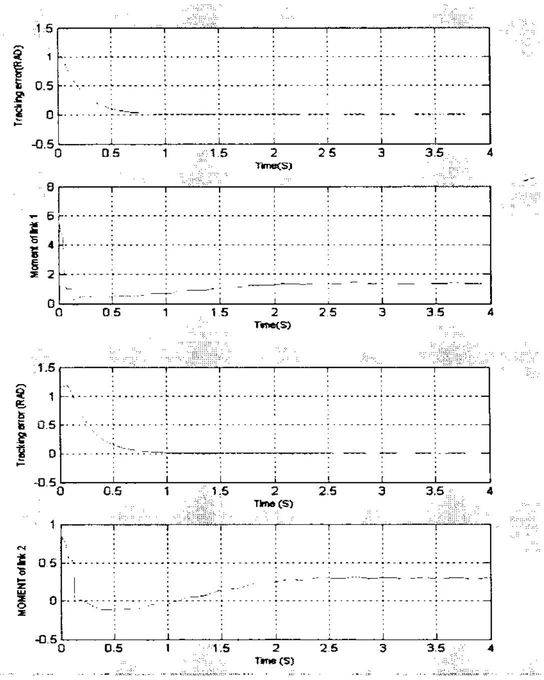

Fig 4.5: The tracking errors and control inputs of the two links when using SMC with the proposed saturated proportional-integral functions in the boundary layers and $\alpha = 0.1$; $C_1 = C_2 = 5$, $\Phi_1 = \Phi_2 = 0.5$, and $K_{I1} = K_{I2} = 75$. Note the errors as e_1=0.000098rad and e_2=0.00029rad at t=2s

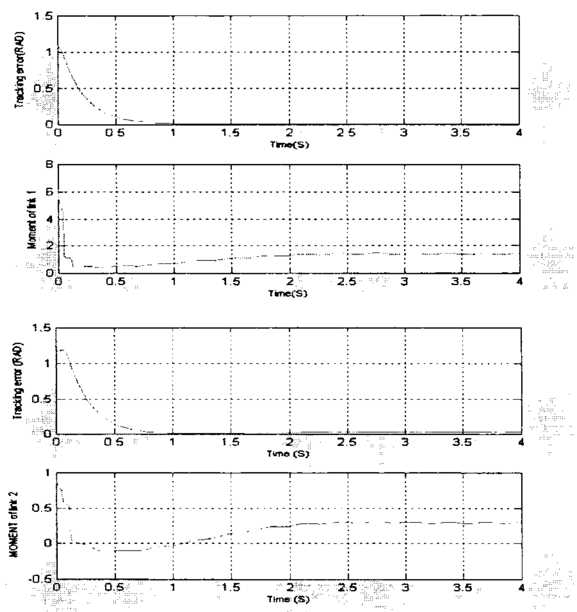

Fig 4.4: The tracing errors and control inputs of 2 links when using conventional SMC with the saturated functions in the boundary layers and $\alpha = 0.1$, $C_1 = C_2 = 5$, and $\Phi_1 = \Phi_2 = 0.5$. Note the errors as e_1=0.00833rad and e_2=0.0222rad at t=2.2s

5 Conclusion

In this paper, a systematic and feasible technique is proposed for the design of VSS controllers for robust tracking of robot manipulators. An improvement of the boundary layer method is achieved by using a proportional-integral combination of the sliding functions. The simulation results show that when the payload of the robot manipulator changes and its disturbances vary within a defined range, the propose control system can maintain its stability and attenuate well the tracking errors while eliminate completely the chattering of the control forces.

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7 References

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