A General Electromagnetic Field-Circuit Coupling Method Based on Time-Stepping FEA for Performance Analysis of PWM Switching Converters Considering Hysteresis Effects

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Abstract

Considering the special characteristics existing in the PWM switching converter, a general method for the time-stepping finite element analysis (FEA) based electromagnetic field coupling with its feeding circuit used in the analysis of PWM switching converter considering hysteresis effects is introduced in this paper. Comparing with the electromagnetic field-circuit indirect coupling method (ICM), the proposed method has overcomed the drawback that the ICM cannot take the hysteresis effects into account. Compared with the electromagnetic field-circuit direct coupling method (DCM), the proposed method has the similar accuracy but higher efficiency. Further more, like the ICM, the proposed method also divides the system with higher state dimensions produced by the DCM into two sub-systems with lower state dimensions; this may reduce the algorithm convergence problem which often happens in high dimensional systems.

Index Terms

General field-circuit coupling method, indirect coupling method, direct coupling method, time-stepping finite element analysis (FEA), PWM switching converter, hysteresis effect.

I. INTRODUCTION

Compared with conventional power converters, the PWM switching converter with high frequency has higher power density and higher efficiency, and is widely used in different applications.[1] In a converter, a transformer or an inductor with soft magnetic materials is usually employed and fed by both the input circuit and output circuit.[2] To predict its dynamic performance, the time-stepping finite element analysis (FEA) based electromagnetic field-circuit direct coupling method (DCM) is often used.[3] As the FEA is executed in each step as its external circuit being executed by the DCM, compared with the electromagnetic field-circuit indirect coupling method (ICM), it has higher accuracy but lower efficiency in most applied fields.[4,5] The other characteristic of ICM is that it can divide the system with higher state dimensions produced by the DCM into two sub-systems with lower state dimensions; this may reduce the algorithm convergence problem which often happens in high dimensional systems.

The DCM may be necessary for performance analysis of electromagnetic devices with moving parts (e.g. electrical machines) as the position of some elements is variable against time. For the static electromagnetic device of converter, as the position of all the elements is fixed and the eddy current loss can be ignored for the low conductivity magnetic materials, the ICM can be used with the same accuracy as the DCM but higher efficiency. For the performance analysis, the traditional ICM cannot include the hysteresis effects, as the equivalent circuit parameters such as differential inductance of the transformer vary with historical operational point and the present exciting current under the hysteresis effect and they cannot be predicted during the execution of its external circuit.

In order to achieve satisfactory accuracy, the time of each step is often chosen smaller than the period of PWM; this causes the low efficiency of DCM. In fact, the switching state of PWM converters is decided by the external circuit and will change once at most during the period of one PWM, so the FEA only needs to be executed for limited times to obtain the equivalent circuit parameters against different current; this will decrease the number of FEAs being executed. For that, an improved coupling method based on the system switching state observer, which takes advantage of both the direct and indirect methods, is proposed in this paper. In the proposed model, an improved scalar Preisach model which only needs the limiting B-H loop is used to model the hysteresis of each element of soft magnetic cores.[6] The complete system model is implemented in Matlab/Simulink, which is widely used by electrical, electronic and control engineers. This makes the proposed model very convenient and flexible to be coupled with external circuits. As an example, an existing flyback dc/dc converter is simulated. The switching frequency of flyback dc-dc converter is 67 kHz, and soft ferrite core (EE-25, TDK PC-40) is used for the transformer. The effects of magnetic hysteresis on the converter performance are clearly shown by the simulation results with and without incorporating the Preisach hysteresis model.

II. COUPLING FEA WITH HYSTERESIS MODEL AND FEEDING CIRCUIT

A. Governing Equation of the Transformer

The general equation for a soft magnetic material can be expressed as

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \mathbf{M}/\mathbf{H})\mathbf{H} = \mu_0\mu_r\mathbf{H} \tag{1}$$

$$\mu_\mathrm{r} = 1 + \mathbf{M}/\mathbf{H} \tag{2}$$

where **B** is the magnetic flux density, μ_0=4π×10^{-7} (H/m) the permeability of free space, and μ_r is the relative permeability and can be determined by the Preisach model.

When the eddy currents are ignored, the magnetic field in the transformer is governed by the Maxwell's equation

$$\nabla \times \mathbf{H} = \mathbf{J}_s \tag{3}$$

where $\mathbf{J_s}$ is the applied current density. Defining a vector magnetic potential **A** by

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{4}$$

We can rewrite (3) in terms of **A** as

$$\nabla \times \left(\frac{1}{\mu_r} \nabla \times \mathbf{A} \right) = \mu_0 \mathbf{J}_s \tag{5}$$

For the 2-D case, this can be expressed as

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_\mathrm{r}} \frac{\partial A_\mathrm{z}}{\partial x} \right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu_\mathrm{r}} \frac{\partial A_\mathrm{z}}{\partial y} \right) = \mu_0 J_\mathrm{z} \tag{6}$$

where A_z and J_z are the z components of **A** and $\mathbf{J_s}$.

Applying the Galerkin's weighted residual approach to (6), we obtain the matrix system equation that describes the magnetic field of the transformer with a nonlinear soft magnetic core as

$$[\mathrm{K}] \cdot \{\mathrm{A}\} = \{\mathrm{F}\} \tag{7}$$

where

$$\begin{cases} K_{ij} = \int_\Omega \frac{1}{u_r} (\nabla N_i \cdot \nabla N_j) \cdot d\Omega \\ F_i = \int_\Omega (N_i \mu_0 J_z) \cdot d\Omega \end{cases} \tag{8}$$

and N_i is the shape function of a finite element.

Solving (7) with the relative permeability determined by the Preisach model and the applied current density steps in time, we can obtain the magnetic field distribution in the transformer in each time step. To the flyback converter, its differential inductance of the transformer primary winding can then be calculated by

$$\begin{cases} L_p = L(i^k) = \dfrac{d\psi^k}{di^k} = \dfrac{\psi^k - \psi^{k-1}}{i^k - i^{k-1}} \\ \psi^k = 2W_k / i^k \end{cases} \tag{9}$$

where ψ is the flux linkage of the primary winding, i the winding current, W the magnetic energy stored in the transformer, and the superscript k refers to the k-th time step.

B. Hysteresis Modeling

Among various magnetic hysteresis models, the Preisach model appears to be practical and easy-to-use.[6] By an improved Preisach model,[7] the magnetization, **M**, corresponding to magnetic field strength, **H,** can be derived as

$$M(H) = T(H, -H), \quad \text{(initial curve)} \tag{10}$$

$$M(H) = M(H^k) - 2T(H^k, H), \text{ (downward trajectory)} \tag{11}$$

$$M(H) = M(H^k) + 2T(H, H^k), \text{ (upward trajectory)} \tag{12}$$

where H^k is the field strength of the last or k-th reversal point,

$$T(\alpha, \beta) = \frac{M_u(a) - M_d(\beta)}{2} + F(a)F(-\beta) \tag{13}$$

$$F(a)=\begin{cases}[M_d(a)-M_u(a)]/\sqrt{4M_d(a)} & ;(a\geq 0)\\ \sqrt{M_d(-a)} & ;(a<0)\end{cases} \tag{14}$$

and M_u and M_d are the upward and the downward trajectories of the limiting loop, respectively.

As the magnetization *M(H)* is determined by not only the current applied field *H* but also the history, $M(H_k)$, (10)-(14) can be expressed symbolically as

$$M(H)=f(H,H^k,M^k) \tag{15}$$

C. Detailed Switching Model of Feeding Circuit

According to the principle of building the detailed switching model, the state-space variable equations in a uniform format can be given as follows:

$$\begin{cases}s(t)=f(X,U,t)\\ \dot{X}(t)=A(s)X(t)+B(s)U(t)\\ Y(t)=C(s)X(t)+D(s)U(t)\end{cases} \tag{16}$$

where *s(t)* is the switch state space variable vector in which all switching parts such as MOSFET-M and diode-D are included (its value being either 0 or 1), *X(t)* the state variable vector of system, *Y(t)* the output variable vector of system, *U(t)* the input variable vector in which all the given input voltage, the constant voltage of diode and

PWM signal are included, *A(s), B(s), C(s)* and *D(s)* are the corresponding coefficient matrices controlled by the states of the switches.

It should be pointed out that in (16), *s(t)=f(X, U, t)* is also the system switching state observer. By using the discrete method, the discrete equation of the system switching state observer is obtained.

$$s(t)^k = f(X^{k-1}, U^{k-1}, t^{k-1}) \tag{17}$$

where the superscripts *k* and *k*-1 refer to the *k*-th and *(k-1)*-th steps, respectively. This switching state observer is executed step by step synchronously to each step of the external circuit solution.

D. Coupling Method

Firstly, the converter circuit topology is determined by the operational state of switching (power MOSFET-M and diode-D). According to (17), all the switching states at present can be obtained. Among the switches, there is a main switch which controls the operational state of the transformer. When the main switch turns on, the magnetic part operates in the excited magnetic state.

By using the Preisach model, the *M-H* curve of each finite element is obtained under the given excitation current, and then by using the nonlinear FEA, some equivalent circuit parameters are obtained. As the maximum number that the switching state changes in one PWM duty cycle is two, the hysteresis model will be executed twice in one PWM duty cycle at most, which is much less than the times that the external circuit solution is executed. The number of FEA being executed under the given excitation current, which is also much less than the times that the external circuit solution is executed, will be decided by the accuracy of calculation. Solving the external circuit model (17) with the equivalent circuit parameters which are obtained through the FEA with the Preisach model and stored in the form of a look-up table, the transient performance of the converter can be obtained.

III. SIMULATION AND EXPERIMENT RESULTS

An existing flyback dc-dc converter is simulated as an example. Table I shows the detailed specifications of the converter.

TABLE I

SPECIFICATIONS OF THE FLYBACK CONVERTER

Input voltage: 370 Vdc	Frequency: 67 kHz
Output voltage/ current: 5V/3.6A	Duty ratio: 0.33
Number of turns of the primary winding: 96	Number of turns of the secondary winding: 8
Material of the core of transformer: TDK PC40-EE25 ferrite core	

The model is implemented in the MATLAB/Simulink environment and the key simulation blocks are shown in Fig.1. By inputting the parameters and running the proposed model, the transient performance of the converter is obtained and the results are depicted in Figs. 2 to 5. Fig.2 shows the magnetization trajectory in an element in the middle of the transformer core, and Fig.3 shows the differential inductance versus current.

To examine the effects of magnetic hysteresis on the converter performance, two simulations are conducted with and without incorporating the Preisach model of magnetic hysteresis. In Fig.2, it can be seen that in the PWM switching mode, the magnetic operating point of transformer core follows a biased minor loop, with the minimal equivalent magnetic flux density of B greater than zero. This agrees with the phenomenon that the minimal current shown in Fig.5 is also greater than 0. In Fig.4, it can also be seen that the effects of magnetic hysteresis on the converter performance is to reduce the duty cycle of PWM and may cause an over shoot current. Fig.5 depicts the output voltage and current, which agree very well with the actual performance of the converter.

V. CONCLUSION

In this paper, a general method of the time-stepping FEA combined with the magnetic hysteresis and external circuits for performance analysis of PWM switching converters based on the system switching state observer is presented in detail. A MATLAB/Simulink based simulation model of the converter is built. As an example, an existing flyback dc/dc converter is simulated. The effects of magnetic hysteresis on the converter performance are clearly shown by the simulation results with and without incorporating the Preisach model of magnetic hysteresis.

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Figure captions

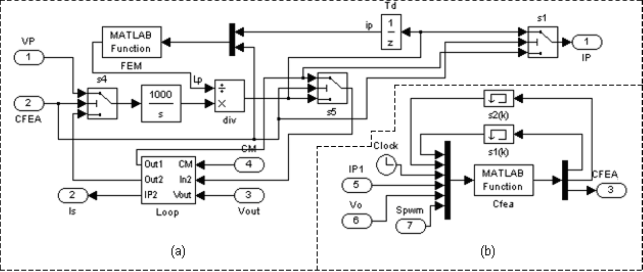
FIG.1. Key simulation blocks: (a) simulation model of time-stepping FEA coupling with the external circuits; (b) the system state observer.

FIG.2. The equivalent B-H loop (outer), and the operational B-H loop (inner)

FIG.3. Differential inductance at different currents

FIG.4. Red (inner) line is obtained by considering hysteresis; blue (outer) line is obtained by considering the initial B-H curve only

FIG.5. Converter performance: Output voltage V_O and output current I_O



Td
ip
1
z
s1
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s4
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CFEA
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Out2
In2
2
Is
IP2
Vout
3
Vout
Loop
s2(k)
s1(k)
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IP1
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MATLAB
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CFEA
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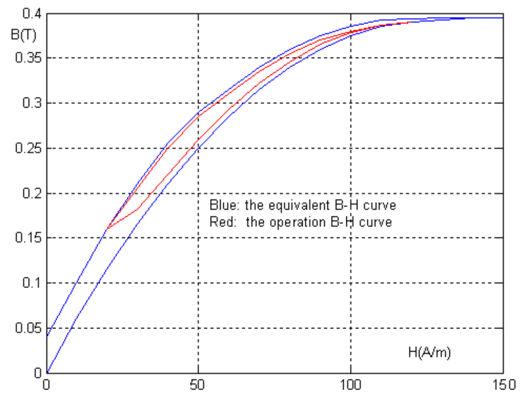
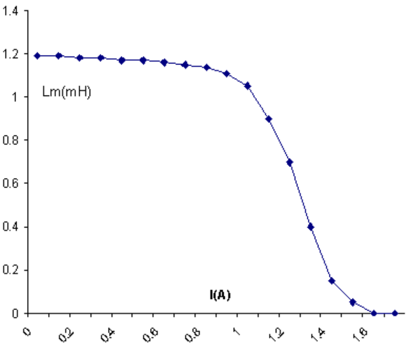
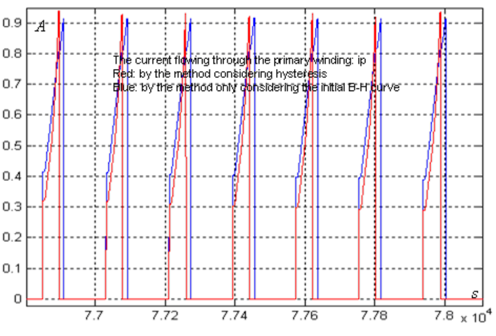
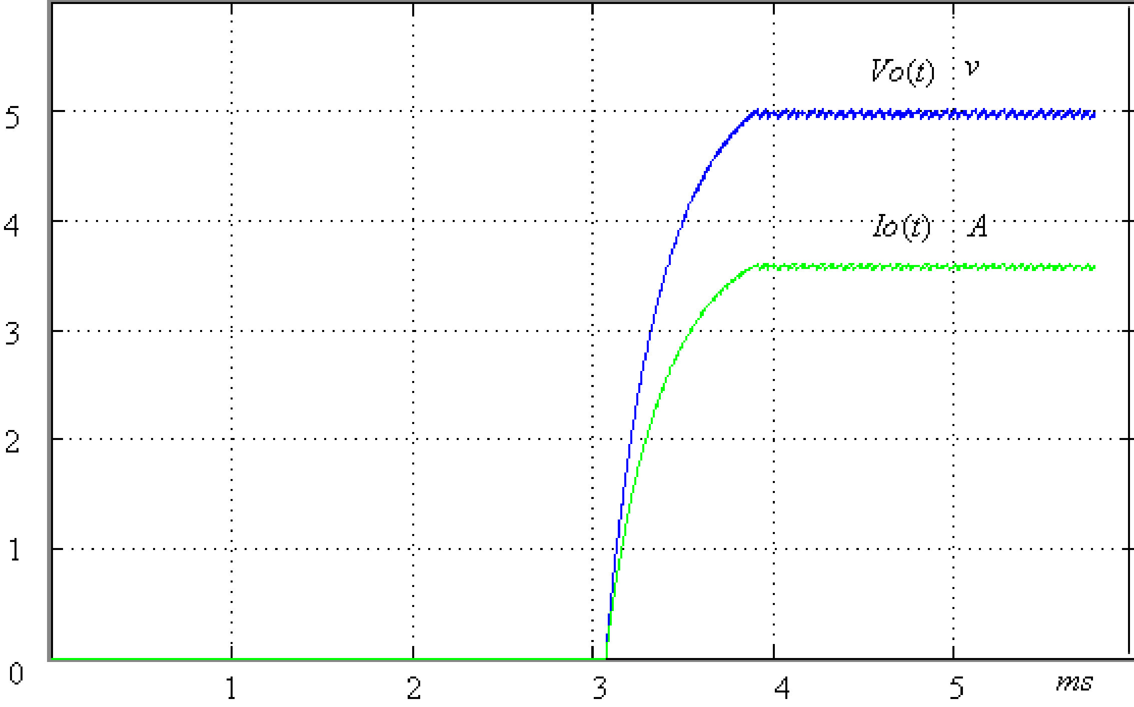
Vo
6
Spwm
7
(a)
(b)


B(T)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Blue: the equivalent B-H curve
Red: the operation B-H curve
H(A/m)
0
50
100
150



1.4
1.2
1
0.8
0.6
0.4
0.2
0
Lm(mH)
I(A)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


A
The current flowing through the primary winding: ip
Red: by the method considering hysteresis
Blue: by the method only considering the initial B-H curve
s
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
7.7
7.72
7.74
7.76
7.78
7.8
x 10^4



Vo(t) v
Io(t) A
0
1
2
3
4
5
1
2
3
4
5
ms