Distributed State Estimation Over Unreliable Communication Networks with an Application to Smart Grids

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Abstract—In contrast to the traditional centralised power system state estimation methods, this paper investigates the interconnected optimal filtering problem for distributed dynamic state estimation considering packet losses. Specifically, the power system incorporating microgrids is modelled as a state-space linear equation where sensors are deployed to obtain measurements. Basically, the sensing information is transmitted to the energy management system (EMS) through a lossy communication network where measurements are lost. This can seriously deteriorate the system monitoring performance and even lose network stability. Secondly, as the system states are unavailable, so the estimation is essential to know the overall operating conditions of the electricity network. Availability of the system states provides designers an accurate picture of the power network, so a suitable control strategy can be applied to avoid massive blackouts due to losing network stability. Particularly, the proposed estimator is based on the mean squared error between the actual state and its estimate. To obtain the distributed estimation, the optimal local and neighbouring gains are computed to reach a consensus estimation after exchanging their information with the neighbouring estimators. Then the convergence of the developed algorithm is theoretically proved. Afterwards, a distributed controller is designed based on the semidefinite programming approach. Simulation results demonstrate the accuracy of the developed approaches under the condition of missing measurements.

Index Terms—Distributed controller, distributed dynamic estimation, energy management system, packet losses, linear matrix inequality, smart grid communication.

I. INTRODUCTION

Generally speaking, the industrial domain application becomes more and more distributed due to advanced information and communication technology [1], [2]. In other words, the automation system is mostly designed based on the distributed architecture and its signal processing algorithms. As the measurements are locally processed, it can accurately handle more data, offer flexible communication infrastructure, deliver required functionality and services in sustainable and efficient ways, for example, monitoring and controlling the power system incorporating microgrids in a distributed way. The main reasons of incorporating microgrid including distributed energy resources (DERs) into the grid are due to the low green house gas emissions, reduced transmission losses and cost [3], [4], [5]. Unfortunately, their intermittent nature of the power generation pattern brings critical challenges for power system operation and stability [6]. As the power substations and energy management system (EMS) are generally far away, so the measurements are normally lost in the communication channel [7], [8]. It is therefore imperative to estimate the power system states and apply a suitable control strategy, so the system can operate properly [9]. In other words, the power network intrinsically requires to expect stability over a lossy communication channel between the microgrid and EMS. This can only be archived if the EMS knows the microgrid states as its states are generally unavailable and affected by uncertainties [10]. For instance, in a smart grid the sensing devices such as sensors may be geographically far away from the estimator and controller which inevitable leads to packet dropouts in the communication network.

A. Related Literature

There is a wealth of research related to the power system state estimation. To begin with, a distributed weighted least square estimation method using additive Schwarz domain decomposition technique is proposed in [11]. This decomposition divides the data set into several subsets to reduce the execution time. Unfortunately, it is assumed that the communication is perfect with no measurement losses. A Kalman filter (KF) based state estimation via wireless sensor networks over fading channels is presented in [12]. This kind of centralized estimation technique is generally not only in need of huge amount of communication and computation resources but also vulnerable to the central point failures which may lead to massive blackouts. To deal with the communication impairments, a distributed fusion based KF algorithm for sensor networks is developed in [13], [14]. The fusion centre linearly combines the local estimators through a set of designed weighting factors. In order to obtain a suitable weighting factor, a weighted density function based recursive algorithm is purposed under the condition of reliable communication channels [15]. In order to accommodate the effects of random delay in measurements, an extended KF based power system state estimation method is proposed in [16]. All of the aforementioned papers consider the centralised estimation or reliable communication channel.

There are many different feedback control techniques available for the power system stability in the literature. First of all, a linear quadratic Gaussian control strategy under the
condition of packet losses is presented in [9]. This networked control system is suitable for the centralized state estimation and its stability analysis. Recently, the time-delay switch attack based on the simple proportional integral derivative (PID) centralized controller is adapted in the context of smart grids [17]. It is considered that delays can be introduced in the sensing loop or control lines so the packet may be lost. Each controller only communicates information with its neighbours in the distributed control strategy [18], [19]. It is therefore very effective for performing the wide-area distributed computation, specifically in the emerging smart grid. Recently, a unified distributed control strategy for the DC microgrid is proposed in [6]. It shows that the standard distributed PI voltage controllers are no longer able to regulate the average DC microgrid bus voltage, so the distributed voltage controllers are replaced by double integrator controllers. Usually, the partial system state information is only available, so the estimation with controller design remain an open question in the signal processing, control and smart grid communities. In [20], the authors have developed a novel distributed observer where each local area has a local controller without any sharing of information between them. The designed distributed functional observers are of reduced order and dynamically decoupled. Basically, after estimating the system states in a distributed way, the distributed controller generally allows a sparse communication to reduce communication and computation resources while maintaining the system stability. Inspiring by the above discussions and analysis, this paper designs a distributed state estimation and distributed controller under the condition of packet losses.

B. Key Contributions

The main contributions of this research are the following:

• Based on the mean squared error principle, the optimal local and neighbouring gains are determined to obtain a distributed dynamic state estimation in the context of smart grids. Each estimator exchanges information with the neighbouring estimators for reaching a consensus estimation even though there are unmeasurable states and packet losses.

• The convergence of the developed approach is theoretically proved based on the continuous-time domain analysis due to its mathematical simplicity. It shows that the error function is gradually decreased over time, therefore the estimated states converge to the actual states.

• For proper operation and maintaining the stability of the microgrid, a distributed controller is proposed based on the semidefinite programming (SDP) approach. The designed sparse feedback gain is calculated by iterative optimization process which is less conservative as it effectively and efficiently computes the Lyapunov matrix \( P \) with no structure constraints on \( P \).

Notations: Bold face upper and lower case letters are used to represent matrices and vectors respectively. Superscripts \( \text{T} \) denotes the transpose of \( x \), \( \text{diag}(x) \) denotes the diagonal matrix, \( E(\cdot) \) denotes the expectation operator and \( I \) denotes the identity matrix.

II. System and Packet Loss Models

In order to develop a distributed estimator and controller, consider the following discrete-time system:

\[
x_{k+1} = A_d x_k + B_d u_k + n_k,
\]

where \( x_k \) is the system state at time instant \( k \), \( u_k \) is the control effort, \( n_k \) is the control effort and \( n_k \) is process noise whose covariance matrix is \( Q_k \). Basically, \( A_d \) describes the system operating conditions, and \( B_d \) represents the system input. These matrices can be obtained by the set of algebraic equations, which are obtained using Kirchhoff’s laws.

The system measurements are obtained by a set of voltage sensors as follows:

\[
z^i_k = C^i x_k + w^i_k,
\]

where \( z^i_k \) is the measurement voltages by the i-th estimator and \( C^i \) is the observation matrix, and \( w^i_k \) is the measurement noise whose covariance matrix is \( R^i_k \). Realistically, the sensing measurements transmit through a lossy communication network which causes packet dropouts. This is due to the fact that the power network and EMS are far way from each other. Taking into account the packet loss, (2) can be written as follows:

\[
y^i_k = \alpha^i_k C^i x_k + \alpha^i_k w^i_k,
\]

where \( y^i_k \) is the measurement voltages under the condition of packet losses, and \( \alpha^i_k \) is the Bernoulli distribution modelled as follows [8]:

\[
\alpha^i_k = \begin{cases} 1, & \text{with probability of } \lambda^i_k, \\ 0, & \text{with probability of } 1 - \lambda^i_k, \end{cases}
\]

where \( \lambda^i_k \) is the packet arrival rate reaching at the estimator. Inspired by [21], [22], [23], and for the sake of mathematical simplicity, it assumes that the observation matrices and packet loss distribution are identical with each other [24]. The assumptions are probably due to the fact that the distributed estimators are not far way from the power substations but as usual information transmits through an unreliable network. Secondly, the sensors have limited power and processing capability. Thirdly, the service provider deploys similar kinds of sensors in the distribution power network. Finally, from Eq. (3) it can be seen that the measurement noises are different from each observation point, so only for the sake of mathematical simplicity in Eq. (7), it assumes that the observation matrices are identical with each other.

III. Proposed Distributed State Estimation Algorithm

Generally speaking, the filtering infrastructure is interconnected to each other to know the operating conditions of the distribution power network [24]. For instance, the proposed interconnected filtering scheme is depicted in Fig. 1. It can be seen that the the sensing information at the subsystems can be shared with the connected estimators. Considering the
packet losses the proposed distributed dynamic state estimator is written as follows:

\[
\begin{align*}
\hat{x}_{k|k-1}^i &= \hat{x}_{k-1|k}^i + K_k^i [y_k^i - \alpha_k C \hat{x}_{k-1|k}^i] + \\
L_k^i \sum_{j \in N_k^i} [y_j^i - \alpha_k C \hat{x}_{k-1|k}^j].
\end{align*}
\]

(4)

Here, \(\hat{x}_{k|k-1}^i\) is the updated state estimation at the \(i\)-th estimator, \(\hat{x}_{k-1|k}^i\) is the predicted states estimate, \(K_k^i\) is the local gain, \(L_k^i\) is the neighbouring gain and \(N_k^i\) denotes the set of neighbouring estimators. The second term in (4) is used for local estimation while third term is used to exchange information with neighbouring estimators to reach a consensus estimation. Based on the aforementioned modelling structure, our first problem is to design the optimal gains \(K_k^i\) and \(L_k^i\), so that the estimated state converges to the actual system state.

Let \(e^i\) denote the estimation error between actual state and estimated state of the \(i\)-th estimator, which can be expressed as follows:

\[
\begin{align*}
e^i_{k|k-1} &= x_k - \hat{x}_{k|k-1}^i, \\
e^i_k &= x_k - \hat{x}_{k|k}^i.
\end{align*}
\]

(5)

(6)

Let \(n_k^i = n_k^i(N_k^i)\) represents the cardinality of \(N_k^i\). Now substituting (4) into (6), and using (3) one can obtain the following expression:

\[
\begin{align*}
e^i_{k|k} &= x_k - \hat{x}_{k|k-1}^i - K_k^i [y_k^i - \alpha_k C \hat{x}_{k-1|k}^i] - \\
&= [I - \alpha_k K_k^i C - n_k^i \alpha_k L_k^i C] [x_k - \hat{x}_{k-1|k}^i] - \alpha_k K_k^i w_k^i - \alpha_k L_k^i \sum_{l \in N_k^i} w_l^i \\
&= [I - \alpha_k K_k^i C - n_k^i \alpha_k L_k^i C] e_{k|k-1}^i - \alpha_k K_k^i w_k^i - \\
&= \alpha_k L_k^i \sum_{l \in N_k^i} w_l^i.
\end{align*}
\]

(7)

Now the estimation error covariance matrix \(P_k^i|k\) is defined by:

\[
P_k^i|k = E[e_k^i e_k^i].
\]

(8)

where \(E(\cdot)\) is the expectation operator. Substituting (7) into (8), one can obtain:

\[
P_k^i|k = \lambda_k [I - K_k^i C - n_k^i L_k^i C] [x_k^i - \hat{x}_{k-1|k}^i] - \\
(I - \lambda_k) P_k^i|k-1 + \lambda_k K_k^i R_k L_k^i + \lambda \sum_{l \in N_k^i} R_k^i L_k^i.
\]

(9)

Here, \(P_k^i|k-1 = E[e_k^i e_k^i]\). In order to find the optimal gain \(K_k^i\), taking the partial derivative of \(P_k^i|k\) in (9) with respect to \(K_k^i\) yields:

\[
\frac{\partial [\text{tr} P_k^i]}{\partial K_k^i} = -2 \lambda_k (I - n_k^i L_k^i C) P_k^i|k-1 C' + 2 \lambda_k K_k^i (C P_k^i|k-1 C' + R_k).
\]

(10)

Now putting \(\frac{\partial [\text{tr} P_k^i]}{\partial K_k^i} = 0\) in (10), the optimal local gain \(K_k^i\) is given by:

\[
K_k^i = \frac{[P_k^i|k-1 C' - n_k^i L_k^i C P_k^i|k-1 C']}{[C P_k^i|k-1 C' + R_k]}.
\]

(11)

Similarly, take the partial derivative of (9) with respect to \(L_k^i\):

\[
\frac{\partial [\text{tr} P_k^i]}{\partial L_k^i} = -2 n_k^i \lambda_k (I - K_k^i C) P_k^i|k-1 C + 2 (n_k^i)^2 \lambda_k L_k^i C P_k^i|k-1 C' + 2 \lambda_k L_k^i \sum_{l \in N_k^i} R_l^i.
\]

(12)

Setting \(\frac{\partial [\text{tr} P_k^i]}{\partial L_k^i} = 0\) in (12), \(L_k^i\) is derived as follows:

\[
L_k^i = \frac{[P_k^i|k-1 C' - n_k^i K_k^i C P_k^i|k-1 C']}{(n_k^i)^2 C P_k^i|k-1 C' + \sum_{l \in N_k^i} R_l^i].
\]

(13)

For simplicity, define \(H_k^i = C P_k^i|k-1 C', \ F_k^i = (H_k^i + R_k)^{-1}, \) and \(G_k^i = [(n_k^i)^2 H_k^i + \sum_{l \in N_k^i} R_l^i]^{-1}\). Then (11) and (13) can be rewritten as follows:

\[
K_k^i = \frac{[P_k^i|k-1 C' - n_k^i L_k^i C P_k^i|k-1 C']}{[F_k^i]},
\]

(14)

\[
L_k^i = \frac{[P_k^i|k-1 C' - n_k^i K_k^i C P_k^i|k-1 C']}{[F_k^i]},
\]

(15)

In order to obtain the optimal gain \(K_k^i\), substituting (15) into (14) leads to:

\[
K_k = \frac{[P_k^i|k-1 C' F_k^i - n_k^i [P_k^i|k-1 C' G_k^i - n_k^i K_k^i H_k^i G_k^i] F_k^i]}{[F_k^i]},
\]

(16)

Similarly,

\[
L_k^i = \frac{[n_k^i P_k^i|k-1 C' G_k^i - n_k^i P_k^i|k-1 C' F_k^i]}{[I - (n_k^i)^2 H_k^i G_k^i] F_k^i}.
\]

(17)
In summary, after initialization the system parameters such as \( P_{k|k-1}^i \) and \( x_{k|k-1}^i \) through the KF based prediction step, each estimator computes the optimal local and neighbouring gains by (16) and (17). \( \hat{x}_{k+1|k}^i \) and \( \hat{P}_{k+1|k}^i \) are given by:

\[
\hat{x}_{k+1|k}^i = A_x \hat{x}_{k|k}^i + B_u u_k, \tag{18}
\]

\[
\hat{P}_{k+1|k}^i = A_x \hat{P}_{k|k}^i A_x' + Q_k. \tag{19}
\]

Afterwards, each estimator computes the state estimation and its update covariance matrix by (4) and (9).

**IV. Consensus Analysis**

From the engineering perspective, the discrete-time system is easy to implement in the digital platforms, while the continuous system is easy to analyze from the mathematical point of view [25]. Motivated by this realistic dilemma and similar to [26], the consensus analysis of the proposed algorithm is completed based on the consensus analysis of the continuous system. Similar to the discrete-time case, the estimator applies the following step [24]:

\[
\hat{x}_i(t) = x_i - \hat{x}_i(t). \tag{20}
\]

The estimation error \( e_i(t) \) can be expressed as follows:

\[
e_i(t) = \hat{e}_i(t) = x_i - \hat{x}_i(t). \tag{21}
\]

By direct differentiation of (21), the estimation error dynamics is in the following form:

\[
\dot{e}_i = \hat{x}_i - \dot{\hat{x}}_i
\]

\[
= \dot{x}_i - A \hat{x}_i - Bu + K_i[y_i - \alpha C \hat{x}_i] - L_i \sum_{j \in N_i} [y_j - \alpha C \hat{x}_j]
\]

\[
= A \hat{x}_i + Bu + n - A \hat{x}_i - Bu - \alpha K_i [C \hat{x}_i + w_i - C \hat{x}_i] -
\]

\[
L_i \sum_{j \in N_i} [y_j - \alpha C \hat{x}_j]
\]

\[
= (A - \alpha K_i C - n_i \alpha L_i C)x_i - (A - \alpha K_i C - n_i \alpha L_i C)\hat{x}_i +
\]

\[
n - \alpha K_i w_i - \alpha L_i \sum_{l \in N_i} w_l
\]

\[
= (A - \alpha K_i C - n_i \alpha L_i C)(x_i - \hat{x}_i) + n - \alpha K_i w_i - \alpha L_i \sum_{l \in N_i} w_l
\]

\[
= (A - \alpha K_i C - n_i \alpha L_i C)e_i + n - \alpha K_i w_i - \alpha L_i \sum_{l \in N_i} w_l.
\]

The error covariance matrix is written as follows [24]:

\[
\dot{P}_i = (A - \alpha K_i C - n_i \alpha L_i C)P_i + P_i (A - \alpha K_i C - n_i \alpha L_i C)^T + Q + \lambda K_i R_i K_i^T + \lambda L_i \sum_{l \in N_i} R_l L_l^T
\]

\[
= \dot{P}_i + P_i A_i + Q - \lambda K_i C P_i - \lambda P_i C' K_i^T + \lambda K_i R_i K_i^T
\]

\[- n_i \lambda L_i C P_i - n_i \lambda P_i C' L_i^T + \lambda L_i \sum_{l \in N_i} R_l L_l^T. \tag{23}
\]

Taking the partial derivative of (23) with respect to \( \dot{K}_i \) yields:

\[
\frac{\partial \text{Tr} \dot{P}_i}{\partial K_i} = -2 \lambda P_i C' + 2 \lambda K_i R_i. \tag{24}
\]

Setting \( \frac{\partial \text{Tr} \dot{P}_i}{\partial K_i} = 0 \) in (24), then the gain matrix is given by:

\[
K_i = P_i C'(R_i)^{-1}. \tag{25}
\]

Taking the partial derivative of (23) with respect to \( L_i \):

\[
\frac{\partial \text{Tr} \dot{P}_i}{\partial L_i} = -2 n_i \lambda P_i C' + 2 \lambda L_i \sum_{l \in N_i} R_i. \tag{26}
\]

Putting \( \frac{\partial \text{Tr} \dot{P}_i}{\partial L_i} = 0 \) in (26), then the gain matrix is obtained as follows:

\[
L_i = n_i P_i C'(\sum_{l \in N_i} R_l)^{-1}. \tag{27}
\]

Substituting (25) and (27) into (23), one can obtain:

\[
\dot{P}_i = AP_i + P_i A_i + Q - \lambda P_i C'(R_i)^{-1} C P_i -
\]

\[
\lambda P_i C'(R_i)^{-1} C P_i - \lambda P_i C'(R_i)^{-1} C P_i -
\]

\[
n_i \lambda P_i C'(\sum_{l \in N_i} R_l)^{-1} C P_i -
\]

\[
(n_i^2 - 2 n_i) \lambda P_i C'(\sum_{l \in N_i} R_l)^{-1} C P_i +
\]

\[
(n_i^2 - 2 n_i) \lambda P_i C'(\sum_{l \in N_i} R_l)^{-1} C P_i +
\]

\[
[(n_i^2 - 2 n_i) \lambda P_i C'(\sum_{l \in N_i} R_l)^{-1} C P_i]. \tag{28}
\]

In order to analyze the stability of the developed approach, define \( \bar{e}_i = E(e_i) \), and take the expectation on both sides of (22) to obtain:

\[
\bar{e}_i = (A - \lambda^2 K_i C - n_i \lambda L_i C)e_i. \tag{29}
\]

Consider the following Lyapunov function [24]:

\[
V = \sum_{i=1}^{M} \bar{e}_i^T (P_i)^{-1} \bar{e}_i. \tag{30}
\]

Now taking the partial derivative and expectation of (30), and
using (25), (27), (28) and (29), we have [24]:
\[
\dot{V} = \sum_{i=1}^{M} \left( \dot{e}^i (P^i)^{-1} e^i + \dot{e}^i (P^i)^{-1} e^i - e^i (P^i)^{-1} \dot{P}^i (P^i)^{-1} e^i \right)
\]
\[
= \sum_{i=1}^{M} \left[ (A - \lambda K^i C - n^i \lambda L^i C)^{-1} + (P^i)^{-1} (A - \lambda K^i C - n^i \lambda L^i C) - n^i \lambda L^i C - (P^i)^{-1} A - A'(P^i)^{-1} - (P^i)^{-1} Q(P^i)^{-1} + \lambda C'(R^i)^{-1} C - [(n^i)^2 - 2n^i] \lambda C'(\sum_{l \in N^i} R^l)^{-1} C] e^i \right]
\]
\[
= \sum_{i=1}^{M} \dot{e}^i [-(\lambda(P^i)^{-1} K^i C - \lambda C^i K^i (P^i)^{-1}) - n^i \lambda(P^i)^{-1} L^i C - n^i \lambda C^i L^i (P^i)^{-1} - (P^i)^{-1} Q(P^i)^{-1} + \lambda C'(R^i)^{-1} C - [(n^i)^2 - 2n^i] \lambda C'(\sum_{l \in N^i} R^l)^{-1} C] e^i \right]
\]
Consequently, the estimated state \( \dot{x}^i \) converges to the actual system state \( x \). After estimating the system states such as bus voltages, the designer needs to apply a suitable distributed control technique for maintaining the stability of the network.

V. PROPOSED DISTRIBUTED VOLTAGE CONTROLLER

The feedback controller is employed to regulate the microgrid states such as bus voltages. The feedback controller is given by [24]:
\[
u_k = Fx_{k|k}.
\]
Here, \( F \) is the distributed feedback gain matrix to be designed. If there is no connection between subsystem/estimator and controller then the corresponding element of \( F \) is zero. For instance, from the Fig. 1 the designed gain matrix \( F \) belongs to the following structure set (assuming \( M = 4 \)):
\[
F^o = \{ F | F = \begin{bmatrix}
F_{11} & F_{12} & 0 & 0 \\
0 & F_{22} & F_{23} & 0 \\
F_{31} & 0 & F_{33} & F_{34} \\
0 & 0 & 0 & F_{44}
\end{bmatrix} \}.
\]
Here, the feedback element \( F_{NM} \) is the connection between subsystem sensor N and controller M. According to the separation principle [27, p. 427], we can implement the control law \( u_k = Fx_k \). Using \( u_k \), (1) can be written as follows:
\[
x_{k+1} = \tilde{A}_d x_k + n_k,
\]
where \( \tilde{A}_d = A_d + B_d F \) is the closed loop state matrix. If there exists a stabilizing gain matrix \( F \in F^o \), then the following LMI holds:
\[
(\beta A_d)P(\beta A_d) - P < 0,
\]
where \( \beta = 1/|\gamma \max \{ \text{eig}(A_d) \}| \), \( \gamma > 1 \) is a free parameter and \( \max \{ \text{eig}(A_d) \} \) is the maximum eigen values of \( A_d \). The quantity \( \gamma \) ensures eigenvalues of the scaled close loop system strictly less than one [24]. Now according to the standard Schur’s complement, (36) can be transformed into the following LMI form:
\[
\begin{bmatrix}
-P & \beta A_d P \\
\beta P A_d & -P
\end{bmatrix} < 0.
\]
(37)

After computing \( P \) in (37) and with the help of (35), one can obtain \( F \in F^o \) by considering the following optimization problem:
\[
\begin{aligned}
& \text{minimise} & & \zeta \\
& \text{subject to} & & (A_d + B_d F)P(A_d + B_d F) - P + \zeta I < 0.
\end{aligned}
\]
(38)

where \( \zeta \) is the semidefinite programming variable. Given \( P \), applying the Schur’s complement to (38) yields:
\[
\begin{bmatrix}
-P + \zeta I & (A_d + B_d F)P \\
(P(A_d + B_d F)) & -P
\end{bmatrix} < 0.
\]
(39)

Finally, one can formulate the proposed optimization problem as follows:
\[
\begin{aligned}
& \text{minimise} & & \zeta \\
& \text{subject to} & & \text{Hold (39), } F \in F^o.
\end{aligned}
\]
(40)

In summary, the proposed feedback gain is designed by solving (40).

VI. APPLICATION TO THE DISTRIBUTION POWER SYSTEM AND SIMULATION RESULTS

In order to demonstrate the effectiveness and robustness of the proposed algorithm, the microgrid is considered.

A. Distribution Power System Incorporating Multiple DERs

Due to the climate change and limited energy resources, the renewable microgrid incorporating DERs is integrated into the main grid at the point common couplings (PCC). As their power generation pattern are generally intermittent in nature, so it needs to monitor the PCC voltages and keep it at a reference value by applying a suitable control technique [28]. Generally speaking, the microgrid may also be installed in the remote and mountain areas, so its monitoring and stability should be managed in a distributed way.
Fig. 2 shows IEEE 4-bus distribution test feeder that is interfaced to the local load through the converter [28]. In addition, the model of multiple DERs connecting to the power network is shown in Fig. 3. The considered N DERs are connected to the main grid. It is assumed that N=4 solar panels are connected through the IEEE-4 bus distribution system shown in Fig. 3 [28], [30]. Here, the input voltages are defined by 
\[ v_p = (v_{p1} \ v_{p2} \ v_{p3} \ v_{p4})', \]
where \( v_{pi} \) is the i-th DER input voltage. It can be seen that the considered four DERs are connected to the power network at the corresponding PCCs whose voltages are defined by 
\[ v_s = (v_1 \ v_2 \ v_3 \ v_4)', \]
where \( v_i \) is the i-th PCC voltage. In order to maintain the proper operation of DERs, these PCC voltages need to be kept at their reference values. A coupling inductor exists between each DER and the rest of the electricity network. Note that the system in Fig. 3 is not restrictive, and can be more general cases; the methods proposed in this paper are independent of the type and the size of microgrids.

The nodal voltage equation is obtained by applying the Laplace transformations as follows:
\[ Y(s)v_s(s) = \frac{1}{s}L_c^{-1}v_p(s), \quad (41) \]
where \( L_c = \text{diag}(L_{c1}, \ L_{c2}, \ L_{c3}, \ L_{c4}) \) and \( Y(s) \) is the admittance matrix of the entire power network incorporating four DERs, and the admittance matrix considering the IEEE 4-bus system parameters is given in [30] [28], [29].

Now we can convert the transfer function into the following form [28]:
\[ \dot{x}_t = Ax_t + Bu_t + n_t, \quad (42) \]
Here, the system state \( x_t = v_s - v_{ref} \) is the PCC voltage deviation, \( v_{ref} \) is the PCC reference voltage, \( u_t = v_p - v_{pref} \) is the DER control effort deviation, \( v_{pref} \) is the reference control effort, \( n_t \) is the process noise whose covariance matrix is \( Q_t, A \) is the system state matrix, and \( B \) is the input matrix.

Fig. 3. DERs are connected to the IEEE 4-bus distribution network [28].

![Diagram](image_url)

Fig. 2. An illustration of the IEEE 4-bus distribution system [29].

![Diagram](image_url)

After applying Kirchhoff's laws with the given IEEE 4-bus specifications in [29], [28], [30], the matrices \( A, B \) are given by [28], [30]:
\[ A = \begin{bmatrix} 1.759 & 1.768 & 5.11 & 1.036 \\ -3.50 & 0 & 0 & 0 \\ -5.442 & -4.748 & -4.088 & -8.288 \\ -1.197 & -5.546 & -9.688 & -1.0775 \end{bmatrix}, \]
\[ B = \begin{bmatrix} 0.008 & 3.342 & 5.251 & -10.36 \\ -3.50 & 0 & 0 & 0 \\ -0.693 & -0.661 & -4.201 & -8.288 \\ -4.349 & -4.142 & -1.087 & -10.775 \end{bmatrix}. \]

Now the system model (42) is expressed as a discrete-time state-space linear equation (1) where system state matrix \( A_d = I + A\Delta t, I \) is the identity matrix, \( \Delta t \) is the sampling period and \( B_d = B\Delta t \) and \( n_d = n_t\Delta t \) with covariance matrix \( Q = Q_t\Delta t^2 \). Based on the aforementioned algorithm and power network, the simulation is carried out for validity of the theoretical analysis.

B. State Estimation Performance Analysis

The simulation is conducted through Matlab and YALMIP and the parameters are shown in Table I.

<table>
<thead>
<tr>
<th>Parameters for the simulation using Matlab [24]</th>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>0.0000001 * I_4</td>
<td>R^2</td>
<td>0.0000001 * I_4</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0000002 * I_4</td>
<td>R^3</td>
<td>0.0000003 * I_4</td>
<td></td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.0000004 * I_4</td>
<td>( \lambda_K )</td>
<td>0.90-0.95</td>
<td></td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>2</td>
<td>( \Delta t )</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

From the simulation, the system state versus time step results are demonstrated in Figs. 4–5. It can be observed that the packet loss significantly affects the system states but the proposed algorithm can well estimate the system states. This is due to the fact that the proposed algorithm can find the optimal
gains to extract the system state information from adversaries [24]. It can also be seen that it requires only 0.15 seconds ($k \times \Delta t$) to estimate the system states.

In practical situation, there may be communication delay in the measurements, i.e., $z_k = Cx_{k-d} + w_k$, where $d$ is the number of delay samples [31]. The simulation results are presented in Figs. 6-7. It can be seen that the results are greatly affected by communication delay. As the developed algorithm is delay free, considering delay in the simulation does not reflect the accurate and reliable estimation results. Including the communication delay in the measurement indicates that the developed algorithm does not trace back to the original system in the optimal sense. In order to get a reliable and accurate estimation, the delay must be consider to develop the algorithm [32].

C. Controller Performance Analysis

After applying the proposed distributed control method, it can be seen from Fig. 8 that the proposed controller is able to keep the voltage deviations to zero in approximately 0.01 seconds ($k \times \Delta t$), which acts as a precursor for stability and microgrid operations. Technically, it means that the developed approach requires much less time to keep the voltage as a reference value compared with the standard stability time frame $1 - 5$ seconds [33], [24].

VII. CONCLUSION

This paper presents a distributed state estimation and control method considering packet losses. The developed distributed consensus estimator is based on the mean squared error, so it can accurately compute the optimal gains to extract the actual system states. Finally, in order to regulate the system states, this study proposes a semidefinite programming based distributed controller in the context of smart grid communication. The proposed distributed control framework could properly determine the sparse gain such that the system states will be stabilized in a fairly short time. These approaches can help to design the future smart EMS under the condition of uncertainties. In future, we will consider the different observation matrices and develop the distributed state estimation algorithm.