Dynamic Programming Approach to Image Segmentation and Its Application to Pre-processing of Mammograms

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Abstract

Image segmentation is an important component of image processing since significant time can be saved if a region of interest is extracted by an efficient segmentation algorithm. A dynamic programming image segmentation algorithm is presented. The algorithm is applicable to images with a large matrix of gray levels of pixel values and generates a path separating the object from the background. The report of an application of the proposed algorithm to digitised mammograms complements its description.

Keywords: digitised mammography, image segmentation, dynamic programming, image border detection

1. Introduction

Digital image processing plays an increasing role in medical applications. In such applications image processing normally includes preprocessing, segmentation and summarization such as classification. Segmentation of an image into several medically meaningful objects is achieved by boundary and texture analysis, where a boundary of an image is defined as a narrow region where changes in texture occur. Edge enhancement and border detection are important components of image segmentation.

A common approach in digital processing of a mammogram containing an image of breast tissue is the utilization of a so-called mask. A mask is a data file specifying for each pixel either one or zero. Value one indicates that the corresponding pixel is relevant to the image of the breast tissue, whereas zero specifies a pixel relevant to the background. Utilization of a mask allows an algorithm to focus on the analysis of the actual image and significantly improves and speeds up the classification of benign and malignant cases. The generation of a mask is based on the location of the image boundary. In this paper this task is viewed as a two-stage procedure comprising edge enhancement and boundary detection.

Section 2 presents a brief survey of relevant publications. Section 3 describes an image segmentation algorithm based on the idea of dynamic programming. Section 4 reports the results of implementation of this algorithm to pre-processing of digitised mammograms and Section 5 indicates the direction of further research.

2. A Brief Literature Survey

Literature on digital processing of mammograms spans several decades. As early as in 1976, Sklansky [7] pointed out the importance of boundary detection as a method allowing (a) to reduce the area of subsequent search and processing (b) to facilitate the computerized normalization and equalization of gray level within the breast tissue region, and (c) to facilitate the comparison of the corresponding regions of the left and right breasts.

Chandrasekhar [5] suggested that the detection of the boundary of the breast should be followed by locating the image of the nipple, which significantly facilitates the subsequent search for breast cancer. Yin *et al* [9] have presented a method of locating the nipple on mammograms that relies on the average intensities of small image regions along the breast border.

Several methods have been developed for the detection of the boundary of breast image. Suckling *et al* [8] have published a method of segmentation of mammograms utilizing multiple linked self-organizing neural networks. Their algorithm can separate the whole mammogram into four major components: background (nonbreast area), pectoral muscle, fibroglandular region (parenchyma), and adipose region. This is a quite complicated algorithm involving training on sample data taken from a mammogram. Bick *et al* [3] have used a local grey-value range and a modified global histogram to outline the breast border on mammograms. Chandrasekhar and Attikiouzel [6] suggested an algorithm requiring testing of several polynomials and choosing one which gives the best result by visual inspection.

The algorithm presented in this paper is based on a dynamic programming approach. Undertaken computation experiments have proven its high efficiency for tracing the boundary of the breast image. The proposed algorithm can be easily implemented with any high level programming language.

3. Description of Algorithm

Although the approach described below can be applied to various image processing problems, in what follows we describe this algorithm in the context of boundary detection for a breast image in digitised mammograms. Normally the mammogram images are taken from the two othogonal views, i.e., cranio caudal (CC) and medio lateral (ML). The resulting appearances are quite different. In the case of the ML view, the curve of the boundary of the breast image is not necessarily monotonically increasing from the bottom of the mammogram to its top. Correspondingly our algorithm locates monotonically increasing segments of the boundary and subsequently combines them into the complete boundary. Due to anatomical reasons the algorithm produces two such monotonical segments. The algorithm also assumes that the breast image is located to the left from its boundary. This assumption is not a restriction because either a similar algorithm can be applied to the opposite orientation of the image, or the latter can be transformed into the desired orientation by simple manipulations with the pixel matrix. Drawing of each monotonic segment of the boundary is achieved in two stages - edge enhancement and curve (boundary) detection. These two stages of the processing are described in detail in the following sub-sections.

3.1 Edge Enhancement

The purpose of edge enhancement is to make the image boundary more evident by trying to assign high gray level values only to the pixels belonging to the boundary. In order to do so, most of these methods consider for each pixel its neighbourhood and assign the resulting value based on the analysis of the gray levels associated with the pixels in this neigbourhood. Larger variation of the gray levels in the neighbourhood leads to a higher value assigned to the considered pixel. Therefore, different methods of edge enhancement differ in neighbourhood definition and in calculation of the resulting value for the considered pixel. Computational experiments led us to the conclusion of combining the following two techniques in a two-step edge enhancement procedure.

Let A be an $m \times n$ matrix of data in a digitised mammogram, i.e. each entry A_{ij} in matrix A is the gray level of image. The first step of edge enhancement is the calculation of $m \times n$ matrix C, where the first column of C is a zero vector and any other ith column of C is the absolute value of the difference between the ith and the $(i-1)$st columns of A. Calculating matrix C we expect that relatively large gray values will be obtained at the image boundary due to the abrupt change of gray level values in adjacent columns in matrix A. Further edge enhancement is achieved by applying to C the gradient modulus technique recommended by Ballard and Sklansky [1]. The method generates a new matrix, where the (i,j)th entry is

$$\begin{aligned}\{[C_{(i+k)j} &- C_{(i-k)j}]^2 + \\ [C_{i(j+k)} &- C_{i(j-k)}]^2\}^{\frac{1}{2}}.\end{aligned}$$

It uses a variable k which specifies how far apart on the picture grid the differences are taken. The quantity $(2k+1)$ is known as the span of the gradient. The drawback of this method is that it produces a result which is smaller than the original image, i.e. the method does not specify how to calculate the first k and the last k rows and similarly for columns of the resulting matrix. In order to produce an image which has the same size as the original one, an image extension technique called symmetric reflection of the image about the edge is used [4]. This technique generates a $(m+2k) \times (n+2k)$ matrix D by duplicating some columns and rows of C, which is subsequently used in calculating an $m \times n$ matrix H, where each (i,j)th entry is

$$\begin{aligned}H_{ij} = \{&[D_{(i+k)j} - D_{(i-k)j}]^2 + \\ &[D_{i(j+k)} - D_{i(j-k)}]^2\}^{\frac{1}{2}}.\end{aligned}$$

3.2 Curve (boundary) Detection

The selection of an algorithm for the curve detection depends on the definition of the image boundary. It was found that the definition below leads to particularly good results of computational experiments. Any segment s of the image boundary can be viewed as a sequence of pixels $p_1, ..., p_{\mu(s)}$, where $\mu(s)$ is the number of pixels in the considered segment. If two pixels, p_e and p_{e+1}, are successive elements of such a sequence, they should be in some sense close to each other. This closeness can be defined by introducing for each pixel p a neighbourhood $N(p)$ - the set of all pixels that can be immediate predecessors of p in such a sequence. In other words, the sequence s should satisfy the condition $p_e \in N(p_{e+1})$. Moreover, a segment of image boundary should start with an element belonging to some pre-specified set of possible initial (starting) pixels and should terminate with a pixel belonging to some pre-specified set of possible terminal pixels. Let I and T be the sets of initial and terminal pixels, respectively, where $I \cap T = \emptyset$ and $N(p) = \emptyset$ for any $p \in I$. As above, the elements of any sequence s of pixels will be denoted by $p_1, ..., p_{\mu(s)}$. Observe that the number of elements $\mu(s)$ in the sequence s may vary from sequence to sequence. Denote by B the set of all sequences s such that

(s1) $p_1 \in I$;

(s2) $\{p_1, ..., p_{\mu(s)}\} \cap T = \{p_{\mu(s)}\}$;

(s3) $p_e \in N(p_{e+1})$ for all $1 < e < \mu(s)$.

Let $H_{i_p j_p}$ be the entry of the matrix H associated with a pixel p and let for any $s \in B$

$$G(s) = \sum_{p_e \in s} H_{i_{p_e} j_{p_e}}.$$

We assume that the system of neighbourhoods is consistent, that is all elements of any $s \in B$ are distinct. This assumption is quite natural and does not impose a serious restriction because we consider only segments of the image boundary. For example, it is often convenient to view the image boundary as a composition of segments, where for any two pixels p' and p'' which are successive pixels in some segment, $i_{p'} \leq i_{p''}$ and $j_{p'} \leq j_{p''}$ and at least one of these two inequalities is strict. Then the segment of image boundary can be defined as an element in B with the largest value of $G(s)$.

We will say that a pixel p is reachable if either $p \in I$ or there exists a sequence p_1, ..., p_q such that

(r1) $p_1 \in I$;

(r2) $p_i \notin T$ for all $1 \leq i \leq q$;

(r3) $p_i \in N(p_{i+1})$ for all $1 \leq i < q$;

(r4) $p_q \in N(p)$.

The set of all reachable pixels will be denoted by R. For any $p \in R$, the largest possible number of elements in a sequence satisfying (r1)-(r4) will be denoted by $l(p)$. Observe that $l(p) = 0$ implies $p \in I$, and $R \cap T = \emptyset$ implies that the desired segment does not exist.

Let R_h be the set of all $p \in R$ such that $l(p) = h$ for all $0 \leq h \leq \max_{v \in R} l(v)$. Then using the forward recursion of dynamic programming [2], a segment of the image boundary can be found by considering the sets R_h in the increasing order of indices h, that is we first consider all pixels from $R_0 = I$, then all pixels from R_1, and so on. For each pixel $p \in R_0$ we assign

$$\varphi(p) = H_{i_p j_p},$$

and for each pixel $p \in R_h$, where $h > 0$, we calculate

$$\varphi(p) = H_{i_p j_p} + \max_{g \in N(p)} \varphi(g)$$

and record pixel g, denoted by $r(p)$, on which $\max_{g \in N(p)} \varphi(g)$ has been attained. Let p^* be a pixel such that

$$\varphi(p^*) = \max_{g \in T} \varphi(g),$$

then the sequence p_1, ..., p_u, where $p_u = p^*$, $p_1 \in I$ and $p_e = r(p_{e+1})$ for all $e < u$, is a segment of the image boundary. Moreover, denoting this sequence by s^*, we have

$$\varphi(p^*) = G(s^*) = \sum_{p_e \in s^*} H_{i_{p_e} j_{p_e}}.$$

3.3 Two-segment Structure of a Breast Image

In order to obtain the boundary of breast image, the enhancement technique and the dynamic programming tracing approach described in sections 3.1 and 3.2 were applied twice to a digitised mammogram. In other words, the image boundary was considered as a combination of two segments. For the first segment, the set I was chosen as the bottom row in the matrix H and the set T was the top row in this matrix. The neighbourhood of a pixel corresponding to the entry H_{ij} was the set of all pixels corresponding to entries $H_{(i+1)y}$ such that $\max\{1, j-w\} \leq y \leq \min\{n, j+w\}$. The selection of the value for w depended on resolution of the image where n is the number of columns in H. We have found that the generated first segment detects the boundary of breast images in most of the cases of CC view but not necessarily in the cases of the ML view due to anatomical reasons. In the latter case this first segment identifies a point which has the furthest distance from the left edge of the image. This point was then used to locate a horizontal reference line dividing the original image into 2 portions. The reference line corresponds to a row, say row z, in the matrix H, which also partitions this matrix into lower and upper portions. This reference line belongs to the lower portion. The lower portion of H was then considered separately. For the lower portion of H the set I was the first column of this portion of H and the set T was the last column of the lower portion. The neighbourhoods were defined in a similar way: the neighbourhood of a pixel corresponding to the entry H_{ij} was the set of all pixels corresponding to entries $H_{y(j-1)}$ such that $\max\{z, i-w\} \leq y \leq \min\{m, i+w\}$ where m is the number of row in H. The two obtained segments were then combined into the whole image boundary.

4. An Example

Results for two images (CC and ML) are presented using the algorithm as described. The original images are shown in Figures 1(a) and 2(a) respectively and Figures 1(b) and 2(b) demonstrate the edge enhancement results. The results of two-stage processing are shown in Figures 1(c) and 2(c). Finally, Figures 1(d) and 2(d) show the original images together with the resulting boundary.

5. Conclusion

The results for two images given in the example demonstrate feasibility of the presented approach for image segmentation as a preprocessing of mammograms. The attractive feature of this method is that it provides the coordinates of breast boundary points. This crucial information takes up very little file space and a mask of the breast image is easily generated using this coordinate information. Further digitised mammogram images are being processed and we intend to compare our detection results with hand tracing results by an experienced radiologist on the same set of images.

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