

Inferential Expectations*

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Abstract

We propose that the formation of beliefs be treated as statistical hypothesis tests, and label such beliefs inferential expectations. If a belief is overturned through the build-up of evidence, we assume agents switch to the rational expectation. Thus, if the test size is unity, agents hold rational expectations. We solve a Dornbusch-style model of exchange rates under rational expectations and inferential expectations. Under the latter we prove that the regression tests of Uncovered Interest Parity and the rational expectations version of the term structure display a downward bias. The model also explains delayed overshooting and sharp changes in exchange rates.

KEYWORDS: expectations, macroeconomics, rationality, Uncovered Interest Parity, term structure, exchange rate

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“While I was aware a lot of these [sub-prime] practices were going on, I had no notion of how significant they had become until very late. I didn’t really get it until very late in 2005 and 2006.”

Alan Greenspan CBS “60 minutes” 16 Sept. 2007

1 Introduction

Recent decades have seen a theoretical backing away from Rational Expectations (RE).¹ Examples include: (1) near rationality (Akerlof and Yellen, 1985), (2) agents as econometricians (Sargent, 1993), (3) parameter uncertainty and econometric learning (Evans and Honkapohja, 2001), (4) model uncertainty and robustness (Hansen and Sargent, 2001), (5) infrequent rational expectations, when inattentive agents consult experts (Carroll, 2003), (6) information processing constraints and ‘rational inattention’ (Sims, 2003) and (7) utility-based beliefs, or ‘optimal’ expectations (Brunnermeier and Parker, 2005).²

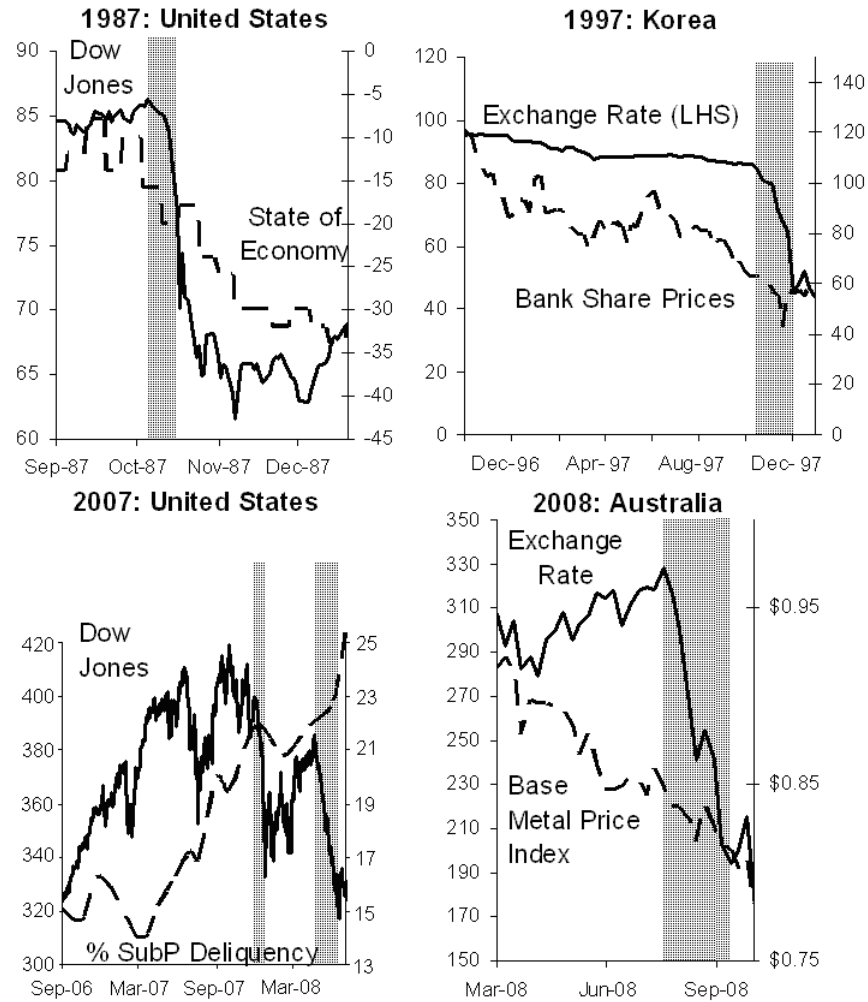
In this paper, we offer another alternative to RE, the insight for which is captured in Alan Greenspan’s quote: evidence must build up to a ‘significant’ threshold before agents ‘get it’. This seems reasonable to us for three reasons.

First, asset prices sometimes move far more sharply than the underlying fundamentals at the onset of a crisis, when beliefs are typically being revised. Figure 1 shows evidence from three decades. In 1987, the ABC State of the Economy Index declined gradually over the whole of the last quarter, yet the Dow Jones did not collapse until October 19. In 1997, gradually declining Korean bank share values heralded the need for substantial macroeconomic adjustment (Burnside, Eichenbaum, and Rebelo, 2001), yet the currency held up until the final quarter of that year (Radelet and Sachs, 2000). In 2007 delinquent sub-prime loans rose steadily over the whole of 2007, but sharp falls in the Dow were not observed until early and mid-2008. Finally, steady declines in base metal prices over 2008 preceded the sharp depreciation of the Australian dollar by nearly half a year.

¹RE has equivocal support in the experimental literature. While its predictions are not rejected as null hypotheses in some contexts (see Dwyer, Williams, Battalio, and Mason, 1993), the most common outcome is that individuals do not hold RE (e.g., Schmalensee, 1976; Blomqvist, 1989; Beckman and Downs, 1997; Swenson, 1997). In addition, experimental research often finds either under-utilization or over-utilization of priors (Camerer, 1995)

²Near rationality has been applied to the failure of UIP in Gruen and Menzies (1995) and more recently in Bacchetta and Wincoop (2006). Econometric learning has been applied to the failure of UIP in Chakrabortya and Evans (2006).

Figure 1: Sudden Asset Price Movements in Three Decades of Crises



Notes: 1987 US: Dow Jones 600 Index and ABC News State of the Economy Index from Datastream. 1997 Korea: USD Korean Won index (1 Aug 96 = 100) and Korean Bank Institution Stock Price Index from Datastream. 2007 US: Dow Jones 600 Index from Datastream and delinquency rates for sub-prime loans originated between Jan 04 and Aug 08 excluding home equity loans. Sub-prime loans are either identified as such by the servicer, or have an original FICO score of less than 620; data from LPS Analytics. 2008 Australia: Westpac Base metals sub-index and USD exchange rate from Datastream.

Our explanation for the mismatch between the *gradual* erosion of fundamentals and the *rapid* asset price declines is that the fundamental factors are noticed by agents as they form expectations. But, as described in the quote, they take some time to ‘get it’ before dramatically downgrading their forecasts. At that instant, asset prices decline sharply.

While the evidence from Figure 1 is clearly only anecdotal, RE has not stood up well to more careful econometric scrutiny. Under the joint hypotheses of RE, risk neutrality and zero transaction costs, the slope coefficients in a regression test of Uncovered Interest Parity (UIP) and the RE version of the term structure should both be unity. In the following regressions S , $I - I^*$ and R represent the log nominal exchange rate, the short interest differential and the long (here two period) interest rate. The errors u and v are both white noise.

$$\Delta S_{t+1} = \beta(I_t - I_t^*) + u_t \quad (1)$$

$$\frac{\Delta I_{t+1}}{2} = \gamma(R_t - I_t) + v_t \quad (2)$$

Typically, the estimated coefficient in the UIP regression, (1), is less than unity, and sometimes it is even negative (Frankel and Rose, 1995; Froot and Thaler, 1990). Evidence based on (2) has not been as damning, but RE remains a seriously contested hypothesis. For example in a study of 3-, 6- and 12-month euro-rates for 17 countries, Gerlach and Smets (1997) found that the 76 per cent of regression coefficients were less than unity (Figure 2).

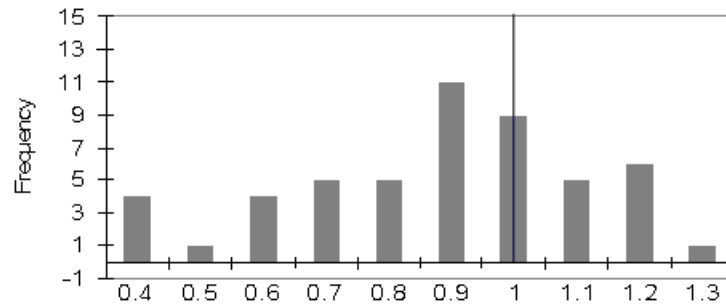
The failure of UIP is particularly disturbing since it is a *predictive* failure. Friedman (1953) argued that a theory may be valid even if the assumptions are unrealistic, provided it predicts better than an alternative. The rational representative agents of financial economics do indeed appear unrealistic,³ but the predictive failure of UIP erodes the justification for RE as an ‘as if’ assumption, at least in the foreign exchange market (although we do not deny the fact that the problem may have other sources).⁴

³Bacchetta and Wincoop (2006) note that the extent of trading on the basis of interest differentials is small relative to cross border wealth, that foreign exchange traders use a variety of forecasting tools since they know that they are not likely to improve upon a random walk (Meese and Rogoff, 1983; Cheung, Chinn, and Pascual, 2005), that agents trading equities respond to information with a significant lag (Froot, O’Connell, and Seasholes, 2001), that mutual funds trade under restrictions of asset classes (Lyons, 2001) and that many large investment companies fail to adjust their portfolios for long periods (Investment Company Institute, 2002).

⁴A vast literature in the 1980s and 1990s routinely rejected UIP. Boudoukh, Richardson, and Whitelaw (2005, pg. 1) claim to have counted ‘well over a hundred papers’ that document its failure, and say that this result ‘is one of the more robust puzzles in finan-

Figure 2: Coefficients for RE Version of the Term Structure Tests

(Under RE Slope=1)



Source: Table 1 in Gerlach and Smets (1997)

Notes: The histogram pools together the slope coefficients of regression of the change in short rates on the interest differential. The regressions have a different form from (2) for 3- 6- and 12-month securities, but under RE they should all be unity. Individually, the sample means of the coefficients for each maturity are significantly lower than unity with a maximum p-value of 0.03. The country list comprised European economies, plus Canada, Japan and the US.

Our second reason is based on the experiment of Menzies and Zizzo (2005). Undergraduates were shown the contents of two urns, each with different combinations of white and orange balls. One urn was then selected randomly, and subjects received signals about its contents by the means of random ball draws with replacement from the chosen urn. The prior probability of a particular urn being chosen was 0.5 at the start of the experiment, but should then have evolved under RE according to Bayes' rule, as *each* new ball was drawn. In

cial economics.' Although Chinn (2006) has shown that the failure is not as pronounced for long-term securities, he also shows that the extent of the downward bias for short to medium run securities is not disappearing with the passage of time. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006) note that the literature has explored time varying risk premia (perhaps as a result of endogenous market segmentation), the interaction of risk premia and monetary policy (as in McCallum, 1994), statistical considerations such as peso problems and non-cointegration of forward and spot rates, learning, biases in expectations, and, the cost of actively managing foreign exchange portfolios. Unlike us, some of these approaches assume RE, and blame the failure of UIP on auxiliary assumptions.

fact, agents displayed belief conservatism. Even allowing for rounding, agents failed to switch their probability guess far more often - over 30 per cent of the time - than would be expected under RE. The evidence suggests that subjects often noticed the evidence without changing their mind (further, less direct experimental evidence for belief conservatism is discussed in the exchange rate experiment by Menzies and Zizzo, 2007).

The third reason comes from the philosophy of science. While economists have become uneasy with RE in recent decades, a parallel debate has been carried on in the physical sciences on the status of hypothesis testing as a model of belief formation. Mayo (1996) and Mayo and Spanos (2006) have argued that there is a paradoxical reluctance of scientists to describe their own belief changes in terms of hypothesis testing. The key point here is that, if we, as social scientists, habitually use hypothesis testing to help form our own beliefs,⁵ it would be natural to assume that our representative agents also use hypothesis tests.

A hypothesis test is a good model for belief formation of the kind described by Greenspan. In particular, until the rejection region is reached, *agents notice information without changing their minds*. We call our model ‘inferential expectations’ (IE) where ‘inferential’ refers to classical Neyman-Pearson inference.

We assume that, when a belief is overturned, agents switch to RE.⁶ Thus, RE is a special case of IE if agents are unconcerned about mistakenly changing their beliefs (the test size α equals unity). Seen in this way, α becomes a metric for rationality: when it is zero agents are completely unresponsive to evidence, and when it is unity they are completely untainted by belief conservatism. By linking RE to IE in this way, we ground our expectations theory ultimately in the structure of the model, which provides modeling discipline. Furthermore, if this modelling practice is adopted consistently backwards through time, then it follows that every null hypothesis was once an RE belief.⁷

We do not explore possible micro-foundations for IE in this paper, though one could seek to justify IE by relative inattention, as implied for example in

⁵One author received criticism from a referee of a top-tier journal because significance was accorded to a test in the submitted paper with a p -value of 0.052.

⁶An implication of this is that Bayesian methods and IE need not conflict: agents could be infrequent Bayesians, with the time interval between updates determined by a hypothesis test.

⁷That is, unless the initial null is arbitrarily chosen. In an empirical application of IE, it would be possible to define the alternative hypothesis, or the null, in a way unrelated to model-consistency. It could, for example, be made dependent on framing effects (e.g., Tversky and Kahneman, 1986). We leave this matter for future research, but note that it would throw the treasured discipline of RE to the wind.

Mankiw and Reis (2003). Scott and Nowak (2005) describe certain environments where hypothesis testing is an optimal way to learn. Bacchetta and Wincoop (2006) show how infrequency of belief adjustment can be formally related to the size of the adjustment cost. Another rationale for IE is that it can be considered as a ‘fast and frugal heuristic’ (see Gigerenzer, Todd, and The ABC Research Group, 1999) of bounded-rational belief formation, characterized by information-gathering and information processing costs.

How fruitful can it be to make our representative agents use hypothesis tests? We answer this question by building a Dornbusch style model for the exchange rate, and then comparing its solution under IE and RE. Under IE, our model generates downward bias for both (1) and (2), in keeping with the empirical evidence.

Other anomalies in forex markets exist, such as the so called delayed overshooting puzzle. Unanticipated US monetary expansions are associated with a persistent decline in US interest rates and an initial depreciation of the dollar followed by an appreciation several months later (Eichenbaum and Evans, 1995). We show that our model can also replicate these anomalies.

The rest of this paper is structured as follows. Section 2 presents our model of exchange rates. Section 3 connects our theory of inferential expectations to related research and concludes.

2 Exchange Rate Determination and Term Structure of Interest Rates

We now append IE to a simplified ‘overshooting’ model (Dornbusch, 1976) inhabited by identical same- α agents. This widely-studied and well-understood framework is a classic application of RE, so it makes for an instructive comparison with IE.⁸

One noteworthy feature of Dornbusch’s model, which makes for an elegant solution, is its stark version of RE. Agents have perfect foresight about the future actions of the monetary authorities, and do not have to assign probabilities to particular future contingencies.

⁸Difficulties in establishing the relationship between the real exchange rate and interest differentials (Campbell and Clarida, 1987) and in forecasting nominal exchange rates (Meese and Rogoff, 1983) count as evidence against this class of models. However, there is some evidence that short-run volatility dominates the unfavourable empirics (e.g., Baxter, 1994; Chinn and Meese, 1995). In the absence of a clear alternative, this class of models may be useful when there are large changes in policy (such as the Volcker deflation).

For the purposes of comparison, our treatment of IE will be stark too. Agents choose between a belief that a monetary expansion is permanent, and a belief that it will be reversed next period forever. Many other setups are of course possible, but take us away from analytic solutions (see the appendix for one example).

2.1 The Equations of an Overshooting Model

$$M - P_t = B_y \bar{Y} - B_i I_t \quad \text{LM}$$

$$P_{t+1} - P_t = -B_p(P_t - E[P_\infty]) \quad 0 < B_p < 1 \quad \text{prices}$$

$$E[S_{t+1}] - S_t = I_t - I^* \quad \text{UIP}$$

$$R_t = \frac{1}{2}(I_t + E[I_{t+1}]) \quad \text{RE term structure}$$

$$S_t - P_t = 0 \quad \text{for } t = 0^- \quad \text{and } \infty \quad \text{PPP}$$

All parameters are positive and all variables are in logs, except the nominal short and long (two-period) interest rate I and R . The exchange rate, money, prices, output and foreign interest rates are S , M , P , \bar{Y} and I^* . P_∞ is the log of the domestic price in the infinite future. The log foreign price is normalized to zero, so purchasing power parity (PPP) implies $S = P$. Time is indexed $t = 0, 1 \dots \infty$, with time zero divided into an initial steady state 0^- and a post shock 0^+ . The expectations operator $E[\cdot]$ refers to either RE or IE at time t , and all agents have the same expectation. The LM curve can be interpreted as a quasi-Taylor rule, if it is re arranged with I as the subject and M as the nominal income target. The price equation says that agents approach the expected long run price level using a partial adjustment mechanism.

We define an initial steady state where M_t is zero for all $t \geq 0$. In the absence of any reason for prices to change, $E[P_\infty] = P_{0^-}$. The exchange rate is stable in the steady state; therefore, UIP implies $I_{0^-} = I^*$, money demand implies $P_{0^-} = -B_y \bar{Y} + B_i I^*$, PPP implies $S_{0^-} = P_{0^-}$ and the RE term structure implies $R_{0^-} = I^*$. The model is re-written in deviations from this initial steady state, using lower case letters. To anticipate the next section, we allow money to depart from its initial value (zero) by m .

$$m - p_t = -B_i i_t \quad (3)$$

$$p_{t+1} - p_t = -B_p(p_t - E[p_\infty]) \quad 0 < B_p < 1 \quad (4)$$

$$E[s_{t+1}] - s_t = i_t \quad (5)$$

$$r_t = \frac{1}{2}(i_t + E[i_{t+1}]) \quad (6)$$

$$s_t - p_t = 0 \quad \text{for } t = 0^- \quad \text{and} \quad \infty \quad \text{PPP}$$

The assumption that expanded money does not lead directly to price pressures (apart from its effects on expectations) is realistic if the length of a period is relatively small in real time. One might expect this to be the case during a currency crisis.

2.2 The RE Solution

We now consider the RE solution for a permanent money shock m at time 0^+ . A new steady state occurs when $t = \infty$, where UIP implies $I_\infty = I^*$, PPP implies $S_\infty = P_\infty$, the RE term structure implies $R_\infty = I^*$ and money demand implies $P_\infty = m - B_y \bar{Y} + B_i I^*$. (Therefore in (4), $p_\infty = P_\infty - P_{0-} = m$.) The standard Dornbusch assumption of sticky prices is adopted ($P_{0+} = P_{0-}$ so $p_{0+} = P_{0+} - P_{0-} = 0$). RE implies perfect foresight so $E[s_{t+1}] = s_{t+1}$, $E[i_{t+1}] = i_{t+1}$ and $E[p_\infty] = p_\infty = m$. Making the latter substitution into (4) we obtain a solution for prices.

$$\begin{aligned} \text{from (4)} \quad p_{t+1} - p_t &= -B_p(p_t - m) \\ \Rightarrow \quad p_{t+1} - m &= (1 - B_p)(p_t - m) \\ p_t - m &= (p_{0+} - m)(1 - B_p)^t \\ &= -m(1 - B_p)^t \end{aligned}$$

The eigenvalue of the system is clearly $(1 - B_p)$. To find i , we put it on the left hand side of (3), and substitute the above solution for p .

$$\begin{aligned} m - p_t &= -B_i i_t \\ i_t &= \frac{p_t - m}{B_i} = \frac{-m(1 - B_p)^t}{B_i} \end{aligned}$$

Clearly, $i_{t+1} = (1 - B_p)i_t$. To obtain s , (5) is iterated forward to infinity (when $s = m$) and the infinite geometric series, with ratio $(1 - B_p)$, is summed.

$$\begin{aligned} s_t &= -(i_t + i_{t+1} + i_{t+2} \cdots) + s_\infty \\ &= -(i_t + (1 - B_p)i_t + (1 - B_p)^2 i_t + \cdots) + m = \frac{-i_t}{B_p} + m \end{aligned}$$

The full solution is given by equations (7)-(9).

$$i_{t+1} = (1 - B_p)i_t$$

$$p_t = -m(1 - B_p)^t + m \quad (7)$$

$$i_t = \frac{-m}{B_i}(1 - B_p)^t \quad (8)$$

$$s_t = \frac{-i_t}{B_p} + m \quad (9)$$

2.3 The IE Solution

As a matter of definition, an IE solution needs two *hypotheses*, H_0 and H_1 , and a *signal* to help choose between them. IE employs the standard statistical terminology in describing how the signal is processed. That is, a *sampling distribution* of a *test statistic* is calculated under the null, where the test statistic is some function of the signal. The test size α then implies a *rejection region*.

With regard to the *hypotheses*, H_0 is the belief that next period ($t + 1$) the central bank will undo the expansion that has been in operation from 0^+ to t , in such a way that the exchange rate will revert to the pre-shock value forever. This requires that all *future* interest differentials (but not the current one) are zero (to fulfil (5) iterated one period forward). Thus we have the following hypotheses.

- H_0 : The monetary expansion is not permanent $\Rightarrow s_{t+j} = i_{t+j} = 0, j > 0$ ⁹
- H_1 : The monetary expansion is permanent; $m_{t+j} = m, j \geq 0$

The null hypothesis is equivalent to believing that the central bank is engaging in a managed float, and that the initial steady-state exchange rate (S_0) is its target level of the currency.¹⁰ The switch between H_0 and H_1 describes a situation where forex traders *suddenly* discard their belief that a currency target will be attained from next period onwards, when the credibility

⁹The two hypotheses do not exhaust all possibilities, such as the central bank undoing the shock in, say, $t + 3$. The text develops the simplest possible stochastic environment that will illustrate the impact of IE on this model.

¹⁰This target can only be met by setting the appropriate level of interest rates, and departures from the target are possible. Therefore, the institutional setup of the model is a managed float (rather than a fixed rate regime) where the instrument is the interest rate (rather than foreign exchange intervention).

strain on the central bank of continually-observed higher liquidity becomes too great. This suddenness is driven by an initial reluctance by forex dealers to change their minds, despite the evidence of higher liquidity and lower interest rates, mimicking Greenspan not ‘getting it’.¹¹ After the belief change, the solution under H_1 is calculated using RE, giving (7) through (9).

We will now show that the above H_0 is model consistent, in the sense that if everyone believes H_0 , and m does in fact return to zero next period, their beliefs will be vindicated. Things are simplified greatly by the fact that agents do not believe the expansion is permanent under H_0 , so $E[p_\infty] = 0$. With no reason to expect prices to change in the future, $p_{t+1} = 0$ and the full solution follows immediately (note that from $t + 1$ onwards $m = 0$ under H_0):

$$\begin{aligned} E[p_\infty] = 0 &\Rightarrow p_t = 0 && \text{from (4)} \\ &\Rightarrow i_t = -\frac{m}{B_i} && \text{from (3)} \end{aligned}$$

$$\begin{aligned} s_t &= -(i_t + i_{t+1} + i_{t+2} \cdots) + s_\infty && \text{from (5)} \\ &= -i_t \quad \text{under } H_0 \\ &= \frac{m}{B_i} \end{aligned}$$

$$r_t = \frac{i_t}{2} = -\frac{m}{2B_i} \quad \text{from (6)}$$

Agents observe prices, money and interest rates. Prices are uninformative, and money is a sufficient statistic for the remaining two, so it is the *signal*. Under H_0 , we assume a distribution of elapsed time before the expansion is undone (and the exchange rate target is hit), with longer temporary expansions being less likely. The number of periods that money is at m is therefore the *test statistic*. After money has been equal to m for a ‘large’ number of periods - with ‘large’ determined by the test size α - agents realize the truth and jump from the belief that the expansion is temporary to the realization that

¹¹Gourinchas and Tornell (2004) suggest that in a US context the uncertainty surrounding the duration of a monetary shock may be due to inaccurate forecasts by the Fed, or, by a temporary shift in the balance of power among the members of the Fed Open Market Committee. Lewis (1989) takes a contrasting approach. In her model, departures from UIP occur as agents learn gradually and ‘rationally’ (in the sense of using information optimally, not in the sense of having zero-mean i.i.d. forecast errors) about money demand. Commenting on this, Gourinchas and Tornell make two claims. First, Bayesian learning would lead to a rapid convergence to the true model which conflicts with their econometric evidence against interest-differentials learning. Second, Lewis’ model would not explain a negative coefficient in the UIP regression.

it is permanent. For example, if the *sampling distribution* of the elapsed time before the central bank undoes a temporary money shock under H_0 is e^{-t} then the p -value for the test is also e^{-t} .

$$\int_t^\infty e^{-t} dt = [-e^{-t}]_t^\infty = e^{-t}$$

From this p -value, it is clear that if the support of the density is 0 to ∞ , the probability density function integrates to unity. Finding the critical value t^* (the time elapsed before agents reject H_0) is a matter of solving $e^{-t^*} = \alpha$, or $t^* = -\ln(\alpha)$. For example, if $\alpha = .05$, $t^* = 3$. This sampling distribution is for illustrative purposes only, and the remaining analysis does not rely on it.

The rejection region is: reject H_0 if $t \geq t^*$. As in all classical hypothesis testing, the *rejection region* is decided upon prior to the collection of any data, i.e. in period 0-. We have now defined all the elements of the IE solution. After H_0 is rejected, the model evolves according to the RE solution given by (7)-(9). However, as was noted earlier, the powers of t are effectively set back to zero since the solution must be identical to the RE solution at 0^+ . Equivalently, t is replaced with $t - t^*$ in (7)-(9).

$$i_{t+1} = (1 - B_p)i_t$$

$$p_t = -m(1 - B_p)^{t-t^*} + m \quad (10)$$

$$i_t = \frac{-m}{B_i}(1 - B_p)^{t-t^*} \quad (11)$$

$$s_t = \frac{-i_t}{B_p} + m \quad (12)$$

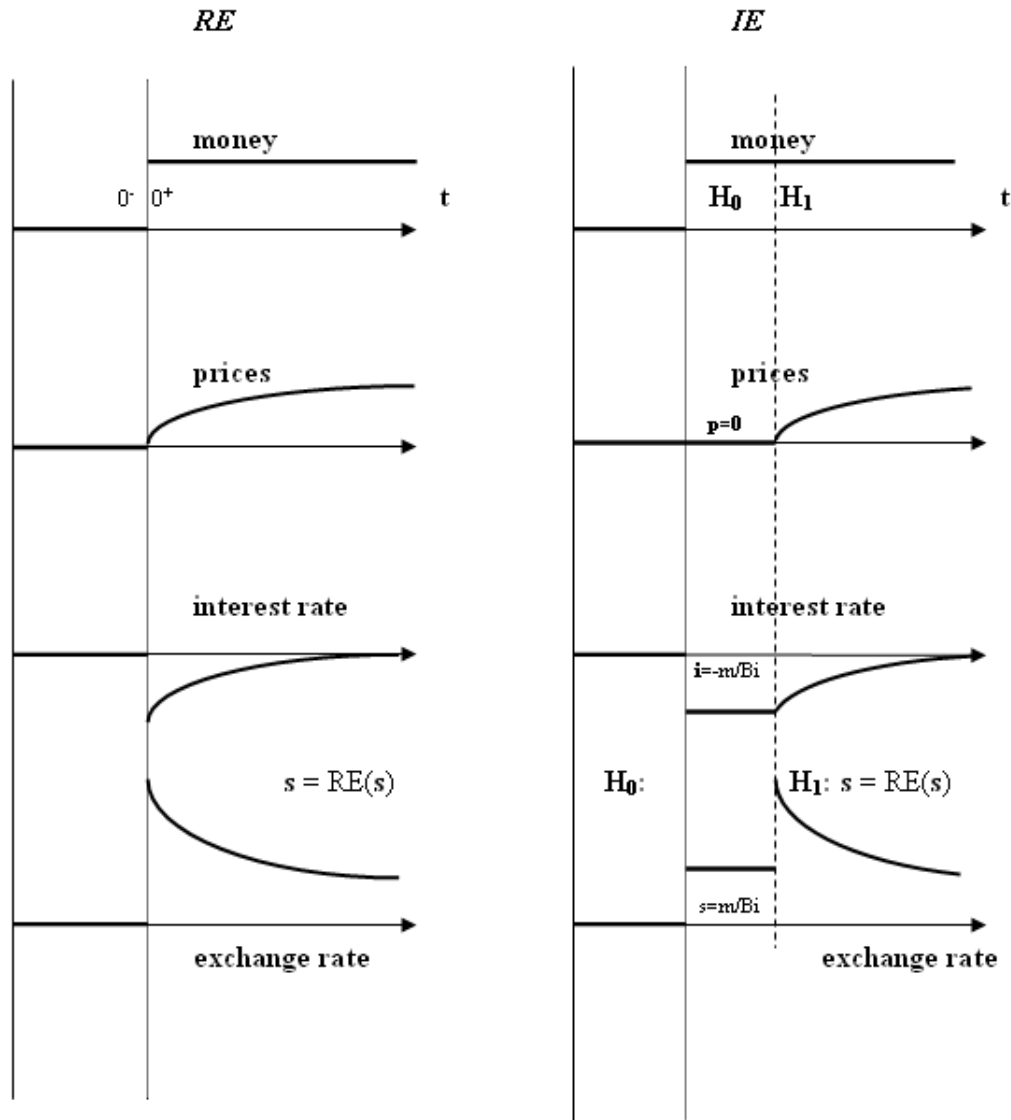
Importantly, IE has the capacity to deliver a sudden, and potentially large, change in a model variable - here the exchange rate - for a small increment of information, at the instant H_0 is rejected. This can be illustrated with a diagram.

The pattern in the right panel of Figure 3 is consistent with the so-called delayed overshooting puzzle by Eichenbaum and Evans (1995). While inconsistent with RE and with the Bayesian model presented in the appendix for reasonable parameter values, the delay in the exchange rate appreciation is consistent with IE.¹²

¹²IE has the property that adjustments need not be gradual (depending upon the implications of H_0 and H_1), as in the diagram. So the comment in the text must be interpreted in terms of a broad pattern. Being a VAR, the impulse response functions of Eichenbaum and Evans had smooth depreciations and appreciations by construction.

Figure 3: Dornbusch Setup with RE and IE

(Under H_1 Left & Right Panels Identical)



Notes: The left-hand panel shows the standard dynamics from Dornbusch (1976) for a monetary expansion. The right hand panel show the IE dynamics. Prior to H_0 being overturned, agents in the foreign exchange market ignore future (negative) interest rate differentials, and the depreciated (lower) long run exchange rate. When H_0 is discarded for H_1 the exchange rate jump depreciates s to the RE solution.

2.4 Downward Bias in Uncovered Interest Parity and RE Term Structure Regressions

We now show that data on S and R generated by this model with IE will, when placed in the OLS regressions (1) and (2), create downward bias in the parameters.

Theorem 1 *If H_0 is believed for at least one period, the OLS coefficient from a regression of ΔS_{t+1} on $I_t - I^*(= i_t)$ will be less than unity.*

Proof: Let t^* be the time period in which H_0 is rejected. The numerator of the OLS coefficient can be decomposed into three terms; the cross product summed prior to, at, and after $t^* - 1$. It will prove useful to express all the cross products as a function of i^2 .

$$\hat{\beta} = \frac{\sum_{t=0}^{n-1} \Delta S_{t+1} (I_t - I^*)}{\sum_{t=0}^{n-1} (I_t - I^*)^2} = \frac{\sum_{t=0}^{n-1} \Delta S_{t+1} i_t}{\sum_{t=0}^{n-1} i_t^2} = \frac{\sum_{t=0}^{t^*-2} \Delta S_{t+1} i_t + \Delta S_{t^*} i_{t^*-1} + \sum_{t=t^*}^{n-1} \Delta S_{t+1} i_t}{\sum_{t=0}^{n-1} i_t^2} \quad (13)$$

We evaluate each quantity in the numerator separately. Taking the first term, H_0 exchange rates are fixed at m/B_i .

$$\begin{aligned} \Delta S_{t+1} &= s_{t+1} - s_t = 0 \\ \therefore \sum_{t=0}^{t^*-2} \Delta S_{t+1} i_t &= \sum_{t=0}^{t^*-2} 0 \cdot i_t^2 \end{aligned} \quad (14)$$

Parenthetically, this establishes Theorem 1 for the special case of $\alpha = 0$. Agents never switch from H_0 and an OLS regression of ΔS on i will fit perfectly, with a zero coefficient.

The middle term of the numerator in (13) involves a change in the exchange rate from the H_0 level at time $t^* - 1$ to the H_1 path at time t^* (see Figure 3.) The H_1 exchange rate at t^* is (9) with $t - t^*$ replacing t . It equals the 0^+ RE

solution, as prices don't change from 0^+ to t^* .

$$\begin{aligned}
 \Delta s_{t^*} &= s_{t^*} - s_{t^*-1} \\
 &= \left(-\frac{i_{t^*}}{B_p} + m \right) - \frac{m}{B_i} \\
 &= \left(-\frac{(m/B_i)(1 - B_p)^{t^*-t^*}}{B_p} + m \right) - \frac{m}{B_i} \\
 &= m \left(\frac{1}{B_i B_p} + 1 - \frac{1}{B_i} \right) \\
 \therefore \Delta s_{t^*} i_{t^*-1} &= m \left(\frac{1}{B_i B_p} + 1 - \frac{1}{B_i} \right) \left(-\frac{m}{B_i} \right) \\
 &= -B_i \left(\frac{1}{B_i B_p} + 1 - \frac{1}{B_i} \right) \left(-\frac{m}{B_i} \right)^2 \\
 &= \left(1 - B_i - \frac{1}{B_p} \right) i_{t^*-1}^2
 \end{aligned} \tag{15}$$

Since B_p is no larger than unity, $1 - 1/B_p$ is non-positive and hence $1 - B_i - 1/B_p$ is strictly negative. The last term on the numerator of (13) is evaluated under H_1 so that (5) holds, with $E[s_{t+1}] = s_{t+1}$. Clearly:

$$\sum_{t=t^*}^{n-1} \Delta s_{t+1} i_t = \sum_{t=t^*}^{n-1} i_t^2.$$

The OLS numerator is thus a weighted average of i_t^2 , divided by the sum of i_t^2 , with weights prior to the observations on the H_1 path strictly less than unity (in fact, non-positive). Q.E.D.

$$\hat{\beta} = \frac{\sum_{t=0}^{n-1} w_t i_t^2}{\sum_{t=0}^{n-1} i_t^2} \quad w_t = \begin{cases} 0 & t < t^* - 1 \\ 1 - B_i - \frac{1}{B_p} < 0 & t = t^* - 1 \\ 1 & t > t^* - 1 \end{cases} \tag{16}$$

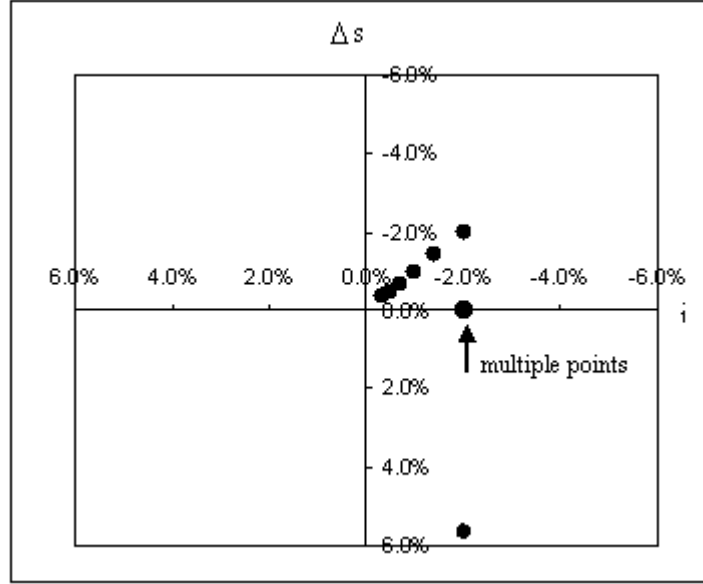
Lemma 1 *The OLS estimate could be negative.*

Proof: A large enough $|w_{t^*-1}|$ could render the numerator sum negative. Q.E.D.

Intuitively, agents are not factoring in the depreciation of the long run exchange rate, which restrains the 0^+ exchange rate (Figure 3). Under H_0 ,

Figure 4: Scatter Plot of ΔS and i

(1% M expansion, $B_i = 0.5$, $B_p = 0.3$, reversed axes)



agents (incorrectly) believe future i_{t+j} and s_{t+j} are zero. Unchanged p_t implies that i_t is fixed (Figure 3), so s_t is too, from (5). The data in a scatterplot of ΔS vs. i (Figure 4 is based on a simulated model) will therefore have a slope coefficient of 0 under H_0 and unity under H_1 . When the exchange rate jump between H_0 and H_1 occurs, the interest differential is negative but the exchange rate depreciates in the next period. If the jump is large enough, Lemma 1 says the OLS slope can be negative.

We now turn to (2), the term structure regression.

Theorem 2 *If H_0 is believed for at least one period, the OLS coefficient from a regression of $\Delta I_{t+1}/2$ on $R_t - I_t$ will be biased downwards (strictly less than unity).*

Proof: First note that $\Delta I_{t+1} = \Delta i_{t+1}$ and $R_t - I_t = r_t - i_t$. As before, it is convenient to express everything in terms of $(r - i)^2$. Under H_0 :

$$\begin{aligned} i_{t+1} - i_t &= 0 \\ \therefore \frac{(i_{t+1} - i_t)}{2}(r_t - i_t) &= 0 \cdot (r_t - i_t)^2. \end{aligned}$$

Note that the regression uses the *actual* change in the interest differential, not the forecast one. The *expected* change from i_t to i_{t+1} is not zero, because H_0 agents believe $i_{t+1} = 0$. Each period, they are disappointed, which is a corollary of being mistaken about the duration of the monetary expansion. Under H_1 , equation (6) holds (with $E[i_{t+1}] = i_{t+1}$) and so:

$$\frac{(i_{t+1} - i_t)}{2}(r_t - i_t) = 1 \cdot (r_t - i_t)^2.$$

Between $t^* - 1$ and t^* interest rates don't change even though we move from H_0 to H_1 .

$$\Delta i_t^* = i_t^* - i_{t-1}^* = \frac{-m}{B_i}(1 - B_p)^{t^*-t^*} - \frac{-m}{B_i} = 0$$

The weighted average argument works again for the OLS numerator.

$$\begin{aligned} \hat{\gamma} &= \frac{\sum_{t=0}^{n-1} \frac{\Delta i_{t+1}}{2}(r_t - i_t)}{\sum_{t=0}^{n-1} (r_t - i_t)^2} \\ &= \frac{\sum_{t=0}^{t^*-2} \frac{\Delta i_{t+1}}{2}(r_t - i_t) + \frac{\Delta i_t^*}{2}(r_{t^*-1} - i_{t^*-1}) + \sum_{t=t^*}^{n-1} \frac{\Delta i_{t+1}}{2}(r_t - i_t)}{\sum_{t=0}^{n-1} (r_t - i_t)^2} \\ &= \frac{\sum_{t=0}^{t^*-2} 0 \cdot (r_t - i_t)^2 + 0 \cdot (r_{t^*-1} - i_{t^*-1})^2 + \sum_{t=t^*}^{n-1} 1 \cdot (r_t - i_t)^2}{\sum_{t=0}^{n-1} (r_t - i_t)^2} < 1 \quad (17) \end{aligned}$$

Q.E.D.

Clearly, a negative $\hat{\gamma}$ is impossible, which is in keeping with the less overwhelming rejection of the RE version of the term structure, compared with UIP.

In concluding this section, we note that (16) and (17) highlight an implicit link - first spelt out by Gourinchas and Tornell (2004) - between interest-rate forecast errors and coefficient bias for the tests in (1) and (2).

Recall the right panel of Figure 3. Under H_0 agents mistakenly believe that the interest differential will return to zero in the next period. However, once H_0 is discarded and the solution jumps to the RE path, there are no interest-differential forecast errors (if it were not a perfect-foresight model the forecast errors on the RE path would be i.i.d. errors). It therefore follows that a lower α , implying a higher t^* , will worsen the forecasts for interest differentials.

The conjecture of Gourinchas and Tornell is that poor interest-differential forecasting is responsible for the failure of UIP. In our model, it is obvious from (16) and (17) that a higher t^* will reduce the proportion of ones in the weighted averages, leading to a predominance of zeros in (17) and/or $1 - B_i - (1/B_p)$ in (16). Both these effects will unambiguously reduce OLS coefficients in (1) and (2). Thus, IE provides an ‘expectations microfoundation’ for the failure of both efficiency conditions, whilst exhibiting the secondary cause put forward by Gourinchas and Tornell.

3 Discussion and Conclusions

In this paper we have proposed a new theory of expectation formation, which we have labelled *inferential expectations*, which combines the model consistency of RE with greater empirical plausibility. In the context of a Dornbusch-style model of exchange rates, we proved that the regression tests of Uncovered Interest Parity and the rational expectations version of the term structure both display downward bias. By its nature, the model can also explain sharp changes in the currency values, as one might expect in financial crises.

However, the idea of IE goes beyond the particular application we have provided, and we wish to conclude by drawing links to existing literature. The overlap of IE with other modelling approaches portends well for its future applicability.

IE is close in spirit to Goldberg and Frydman (1996) and Frydman and Goldberg (2003), who allow agents to conduct hypothesis tests over models. Their program, in turn, can be traced back to an early discussion by Rappoport (1985). IE is also related to Foster and Young (2003) game-theoretical work on hypothesis testing by bounded-rational agents on their opponents’ repeated games strategies.

IE is consistent with both empirical and theoretical work on infrequent adjustment. On the former, near rationality has been shown to have small expected utility costs (e.g., Gruen and Menzies, 1995), and, on the latter, models of sticky expectations assume barriers to continual adjustment. However, IE sits firmly in the class of state-dependent infrequent adjustment (as does

Woodford, 2008), whereas much of the existing literature is time dependent.

There is also a potential connection between IE and reference-dependent preferences, which have been explored in a number of papers to explain for example labor supply and consumption decisions (e.g., Tversky and Kahneman, 1991; Sugden, 2003; Farber, 2008). Reference-dependent models need to reconcile decisions which assume that the reference point is unchanged in the short run with the plausible assumption that the reference point does change through time, and just assuming that the reference point is the status quo is clearly unsatisfactory. Tversky and Kahneman (1991) note that the reference point may depend on expectations, and Közegi and Rabin (2006) make this dependence endogenous by assuming RE. An alternative approach of making reference-dependent models work is by assuming that the reference point remains fixed while the null hypothesis holds, and shifts when the null hypothesis is rejected (e.g., the agent ‘feels’ richer, therefore her reference point in terms of wealth changes).

Finally, IE actually encompasses econometric learning (Evans and Honkapohja, 2001; Orphanides and Williams, 2005). Learning, like IE, gains some legitimacy from the practice of economists, and is a sub-case of IE. If we take account of *both* the recursive least-squares estimates *and their standard errors*, then regression-based IE with $\alpha = 1$ becomes econometric learning.

A Appendix: A Bayesian Version of RE

A well-known result is that Bayesian updating can imply downward UIP bias (see Lewis, 1989). We now verify this for our model, but show that delayed overshooting does not occur for reasonable parameters. In period 0, agents believe a permanent or temporary expansion is equi-probable. As time passes, they update the probability the expansion is permanent (θ) using a discrete pdf that a temporary expansion is unwound in period t , along the lines of section 2.3:

$$\Pr(t_{\text{unwind}}) = \lambda e^{-t} \quad \lambda = \frac{e-1}{e} \quad t \geq 0 \quad (18)$$

The time-evolving posterior θ_t that the expansion is permanent is given by

Bayes' rule:

$$\begin{aligned}
 \theta_t &= \frac{\Pr(\text{shock survives to } t | \text{permanent})}{\Pr(\text{shock survives to } t)} 0.5 \\
 &= \frac{1}{\left[\frac{\Pr(\text{shock survives to } t | \text{permanent}) \cdot 0.5}{+ \Pr(\text{shock survives to } t | \text{temporary}) \cdot (1 - 0.5)} \right]} 0.5 \\
 &= \frac{1}{1 + \Pr(\text{shock survives to } t | \text{temporary})} \\
 &= \frac{1}{1 + \sum_{s=t+1}^{\infty} \lambda e^{-s}} = \frac{1}{1 + e^{-(t+1)}} \quad (19)
 \end{aligned}$$

Expected money is a convex combination of its permanently-higher value and its zero value:

$$\hat{m}_t = \theta_t \cdot m + (1 - \theta_t) \cdot 0 = \theta_t m \quad t \geq 0 \quad (20)$$

Forward looking agents use \hat{m}_t as the certainty-equivalent value of m from $t+1$. So, their expectations are based on the RE solution with \hat{m}_t replacing m in section 2.2. As time advances, \hat{m}_t evolves with θ_t , and agents update their expectations. As in the RE and IE solutions, the p_0 is zero. From p_1 onwards, prices are determined by (21):

$$\text{from (4)} \quad p_{t+1} = p_t - B_p(p_t - \hat{m}_t) \quad t \geq 0 \quad (21)$$

The interest rate clears the money market, with supply given by the actual money stock:

$$i_t = \frac{p_t - m}{B_i} \quad t \geq 0 \quad (22)$$

Expected interest rates (from $t+1$ forward) are based on (22) with expected money \hat{m}_t :

$$i_{t+j} = \frac{p_{t+j} - \hat{m}_t}{B_i} \quad j \geq 1 \quad (23)$$

The solution for the exchange rate relies on the expected path of interest rates and money. As in the RE solution, (21) and (23) together imply $i_{t+1+j} = (1 - B_p)^j i_{t+1}$:

$$\begin{aligned}
 s_t &= -(i_t + i_{t+1} + i_{t+2} \cdots) + s_{\infty} \\
 &= -i_t - (i_{t+1} + (1 - B_p)i_{t+1} + (1 - B_p)^2 i_{t+1} + \cdots) + \hat{m}_t \\
 &= -i_t - \frac{i_{t+1}}{B_p} + \hat{m}_t \quad (24)
 \end{aligned}$$

The period t exchange rate therefore requires an expression for i_{t+1} :

$$\begin{aligned} p_{t+1} - p_t &= -B_p(p_t - \hat{m}_t) \Rightarrow p_{t+1} - \hat{m}_t = (1 - B_p) \cdot (p_t - \hat{m}_t) \\ \therefore i_{t+1} &= \frac{p_{t+1} - \hat{m}_t}{B_i} = \frac{(1 - B_p) \cdot (p_t - \hat{m}_t)}{B_i} \\ \therefore s_t &= -i_t - \frac{i_{t+1}}{B_p} + \hat{m}_t = \frac{1}{B_i} \left[(m - \hat{m}_t) + \frac{\hat{m}_t - p_t}{B_p} \right] + \hat{m}_t \quad (25) \end{aligned}$$

For the simulated solution, $p_0 = 0$ and the expected monetary shock for the future is calculated using (19) and (20) with $t = 0$. Then (21) gives the price level for the next period. Equation (22) gives the current interest rate, again for $t = 0$, and (25) the current exchange rate. In period 1 the probability the shock is permanent is updated using (19), and so on.

When this model is simulated for reasonable values, it turns out that a bias of around 0.5 robustly occurs in both the UIP and term structure regressions.¹³ However, unlike with the IE model, negative coefficients on the UIP regression cannot be found, and delayed overshooting (Eichenbaum and Evans, 1995) does not occur. Naturally, given the gradual nature of Bayesian updating, sharp changes in the currency values - as observed in Figure 1 - are also ruled out in the model, while they are predicted by IE.

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¹³We ran regressions (without a constant term and with 30 observations) for (B_i, B_p) pairs (.2, .2) (.2, .5) (.5, .2) and (.5, .5). For a quarterly model, B_p values of 0.5 and 0.2 imply that 75% of adjustment occurs by 2 and 7 quarters. The UIP biases are -.45, -.38, -.48 and -.42, and, the term structure biases are -.54, -.38, -.54 and -.38.

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