Efficient coupling into slow light photonic crystal waveguide without transition region: role of evanescent modes

C. Martijn de Sterke[1*], K.B. Dossou[2], T.P. White[3], L.C. Botten[2] and R.C. McPhedran[1]

[1] *CUDOS, School of Physics, University of Sydney, NSW 2006, Australia*
[2] *CUDOS, School of Mathematical Sciences, University of Technology, Sydney, NSW 2007, Australia*
[3] *School of Physics and Astronomy, University of St Andrews, St Andrews, Fife, KY169SS UK.*
[*] *m.desterke@physics.usyd.edu.au*

Abstract: We show that efficient coupling between fast and slow photonic crystal waveguide modes is possible, provided that there exist strong evanescent modes to match the waveguide fields across the interface. Evanescent modes are required when the propagating modes have substantially different modal fields, which occurs, for example, when coupling an index-guided mode and a gap-guided mode.

OCIS codes: (130.5296) Photonic crystal waveguides; (350.4238) Nanophotonics and photonic crystals

References and links

1. J.B. Khurgin and R.S. Tucker, *Slow light: science and applications* (CRC Press, Boca Raton, Fla 2008).
2. T.F. Krauss, "Why do we need slow light?" Nature Photonics **2**, 448-450 (2008).
3. T. Baba, "Slow light in photonic crystals," Nature Photonics **2**, 465-473 (2008).
4. J. P. Hugonin, P. Lalanne, T. P. White, and T. F. Krauss, "Coupling into slow-mode photonic crystal waveguides," Opt. Lett. **32**, 2638-2640 (2007).
5. T.P. White, L.C. Botten, C.M. de Sterke, K.B. Dossou, and R.C. McPhedran, "Efficient slow-light coupling in a photonic crystal waveguide without transition region," Opt. Lett. **33**, 2644-2646 (2008).
6. M. L. Povinelli, S. Johnson, and J. Joannopoulos, "Slow-light, band-edge waveguides for tunable time delays," Opt. Express **13**, 7145-7159 (2005).
7. P. Velha, J. P. Hugonin, and P. Lalanne, "Compact and efficient injection of light into band-edge slow-modes," Opt. Express **15**, 6102-6112 (2007).
8. C. Li, H. Tian, C. Zheng and Y. Ji, "Improved line defect structures for slow light transmission in photonic crystal waveguide," Opt. Comm. **279**, 214-218 (2007).
9. L.C. Botten, T.P. White, A.A. Asatryan, T.N. Langtry, C.M. de Sterke, and R.C. McPhedran, "Bloch mode scattering matrix methods for modeling extended photonic crystal structures. 1. Theory," Phys. Rev. E **70**, 056606 (2004).
10. M. Notomi, K. Yamada, A. Shinya, J. Takahashi, C. Takahashi, and I. Yokohama, "Extremely large group-velocity dispersion of line-defect waveguides in photonic crystal slabs," Phys. Rev. Lett. **87**, 253902 (2001).
11. J.D. Joannopolous, S.G. Johnson, J.N. Winn, and R.D. Meade, *Photonic crystals* 2nd Ed (Princeton University Press, Princeton, 2008), Ch. 8.

1. Introduction

Slow light has many interesting properties and applications, for example in nonlinear optics [1,2], and has been observed in a number of different geometries. Of these, photonic crystal (PC) waveguides are particularly interesting because they allow for the precise manipulation of the dispersion relation of the modes [2,3]. A recurring problem with this geometry is the difficulty in coupling light into slow modes, for example from another PC waveguide. It is usually assumed that such coupling is inefficient, necessitating the use of coupling regions, for example a taper. Recently, however, it was demonstrated numerically by Hugonin *et al.* [4] that efficient coupling into a slow mode is possible without a coupling region, for example near an inflection point in the waveguide mode's dispersion relation, where the coupling efficiency can be over 99%. At such a point, the structure supports weakly evanescent modes in addition to the slow propagating mode; one of these evanescent modes plays a key role in matching the transverse field components at the boundary, but does not carry energy [5]. Another example occurs near the quadratic band edge of the dispersion relation, which occurs at the edge of the Brillouin zone [4]. Although there are no weakly evanescent modes in this situation, the coupling from a mode with $n_g \approx 5$ into a slow mode with $n_g=100$ is as high as approximately 77% [4]. While lower than in the first example, such efficient coupling near a quadratic band edge is surprising. Here we analyze this phenomenon, show that it is also associated with evanescent modes, and indicate the conditions when one might expect this to arise.

We first briefly review the argument which might lead us to conclude that this coupling is inefficient [5,6,7]. It relies on the relation $S = U\, v_g$, with S the energy flow, U the energy density and v_g the group velocity. This relation implies that, for a fixed energy flow, a slow mode has a much larger energy density, and therefore much higher field strengths, than a fast mode. Therefore, in order to match the transverse field component at the interface between the slow and the fast mode region, the energy flow in the slow region must be much lower than in the fast region—in other words, the coupling is poor. This argument does not account for the transverse mode profile and thus it implicitly applies to structures in which the field profiles of the fast and slow modes are essentially the same [4,5].

In Section 2 we consider an example for a square lattice where this simple argument applies and the coupling into a slow mode is poor. Then, in Section 3, we contrast this with the behavior in a hexagonal lattice discussed by Hugonin *et al* [4], with substantial coupling into a slow mode. We conclude in Section 4.

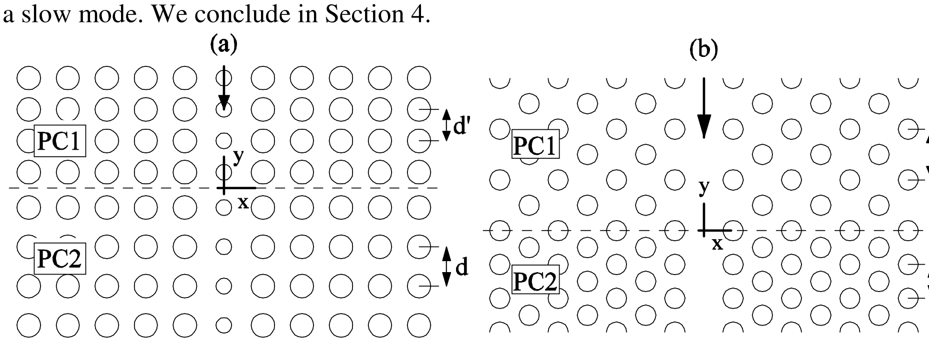

Fig. 1. Schematic of the geometries: light is incident from PC1 into PC2, which supports a slow mode. (a) PC2 has a square lattice; PC1 is similar but the lattice is compressed by 10% in the direction parallel to the waveguide (Sect. 2). (b) PC2 has a hexagonal lattice; PC1 is similar, but its lattice is stretched by 7.1% parallel to the waveguide (Sect. 3). Stretching and compression are exaggerated for clarity.

2. Square lattice—poor slow mode coupling

We first consider a PC similar to that of Li *et al.* [8]: it has a square lattice of period d consisting of rods with refractive index n=2.83 and radius a=0.3d. The waveguide is formed by changing the radius of a row of holes along the ΓX direction to a=0.2d. For TM polarization, this structure (PC2) has a slow mode below the light line near the Brillouin zone (BZ) edge, reaching the edge at $d/\lambda \approx 0.3$. We operate at the frequency $d/\lambda=0.316$ where the group velocity $v_g/c=0.01$ (n_g=100). The light is coupled from a waveguide in PC1 (see Fig. 1(a)) in which the lattice has been compressed by 10% in the direction parallel to the waveguide. Thus it has a rectangular lattice with lattice constant d in the direction orthogonal to the guide and d'=0.9d parallel to it. At the frequency of operation, the waveguide in PC1 supports a guided mode with v_g/c=0.216 (n_g=4.629), and is thus not slow. The transmission of the light from PC1 into PC2 is 16.2%, very close to the result based on the effective medium approximation $R=\left(\left(n_{g1}-n_{g2}\right)/\left(n_{g1}+n_{g2}\right)\right)^2$ which leads to T=1−R=16.9%.

To understand this result better, Figs 2 show the modulus of the electric field in the region surrounding the interface. The fields in this figure, and all calculations reported in this paper, were performed using a highly accurate, transfer matrix calculation [9], which takes into account the significant Bloch modes, both propagating and evanescent. Figures 2(a)–2(d) refer to PC1 and Figs 2(e)–2(g) refer to PC2. A common scale is used in all figures so as to be able to compare the magnitudes of the different contributions to the field. In particular, (a) shows the total field, while (b), (c) and (d) show the evanescent component, and the forward and backward propagating components, respectively. Similarly, (e) shows the total field, with (f) and (g) the evanescent and forward propagating components, respectively (by assumption there is no backward propagating wave in PC2). Note first that the propagating modes in PC1 and PC2 have a similar structure with most of the field confined to the waveguide region. Note also that the amplitude of the evanescent field component is much smaller than that of the propagating fields in both PC1 and PC2.

Since the evanescent fields are weak in PC1, the field is essentially a superposition of forward and backward propagating modes. Similarly, in PC2 the evanescent field component is also small, and thus the total field consists essentially entirely of the field of the forward propagating slow mode. Since the evanescent fields are weak, the argument from Section 1 applies, and the coupling into the slow mode is poor with most light being reflected at the PC1-PC2 interface.

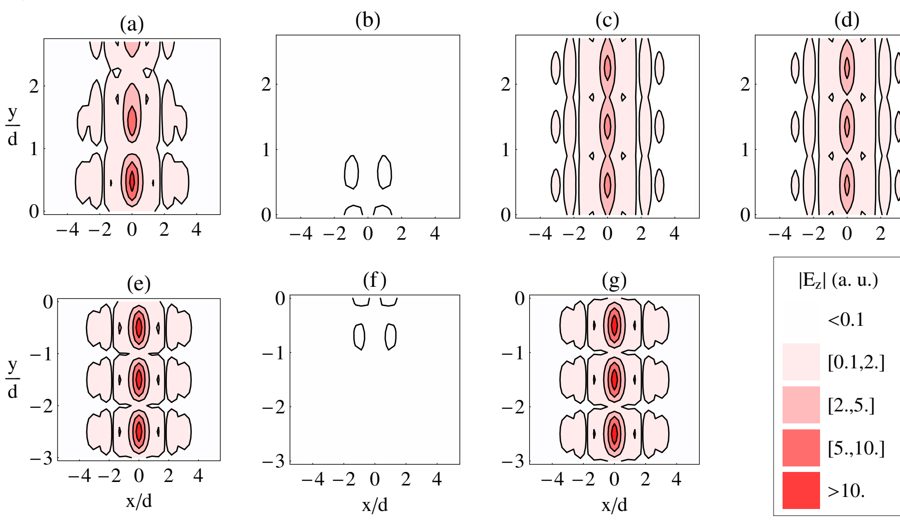

Fig. 2. Modulus of the electric field vs. position. Shown is: (a) total field in PC1; (b) evanescent field in PC1; (c) forward propagating field in PC1; (d) backward propagating field in PC1; (e) total field in PC2; (f) evanescent field in PC2; (g) forward propagating field in PC2.

3. Hexagonal lattice—efficient slow mode coupling

We now consider the example given by Hugonin *et al.* [4]. The geometry is shown in Fig. 1(b): light is incident from PC1 into PC2, both of which consist of air holes of radius a in a background with n=2.83. PC2 has a hexagonal lattice with period d, and a/d=0.3. A waveguide is formed by removing a row of holes. PC1 is similar to PC2, except that the lattice is stretched by 7.1% in the direction parallel to the waveguide (see Fig. 1(b)).

Figs 3(a) and 3(b) show the projected band structures of PC1 and PC2, respectively, for TE polarization, the polarization in which this structure operates. They are similar to those studied earlier by Notomi *et al*. [10] and Joannopoulos *et al*. [11]: they show a variety of waveguide modes inside the photonic bandgap. These include index-guided modes, which rely for their guidance on the higher average refractive index inside the guiding region compared to that outside the region, and gap-guided modes, modes which are guided by Bragg reflection from the periodic structure. Some of these modes are symmetric with respect to the waveguide axis, and some are anti-symmetric. The latter play no role in the current problem since they do not couple to the symmetric modes. At d/λ=0.260 (dotted lines in Figs 3), PC2 has a slow mode with $n_g \approx 100$, while the associated mode in PC1 has $n_g \approx 4.3$. Direct calculations show that the coupling efficiency into the slow mode in PC2 from PC1 is as large as 77% in spite of the large difference in group velocity and in contrast to the geometry in Section 2. It is clear that the naïve argument outlined in Section 1, and which holds for the example in Section 2, does not apply in this instance.

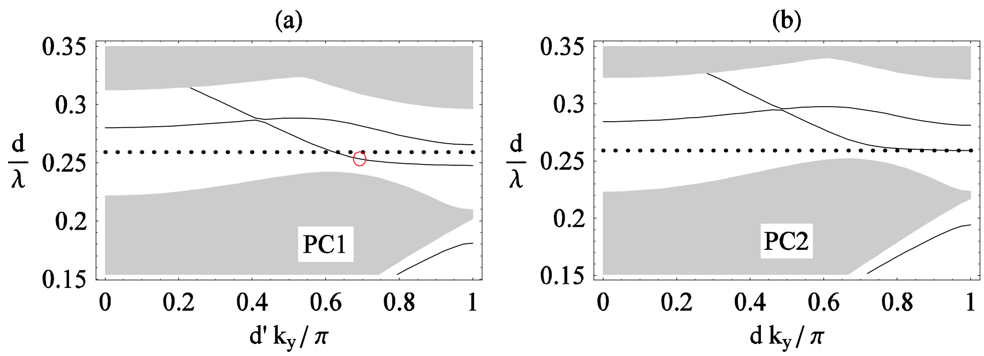

Fig. 3. Projected band structures of (a): PC1, and (b) PC2. The horizontal dotted line gives the frequency of operation. Curves show the waveguide modes. The circle in (a) indicates the position of the anti-crossing between the gap-guided modes and index-guided modes.

The first step in analyzing the coupling is to consider the fields of the single propagating mode in each waveguide at the operating frequency. Figures 4, like Figs 2, show contour plots of field profiles of the various components of the total field. The only difference, because of the orthogonal polarization, is that we display the modulus of the *magnetic* field in Figs 4 (rather than that of the electric field in Figs 2). The fields are again shown on a common scale. As before, Figures 4(a)–4(d) refer to the fields in PC1, whereas Figs 4(e)–4(g) refer to PC2. Fig. 4(a) shows the total field, while (b), (c) and (d) show the evanescent component, and the forward and backward propagating components, respectively. Similarly, (e) shows the total field, with (f) and (g) the evanescent and forward propagating components, respectively. Note that the propagating fields in PC1 and PC2 are quite different: the field in PC1 is almost entirely concentrated in the waveguide region, whereas the mode in PC2 extends significantly into the surrounding periodic structure. Clearly, the forward and backward propagating modes in PC1 would never be able to match the forward propagating mode in PC2. Matching of the fields can occur only when evanescent modes in each of PC1 and PC2 are included. This is confirmed in Figs 4(b) for PC1 and Fig 4(f) for PC2. On the PC1-side of the interface, the field outside the waveguiding region is almost entirely due to evanescent field components. Away from the interface the evanescent components disappear and only the propagating field components are left.

The situation on the PC2-side of the interface is possibly more striking. At the interface, the total field near the waveguide's centre is composed of approximately equal propagating and evanescent components. The phases of these components are such that at the interface the total field is smaller than each of these components, and matches the field in PC1. When moving away from the interface, the evanescent components again disappear, and only the sizeable propagating component is left, so that the total field actually increases in strength.

We return to this below. The reason the simple argument from Section 1 does not apply, and the reason the transmission into the slow mode can be so high, is now clear: the transverse fields at the interface cannot be matched using the propagating modes only; rather, it is possible only through significant contributions of the evanescent modes which partly cancel naturally large fields associated with the slow mode at the interface. Even though these fields contribute to the field matching, they decay away from the interface and so do not contribute to the steady-state energy flow.

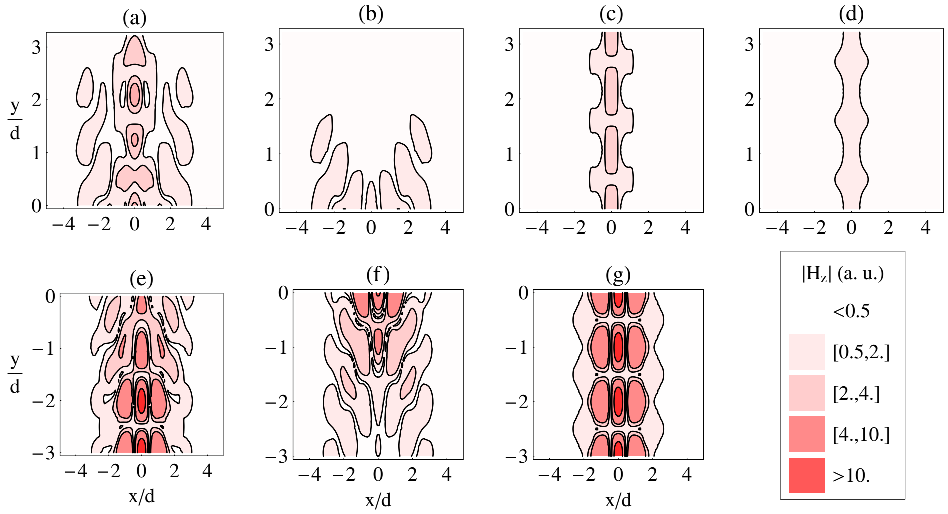

Fig. 4. As in Fig 2, but for the hexagonal lattice and the modulus of the magnetic field.

Figure 5(a) shows the longitudinal cross-section of the modulus of the total magnetic field through the waveguide centre. The fact that the field increases to its maximum value in a few periods below (i.e, behind) the interface is consistent with the argument in the previous paragraph: the evanescent modes help satisfy the boundary conditions at the interface by partly canceling the naturally strong fields of the slow mode. A few periods beyond the interface, however, the evanescent waves have decayed away, and only the high field strength associated with the slow propagating mode remains. This is similar to the observation of White *et al*. for a slow mode near a cubic inflection point [5], except that in the case considered there the evanescent modes have decay lengths of tens of periods. Figure 5(b) displays the corresponding modulus profile for the square lattice example of Section 2. The strong beating that is apparent in PC1, and which does not occur for the hexagonal example in Fig 5(a), is characteristic of a significant reflection associated with imperfect coupling in that case. The fact that the amplitude of the total field in PC2 is approximately constant (also seen in Fig 2(e)) is consistent with the earlier observation that evanescent fields are not important in this example.

From our discussion, it is clear that the simple argument outlined in Section 1 is invalid because of the significant amplitudes of the evanescent modes, which, in turn, are necessitated by the different modal fields in PC1 and PC2. The final issue to be discussed is why the modal fields in PC1 and PC2 are so different, even though the PCs are very similar. The explanation for this can be seen in Figs. 3: according to the definition of Notomi *et al*. [10], the slow mode in PC2 is a gap-guided mode. This is consistent with the mode profile in Fig. 4(g) which shows that the field penetrates significantly into the background photonic crystal surrounding the waveguide. In contrast, as can be seen in Fig. 3(a), the mode in PC1 is near a feature in the dispersion relation corresponding to an anticrossing [10,11] with the lowest band of the bulk PC, indicated by the circle in Fig 3(a). While traversing the anticrossing, the character of the mode changes from being gap-guided to being index-guided [10]. Indeed, Fig. 4(c) is consistent with the modal profile of an index-guided mode as the field is essentially confined to the waveguide region.

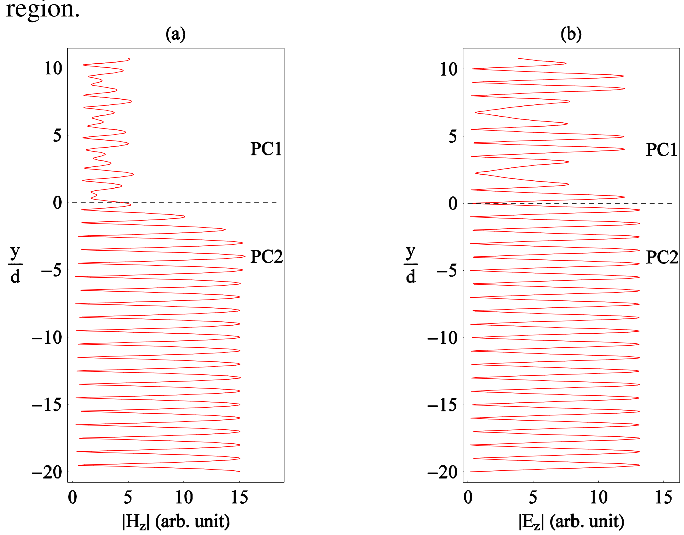

Fig. 5. (a) Longitudinal profile of $|H_z|$ through the hexagonal lattice waveguide centre. The transmittance from PC1 (n_g=4.3) into PC2 (n_g=100) is T=77%. (b) Similar, but for the field $|E_z|$ in the square lattice waveguide studied in Sect. 2. Here the transmittance from PC1 (n_g=4.629) into PC2 (n_g=100) is T=16%. The dashed line indicates the interface.

4. Discussion and Conclusion

In summary, we have shown that coupling into a slow mode of a PC waveguide can be efficient without a transition region provided that evanescent modes with significant amplitudes are available to match the fields across the interface. In our example, these evanescent modes are required because the propagating modes have very different modal fields. *Thus, our counterintuitive conclusion is that it is necessary for the transverse mode profiles to be different in order to get the efficient coupling.*

In contrast, when the fast and the slow modes have similar modal fields, evanescent modes are not required, and the coupling is poor, as illustrated in the example in Section 2. Of course the presence of evanescent modes does not guarantee efficient coupling—however, without the substantial presence of these modes the coupling is necessarily inefficient.

The observation that evanescent modes are needed for efficient coupling into slow-mode waveguides was earlier reported [5]. However, this previous instance referred to the special case in which the slow light derived from a dispersion relation with a cubic inflection point, and which therefore had weakly evanescent modes. The analysis here is much more general, and shows that the involvement of evanescent modes is crucial in many mode matching situations. While we here adjusted the properties of the waveguide modes by stretching or compressing the lattice, our analysis equally applies when this is done by other means, for example by adjusting the size of the holes.

Acknowledgments

This work was produced with the assistance of the Australian Research Council under the ARC Centres of Excellence Program. T.P. White is supported by the EU FP6 Marie Curie Fellowship “SLIPPRY”.