

AN APPLICATION OF A GENERALISED JAKES MODEL FOR MIMO CHANNELS

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ABSTRACT

Jakes model for frequency flat fading processes in mobile radio systems is extended to allow for better modelling of a space-time Rayleigh fading multiple-input multiple-output (MIMO) channel. A conventional four transmit and two receive (4,2) MIMO radio channel is analysed, and a ring of scatterers model is used to find the fading channel distortions. A base station antenna spacing for appropriate partially correlated fading is assumed. Differential space-time modulation is used over the channel. A general trend is presented showing optimality of a range of frame lengths over the MIMO channel for rapid decoding in this differential space-time application. The importance of restricting the frame lengths within certain limits is also demonstrated.

INTRODUCTION

Diversity is one effective means for providing performance improvements over fading channels, principally by mitigating the fading that occurs. The combination of temporal and spatial diversity can significantly improve the communication quality in a rich scattering environment. This combination has been demonstrated through proposals for space-time coded modulations. The majority of these proposals have relied on accurate channel state information for decoding.

When assuming space-time modulation it is often assumed that spatial channels are uncorrelated when considering time-varying fading; a condition which is hard to satisfy. Based on this a space-time model for a two transmit and one receive (2,1) multiple-input single-output (MISO) channel is further generalised to a typical (4,2) MIMO radio channel in a partially correlated fading environment [1]. The model used is a space-time generalisation of Jakes model [2].

For this application space-time modulation is used based on recent proposals for differential space-time modulation schemes which require no channel-state information. Some of these use linear dispersive diagonal codes that can be easily generated [3]. It has recently been shown how to rapidly decode these differential modulation schemes over flat fading channels [4]. This paper demonstrates optimal transmission frame lengths for newly developed rapid approximate differential decoding over a Rayleigh fast-fading MIMO channel. The optimal transmission frame lengths are demonstrated relating to non-optimised block error rates for flat fading channels, using the generalised Jakes model developed.

CHANNEL MODEL

A typical scenario for Rayleigh fading MIMO channels is as follows; two, $N_R = 2$, co-located mobile station (MS) antennas and four, $M = 4$, base station (BS) antennas modelled as Fig. 1, where a scatterer ring is placed around the MS to model the multipath reflectors. The scatterers are uniformly distributed on the ring, and each scatterer has an independent, uniformly distributed initial phase over $[-\pi, \pi)$. The model is meant to represent an average channel for the purpose of macroscopic system design, and not to describe individual channel realisations. The antennas in the model are assumed to be omnidirectional. The model is also not designed to apply to a fixed wireless system where the Doppler spread is mainly due to the motion of the scatterers. It is possible, however, that this model could be extended to the case of the fixed system.

For the purposes of this analysis the co-located mobile antennas are spatially decorrelated. The general expression for the flat fading channel distortions from each of the BS transmit antennas, $m = 1 \dots 4$, to either of the MS receive antennas, $n_r = 1, 2$, without normalisation allowing for the number of scatterers (i.e. multipath reflectors) is given by:-

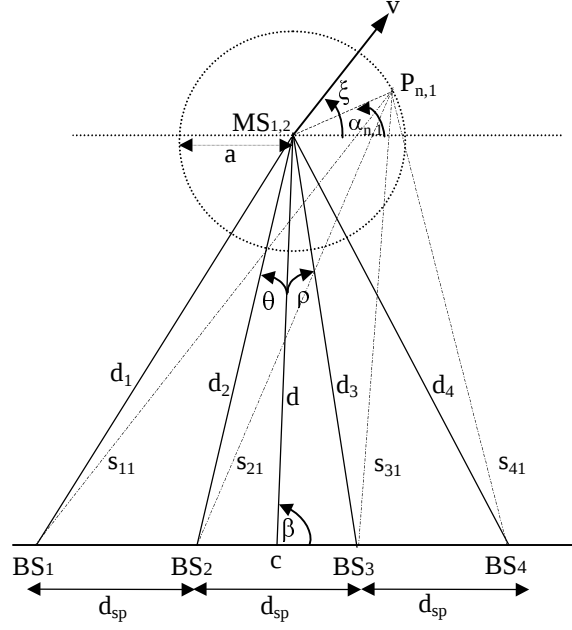


Fig.1. The MIMO transmission model between mobile and the base station

$$cs_{m,n_r}(t) = \sum_{n=1}^N \exp\{j[2\pi f_D t \cos(\xi - \alpha_{n,n_r}) + \phi_{n,n_r} - \Delta\phi_{n,m,n_r}]\} \quad (1)$$

where, in reference to Fig. 1, N is the number of reflectors, f_D is the Doppler spread caused by vehicle movement, α_{n,n_r} is the angle of the n th reflector on the scatterer ring related to the n_r th MS antenna; $\alpha_{n,1} = 2\pi n/N$, $\alpha_{n,2} = 2\pi(n+0.5)/N$, ξ is the angle of mobile motion, ϕ_{n,n_r} is the initial phase of the n th scatterer related to either MS antenna received at the second BS antenna, and $\Delta\phi_{n,m,n_r}$ is the phase difference caused by the path length difference from the n th reflector to the m th BS antenna with respect to the second BS antenna. The $\Delta\phi_{n,m,n_r}$ are deterministic and can be evaluated by $\Delta\phi_{n,m,n_r} = 2\pi(s_{2,nr} - s_{m,nr})/\lambda$, these are defined in Appendix 1. Both phases $\phi_{n,1}$ and $\phi_{n,2}$ will be modelled as uniformly distributed on $[-\pi, \pi)$, but are decorrelated due to offset.

Due to approximations in the (4,2) MIMO model used, and the generality of the macroscopic system model, in order to model each of the fading channel distortions the variance of the expression given in (1) for each of the fading channel distortions is not exactly N . Obtaining the variances for approximate spatial decorrelation, at the appropriate BS antenna spacing, d_{sp} , for each of the channel distortions enables better channel modelling. This gives an approximate $CN(0,1)$ model of the channel by normalisation by these variances when the fading coefficients between the antennas are spatially independent. This variance is denoted $\text{var}(cs_d)_{m,n_r}$; giving the proper normalised fading channel distortions as

$$c_{m,n_r}(t) = \sqrt{\frac{\sigma^2}{\text{var}(cs_d)_{m,n_r}}} cs_{m,n_r}(t) \quad (2)$$

where the variance of the channel, $\sigma^2 = 1$ in this paper. Equation (2) allows for the modelling of coherent detection, although this is not required in the following sections, in which non-coherent detection is analysed.

The space-time cross-correlation between adjacent BS antennas can be found in a manner similar to [1] for the case of the MIMO channel. When $\tau = 0$, the approximate optimal spatial correlation can be derived for independent fading; similarly an appropriate d_{sp} for partial spatial correlation can be found. If one assumes $d = 1000\lambda$, $a = 25\lambda$, $\beta = \pi/4$, $\xi = \pi/4$, the optimal BS antenna spacing is found to be $\approx 22\lambda$. For a spatial correlation of 0.7 the BS antenna spacing d_{sp} is $\approx 10\lambda$, for a spatial correlation of 0.5, $d_{sp} \approx 14\lambda$. In reference to Fig. 1, β is the mobile position angle with respect to the end-fire of the BS antennas and d is the distance from the mobile to the centre of the four BS antennas.

DIFFERENTIAL SPACE-TIME MODULATION

In the introduction a modulation scheme which requires no channel-state information was discussed briefly. The modulation scheme, called unitary space-time modulation, is ideally suited for Rayleigh fast fading environments. It does not require the receiver to know or learn the propagation coefficients. The complex-valued signals are orthonormal with respect to time among the transmitter antennas. Importantly, when viewed as vector functions of time, the signals carry the message information entirely in their directions [3,5].

The signals transmitted over the M antennas in the MIMO channel will be grouped in time blocks of size M , as in [3,4]; τ will be used to index the time blocks. The transmitted signals are organised in an $M \times M$ matrix S_τ where the column indices represent the different antennas and the row indices represent the time samples $t = \tau M, \dots, \tau M + M - 1$. The matrices are power normalised so that the total transmitted power does not depend on M . The model for the channel can be written compactly as

$$X_\tau = \sqrt{\rho} S_\tau H_\tau + W_\tau \text{ for } \tau = 0, 1, \dots \quad (3)$$

where X_τ is the $M \times N_R$ matrix of received signals, W_τ is an $M \times N_R$ matrix of additive independent $CN(0,1)$ receiver noise. The $M \times N_R$ matrix H_τ contains the fading coefficients which are given by the flat fading channel distortions (2) in the previous section, note that across blocks $h_{\tau;m,nr}$ and $h_{\tau';m,nr}$ are not independent and the fading coefficients are also not time-invariant within a block, ρ is the effective SNR at the receiver.

One block takes up M uses of the channel, so that a rate R requires $L = 2^{RM}$ different signals. Each signal is an $M \times M$ unitary matrix V_l from a constellation $\mathcal{V}$ of L such matrices. The bits to be transmitted are packed into an integer data sequence $z_1 \dots z_\tau \in \{0, 1, \dots, L-1\}$. The relevant transmission equation, [3], is

$$S_\tau = V_{z_\tau} S_{\tau-1}, \tau = 1, 2, \dots, \text{ with } S_0 = I_M \quad (4)$$

The specified constellations in [3,4], with full diversity and high diversity product, are used, which are given by

$$V_l = V_1^l, \text{ where } V_1 = \text{diag}[e^{j2\pi u_1/L}, e^{j2\pi u_2/L} \dots e^{j2\pi u_M/L}], \quad 0 \leq l < L \quad (5)$$

where u_m are integers between 0 and $L-1$ and $u_1 = 1$. The constellation is thus entirely defined by $u_2 \dots u_M$. Because $\mathcal{V}$ forms a group every transmitted matrix S_τ belongs to $\mathcal{V}$. This implies that at any given time only one antenna transmits a phase-shift keying (PSK) symbol.

Differential space-time modulation is typically decoded using maximum-likelihood (ML) decoding, which in this case is given by

$$(\hat{z}_\tau)_{ml} = \arg \min_{l=0, \dots, L-1} \|X_\tau - V_l X_{\tau-1}\|_F \quad (6)$$

where F represents the Frobenius norm, $\|A\|_F^2 = \text{tr}(A^\dagger A)$ and † represents the adjoint operator.

TRANSMISSION FRAME LENGTHS FOR RAPID DIFFERENTIAL DECODING

Using the nature of the transmitted signals one can obtain a good approximation of ML decoding by a form of pseudo-Differential PSK (DPSK) decoding. This is based on the approximation of the cosine representation of the ML decoder used for fast lattice decoding [4]. The cosine representation is

$$\hat{z}_\tau^{\text{ML}} = \arg \max_l \sum_{n_r=1}^{N_R} \sum_{m=1}^M A_{m,n_r}^2 \cos((u_m l - \varphi_{m,n_r}) 2\pi/L) \quad (7)$$

where $A_{m,nr} = |x_{\tau,m,nr} \cdot x_{\tau-1,m,nr}|^{1/2}$, $|\cdot|$ represents the geometric mean of the modulus of the signals, and $\varphi_{m,nr} = \arg(x_{\tau,m,nr}/x_{\tau-1,m,nr})L/(2\pi)$, where $\arg(\cdot)$ has the range $[-\pi, \pi)$, represents their phase-difference in units of $2\pi/L$. However this is not a fast lattice decoding procedure, [4,6], but is an even quicker procedure because the process of basis reduction is not required. A better decoder output, $\hat{z}^{\text{dec}}$, is obtained (with little computational overhead) for large symbol constellations, by checking a much smaller subset of symbols by ML decoding around the original decoder answer [6].

The differential space-time modulation schemes described in the previous section have greater utility at higher signal-to-noise ratios (SNR). The optimality of transmission frame lengths can be found using rapid differential decoding, which closely approximate to those for a modification of fast lattice decoding to MIMO channels [6]. This optimality can be demonstrated by the general trend of the block-error rate versus transmission frame lengths at a fixed SNR, in Fig. 2 this is 17.5 dB. The block-error rate is the number of times that $\hat{z}^{\text{dec}}$ is not equal to z_r . The effect of varying the SNR on the optimality of three particular transmission frame lengths is shown in Fig. 3; block-error rate is again used as the performance measure.

In both Fig. 2 and Fig. 3 the fading parameter, $f_d T = 0.0025$ (where T is the symbol period). The channel model used is the generalised Jakes (4,2) MIMO channel model as described in the previous section, with the same model parameters used as specified for spatial correlation, $|\rho| \approx 0.5$. The scatterer ring contains $N = 34$ reflectors. A rate $R = 2$ differential unitary space-time code is used for an $L = 256$ symbol constellation. The pseudo-DPSK scheme is used for rapid decoding. In Fig. 3, three different frame lengths are used, 200, 212 and 220, corresponding to 50, 53 and 55 blocks being transmitted in one frame, for a range of SNR from 12.5 dB to 30 dB.

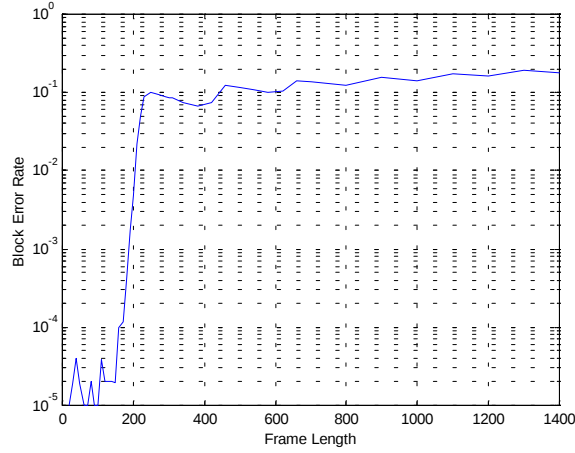


Fig. 2. Performance Trend of Frame Lengths for Rapid Differential Decoding over a MIMO channel, SNR = 17.5 dB, using Block Error Rate as Performance Measure

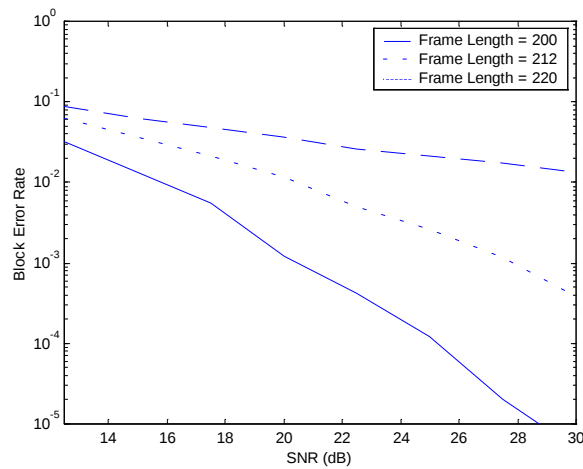


Fig. 3. Performance of three particular Frame Lengths for Rapid Differential Decoding over a MIMO channel, for high SNR.

It is clear that there is a significant rapid increase in block-error rate for frame lengths from ~ 160 to ~ 240 in Fig. 2, i.e. for number of blocks transmitted from ~ 40 to ~ 60 . The general trend of transmission frame length optimality fluctuates, i.e. the curve of block-error rate shown in Fig. 2 is not always increasing. This is despite the general trend of degradation in block-error rate as might be expected. The results in Fig. 2 show clearly the effect of a wide range of transmission frame lengths in a typical MIMO channel using rapid differential decoding. They show the importance of restricting the frame lengths, and thus the numbers of blocks transmitted within certain limits to optimise the performance of the system.

Fig. 3 provides a comparison with the results of Fig. 2, demonstrating what happens at three critical transmission frame lengths in this differential space-time decoding application. A high range of SNR is used for purposes of valid comparison; it is clear that the performance diminishes rapidly as the block length (number of blocks in one frame) increases from 50 to 55 for $M = 4$ transmit antennas. The improvement in performance as the SNR increases also diminishes significantly as the frame length increases. It should be noted that the block-error rate is orders of magnitude larger than what the corresponding bit-error rate would be, and is thus a strict performance measure.

CONCLUSIONS

The application of a space-time generalisation of Jakes model for a typical Rayleigh fading MIMO channel to fast decoding of differential unitary space-time modulation has demonstrated some significant trends. These trends give some guidelines and insight in relation to desirable transmission frame lengths for this application. This should be an important consideration in the design of a number of applications using space-time coding. It is clearly demonstrated that transmission frame lengths for this typical (4,2) MIMO channel should be restricted to less than 200 for good performance at high SNR. For better performance, frame length should be restricted to less than 160. For further optimisation of performance below a frame length of 160, some consideration could also be given to fluctuations in performance at different frame lengths.

The results from the generalised Jakes model applied to a (4,2) MIMO channel has possibly uncovered some limitations in the uses of fast differential decoding of unitary space-time modulation. Further, the statistical complexity of this new space-time generalised Jakes model is still yet to be fully understood, and warrants further analysis. It is postulated that it is more rigorous than previous applications of Jakes model to MIMO or MISO channels, hence better suited to obtain good performance measures. However the level of rigour in this MIMO channel model requires further investigation.

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APPENDIX 1

In this appendix the expressions for path-length differences from the n th reflector in the scatterer ring around the co-located MS antennas to the four BS antennas is given, with reference to Fig. 1. From these the relevant phase-differences, $\Delta\phi_{n,m,nr}$, (1), can be derived.

By using far-field assumptions, which are typical for wireless communications, whereby $d_1, d_2, d_3, d_4 \gg a$, $\theta \approx \rho$, $\cos\theta \approx 1$, $(d_2 + d_3) \approx 2d$ and $(d_2 - d_3) \approx d_{sp}\cos\beta$, and making further appropriate simplifications, we obtain the following approximations for the path length differences

$$s_{2,nr} - s_{1,nr} \cong - \left(d_{sp} \cos \beta + \frac{d_{sp}^2}{d} \sin^2 \beta + \frac{ad_{sp}}{d} \sin \beta \sin(\beta - \alpha_{n,nr}) \right) \quad (8)$$

$$s_{2,nr} - s_{3,nr} \cong d_{sp} \cos \beta + \frac{ad_{sp}}{d} \sin \beta \sin(\beta - \alpha_{n,nr}) \quad (9)$$

$$s_{2,nr} - s_{4,nr} \cong 2d_{sp} \cos \beta - \frac{d_{sp}^2}{d} \sin^2 \beta + \frac{2ad_{sp}}{d} \sin \beta \sin(\beta - \alpha_{n,nr}) \quad (10)$$

$\Delta\phi_{n,m,nr}$ can then be evaluated by $2\pi(s_{2,nr} - s_{m,nr})/\lambda$.