

APPROXIMATION OF PRICES FOR
AVERAGE-TYPE OPTIONS VIA BOUNDS



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Declaration of authorship

I certify that the work in this thesis has not previously been submitted for a degree nor been submitted as part of requirements for a degree. I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Scott Alexander

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Dedication

For my mother Denise, father John and brother Tim.

Abstract

The problem of pricing multi-dimensional arithmetic average-type options is a complex problem both analytically and numerically, analytically because the distribution of the average on which the payoff function depends is unknown in closed-form and numerically because of the high dimensionality of the problem. In this thesis we develop methods that avoid these issues by providing price approximations in the form of lower and upper bounds. We do so by approximating the event that the option finishes in-the-money with a closely related proxy and use an optimisation procedure to tighten up the bounds. The result is a pricing tool that provides accurate approximations without suffering from the exponential curse of dimensionality associated with other techniques. The method is developed for underlying assets modelled as exponential Lévy processes and numerical examples are provided under a variety of these models.

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