ESTIMATING THE INTENSITY OF BUY AND SELL ARRIVALS IN A LIMIT ORDER BOOK MARKET

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ABSTRACT. In this paper, we model the simultaneous buy and sell trade arrival process in a limit order book market. The buy and sell process is modelled based on a bivariate intensity model. In this context we extend the autoregressive conditional intensity (ACI) model by allowing for time varying covariates. Using limit order book data from the SEATS system of the Australian Stock Exchange (ASX), we include variables that reflect the state of the order book with respect to market depth, tightness, as well as, the cumulated volume in the ask and bid queue. Moreover, changes of the order book induced by limit order arrivals are captured as time varying covariates. We show that the state of the order book as well as the observed trading process has a significant impact on the bivariate buy and sell intensity, and thus, influences traders' decision when to trade and on which side of the market.

1. INTRODUCTION

A large amount of theoretical and empirical market microstructure research has been generated as a result of the development and distribution of transaction based financial data. Intertemporally aggregated data cannot properly address empirical market microstructure issues since these questions involve the dynamic characteristics of individual quotes and resulting trades. Our primary econometric interest is the dynamics of the quote arrival process and its implications for the dynamics of trade arrival process, the dynamics of the price process and the dynamics of the trading volume process. The quote arrival times and transaction times are not equally spaced in time, and there is strong empirical evidence

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that these arrival rates are intertemporally correlated, so that quotes and trades tend to cluster in time in both a deterministic and stochastic manner. As illustrated in recent papers (see, for example Renault and Werker, 2002, or AÏT-Sahalia and Mykland, 2003), trade durations are a major source of price volatility. Therefore, understanding the determinants of the trade arrival process is an important requirement of the understanding of volatility.

The main objective of the paper is to study the buy and sell arrival process in a limit order book market. The analysis of electronic limit order books is an ongoing topic in financial econometrics literature. Particularly, this effect enforced by the growing importance of electronic trading at worldwide exchanges. However, the availability of detailed limit order book data opens up the possibility for a wide range of empirical studies. A well known study has been made by Biais, Hillion, and Spatt (1995) for the order flow in the limit order book of the Paris bourse. Their results are consistent with the existence of traders which monitor the current state of the limit order book and exploit situations in which the execution of a particular order is favorable. Lo, MacKinlay, and Zhang (2002) examine the execution times of limit orders in the hybrid trading system of the NYSE. Coppejans and Domowitz (2002) use ACD specifications with time-varying covariates to model the transaction and quote process in the electronic stock index trading of the OMLX. Giot and Grammig (2002) investigate liquidity risk by using limit order book data from the German XETRA trading.

Nonetheless, relatively less is known with respect to the impact of the state of the limit order book on the trading intensity on the particular sides of the market. Therefore, in this study we analyze whether the state of the order book has a significant impact on a traders' decision *when* to trade and *on which side of the market* to trade.

A unique data set, extracted by replicating the execution engine of the Australian Stock Exchange (ASX), allows the reconstruction of all quotes and resulting trades, delivers time stamped transaction data that identifies whether the trade is buy or sell initiated as well as almost complete information about the state of the order queues.

The econometric analysis consists of two parts. In the first part we run a simple probit regression in order to examine the factors driving the buy/sell decision. In the second part we jointly model the intensity of the buy and sell arrival process, using the information on the state of the order queues as predetermined or weakly exogenous time-varying variables or marks. In this context we apply an augmented version of the autoregressive conditional intensity (ACI) model proposed by Russell (1999) which is extended to allow for time varying covariates. This model allows us to investigate which factors drive the intensity of

the buy and sell arrival process, i.e. when do market participants trade and on which side of the market. We use three groups of variables, consisting of variables associated with characteristics of the previous transaction, variables that reflect the state of the order book after the previous transaction and variables that reflect the limit order arrival process and thus induced changes of the order book between two transactions (time varying covariates). Our empirical results indicate that the state of the order book has a significant impact on the buy/sell intensity. We observe that market participants seem to infer from the order book with respect to the existence of information. However, we only find weak evidence for the hypothesis that traders strategically tend to exploit situations of high tightness and market depth.
prefer to trade in situations characterized by a high depth and tightness of the market. The remainder of the paper is organized in the following way: In Section 2 we discuss economic hypotheses based on market microstructure theory. Section 3 shows the econometric model. In Section 4 we present the data base and the corresponding explanatory variables. Section 5 gives the empirical results while Section 6 concludes.

2. Economic motivation

Traditional microstructure theory provides two major frameworks to explain price setting behavior: inventory models and information-based models[1]. The first branch of the literature investigates the uncertainty in the order flow and the inventory risk and optimization problem of liquidity suppliers [2]. The latter branch models market dynamics and adjustment processes of prices by using insights from the theory of asymmetric information and adverse selection. In the latter framework, a crucial assumption is made regarding the existence of different informed traders. It is assumed that there exist so called 'informed traders', who trade due to private information and 'liquidity traders', who trade due to exogenous reasons, like, for example, due to portfolio adjustments or liquidity aspects. The assumption of heterogeneous groups of traders provides the basis for a plethora of asymmetric information models.[3] In these approaches, uninformed market participants deduce from the trading process the existence of information in the market. Thus, the trading process itself serves as a source of information. Therefore, on the basis of the

[1]For a comprehensive overview see, for example O'Hara (1995), or the recent survey by Madhavan (2000).

[2]See, for example, Garman (1976), Stoll (1978), Amihud and Mendelson (1980) or Ho and Stoll (1981).

[3]See, for example, Bagehot (1971), Copeland and Galai (1983), Glosten and Milgrom (1985), Kyle (1985), Diamond and Verrecchia (1987), Admati and Pfleiderer (1988) or Easley and OHara (1987, 1992) among others.

assumption of diversely informed, heterogeneous traders, information-based market microstructure settings model relationships between the price change, the bid-ask spread, the trading volume and the trading intensity.

In the following we discuss several economic hypotheses reflecting the impact of the trade and limit order process as well as the state of the limit order book on the buy and sell trading intensity. We differentiate between three groups of explanatory factors. The first group of variables is associated with characteristics of the previous trade and consists of:

- the type of the particular transaction
- the buy/sell volume
- the difference between the transaction price and the current midquote
- the spread

According to market microstructure theory, the traded volumes on the particular sides of the market are strong proxies for the existence of information, therefore, we expect a positive impact on the particular intensities. The difference between the transaction price and the current midquote characterizes the depth associated with the last transaction. The higher this difference, the more volume is absorbed from the particular queue which should decrease the probability for the occurrence of a trade of the same type in the next instant. In a market maker market, the spread compensates a market maker for the risk due to adverse selection and thus is positively correlated with the existence of information on the market. Therefore, several market microstructure approaches predict a positive relationship between the magnitude of the spread and the trading intensity. However, in a limit order book market, the implications are less clear. Nevertheless, we would expect a higher spread in informative market periods, thus the spread should have a positive influence on the intensity on both sides of the market. Contrarily, we do not expect any influences of the spread on the buy/sell decision.

The second group of variables contains characteristics associated with the state of the order book at the previous trade:

- the total volume in the bid (ask) queue
- the steepness of the market reaction curve on the bid (ask) side
- the difference between the absolute bid curve slope and the absolute ask curve slope

The total volume in the particular queues characterizes the demand and supply side. For example, a high volume on the bid side indicates a high supply, i.e. traders' wish to sell their positions. This might be associated with "bad" expectations concerning future price movements. Therefore, we expect a negative relationship between the bid (ask) volume and the probability to observe a buy (sell) transaction in the next instant. The steepness of the market reaction curve is measured by the price impact of a (hypothetical) volume. In this study, we measure it by the ratio between the lowest and highest limit price divided by the corresponding total volume. The bid/ask market reaction curves indicate the market depth on the particular sides of the market. The steeper the market reaction curve, the higher the price impact of a hypothetical volume, and thus, the higher the liquidity costs. Therefore, we state the hypothesis that the buy (sell) probability should be the lower the higher the steepness of the market reaction curve. The third variable is associated with imbalances between the bid and ask side. Obviously, a positive difference between the absolute bid and ask slope indicates a higher (relative) liquidity, i.e. lower price impacts, on the ask side of the market. However, the economic implications are less clear: On the one hand we would expect a positive relationship between the relative liquidity and the corresponding trading intensity. A contrary argument arises by the fact that a higher market depth comes along with a higher limit order activity on the corresponding side of the market. Following the argumentation above, a high one-sided limit order activity might be associated with expectations of the market participants with respect to future price movements. Thus, in this case a higher relative market depth should decrease the corresponding trading intensity.

The third group of variables captures the limit order arrival process since the previous transaction:

- the type of the order (bid or ask)
- the bid-volume
- the ask-volume
- an indication whether the limit bid (ask) price is equal or lower than the current bid (ask)

These variables allow to analyze whether current changes of the limit order book induced by the arrival of new orders, influences the buy/sell intensity. These variables enter the model as time-varying covariates (see Section 3).

3. Modelling multivariate Point Processes

3.1. **Notation.** Let t denote the physical (calendar) time. Define $\{\tilde{T}_i\}_{i\in\{1,2,\ldots,\tilde{n}\}}$ as the arrival times of a point process. Moreover, define $\{Y_i\}_{i\in\{0,1,2,\ldots,\tilde{n}\}}$ as a sequence of an integer variable that can take the values $\{0,1,2,\ldots,K\}$. Then, the sequence $\{\tilde{T}_i, Y_i\}_{i\in\{1,2,\ldots,\tilde{n}\}}$ is a $(K+1)$-dimensional point process. Let $\tilde{N}(t)=\sum_{i\geq 1}\mathbb{1}_{\{\tilde{T}_i\leq t\}}$ be the counting function associated with $\{\tilde{T}_i\}$. $\tilde{N}(t)$ is a right-continuous step function with upward jumps (at each

$\tilde{T}_i$) of magnitude 1. By selecting all points for which $Y_i = k$, we obtain the k-th point process associated with the squence of arrival times $\{T_i^k\}_{i\in\{1,2,\ldots,n^k\}}$ and characterized by the counting function $N^k(t) = \sum_{i\geq 1} 1\!\!1_{\{T_i^k \leq t\}}$. Moreover, define $X^k(t)$ as the backward recurrence time for the k-th process, given by $X^k(t) = t - \breve{N}^k(t)$, where $\breve{N}^k(t) = \sum_{i\geq 1} 1\!\!1_{\{T_i^k < t\}}$ is the *left-continuous* counting function that counts the number of events of process k that occur *before* t. The backward recurrence time is the time elapsed since the previous point and is a left-continuous function that grows linearly through time with discrete jumps back to zero after each arrival time T_i^k.

In order to allow for time-varying covariates, we define the process $N^0(t)$ as the arrival times of a covariates process $\{Z_{0,i}\}_{i\in\{1,2,\ldots,n^0\}}$. In our framework, this process is associated with the limit order arrival process. In the following we consider the K-dimensional process of all selected points for which $Y_i > 0$. For this process, we model the (multivariate) intensity function (in our context the buy and sell arrival process) in dependence of the process of time-varying covariates. In the following this process is denoted by $\{T_i\}_{i\in\{1,2,\ldots,n\}}$ with corresponding counting function $N(t)$.

Furthermore, let $\{Z_i\}_{i\in\{1,2,\ldots,n\}}$ be a sequence of marks corresponding to characteristics associated with the arrival times of the process $N(t)$. Marks can be interpreted as time-invariant covariates, that are only observable at the particular points $\{T_i\}_{i\in\{1,2,\ldots,n\}}$ and that do not change between two subsequent points T_{i-1} and T_i.

In the following we assume that the process $N(t)$ is orderly, i.e.

$$\Pr\left[(N(t+\Delta) - N(t)) > 1 \,|\mathcal{F}_t\right] = o(\Delta),$$

hence, all particular K $(k = 1, \ldots, K)$ univariate point processes are orderly as well.

3.2. **The intensity function.** Define

$$\lambda^k(t;\mathcal{F}_t) = \lim_{\Delta\downarrow 0} \frac{1}{\Delta} \mathrm{E}\left[N^k(t+\Delta) - N^k(t)\middle| \mathcal{F}_t\right], \qquad \lambda^k(t;\mathcal{F}_t) > 0,\ \forall\, t, \tag{1}$$

as the $\mathcal{F}_t$-intensity function of the counting process $N^k(t)$, where $\mathcal{F}_t$ denotes the history of the complete (pooled) process up to t. The function $\lambda^k(t;\mathcal{F}_t)$ characterizes the evolution of the point process $N^k(t)$ conditional on some history $\mathcal{F}_t$.

Under the assumption that $N^k(t)$ is orderly, $\lambda^k(t;\mathcal{F}_t)$ can be alternatively written as

$$\lambda^k(t;\mathcal{F}_t) = \lim_{\Delta\downarrow 0} \frac{1}{\Delta} \Pr\left[(N^k(t+\Delta) - N^k(t)) > 0 \,|\mathcal{F}_t\right] \tag{2}$$

which can be associated, roughly speaking, with the conditional probability per unit time to observe an event in the next instant, given the conditioning information.

In point process models the integrated intensity function

$$\Lambda^k(s_0, s_1) = \int_{s_0}^{s_1} \lambda^k(t; \mathcal{F}_t)dt \tag{3}$$

plays a central role. By assuming that

$$\int_0^\infty \lambda^k(t; \mathcal{F}_t)dt = \infty \quad \text{a.s.} \tag{4}$$

it can be shown (see, for example Brémaud, 1981) that

$$\varepsilon_i^k := \Lambda^k(T_{i-1}^k, T_i^k) \sim \text{i.i.d. Exp}(1). \tag{5}$$

This property establishes the relationship between the integrated intensity function and the resulting waiting time between consecutive points. It plays an essential role for diagnostics in point process models.

3.3. **The ACI Model with Time Varying Covariates.** In the following we define the multivariate intensity function as

$$\lambda(t; \mathcal{F}_t) = \begin{bmatrix} \lambda^1(t; \mathcal{F}_t) \\ \lambda^2(t; \mathcal{F}_t) \\ \vdots \\ \lambda^K(t; \mathcal{F}_t) \end{bmatrix}. \tag{6}$$

The Autoregressive Conditional Intensity (ACI) model proposed by Russell (1999) is based on the parameterization

$$\lambda^k(t; \mathcal{F}_t) = \exp(\Psi_{N(t)}^k) h^k(X^1(t), \ldots, X^K(t)) s^k(t), \tag{7}$$

where $\Psi_{N(t)}^k$ denotes an autoregressive process based on the integrated conditional intensity, $h^k(\cdot)$ a function of the K backward recurrence times and $s^k(t)$ is a continuous function of time capturing seasonalities. Define in the following $\Psi_{N(t)}$ as the vector

$$\Psi_{N(t)} = \begin{bmatrix} \Psi_{N(t)}^1 \\ \Psi_{N(t)}^2 \\ \vdots \\ \Psi_{N(t)}^K \end{bmatrix}. \tag{8}$$

Russell proposes to specify $\Psi_{N(t)}$ based on a VARMAX type parameterization. By extending this specification in order to allow for time-varying covariates, we parameterize $\Psi_{N(t)}$ as

$$\Psi_{N(t)} = \left(A^k \tilde{\varepsilon}_{N(t)-1} + B^k \Psi_{N(t)-1} + \gamma' z_{N(t)-1} + \tilde{\gamma}' z_{0,N^0(t)-1} \right) \mathbb{1}_{\{Y_{N(t)-1}=k\}}, \tag{9}$$

where A^k is a $(K \times 1)$ vector, while B^k is a $(K \times K)$-matrix. Moreover, $z_{N(t)}$ and $z_{0,N^0(t)}$ are $(M \times 1)$ and $(M_0 \times 1)$ vectors, respectively, while γ and γ_0 are the corresponding $(M \times K)$ and $(M_0 \times K)$ coefficient vectors, respectively.

The innovation term $\tilde{\varepsilon}_{N(t)}$ is based on the log integrated intensity associated with the most recently observed process, thus

$$\tilde{\varepsilon}_{N(t)} = (-0.57722 - \ln \varepsilon^k_{N(t)}) \mathbb{1}_{\{Y_{N(t)}=k\}}, \tag{10}$$

where

$$\varepsilon^k_{N(t)} = \int_{T^k_{N^k(t)-1}}^{T^k_{N^k(t)}} \lambda^k(t;\mathcal{F}_t)dt = \int_{\tilde{T}_{\tilde{N}(\tau^k(t))}}^{\tilde{T}_{\tilde{N}(t)}} \lambda^k(t;\mathcal{F}_t)dt \tag{11}$$

$$= \sum_{j=0}^{\tilde{N}(t)-\tilde{N}(\tau^k(t))} \int_{\tilde{T}_{\tilde{N}(\tau^k(t))+j}}^{\tilde{T}_{\tilde{N}(\tau^k(t))+j+1}} \lambda^k(t;\mathcal{F}_t)dt, \tag{12}$$

where $\tau^k(t) := T^k_{N^k(t)-1}$. Note that eq. (12) corresponds to the piecewise integration over one spell, where the pieces are determined by the time-varying covariate process. Under correct model specification, $\varepsilon^k_{N(t)}$ is i.i.d. standard exponential distributed. Therefore, $\ln \varepsilon^k_{N(t)}$ follows an extreme value distribution with mean $\mathrm{E}[\varepsilon^k_{N(t)}] = -0.57722$ and $\mathrm{Var}[\ln \varepsilon^k_{N(t)}] = \pi^2/6$. Every of the K particular processes is updated at every occurrence of a new event T_i. Note that we assume that the process is orderly, i.e. only one type of innovation can occur at each instant. For this reason the innovation ε^k_i is a scalar function. However, the impact of this innovation on the particular processes can differ. Note that the innovation term enters eq. (9) negatively. Thus, a lower than expected integrated intensity (corresponding to lower than expected number of events) decreases the intensity. The choice of the parameterization of A^k and B^k determines the dynamic and interdependence structure of the particular processes. For example, as proposed by Russell (1999), a more parsimonious model is obtained by specifying B^k as a diagonal matrix, thus the persistence of one particular process is not influenced by the persistence of the other process. In this case, for $K = 2$, we parameterize B^1 and B^2 as

$$B^1 = \begin{bmatrix} \beta^1_{11} & 0 \\ 0 & 1 \end{bmatrix}, \qquad B^2 = \begin{bmatrix} 1 & 0 \\ 0 & \beta^2_{22}. \end{bmatrix} \tag{13}$$

Note that this structure implies that only the k-th process is updated with new information when an event of type k occurs.

The stationarity of the ACI model depends on the eigenvalues of the matrices B^k. Because the matrices A and B depend on the type of the most recent point, the model admits an interpretation as a regime switching VARMA process for the conditional intensity function. However, this structure renders the derivation of stationarity conditions for the general case difficult. Nonetheless, for important special cases, stationarity can be shown. In particular, for the case $B^1 = \ldots = B^K = B$, Russell (1999) proves that the ACI model is mean-reverting if the conditional survivor function is absolutely continuous with respect to the Lebesgue measure and the eigenvalues of B lie inside of the unit circle. For the more general case, the same result holds for the particular case (13) whenever β_{11}^1 and β_{22}^2 lie inside the unit circle. However, for more general cases, the stationarity conditions are not known.

The functions of the backward recurrence times can be specified in different ways. The most simple way is to assume that $h^k(\cdot)$ is constant but depends on the type of the event which occurred most recently, i.e.

$$(14) \qquad h^k(X^1(t),\ldots,X^K(t)) = \exp(\omega_j^k)\,\mathbb{1}_{\{Y_{N(t)-1}=k\}}, \quad j=1,\ldots,K, \quad k=1,\ldots,K.$$

Alternatively, the backward recurrence time function may be specified as a multivariate Weibull type (see Russell, 1999) or Burr type specification, i.e.

$$(15) \qquad h^k(X^1(t),\ldots,X^K(t)) = \exp(\omega^k)\prod_{j=1}^{K} X^j(t)^{\delta_j^k-1},$$

or

$$(16) \qquad h^k(X^1(t),\ldots,X^K(t)) = \exp(\omega^k)\prod_{j=1}^{K} \frac{X^j(t)^{\delta_j^k-1}}{1+\kappa_j^k X^j(t)^{\kappa_j^k}}.$$

A particular case occurs when the k-th process depends only on its own backward recurrence time, which correponds to $\delta_k^j = 1$ and $\kappa_k^j = 0$, $\forall\, j \neq k$.

3.4. **Inference.** The log likelihood function is given by

$$(17) \qquad \ln \mathcal{L}(\theta|\{\tilde{T}_i, Y_i\}_{i\in\{0,1,2,\ldots,\tilde{n}\}}) = \sum_{i=1}^{n}\sum_{k=1}^{K}(-\Lambda^k(T_{i-1},T_i)) + \ln\left[\lambda^k(T_i;\mathcal{F}_{T_i})\right]\mathbb{1}_{\{Y_i=k\}},$$

where

$$(18) \qquad \begin{aligned}\Lambda^k(T_{i-1},T_i) &= \int_{T_{i-1}}^{T_i} \lambda^k(t;\mathcal{F}_t)dt = \int_{\tilde{T}_{\tilde{N}(\tau(t))}}^{\tilde{T}_{\tilde{N}(t)}} \lambda^k(t;\mathcal{F}_t)dt \\ &= \sum_{j=0}^{\tilde{N}(t)-\tilde{N}(\tau(t))} \int_{\tilde{T}_{\tilde{N}(\tau(t))+j}}^{\tilde{T}_{\tilde{N}(\tau(t))+j+1}} \lambda^k(t;\mathcal{F}_t)dt,\end{aligned}$$

and $\tau(t) := T_{N(t)-1}$.

Under correct specification the residuals $\hat{\varepsilon}_i^k$ should be i.i.d. unit exponential distributed. Hence, model evaluation can be done by testing the dynamic properties of the residual series using e.g. Ljung-Box statistics, and by testing the distribution properties, e.g. against overdispersion. Engle and Russell (1998) propose a test against excess dispersion based on the asymptotically normal test statistic $\sqrt{n^k/8}\,\hat{\sigma}^2_{\varepsilon^k}$ where $\hat{\sigma}^2_{\varepsilon^k}$ is the empirical variance of the residual series. Other checks of the distribution properties of the residuals can be done by the computation of the probability integral transform based on the exponential distribution

$$u_i^k = \int_{-\infty}^{\varepsilon_i^k} \exp(-s)ds = 1 - \exp\left(-\varepsilon_i^k\right). \tag{19}$$

Diebold, Gunther, and Tay (1998) show that under correct model specification, the distribution of the u_i^k series should be i.i.d. uniform.[4] A corresponding test can be based on graphical inspections or on usual χ^2 goodness-of-fit statistics.

4. The data

4.1. The Australian Stock Exchange. The Australian Stock Exchange (ASX) is a continuous double auction electronic market with business (trading) rules similar to other electronic limit order markets such as Paris, Hong Kong and Sao Paulo.

The continuous auction trading period is preceded and followed by an opening call auction. Normal trading takes place continuously between 10:09 am and 16:00 pm Monday to Friday. The market is opened with a call auction market in all stocks. Before the call market opens, traders are allowed to enter public quotes from 07:00 am until the call auction completes and the market is opened for continuous trading. In order to minimize the instantaneous transaction load on the system, the call market is executed in steps by grouping stocks alphabetically into 5 groups and opening these groups 2 minutes and 15 seconds apart. To reduce last second gaming of the opening call auction, the exact opening times are randomized from the nominal opening time for each group (± 15 seconds). At the end of normal trading, another call market is held, usually at 16:05pm (± 15 seconds), and the price of any trades generated by this closing call auction set the last price for the day. This enables any broker wishing to trade at the last price of the day to do so by participating in this closing call auction. From 16:05 pm to 17:00 pm some limited late trading is permitted under restrictive rules. These rules are summarized in Tables 1 and 2.

[4] See, for example, Bauwens, Giot, Grammig, and Veredas (2000) for an application of this technique to the evaluation of ACD models.

Table 1: Daily ASX Market Schedule

Market Phase	Time	Functionality
Market Enquiry	(approx) 3:00 - 7:00	Enquiry on current orders only.
Market Pre-Open	7:00 - 10:00	Can enter, delete and amend orders and enter off-market trades.
Market Opening	10:00 - 10:09 (see below)	Call auction opens continuous trading.
Normal Market	10:09 - 16:00	Can enter, amend and delete orders. Overlapping buy and sell orders execute immediately.
Closing Call Auction	16:05($\pm$ 15 seconds random)	Can enter, delete and amend orders prior to the closing call auction.
Late Trading	16:05 - 17:00	Can enter, delete and amend orders. Late trades permitted with restrictions.
Market Closed	17:00 - 19:00	Can only delete orders.

Table 2: Alphabetic Group Opening Times

Alphabetic Group	A-B	C-F	G-M	N-R	S-Z
Nominal Opening Time	10:00:00	10:02:15	10:04:30	10:06:45	10:09:00

4.2. **Normal Market Trading.** A comprehensive description of the trading rules of the Stock Exchange Automated Trading System (SEATS) on the ASX can be found in the SEATS Reference Manual (The Australian Stock Exchange, SEATS Reference Manual, available at http://www.asxonline.com in the Participants Library). A simplified summary description of the major features of the electronic double auction market during normal market trading is briefly described below.

4.2.1. *Limit Orders, Market Orders and Aggregated Trades.* When the ASX is in normal open mode (10:09-16:00), any buy (sell) order entered that has a price that is greater (less) than existing queued sell (buy) orders, will execute immediately. Trades will be generated and traded orders deleted until there is no more sell (buy) order volume that has a price that is equal to or less than the entered buy (sell) order. Orders that execute immediately are *market orders.* Queued orders (orders entered with a price that does not overlap the opposite order queue) are *limit orders.* Entered orders that partially execute are a *combination* of a market order for the immediately executed volume and a limit order for the remaining volume.

When a market order executes against limit orders, the ASX generates a trade record for each market order - limit order pair of executing orders. Therefore, if a market order executed against several limit orders, several trade records will be generated. However only one logical trade has been executed, and for the empirical research reported here, the multiple trade records generated by a single market order are aggregated into a single trade record.

4.2.2. *Time, Price Priority Rules.* Limit orders are queued in the buy and sell queues according to a strict time-price priority order. Between 7:00 am and 17:00pm, orders may be entered, deleted and modified without restriction. Modifying order volume downwards does not affect order priority. Modifying order volume upwards automatically creates a new order at the same price as the original order with the increase in volume as the volume of the newly created order. This avoids loss of priority on the existing order volume. Modifying order price so that price overlaps the opposite order queue will cause immediate or partial execution of the order according to the rules above (the order is converted to a market order or a market and limit order combination). Modifying price otherwise causes the order to move to the lowest time priority within the new price level.

4.2.3. *Market Information Visibility.* All previous and current orders and trades are always visible to the public. Order prices are always visible, however orders may be entered with an undisclosed (hidden) volume if the total value of the order exceeds $200,000 . Although undisclosed volume orders are permitted, sufficient information is available to unambiguously reconstruct transactions. The identity of the broker who entered an order is not public information, but is available to all other brokers. The efficiencies generated by moving to electronic trading and settlement systems mean that the ASX no longer requires minimum lot sizes to be traded in any stock and single share orders are common.

4.2.4. *Minimum Tick Size.* The minimum tick size for order prices below $0.10 is $0.001, for order prices above$0.10 and below $0.50 is $0.005, while the minimum tick size for stock orders priced $0.50 and above is $0.01. If brokers have a matching buy or sell order they may 'cross' this order in the market. Crossings do not participate in the market because the broker provides both the buy and sell volume in the crossing, the buy and sell limit order queues are unchanged and the trade generated is reported with a special crossing parameter. Crossings should not be confused with the situation where the same broker is both buyer and seller in the normal course of trading, in this case the limit order queues are modified by the trade execution in the usual way.

Trades reported to the market that are not executed through the SEATS system are designated "Off-Market Trades". There are three main generators of off-market trades, late and overnight trading, reporting trades from the "upstairs" telephone market and reporting the exercise of in-the-money exchange traded options (which are stock settled).

Reported trades from the "upstairs" phone market do not participate in the price discovery process and may be priced away from the current market. These trades are called specials and must must have a value greater than $2,000,000. It is a requirement that these trades be reported immediately to the market, however there is no practical way for the ASX to enforce this rule. These trades are executed by private negotiation between the brokers and their institutional clients.

The data extracted from the SEATS system contains time stamped prices, volumes and identification attributes of all transactions, market and limit orders, and reconstructs the state of the order book at any time. We extract a sample consisting of trades and limit orders for National Australia Bank (NAB), the largest capitalised stock trading on the ASX, during the normal trading period between 02/04/02 and 02/25/02. Data from the opening and closing call auctions periods are not utilized and all crossing and off market trades are removed. The limit order arrival during two consecutive trades is captured by time varying covariates. In this context, we summarize the changing state of the order queues as a result of the arrival new limit orders or amendments of existing orders.

4.3. **Explanatory variables.** Our empirical analysis is based on the NAB stock traded at the ASX. According to our economic considerations in Section 2, we generate several explanatory variables. Table 3 in the Appendix gives the definition of the particular explanatory variables used in the empirical study. Table 5 contains descriptive statistics. Table 4 presents descriptive statistics associated with observations on the trade and limit order arrival process considered in this study. We observe a mean trade and limit order duration of 26 and 32 seconds, respectively, indicating a relatively high trading activity. During the analyzed sample period we observe a higher fraction of sell trades compared to buy trades, coming along with a mean buy duration of 60 seconds and a mean sell duration of 46 seconds. The particular Ljung-Box statistics reveal a strong persistence in the trade process as well as in the limit order process which is a well known phenomenom for trade and limit order durations (see, for example, Engle and Russell, 1998, or Russell, 1999).

5. Empirical results

5.1. **Determinants of the buy/sell decision.** The first part of the empirical analysis is devoted to the determinants of the buy/sell decision. Table 6 shows the results of simple probit regressions with the buy/sell indicator (BUY) as dependent variable. We can summarize the main results: First, the type of the previous trade has no significant impact on the buy/sell decision. Second, the smaller the difference between the quoted buy price and the midquote at the previous transaction, the lower the probability to observe a buy. Accordingly, the buy probability decreases the higher the difference between midquote and quoted sell price. Hence, these findings confirm our considerations in Section 2. Third, the impact of total volume on both sides of the market decreases the probability for the occurrence of a buy transaction which is only partly in line with our theoretical hypotheses. Fourth, with respect to the steepness of the market reaction curve we find mixed evidence since a negative relationship of the slopes on both the ask and bid side and the probability for the occurrence of a buy is observed. Fifth, contrary results are obtained for the impact of the relative market depth on the particular sides of the market on the buy probability. Note that the variable $DFF1$, $DFF2$ and $DFF10\_$ measure the slope difference between bid and ask side, thus a positive coefficient of these variables is associated with a higher market depth on the buy side. However, we observe significantly negative coefficients for the variables $DFF1$, $DFF2$ associated with the slope of the market reaction curve between the midquote and the 20% quantile. Nevertheless, the positive coefficient for $DFF10\_$ indicates the existence of nonlinearities. Sixth, surprisingly, we find a significant negative coefficient associated with the spread at the previous trade. Thus, the decision to buy seems to be more sensitive to the current spread as the sell decision which is not confirmed by economic theory. Seventh, a significant impact of limit orders arriving after the previous trade is found. In particular, the fact that the most recent limit order was an ask decreases the buy probability, thus the buy/sell decision of market participants seem to be influenced by current changes in the order book. Not surprisingly, recent changes of the best ask (bid) price, coming along with a lower spread, increase (decrease) the buy (sell) probability.

5.2. **Modelling the buy/sell intensity.** Table 7 gives the results of bivariate ACI model with time-varying covariates. The persistence matrix B is specified according to (13), implying no interdependence of the persistence terms between both processes. The backward recurrence function is specified as in eq. (15). Figure 1 shows the estimated intraday seasonalities of the buy and sell intensity based on linear spline functions. Since both plots are relatively similar, we specify the seasonality functions $s(\cdot)$ in the final regression as a joint seasonality spline function for both series based on 1 hour knots.

We observe a strong persistence in both processes coming along with relatively small innovation coefficients and persistence parameters that are close to one. Such a result is quite typical for trade duration processes. The parameters of the backward recurrence function indicate a negative duration dependence, i.e. a decreasing intensity function over time. In general it turns out that the inclusion of static and time varying covariates improve the goodness-of-fit, as measured based on the BIC as well as based on residual diagnostics. The inclusion of time varying covariates does not substantially change the coefficients of the static covariates which indicates slight evidence for the robustness of our results.

Concerning to our economic hypotheses we can summarize the following results: First, we find asymmetries with respect to the impact of the traded buy volume at the previous trade. In particular, we observe an increasing (decreasing) buy (sell) intensity after observing a high buy (sell) transaction volume which partly confirms our theoretical considerations. Second, the higher the total volume on the ask side of the order book, the lower the buy intensity and the higher the sell intensity. Correspondingly, contrary results are obtained with respect to the total volume on the bid side. This result confirms the findings based on the probit regression and supports economic theory. Thus, a high volume seem to be associated with "bad" expectations concerning future price movements, leading to a decreased buy intensity. Third, the steeper the market reaction curve on the ask side, i.e. the lower the market depth, the higher the intensity of both buy and sell arrivals. Contrarily, the slope of the market reaction curve on the bid side is negatively correlated with both the buy and sell intensity. Obviously, a higher limit order activity on the bid side of the market seem to be associated with "bad news" which decreases the buy intensity. Thus, this effect seems to dominate the advantages of a higher market depth, and thus lower price impacts. Fourth, for the variables $DFF1$ and $DFF2$ we find insignificant parameters, however DFF_10 reveals a significant negative (positive) coefficient with respect to the buy (sell) intensity. Thus, as in the probit analysis we find a significant negative (positive) relationship between the difference between bid and ask slope and the buy (sell) intensity. Fifth, in contrast to the findings based on the probit regression, a significant negative impact of the magnitude of the current spread on the sell intensity is observed. This result is in contrast to general market microstructure theory which derives a positive relationship between the width of the spread and the expected trading intensity.

Furthermore, the arrival of limit orders between two transactions has a significant impact on the intensity of the buy/sell process. This finding illustrates that market participants pay strong attention to changes in the limit order book. With respect to the impact of the quoted volume at the current limit order on the buy/sell intensity, we find no clear results. It turns out that the quoted volume on the ask side has a significant positive influence on the buy intensity while the quoted bid volume decreases the sell intensity. Moreover, a change of the current ask price caused by the arrival of a new order increases the buy intensity and decreases the sell intensity which strongly supports economic theory. However, the results concerning the changes of the current bid price are less clear and somewhat conflictive.

6. Conclusions

In this study, we analyze the simultaneous arrival process of buy and sell trades in an electronic order book market. The econometric analysis consists of two parts: In the first part we run a simple probit regression in order to analyze the determinants of the buy/sell decision. In the second part we specify an extended autoregressive conditional intensity model which allows to model the bivariate buy/sell intensity in dependence of marks and time varying covariates.

By using limit order book data from the ASX we examine the determinants of the buy/sell decision as well as the buy/sell trading intensity in dependence of three groups of explanatory factors. In particular, we consider variables associated with characteristics of the previous transaction, variables that reflect the state of the order book after the previous transaction and variables that reflect the limit order arrival process and thus induced changes of the order book between two transactions. The latter type of variables is modelled as time varying covariates.

Our empirical results show that characteristics associated with previous transactions as well as the current state of the order book have a significant impact on the decision when to trade and on which side of the market. Obviously, traders pay strong attention to the limit order process and the corresponding queues of the order book. Our findings are mostly consistent with economic theory based on information-based market microstructure models. Therefore, market participants seem to infer from the order book with respect to the existence of information in the market. Contrarily, we only find weak evidence for the hypothesis that traders try to exploit the state of the order book with respect to tightness and market depth.

References

Admati, A., and P. Pfleiderer (1988): "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies*, 1, 3–40.

Aït-Sahalia, Y., and P. Mykland (2003): "The effects of random and discrete sampling when estimating continous-time diffusions," *forthcoming Econometrica.*

Amihud, Y., and H. Mendelson (1980): "Dealership Markets: Market Making with Inventory," *Journal of Financial Economics*, 8, 31–53.

Bagehot, W. (1971): "The Only Game in Town," *Financial Analysts Journal*, 27, 12–14, 22.

BAUWENS, L., P. GIOT, J. GRAMMIG, AND D. VEREDAS (2000): "A Comparison of Financial Duration Models Via Density Forecasts," Discussion paper, CORE.

BIAIS, B., P. HILLION, AND C. SPATT (1995): "An Empirical Analysis of the Limit Order Book and the Order Flow in the Paris Bourse," *The Journal of Finance*, 50, 1655–1689.

BRÉMAUD, P. (1981): *Point Processes and Queues, Martingale Dynamics*. Springer, New York.

COPELAND, T. E., AND D. GALAI (1983): "Information Effects and the Bid-Ask Spread," *Journal of Finance*, 38, 1457–1469.

COPPEJANS, M., AND I. DOMOWITZ (2002): "An Empirical Analysis of Trades, Orders, and Cancellations in a Limit Order Market," Discussion paper, Duke University.

DIAMOND, D. W., AND R. E. VERRECCHIA (1987): "Constraints on Short-Selling and Asset Price Adjustment to Private Information," *Journal of Financial Economics*, 18, 277–311.

DIEBOLD, F. X., T. A. GUNTHER, AND A. S. TAY (1998): "Evaluating Density Forecasts, with Applications to Financial Risk Management," *International Economic Review*, 39, 863–883.

EASLEY, D., AND M. OHARA (1987): "Price, Trade Size, and Information in Securities Markets," *Journal of Financial Economics*, 19, 69–90.

ENGLE, R. F., AND J. RUSSELL (1998): "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data," *Econometrica*, 66, 1127–1162.

GARMAN, M. (1976): "Market Microstructure," *Journal of Financial Economics*, 3, 257–275.

GIOT, P., AND J. GRAMMIG (2002): "How large is liquidity risk in an automated auction market?," Discussion Paper 2002-23, University of St. Gallen.

GLOSTEN, L. R., AND P. R. MILGROM (1985): "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics*, 14, 71–100.

HO, T., AND H. STOLL (1981): "Optimal Dealer Pricing Under Transactions and Return Uncertainty," *Journal of Financial Economics*, 9, 47–73.

KYLE, A. S. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53 (6), 1315–1335.

LO, A. W., A. C. MACKINLAY, AND J. ZHANG (2002): "Econometric Models of Limit-Order Executions," Discussion paper, MIT.

MADHAVAN, A. (2000): "Market microstructure: A survey," *Journal of Financial Markets*, 3 (3), 205–258.

O'HARA, M. (1995): *Market Microstructure*. Oxford: Basil Blackwell.

RENAULT, E., AND B. J. M. WERKER (2002): "Stochastic Volatility Models with Transaction Time Risk," Discussion paper, Tilburg University.

RUSSELL, J. R. (1999): "Econometric Modeling of Multivariate Irregularly-Spaced High-Frequency Data," Discussion paper, University of Chicago.

STOLL, H. (1978): "The Supply of Dealer Services in Securities Markets," *The Journal of Finance*, 33, 1133–1151.

7. APPENDIX

Table 3: Definition of explanatory variables used in the empirical study.

	Variables observed at each transaction
BUY	1: if trade is buy, 0: otherwise
$SPRD$	bid-ask spread
$TRVB$	traded buy volume
$TRVS$	traded sell volume
MQ	midquote
D_TRPMQB	difference between quoted buy price and midquote, 0 if last trade was sell
D_TRPMQS	difference between midquote and quoted sell price, 0 if last trade was buy
$AVOL$	total volume on ask queue
$BVOL$	total volume on bid queue
A_x	price associated with $x\%$ quantile of cumulated volume on ask queue
B_x	price associated with $x\%$ quantile of cumulated volume on bid queue
ASL	log steepness of ask queue: $ASL = \ln(A_{100} - A_{10}) - avol$
BSL	log steepness of bid queue: $BSL = ln(B_{10} - B_{100}) - bvol$
$DFF1$	$DFF1 = \|B_{10} - mq\| - \|A_{10} - mq\|$
$DFF2$	$DFF2 = \|B_{20} - B_{10}\| - \|A_{20} - A_{10}\|$
$DFF10\_$	$DFF10\_ = \|B_{100} - mq\| - \|A_{100} - mq\|$
	Time varying covariates associated with limit order process
$QASK$	1: if limit order is ask, 0: otherwise
$QBID$	1: if limit order is bid, 0: otherwise
$QVOLA$	quoted volume if limit order is ask, (0 for bids)
$QVOLB$	quoted volume if limit order is bid, (0 for asks)
$DQPRINDA$	1: if quoted ask is lower or equal than current ask price, 0: otherwise
$DQPRINDB$	1: if quoted bid is higher or equal than current bid price, 0: otherwise

Table 4: Descriptive statistics and Ljung-Box statistics associated with the trade and limit order arrival process of NAB stock. Based on the SEATS system of the ASX. Sample period from 02/04/02 to 02/05/02.

Number of trades	12950
Number of buys	5573
Number of sells	7377
Number of limit orders	10273
Number of bid orders	4571
Number of ask orders	5702
Mean trade duration	26.171
Standard deviation of trade durations	42.868
LB(20) statistic for trade durations	5701
Mean buy duration	60.925
Standard deviation of buy durations	108.468
LB(20) statistic for buy durations	2271
Mean sell duration	45.908
Standard deviation of sell durations	69.540
LB(20) statistic for sell durations	2127
Mean limit order duration	32.939
Standard deviation of limit order durations	52.656
LB(20) statistic for limit order durations	6261
Mean ask order duration	59.288
Standard deviation of ask order durations	88.879
LB(20) statistic for ask order durations	2826
Mean bid order duration	74.045
Standard deviation of bid order durations	116.634
LB(20) statistic for bid order durations	2181

Descriptive statistics in seconds. Overnight spells are ignored.

Table 5: Descriptive Statistics of the explanatory variables used in the empirical study. Based on NAB stock. Data extracted from the ASX trading, sample period February 2002.

	Mean	Std.dev.	0.05q	0.1q	0.5q	0.90q	0.95q
	Variables observed at each transaction						
$SPRD$	2.255	1.653	1	1	2	4	5
$TRVB$	3257	4080	70	151	1961	8714	10000
$TRVS$	2629	3651	58	100	1000	7230	10000
MQ	34.775	.580	33.61	33.695	34.88	35.435	35.53
D_TRPMQB	.096	.541	-.5	0	0	.5	1
D_TRPMQS	.352	1.038	-1	-1	.5	1.5	2
$AVOL$	262799	103324	107320	122733	282293	387871	409504
$BVOL$	188511	53483	119158	130305	176093	270723	293433
ASL	-6.325	.502	-6.962	-6.914	-6.545	-5.596	-5.469
BSL	-5.246	.348	-5.736	-5.649	-5.264	-4.831	-4.450
$DFF1$	-.032	.150	-.23	-.16	-.02	.1	.16
$DFF2$	-.039	.296	-.76	-.36	-.01	.24	.37
$DFF10\_$	6.047	5.882	2.71	2.86	4.76	5.47	26.39
	Time varying covariates associated with limit order process						
$QVOLA$	3460	4839	1	100	1635	10000	10000
$QVOLB$	4359	5757	1	100	2500	10000	13800
$DQPRINDA$	.641	.479	0	0	1	1	1
$DQPRINDB$	.725	.446	0	0	1	1	1

Table 6: Probit regressions for the buy/sell decision. Dependent variable: BUY. Explanatory variables enter the regression as *lagged* values. Based on NAB. Data extracted from the ASX trading, sample period February 2002. P-values are based on robust standard errors.

	(NAB)	
	est.	p-value
BUY	-.106	0.268
$TRVB$	.014	0.189
$TRVS$	.009	0.132
D_TRPMQB	.258	0.000
D_TRPMQS	-.173	0.000
$AVOL$	-.899	0.000
$BVOL$	-3.558	0.000
ASL	-.844	0.000
BSL	-.000	0.000
$DFF1$	-.533	0.000
$DFF2$	-.264	0.000
$DFF10\_$	.010	0.000
$SPRD$	-.115	0.000
$QASK$	-.603	0.000
$QVOLA$	.007	0.189
$QVOLB$	.003	0.615
$DQPRINDA$	.320	0.000
$DQPRINDB$	-.636	0.000
constant	45.281	0.000

Table 7: ML estimates of bivariate ACI(1,1) models with static and time-varying covariates for the NAB stock. Data extracted from the ASX trading, sample period February 2002. Standard errors based on OPG estimates.

	(1)		(2)		(3)	
	est.	p-value	est.	p-value	est.	p-value
			ACI parameters buy trades			
ω^1	-0.5129	0.0000	-0.1557	0.1429	-0.2012	0.0894
δ_1^1	-0.2106	0.0000	-0.1834	0.0000	-0.1826	0.0000
δ_2^1	-0.1260	0.0000	-0.1344	0.0000	-0.1354	0.0000
α_1^1	0.0604	0.0000	0.0587	0.0000	0.0597	0.0000
α_2^1	0.0306	0.0000	0.0439	0.0000	0.0440	0.0000
β^1	0.9748	0.0000	0.9157	0.0000	0.9093	0.0000
			ACI parameters sell trades			
ω^2	-0.2945	0.0000	-0.2529	0.0269	-0.2469	0.0287
δ_1^2	-0.1032	0.0000	-0.1077	0.0000	-0.1089	0.0000
δ_1^2	-0.1486	0.0000	-0.1412	0.0000	-0.1390	0.0000
α_1^2	0.0333	0.0000	0.0307	0.0000	0.0306	0.0000
α_2^2	0.0394	0.0000	0.0479	0.0000	0.0455	0.0000
β^2	0.9823	0.0000	0.9409	0.0000	0.9435	0.0000
			Seasonality parameters			
s_1			0.5230	0.2006	0.5026	0.2075
s_2			-0.0068	0.4981	0.0529	0.4855
s_2			-0.9693	0.2961	-1.0626	0.2777
s_4			-0.3628	0.4215	-0.3098	0.4317
s_5			1.1363	0.2221	1.1347	0.2173
s_6			-0.3267	0.2860	-0.3237	0.2833
			Static covariates buy trades			
$TRVB^1$			0.0069	0.0018	0.0057	0.0147
$TRVS^1$			0.0047	0.1716	0.0062	0.1159
D_TRPMQB^1			-0.0699	0.2842	-0.0825	0.2596
D_TRPMQS^1			0.0003	0.4896	-0.0012	0.4584
$AVOL^1$			-0.0233	0.0176	-0.0250	0.0154
$BVOL^1$			0.0000	0.4612	0.0003	0.3606
ASL^1			0.0106	0.0000	0.0101	0.0000
BSL^1			-0.0143	0.0005	-0.0128	0.0014
$DFF1^1$			0.1190	0.1391	0.1094	0.1516
$DFF2^1$			0.0211	0.0219	0.0199	0.0250
$DFF10\_^1$			-0.0178	0.0177	-0.0182	0.0138
$SPRD^1$			0.0005	0.2540	0.0004	0.3026
			Static covariates sell trades			
$TRVB^2$			0.0088	0.0003	0.0075	0.0027
$TRVS^2$			-0.0187	0.0000	-0.0206	0.0000
D_TRPMQB^2			0.0278	0.4235	0.0000	0.4999
D_TRPMQS^2			0.0023	0.4469	-0.0026	0.4426
$AVOL^2$			0.0010	0.4202	0.0032	0.2815
$BVOL^2$			-0.0023	0.0196	-0.0029	0.0051
ASL^2			0.0090	0.0000	0.0094	0.0000
BSL^2			-0.0019	0.2953	-0.0025	0.2350
$DFF1^2$			-0.1117	0.1992	-0.0781	0.2727
$DFF2^2$			-0.0102	0.2620	-0.0079	0.3048
$DFF10\_^2$			0.0114	0.0085	0.0119	0.0055
$SPRD^2$			-0.0035	0.0001	-0.0036	0.0001

Table 7 continued:

Time varying covariates buy trades						
$QASK^1$					0.0124	0.2551
$QVOLA^1$					0.0351	0.0264
$QVOLB^1$					0.0077	0.1919
$DQPRINDA^1$					0.0150	0.0775
$DQPRINDB^1$					0.0199	0.0008
Time varying covariates sell trades						
$QASK^2$					-0.0059	0.3258
$QVOLA^2$					0.0155	0.0267
$QVOLB^2$					-0.0461	0.0014
$DQPRINDA^2$					-0.0341	0.0100
$DQPRINDB^2$					0.0046	0.2700
Obs	12950		12950		12950	
LL	-19733		-19666		-19648	
BIC	-19738		-19671		-19652	
Diagnostics of ACI residuals for the buy series						
Mean of $\hat{\varepsilon}_i^k$	1.0128		0.9997		0.9995	
S.D. of $\hat{\varepsilon}_i^k$	1.0626		1.0437		1.0465	
LB(20) of $\hat{\varepsilon}_i^k$	68.1537	0.0000	42.3155	0.0025	42.8758	0.0021
Test stat.ex. disp.	3.4054	0.0007	2.3573	0.0184	2.5107	0.0121
$\chi^2(20)$ of u_i^k	105.5968	0.0000	83.2247	0.0000	88.1915	0.0000
LB(20) of u_i^k	68.3898	0.0000	45.7203	0.0009	41.5832	0.0031
Diagnostics of ACI residuals for the sell series						
Mean of $\hat{\varepsilon}_i^k$	1.0112		1.0004		1.0002	
S.D. of $\hat{\varepsilon}_i^k$	1.0599		1.0536		1.0522	
LB(20) of $\hat{\varepsilon}_i^k$	31.4560	0.0495	27.6122	0.1189	25.1131	0.1971
Test stat.ex. disp.	3.7453	0.0002	3.3441	0.0008	3.2504	0.0012
$\chi^2(20)$ of u_i^k	101.3869	0.0000	97.2117	0.0000	98.0847	0.0000
LB(20) of u_i^k	29.3952	0.0803	21.4855	0.3691	21.6837	0.3579

Diagnostics: Log Likelihood (LL), Bayes Information Criterion (BIC), diagnostics (mean, standard deviation and Ljung-Box statistics) of ACI residuals $\hat{\varepsilon}_i^k$.

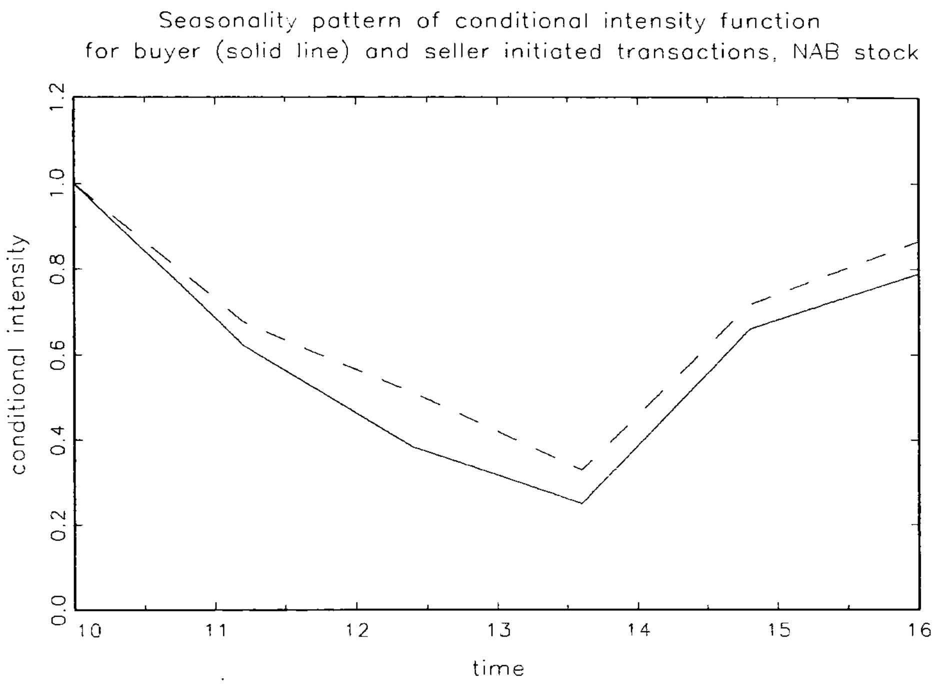

Figure 1: Intraday seasonality function of the buy and sell intensity for the NAB stock. Data extracted from the ASX trading, sample period February 2002.