Robust Optimization in HTS Cable Based on Design for Six Sigma

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The nonuniform ac current distribution among the multilayer conductors in a high-temperature superconducting (HTS) cable reduces the current capacity and increases the ac loss. Various numerical simulation techniques and optimization methods have been applied in structural optimization of HTS cables. While the existence of fluctuation in design variables or operation conditions has a great influence on the cable quality, in order to eliminate the effects of parameter perturbations in design and to improve the design efficiency, a robust optimization method based on design for six sigma (DFSS) is presented in this paper. The optimization results show that the proposed optimization procedure can not only achieve a uniform current distribution, but also improve significantly the reliability and robustness of the HTS cable quality, comparing with those by using the particle swarm optimization.

***Index Terms*—Current distribution, design for six sigma, high-temperature superconducting (HTS) cable, particle swarm optimization (PSO).**

I. Introduction

THE high-temperature superconducting (HTS) cable for large current transmission in general has a multilayer structure consisting of parallel connected tapes, twisted in each layer. Due to the difference of inductances among layers, the currents flowing in these layers are different. One effective method to obtain a uniform current distribution is to alternate the inductive impedances of layers by adjusting the structural parameters of the cable conductors. Some optimization methods, such as the genetic algorithm (GA) and the particle swarm optimization (PSO), have been applied to optimize the structural parameters of HTS cable conductors to achieve the mostly uniform current distribution among layers [1].

Most real-world engineering problems involve at least an element of errors and uncertainties in design process, as well as in manufacturing process. Traditional optimization methods cannot take into account the perturbations, so they may lead to unreliable or nonrobust solutions.

The idea of robust optimization considering both the optimality and the robustness of objective function and constraints has attracted attention for real-world design problems in recent years. There are a number of methods, such as sensitivity-based method [2] and Taguchi method [3], [4], which may deal with the robustness of objectives. The rapidly growing current push in industry with respect to managing uncertainty and seeking quality products is the design for six sigma (DFSS) [5], [6]. The main purpose of DFSS is to prevent defects at design stage instead of fixing them at later stages, and also to improve quality up to 6σ level.

In this paper, the robust design method based on DFSS is introduced for HTS cable optimization. The philosophy of DFSS in quality engineering is applied in this optimization procedure to improve the process quality and design reliability. The optimized results are compared with those obtained by PSO only.

II. Particle Swarm Optimization Method

The particle swarm optimization (PSO) method is a population based stochastic optimization technique developed in 1995 by Kennedy and Eberhart, inspired by the social behavior of birds flocking and fish schooling [7].

Suppose the search space has D-dimensions. The position of the ith particle in the swarm can then be expressed as a vector $\boldsymbol{X}_i(t) = (X_{i,1}(t), X_{i,2}(t), \ldots, X_{i,D}(t))$. The velocity of this particle can be represented by another vector $\boldsymbol{V}_i(t) = (V_{i,1}(t), V_{i,2}(t), \ldots, V_{i,D}(t))$. The ith particle also maintains a memory of its previous best position in the vector $\boldsymbol{pbest}_i = (pbest_{i,1}, pbest_{i,2}, \ldots, pbest_{i,D})$ and in each iteration step, the vector $\boldsymbol{gbest} = (gbest_1, gbest_2, \ldots, gbest_D)$ is designated as the index of the best particle in the swarm. Subsequently, the swarm is manipulated according to the following two equations:

$$V_{i,d}(t) = wV_{i,d}(t-1) + c_1 r_1 \times (pbest_{i,d} - X_{i,d}(t-1)) / \Delta t + c_2 r_2 \times (gbest_d - X_{i,d}(t-1)) / \Delta t \quad (1)$$

$$X_{i,d}(t) = X_{i,d}(t-1) + V_{i,d}(t) \cdot \Delta t \quad (2)$$

where $d = 1, 2, \ldots, D$ is the size of search space, $i = 1, 2, \ldots, M$, M is the size of the swarm, c_1 and c_2 are two positive constants, namely social and cognitive parameters, r_1 and r_2 are two random numbers distributed within the range [0,1], t is the iteration number, $\Delta t = 1$ and w is the inertia weight.

III. Robust Optimization Using Design for Six Sigma

The six-sigma methodology was proposed at Motorola and developed into DFSS at General Electric (GE). DFSS is one of the robust optimization approaches. Here the term "sigma" refers to standard deviation σ, which is a measure of dispersion, and the performance level of 6σ is equivalent to 3.4 defect parts per million (PPM), while at 3σ level (the average sigma level for most companies) the defect ratio is about 66 800 PPM.

In a traditional optimization problem, the objective function $f(\boldsymbol{X})$ of design variable $\boldsymbol{X}$ should be minimized and subjected to constraints $g_k(\boldsymbol{X})$ as follows:

$$\text{Minimize: } f(\boldsymbol{X}) \quad (3)$$

$$\text{subject to: } g_k(\boldsymbol{X}) \geq 0, \quad k = 1, \ldots, \text{Number of constraints}$$

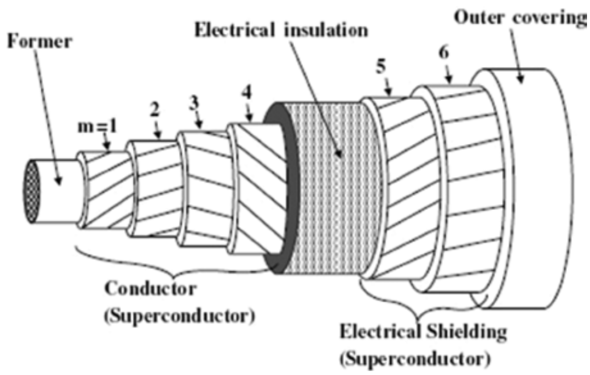

Fig. 1. Schematic diagram of a cold dielectric type HTS cable.

In DFSS, this normal optimization problem is rewritten to the problem in which both the mean value μ and standard deviation σ of $f(\boldsymbol{X})$ should be minimized as follows:

$$\begin{aligned}&\text{Minimize:}\quad F\left(\mu\left(f(\boldsymbol{X})\right),\sigma\left(f(\boldsymbol{X})\right)\right)\\&\qquad\qquad=\mu\left(f(\boldsymbol{X})\right)+\omega_1\sigma\left(f(\boldsymbol{X})\right)\\&\text{subject to:}\quad \mu\left(g_k(\boldsymbol{X})\right)-n\sigma\left(g_k(\boldsymbol{X})\right)\geq 0\\&X_L+n\sigma(x)\leq\mu(x)\leq X_U-n\sigma(x)\\&\mu\left(f(\boldsymbol{X})\right)-n\sigma\left(f(\boldsymbol{X})\right)\geq \text{ Lower specification limit}\\&\mu\left(f(\boldsymbol{X})\right)+n\sigma\left(f(\boldsymbol{X})\right)\leq \text{ Upper specification limit}\end{aligned}\quad (4)$$

where ω_1 is the weighting factor, and n denotes the sigma level. The set of design variables $\boldsymbol{X}$ includes the input design variables. X_L and X_U are the lower and upper threshold of one deign variable x.

The key to implementing this six sigma-based robust optimization formulation is the ability to estimate performance variability statistics to allow the reformulation of objectives and constraints, as in (4). In this paper, the Monte Carlo simulation techniques are implemented by giving the stochastic sampling values of each particle defined in PSO and the statistical natures, such as mean on target, variance of the objectives, and constraints can be assessed. The differences between traditional PSO and DFSS are not only the formulations of objectives and constraints, but also the statistic assessment procedure.

IV. Robust Optimization Model of HTS Cable

A. Model of HTS Cable

The structure of a single-phase cold dielectric type HTS power cable, which consists of four layers of conductors and two layers of shield of Ag/Bi-2223 tapes, is shown in Fig. 1.

An equivalent circuit of the cold dielectric type HTS cable consisting of m layers of superconductors and $(N-m)$ layers of shields was established in [1]. N is the total of layers. The resistance of each layer is determined by the V-I characteristics of the conductor layer. The self-inductance and mutual inductance are calculated based on the computation of electromagnetic field.

B. Optimization Models

As described in [1], the winding angle β, winding direction α, and radius R of each layer are regarded as the design variables. For a cable of N layers, the optimized variables can be expressed as a vector

$$\boldsymbol{X}=[\beta_1,\alpha_1,R_1,\beta_2,\alpha_2,R_2,\cdots,\beta_N,\alpha_N,R_N].$$

Without quenching, the objective function for optimization of the cold dielectric type HTS cable is derived to achieve the uniform current distribution among the conductors and the shields, respectively, and can be described as

Minimize:

$$\begin{aligned}f(\boldsymbol{X})=&\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}|I_{ix}(\boldsymbol{X})-I_{jx}(\boldsymbol{X})|+\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}|I_{iy}(\boldsymbol{X})-I_{jy}(\boldsymbol{X})|\\&+\sum_{i=m+1}^{N-1}\sum_{j=i+1}^{N}|I_{ix}(\boldsymbol{X})-I_{jx}(\boldsymbol{X})|+\sum_{i=m+1}^{N-1}\sum_{j=i+1}^{N}|I_{iy}(\boldsymbol{X})-I_{jy}(\boldsymbol{X})|\end{aligned}\quad (5)$$

where $I_{ix}(\boldsymbol{X})$ and $I_{iy}(\boldsymbol{X})$ are the real and imaginary components of current $I_i(\boldsymbol{X})$ in the i th layer. m is the number of superconductor layers.

The current distribution among layers should become more uniform when $f(\boldsymbol{X})$ is closer to a minimum value. Under the constraints of the mechanical properties and critical current of the tape, the PSO algorithm is employed for structural parameter optimization in cold dielectric type HTS cables [1].

C. Robust Optimization Models

However, the performance of an HTS cable may to a certain degree be affected by the perturbation of parameters possibly caused by imperfect manufacturing or nonideal properties of superconducting tapes, e.g., shrinkage at low temperature [8].

In an attempt to increase the robustness of the HTS cable, the objective function is converted to a six-sigma robust optimization formulation. This robust optimization formulation based on DFSS is given as

$$\text{Minimize:}\quad \mu\left(f(\boldsymbol{X})\right)+\sigma\left(f(\boldsymbol{X})\right)\quad (6)$$

where $f(\boldsymbol{X})$ is defined in (5).

The associated constraints are converted to the following.

1) Constraints of mechanical properties

$$\begin{cases}\mu\left(\arcsin\left(\frac{2R_i\varepsilon_{cb}}{t}\right)^{\frac{1}{2}}-\beta_i\right)\\\quad-n\sigma\left(\arcsin\left(\frac{2R_i\varepsilon_{cb}}{t}\right)^{\frac{1}{2}}-\beta_i\right)\geq 0\quad i=1,2,\cdots,N\\\mu\left(\beta_i-\arcsin\left(\frac{\varepsilon_{ct}+\varepsilon_p-\varepsilon_{fc}}{\varepsilon_p-\varepsilon_r}\right)^{\frac{1}{2}}\right)\\\quad-n\sigma\left(\beta_i-\arcsin\left(\frac{\varepsilon_{ct}+\varepsilon_p-\varepsilon_{fc}}{\varepsilon_p-\varepsilon_r}\right)^{\frac{1}{2}}\right)\geq 0\end{cases}\quad (7)$$

where ε_{cb} and ε_{ct} are the critical bending and tensile strain of the tape at 77 K, ε_p and ε_{fc} are the thermal shrinkages of the winding pitch and the tape, respectively. ε_r is the radial thermal shrinkage of the former, and t is the thickness of the tape.

2) Constraints of radii

$$\begin{cases}\mu\left(R_{i+1}-R_i-(t_f+t)\right)\\\quad-n\sigma\left(R_{i+1}-R_i-(t_f+t)\right)\geq 0\quad i=1,2,\cdots,N-1\\\mu\left(R_1-\frac{D_{\min}}{2}+\left(t_f+\frac{t}{2}\right)\right)-n\sigma\left(R_1-\frac{D_{\min}}{2}+\left(t_f+\frac{t}{2}\right)\right)\geq 0\end{cases}\quad (8)$$

TABLE I
STRUCTURAL PARAMETERS OF COLD DIELECTRIC TYPE HTS CABLE

Layer Index	1	2	3	4	5	6
α	+1	+1	-1	-1	+1	+1
β(°)	27.0	27.0	27.0	27.0	27.0	27.0
R (mm)	10.0	10.45	10.90	11.35	18.50	18.95

Note: Layers 1–4 are the conductors, Layers 5 and 6 are the shields.

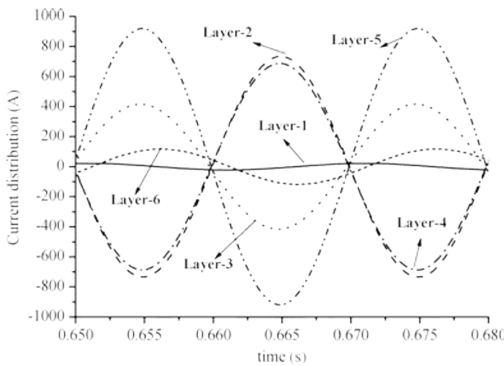

Fig. 2. Current distribution in HTS cable before optimization.

where $D_{\min}$ is used to limit the inner diameters of the cable conductors, and t_f is the thickness of the dielectric between layers.

3) Constraints of critical current

$$\mu(Z_i I_c k_1 k_2 k_3 k_4 - I_i) - n\sigma(Z_i I_c k_1 k_2 k_3 k_4 - I_i) \geq 0$$
$$i = 1, 2, \cdots, N \tag{9}$$

where Z_i is the number of tapes wound on the i th layer, I_c is the mean of critical currents of HTS tapes in the cable, k_1, k_2 and k_3 are the deteriorations of the critical current considering the magnetic field and the temperature, manufacture, and the thermal cycles, respectively, and k_4 is the design safety margin.

V. STRUCTURAL PARAMETER OPTIMIZATION

Taking the cold dielectric type HTS cable with 4-layer of conductors and 2-layer of shields as an example, Table I tabulates the structural parameters of the HTS cable before the structural optimization.

The length of the cable for calculation is chosen as 100 m. The ac current source is 1000 A (rms). Fig. 2 plots the current distributions before the structural optimization. It is found that the currents in different layers before optimization differ greatly in both the amplitude and phase angle.

By using PSO algorithm, in which w is 1.4, c_1 is 2.6, and c_2 is 1.3, the optimized parameters of the HTS cable are shown in Table II. Fig. 3 plots the corresponding current distributions. It is found that the currents become uniform after the optimization.

When the perturbation range of design variables is $\pm$ 0.1%, the robust optimization problems are solved employing DFSS, coupled with PSO algorithm. The Monte Carlo simulation is applied to estimate the mean and standard deviation of all outputs. The structural parameters and the current distributions optimized with DFSS are shown in Table II and Fig. 4, respectively. It is found that the current distributions optimized by PSO and DFSS are almost the same.

Based on the data in Table II, the statistic analysis is conducted in the perturbation range using Monte Carlo descriptive sampling with 500 points. Fig. 5 demonstrates the probability distributions of fitness function F of objective function in different optimization results. It can be seen in Fig. 5(a) that the distribution covers a wider range by using PSO, which means that there are a few samples whose current distributions differ greatly from the expected results, while in DFSS the probability distribution shrinks much thinner. In this way, the DFSS optimization increases the robustness of the design.

A comparison of the quality of the design results is proposed in Table III. With PSO algorithm, the mean value μ_{F} of the fitness function is 101.729, the standard deviation σ_{F} of the fitness function is 56.677 and the reliability is 63.8723%. By using the six-sigma robust optimization, the mean value and the standard deviation of the objective function are reduced to 54.52 and 24.67, respectively. The constraints have almost 0% probability in exceeding their limits. So the reliability of DFSS is much higher than that of PSO.

TABLE II
OPTIMIZED STRUCTURAL PARAMETERS OF HTS CABLE

Layer Index	PSO Algorithm			DFSS Optimization		
	α^{Δ}	β^{Δ}	R^{Δ} (mm)	α^{*}	β^{*}	R^{*} (mm)
1	-1	16.23	9.69	-1	24.15	9.00
2	-1	8.03	10.11	-1	8.00	9.91
3	1	8.92	10.64	1	10.37	10.69
4	1	30.64	10.99	1	30.00	11.55
5	-1	21.94	17.94	-1	22.81	18.05
6	-1	10.59	18.51	-1	14.41	19.21

Note: represents PSO algorithm, and * DFSS optimization

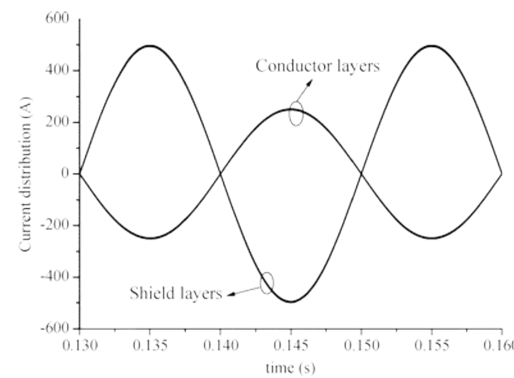

Fig. 3. Current distribution after optimization with PSO algorithm.

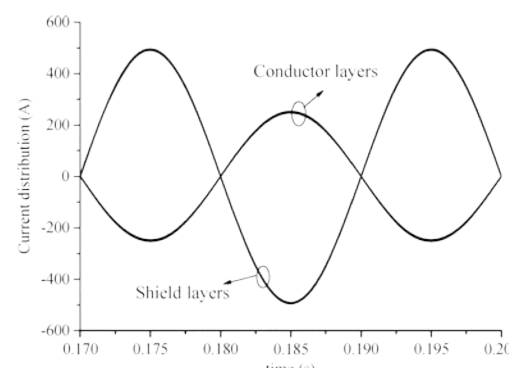

Fig. 4. Current distribution after optimization with DFSS.

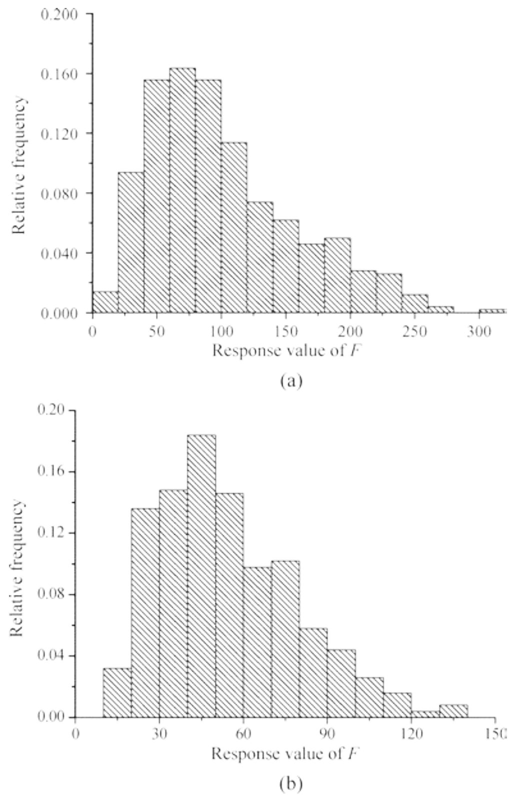

Fig. 5. Histogram of PSO algorithm and robust design for six sigma. (a) PSO algorithm. (b) Robust optimization based on DFSS.

TABLE III
QUALITY IMPROVEMENT FOR THE HTS CABLE

Optimization	μ_F	σ_F	Reliability
PSO	101.729	56.677	63.8723%
DFSS	54.5188	24.6709	~100%

VI. PERTURBATION ANALYSIS OF OPTIMIZED PARAMETERS

The perturbation analysis is applied to evaluate the influence of the distorted structural parameters. The current relative error of the ith layer is introduced to investigate the current distribution with perturbed parameters as follows:

$$E_{cv,i} = \frac{|I_{ave} - I_{err,i}|}{I_{ave}} \times 100\% \tag{10}$$

where I_{ave} is the average value of the currents of the layers obtained through the optimized parameters, and $I_{err,i}$ is the current of the ith layer with the perturbed structural parameters.

The perturbations are performed on the winding angle and the radius in a certain range. The maximum value of E_{cv} in all cases is defined as $E_{\max}$. The curves in Fig. 6 reveal that the robust stabilization of DFSS is higher than that of PSO.

VII. CONCLUSION

In this paper, considering the uncertainties in HTS cable structural design, an optimization algorithm based on design for six sigma combining with PSO is applied to perform a robust design. The comparison between traditional PSO and DFSS-based optimization shows that the robust optimization using DFSS is superior to the PSO algorithm to achieve a higher reliability and quality. The improvement of PSO to deal with the prematurity and the evaluation of efficiency of DFSS will be presented in the future works.

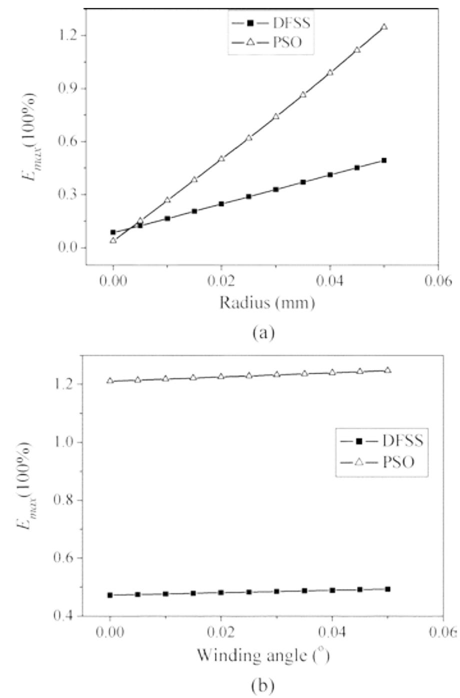

Fig. 6. Influence of parameter perturbation on current distribution. (a) Perturbation of the radius. (b) Perturbation of the winding angle.

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