Taking a CLOSER LOOK at young students' VISUAL IMAGERY

JOANNE MULLIGAN, ANNE PRESCOTT and MIKE MITCHELMORE explore the use of pattern imagery and draw our attention to the relationship between numerical and spatial patterns. Their article highlights the importance of the process of noticing, or being attuned to pattern and structure.

Spatial skills are well recognised as critical to young students' development of mathematical ideas. Spatial visualisation processes such as visual imagery underpin many aspects of early number, measurement, space and data exploration where children construct images from pictures, words and symbols (Brown & Wheatley, 1997). Creating and interpreting mathematical patterns and representations also relies on spatial processes. Many mathematical representations such as simple diagrams and arrays present students with a variety of mathematical representations that show an organised or structured picture of mathematical ideas (Diezmann & English, 2001; Owens & Outhred, 1996).

In the recent Space and Geometry Focus Issue (2003) of this journal, several interesting articles described the role of visual reasoning and visual imagery in enhancing students' spatial skills and understandings. In this article we expand on the theme of visual imagery with examples from young students' use of pattern and structure.

Pattern and structure

Battista (1999) talks about spatial structuring as fundamental to geometric reasoning. He defines spatial structuring as the mental organisation of objects or groups of objects and their components. The structure reflects how a student perceives the nature, shape or form of the mathematical components; for example, when constructing a simple array, a row-by-column structure that locates squares within rectangles or cubes within three-dimensional boxes.

Subitising is the ability to see a number of objects instantly without counting them one by one. Some authors restrict this term to the immediate recognition of the number of objects in a particular pattern (e.g. the dots on dice). However, subitising is important in developing pattern and structure in mathematical representations (Bobis, 1996; Clements, 1999). For example, a student may recognise patterns of two or three dots in a rectangular array of six dots and hence see it as a structure rather than as six individual items.

It has been shown that 7 to 11 year-olds whose early mathematical representations show pattern and structure have a better chance of achieving well in mathematics later on (Mulligan, 2002). However, it is not known exactly how young children in the first few years of schooling develop and apply pattern and structure across different contexts, or whether pattern and structure are essentially mathematical or primarily related to spatial organisational skills.

Our research

In an early numeracy project involving 109 Year 1 students from nine Sydney NSW Department of Education and Training schools, we explored their use of pattern and structure across different strands of the mathematics curriculum (Mulligan, Prescott & Mitchelmore, 2003). The project included a wide range of tasks across the Number, Space, Measurement, Data and Working Mathematically strands of the curriculum which had pattern and structure as their common underlying features. Assessing students' spatial visualisation, visual imagery and problem-solving strategies through individual (videotaped) interviews were key components of this investigation.

Visual imagery refers to the students' ability to create a picture in the mind. Imagery may refer to an abstract image of a concrete object or a dynamic activity where the parts of an image can be taken apart, reconstructed or changed (Owens, 2003). In this project, we were interested in the way students drew their images and the way that they used pattern imagery to count the number of dots in a pattern. We describe examples of students' responses to one such task below.

Visual imagery task: a triangular pattern of circles

The assessment tasks included a Space task where students were briefly shown a triangular pattern of six circles (Figure 1) and then asked to 'draw what you saw'. This task provided critical evidence of many students' difficulties with visual imagery and recording a triangular pattern. The key questions we posed were as follows:

- Do students notice and use features that are related to spatial organisation, such as the three circles evenly spaced along each side of the triangle?
- Do students rely on unitary counting (counting by ones)?
- Do students make connections between numerical and spatial patterns in developing more efficient mathematical strategies? For example, do they count in a pattern 1, 2, 3 representing the rows of the triangle?

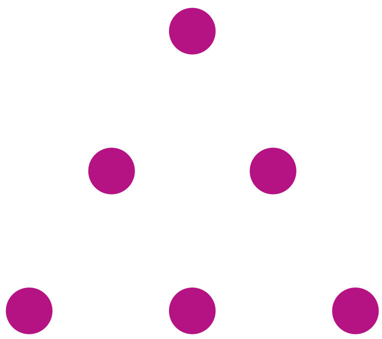

Figure 1. Triangular pattern task.

The task proved very difficult, with only 20% of students giving correct responses. Interestingly, students could often remember the correct number of circles in the pattern but could not reproduce the triangular shape or the spatial pattern of the circles.

The following figures show a range of responses to this task, described according to four categories: no structure

(Figure 2), emergent structure (Figures 3, 4 and 5), partial structure (Figures 6, 7, 8, and 9), and structure (Figures 10 and 11).

Mandy drew a row of circles bearing no relationship to the shape, pattern or quantity of the triangular pattern of circles (Figure 2). Interestingly, Mandy made several attempts to depict the triangular shape but consistently drew a row of circles. She explained to the interviewer that she only saw circles and that she drew a picture of what she remembered.

Figure 2. Mandy's response: no structure.

Mitchell initially drew the correct number of circles, but then added squiggles that are unrelated to the triangular pattern (Figure 3). When asked to explain his response, he mentioned that he had counted six circles. He did not, however, represent the spatial organisation, pattern, or triangular shape of the circles. Since there was some attempt to show the circles as evenly spaced, we infer that some signs of structure are beginning to emerge in Mitchell's drawing.

Figures 3. Mitchell's response: emergent structure.

Clare described her drawing (Figure 4) as a 'Christmas tree'. It shows little awareness of the structure of the pattern or the number of circles. However, some idea of structure was beginning to emerge, with equally spaced markings and some indication of the triangular shape.

Figure 4. Clare's response: emergent structure

Melinda responded with a curved sequence of circles that neither represented the shape nor the number of circles (Figure 5). Interestingly, she explained that the curve resembled a triangle on its side (rotated 90° to the left). It is difficult to assess whether she had actually formed an image of a triangle, but there is some indication of emergent structure in the way she spaced the circles.

Figure 5. Melinda's response: emergent structure.

Figure 6 shows Trisha's response: an array of three rows of four circles. She made an initial attempt to draw a triangular form (as can be seen from the sloping side) but then reverted to a rectangular array, perhaps because it was more familiar. Her drawing does, however, correctly show some important aspects of the required structure: it has distinct rows and columns, and the circles were equally spaced. For this reason, we classify Trisha's drawing as showing partial structure.

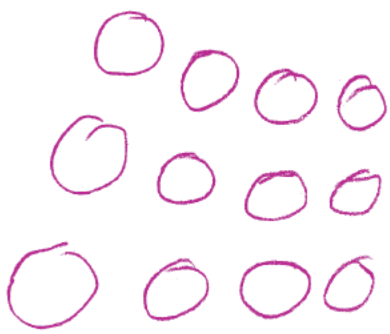

Figure 6. Trisha's response: partial structure

Figure 7 shows Lloyd's drawing. The triangular shape is clear, and the circles around the edge are fairly equally spaced, but he has not represented the numerical pattern or quantity correctly: the circles inside the triangle just seem to be drawn to fill in the space.

Figure 7. Lloyd's response: partial structure.

Figure 8 shows Katie's first drawing, an equilateral triangle. She showed the correct spatial structure (three equal sides) but completely neglected to represent the spatial pattern. At a follow-up interview, she did attempt to show the pattern of circles within an equilateral triangle shape (Figure 9). However, like Lloyd, she seemed to have filled the space inside the triangle with random circles.

Figure 8. Katie's response in Interview 1: partial structure.

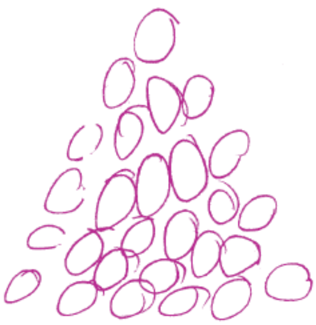

Figure 9. Katie's response in Interview 2: partial structure.

Kirsten's and Mitchell's drawings (Figures 10 and 11) both depict a triangular shape (although not exactly aligned), and the correct numerical pattern of 1, 2 and 3 circles aligned in rows. These students reproduced and described the triangular shape, the spatial structure and number pattern without any hesitation. In follow-up interviews, both these students could extend the pattern of triangular numbers accurately.

Figure 10. Kirsten's response: structure.

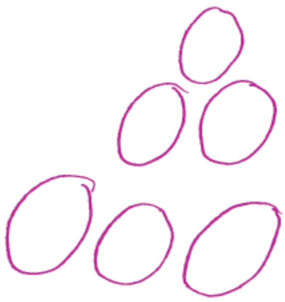

Figure 11. Mitchell's response: structure.

Teaching implications

Although our task reflects only one aspect of underlying mathematical structure and pattern, the responses provide indicators of Year 1 students' developing visual imagery. Most importantly, these representations provide an indication of students' ability to represent a spatial/numerical pattern from memory. We were surprised at the wide range of responses.

There are many tasks that can capture students' ability to use visual imagery (tasks requiring rearrangement of parts of shapes) and transformations (reflection, rotation and translation). The triangular pattern task requires attention to three aspects: the shape of the pattern (a triangle), the arrangement of the parts (equally spaced in lines), and the quantity (6 circles). Recognition of pattern and structure in tasks like this lays the foundation for success in other patterning tasks, such as consecutive odd numbers depicted as a triangle (1, 3, 5, 7…).

There is evidence that students can become more attuned to the importance of pattern and structure if the teacher provides appropriate tasks. The visual memory task would be a good one to begin with. Students may need to focus on each feature (shape, arrangement and quantity) separately, before they can integrate them into one mathematical image. Their attention can be drawn to the shape of the pattern, they can be asked to visualise it in their 'mind's eye', and then they can draw their image and check their drawing against the original pattern. The numerical pattern can be built up gradually through subitising, counting and grouping tasks. Similar visual memory tasks can be devised using a range of two-dimensional and three-dimensional shapes. In this way, students would be encouraged to focus on mathematical aspects of representations rather than distracting features which do not lead to mathematical understanding (Thomas, Mulligan & Goldin, 2002).

More able students can be challenged to develop complex aspects of pattern and structure through open-ended tasks creating and extending spatial and numerical patterns and models. Key questions to extend such students might be the following:

- What do you notice is the same about this pattern (e.g., a triangle of dots) and a numerical pattern (e.g., 1, 2, 3)?
- Can you draw the same pattern in a different way (e.g., upside down)?
- Can you create and draw from memory a similar pattern using a different shape?

Identifying students' use of visual imagery can provide indicators of their mathematical images, and this information could be particularly significant for assessing and assisting gifted and talented students as well as those experiencing learning difficulties.

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