Influence of Temperature Boundary Conditions in the Wire-Coating Process

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Abstract
Influence of temperature boundary conditions on the flow field inside the die of the wire-coating process is investigated numerically, using viscous fluids of a Newtonian type for coating material and a finite element method. It is found that changing the die or wire temperature can have significant effects on the flow velocity, and hence flow rate. On the other hand, the heat transfer coefficient between the coat's free surface and its surroundings has only negligible influence. Also, flow rate variation correlates well with induced pressure gradients.

Introduction
This work is concerned with numerical study of the influence of temperature boundary conditions in the wire-coating process, specifically on the flow field inside the die, when Newtonian fluids are dragged out of it by a wire moving axially under steady conditions.

Wire-coating is a process whereby wires, rods, etc. are provided with layers of coating materials. It is closely related to the extrusion process, and is thus usually discussed under this heading in books dealing with, for example, materials processing [2,6]. It has also been treated rather extensively in literature [3,4,6-9,11-17,19-23]. Due to the difficulty associated with experimental work [7], this process is often investigated via numerical or theoretical methods [7,8,19,etc]. However, in numerical and analytical treatments, either the flow rate or a velocity profile at inlet to the considered flow domain is prescribed, in addition to a prescribed wire velocity [4,7,13-17,20,22,23]. This is rather restrictive, and with regards to velocity profile in non-isothermal flow situation, even unrealistic, since such profile is commonly not known beforehand.

In an effort to provide further understanding of the wire-coating process, a main aim of this work is thus to examine the effects of temperature change on the flow velocity of the process, and hence its flow rate. The imposition of flow rate or velocity profile at inlet to the flow domain mentioned above will be relaxed.

Mathematical Model and Numerical Method
The die-and-wire geometry considered is shown diagrammatically in figure 1. This thus corresponds to the region around the die exit in arrangements shown in, for example, [2].

The mathematical model used is that for a steady, axi-symmetric, non-isothermal flow of incompressible, Newtonian fluids without body forces. The governing equations are those of conservation of mass and momentum, and balance of energy [5]. Using die radius, wire speed and the difference between die wall temperature at locations far upstream of die exit and ambient temperature for scaling factors, a non-dimensionalisation scheme is also applied; the non-dimensional parameters and variables are defined such that the form of the governing equations is unchanged [10].

The coupling between the flow and temperature fields is via the fluid's temperature-dependent viscosity \( \mu \), which is assumed to decrease exponentially with temperature \( T \) according to the formula \( \mu = \mu_0 e^{-\alpha T} \), where \( \alpha \) is a non-dimensional exponential coefficient and \( \mu_0 \) a constant, here set equal to 1. Other fluid properties are assumed to be constant, and the following non-dimensional values are used: density \( \rho = 1.67 \times 10^3 \); thermal conductivity \( k = 0.190 \); specific heat capacity \( c = 7.19 \times 10^2 \). It was shown [10] that when appropriate values are taken for the physical parameters like temperatures, mean flow velocity, tube radius, etc., then the above non-dimensional properties correspond approximately to those of low-density polyethylene under zero shear at about 150°C - 190°C temperature range.

### Boundary Conditions

**Referring to figure 1 and using standard notations, the following non-dimensional boundary conditions are used:**

\[(a) \quad \text{At entrance to the flow domain (at } z = 3\text{): zero radial velocity component and uniform, zero temperature; } u = 0, T = 0 \]

As for axial velocity component, only values on the die wall and wire surface are prescribed (in the following, \( R \) stands for radius).

\[w = 0 \text{ at } r = R_{\text{die}} \]

\[w = w_{\text{wire}} \text{ at } r = R_{\text{wire}}\]

Note that no profile of axial velocity component is assumed, and thus flow rate is not fixed.

\[(b) \quad \text{The die wall is divided into two sections.} \]

\[(i) \quad \text{Section 1, between entrance to the flow domain and a location } z = z_i < 0 \text{ upstream of the die exit: non-slip condition and zero temperature,} \]

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(ii) Section 2, between the location \( z = z_f \) and die exit \( (z = z_{exit} = 0) \), shown as \( \square \) in figure 1: non-slip condition and some imposed die wall temperatures,
\[
\begin{align*}
\boldsymbol{u} = \boldsymbol{w} = 0, \quad T = 0
\end{align*}
\]
In this work, \( z_f \) is taken to be \(-0.6\).

(c) At the “far-downstream” section or exit of the flow domain \( (z = z_{exit}) \) which is sufficiently large so that a uniform axial flow has occurred; here value of \( z_{exit} = 4.4 \) has been used: zero radial velocity and axial stress; \( u = w = 0\). \( t_m = 0\)

No thermal boundary condition is imposed on this end, thus solution attempts to make \( \partial T \partial z = 0 \) here. This is acceptable [18].

(d) Along the wire surface \( (r = R_{wire} = \gamma R_{die} : \gamma < 1) \): zero radial velocity and axial velocity equal to wire speed which is fixed at I, as well as some imposed wire temperatures;
\[
\begin{align*}
\boldsymbol{u} = 0, \quad w = w_{wire} = 1, \quad T = T_{wire}
\end{align*}
\]

(e) On the free surfaces \( (z > 0) \): zero stresses;
\[
\begin{align*}
t_m = t_{na} = 0
\end{align*}
\]

where \( n \) is the (non-dimensional) outward-pointing co-ordinate normal to the surfaces, and \( s \) is the co-ordinate along them. Convective cooling condition is also imposed on the coat free surface;
\[
\begin{align*}
\partial T \partial n = f_{free} (-0.72 T - 28.8)
\end{align*}
\]

where \( f_{free} \) is a simple multiplying factor. Here the heat transfer coefficient \( h \) is related to \( f_{free} \) according to the relation \( h / k = 0.72 f_{free} \). The value of 0.72 has been obtained from Acierno et al. [11] and used in [10,18].

A finite element scheme based on the Galerkin discretisation procedure is used [18]. Grid patterns of 13 quadrilateral elements in the radial direction by 98 elements in the axial direction \((14 \times 99 \text{ grid points})\) are used. Grid convergence tests have been carried out to ascertain the adequacy of these patterns. Also numerical convergence has been ensured to be excellent. Computation is done on a Sun Enterprise 3000 machine, running a UNIX operating system. Double precision \((64 \text{ bits})\) is used throughout.

Results and Discussion

Two radius ratios \( \gamma - R_{wire}/R_{die} \) of 0.7 and 0.9 are considered under a combination of different \( T_{die}, T_{wire} \) and \( f_{free} \) values. The combination results in a total of 18 “series” as listed in table 1. The series are also grouped into 2 groups according to the \( \gamma \) value. Within each series, the fluid viscosity’s exponential coefficient \( \alpha \) is varied, allowing the Nahme-Griffith number \( Na \) to change as a primary changing parameter. The Nahme-Griffith number is defined as
\[
Na = \alpha w_{wire}^2 \mu_{na}/k
\]

where \( w_{wire} \) is the wire velocity. For the cases considered here, \( Na \) is related to \( \alpha \) by the relation

\[
Na = 5.263 \alpha
\]

The Pécelt number, defined as
\[
Pe = \rho c w_{wire}^2 (R_{die} - R_{wire}) / k
\]

is, however, constant within each group. It should be pointed out that \( Pe \) represents the ratio between convective and conductive heat transfers. On the other hand, since \( \alpha - 1 \) can be considered as a characteristic temperature change, \( Na \) provides a measure of the relative change in viscosity due to heat generation and thus determines the amount of coupling between the flow field and temperature field. Below, occasionally, we identify an individual case of a series by the series letter followed by the \( Na \) value; thus for example, 7bl.58 designates a case of series 7b with \( Na = 1.58 \) \((\alpha = 0.30)\).

The local Reynolds number based on a local length scale \( L \) and defined as \( Re = \rho w_{wire} L / \mu \) is always very small. Using the annular gap size for \( L \) and the minimum value in the annular gap for \( \mu \), the maximum \( Re \) attained is about \( 8 \times 10^{-3} \).

<table>
<thead>
<tr>
<th>Group</th>
<th>Wire / Die</th>
<th>Radius Ratio ( \gamma )</th>
<th>Case series</th>
<th>( T_{wire} / T_{die} )</th>
<th>( f_{free} )</th>
<th>Pécelt Number ( Pe )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.7 / 1</td>
<td>(Annular gap size 0.3)</td>
<td>7a</td>
<td>0 / 0</td>
<td>1</td>
<td>18.96</td>
</tr>
<tr>
<td></td>
<td>7b</td>
<td>0 / - 8</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7c</td>
<td>0 / + 8</td>
<td>1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>7d</td>
<td>0 / - 16</td>
<td>1</td>
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<td></td>
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<td></td>
<td>7e</td>
<td>0 / + 16</td>
<td>1</td>
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<td></td>
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<tr>
<td></td>
<td>7f</td>
<td>-8 / 0</td>
<td>1</td>
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<tr>
<td></td>
<td>7g</td>
<td>+8 / 0</td>
<td>1</td>
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<tr>
<td></td>
<td>7h</td>
<td>0 / 0</td>
<td>0</td>
<td></td>
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<tr>
<td></td>
<td>7i</td>
<td>0 / 0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>N</td>
<td>0.9 / 1</td>
<td>(Annular gap size 0.1)</td>
<td>9a</td>
<td>0 / 0</td>
<td>1</td>
<td>6.32</td>
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<tr>
<td></td>
<td>9b</td>
<td>0 / - 8</td>
<td>1</td>
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<td></td>
<td>9c</td>
<td>0 / + 8</td>
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<td>9d</td>
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<td>9i</td>
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Table 1. Cases considered.

Velocity Change

In an isothermal, pure annular drag flow which corresponds to when the wire moves axially and steadily with zero pressure gradient (axial annular Couette flow), velocity distribution in the annulus is given by [12]

\[
w_{drag} = w_{wire} \ln(r/R_{die}) / \ln \gamma
\]

In this work, the wire motion induces significantly pressure gradients which in turn change velocity profiles substantially from the pure drag distribution above. Figure 2 shows axial and radial velocity profiles at die exit from some representative cases for comparison with \( w_{drag} \). It has been noted that profiles of the
axial velocity \( w \) remain fairly well unchanged along the length of the die, so that the profiles shown in figure 2 are also similar to those inside the die. Since an integration of \( w(2\pi r)dr \) gives flow rate, a comparison of \( w \) profiles is also a comparison of flow rate, an important parameter in wire-coating.

Also, from the behaviour of \( w \)-curves in figure 2 (they do not cross each other in any significant way), a single representative value from these curves can be used for comparison of flow rates. Thus figure 3 shows the \( w \)-values at die exit's mid-annulus point \( r = 0.85 \) for group \( S \), 0.95 for group \( N \). The corresponding values for pure drag flow are \( w_{\text{drag}} = 1 \ln(0.85)/\ln(0.7) = 0.4556 \) for group \( S \), and \( w_{\text{drag}} = 1 \ln(0.95)/\ln(0.9) = 0.4868 \) for group \( N \). It can thus be seen that a hot wire (series 7q and 9q) or a cold die (series 7b, 7d, 9b and 9d) can suppress flow rate to well below that of pure drag at sufficiently high \( \text{Na} \). On the other hand, a cold wire (series 7p and 9p) increases flow rate. Furthermore, even though numerical instability has prevented a more visible presentation in figure 3 of results from series 7c, 7e, 9c and 9e, which correspond to hot die walls, it has nevertheless been seen from computer print-out that hot die walls also enhance flow rate: and this effect becomes more pronounced with higher \( \text{Na} \). On the other hand, series 7a and 9a, while giving higher flow rate than pure drag, do not vary significantly with \( \text{Na} \). This is also the case with series 7f, 7z, 9f and 9z, which correspond to varying heat transfer coefficient on the coat's free surface.

The enhancement or suppression of flow rate relative to that of pure drag flow correspond well with induced pressure gradients. An example is the case \( 7d0.974 \) \( (\alpha = 0.185) \). In this case with a cold die wall, an adverse pressure gradient \( \Delta P \Delta z \) is induced at inlet to the flow domain and the flow rate is reduced accordingly. The calculated result for this case is \( \Delta P \Delta z = 11.15 \), which gives the corresponding annular Poiseuille (pressure driven) flow rate of \( Q_{\text{Poi}} = 0.1343 \) [12]; the pure drag (Couette) flow rate calculated from \( Q_{\text{drag}} = 1 \ln(0.95)/\ln(0.9) = 0.4868 \) gives an algebraically combined flow rate \( Q_{\text{sum}} = Q_{\text{Poi}} + Q_{\text{drag}} = 0.5732 \). On the other hand, from the \( w \)-profile presented in figure 2, actual flow rate can be calculated to be 0.5710. The agreement is thus good.

Variation of the radial velocity \( u \) with respect to temperature boundary conditions and \( \text{Na} \) is shown in figures 2 and 4. They show that relatively large changes in \( u \) can occur. However, they also show that die temperature has a much stronger influence than wire temperature in affecting \( u \). Also, from figures 3 and 4 it can be seen that at comparable \( \text{Na} \) values, velocity changes from corresponding series of groups \( S \) and \( N \) are similar. This indicates that the influence of Peclet number is small for the conditions of the present work.

Finally figure 5 shows pressure contours of an example case, case \( 7d0.974 \). The figure shows typical pressure gradients as they are induced in the annulus, while the coat region is largely stress-free. Note that in this case the pressure inside the annulus first increases downstream, reaches a maximum, then decreases.

**Conclusions**

Influence of temperature boundary conditions on flow velocity and hence flow rate in the wire-coating process has been considered. Cooling the die walls reduces significantly flow rate; on the other hand, cooling the wire enhances it. These effects become stronger as \( \text{Na} \) increases. There is also evidence that heating the die walls or the wire produces opposite effects to
cooling them. The heat transfer coefficient at the coat's free surface, however, has very little influence on flow rate. Changes in the radial velocity at die exit can also be substantial. Induced pressure gradients, which are typically not uniform, correlate well with flow rate variation.

References

Figure 4. Representative radial velocity \( u \) versus \( Na \) for all series of group S (a), and group V (b).

Figure 5. Pressure contours of an example case after convergence has occurred. The case is 780 974.
The referees' page

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