Monetary Policy and Exchange Rate Regime: Proposal for a Small and Less Developed Economy

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Abstract

We investigate monetary policy under the assumption that a country's capital market is "open" under the WTO framework while the exchange rate is fixed. Our purpose is to determine if it is possible in this case for the economy to maintain an effective monetary policy for stabilizing the domestic economy. For this, we suggest two institutional restrictions. Given the restrictions, we demonstrate within a macro-dynamic model that monetary policy can still be effective. The implication of such an institutional design for an exchange rate regime is also discussed with special reference to small and less development economies.

Keywords: open economy trilemma, macroeconomic stability, exchange rate regime

JEL: E12, E32, and C62

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1 Introduction

Many central banks have the objective of protecting the value of their domestic currency. This may imply two stabilities: 1) stability against domestic price; 2) stability against foreign currencies. Among the variety of choices, a fixed exchange regime is the best means to achieve stability against foreign currencies.

Yet, it is seemingly impossible to achieve these two stabilities simultaneously under the WTO framework, according to which countries are assumed to open their capital markets. By the theorem of open economy trilemma, if a capital market is open while the exchange rate is fixed, the monetary authority will lose its independence in stabilizing the domestic economy because of its activities in the foreign exchange market. Meanwhile, capital inflow and outflow will strengthen the connection between the domestic interest rate and the interest rate in the world market. This leaves the monetary policy via interest rate mechanism further ineffective for stabilizing the domestic economy. However, if the exchange rate is floating, not only does the monetary authority not have to be concerned with the exchange rate so that its monetary policy can be independent in stabilizing domestic economy, but also the interest rate connection will be weakened because of uncertainty in the exchange rate. This has been the main result since the Mundel-Fleming model was developed in the 1960s.[1] The recent new open economy macroeconomics initiated by the Redux model of Obstfeld and Rogoff (1995) does not change the basic result.[2] The trilemma theorem has also led to the current bipolar view, which basically rejects the fixed exchange regime as a viable possibility for an exchange rate system.[3]

Stability against foreign currency is perhaps more important for small and less developed economies because they are usually more vulnerable to external shocks. Stability is also a requirement for these countries to attract the foreign direct investment that is crucial for their economic development. This paper examines monetary policy for small and less developed economies. On the assumption that a stable exchange rate is still pursued by the government, we investigate whether it is possible for the government to maintain an effective monetary policy for stabilizing the domestic economy under a

[1]See Mundell (1963, 1964), Fleming (1962) and Dornbusch (1976).

[2]For a review on new open economy macroeconomics, see Lane (2001). For the most recent Redux model, see Obstfeld (2006).

[3]See Fischer (2002), Summer (2000) among others.

fixed exchange regime while the capital market is "open" under the WTO framework.

It is well known that the literature that has generated the trilemma theorem implies some form of institution which makes all financial assets perfectly substitutable, and thus the variety of interest rates can be merged essentially into one in terms of volatility.[4] Because one of the interest rates, such as the bond rate, must be equal to the interest rate in the world market under the fixed exchange regime when the capital market is open, the other interest rates, such as the loan interest rate, can hardly be impacted by monetary policy. Monetary policy thus loses its effectiveness in stabilizing the domestic economy via the interest rate mechanism.

This discussion indicates that if there exists some institutional restrictions under which the variety of financial assets are not perfectly substitutable, the interest rates could be varied in different directions, and thus monetary policy could be effective in stabilizing the domestic economy even if the exchange rate is fixed and the capital market is "open" under the WTO framework. In this paper, we suggest the following institutional restrictions:

> *Restriction 1*: There is a block for the loan issued from commercial banks to be used for financial investments, such as buying bonds on the bond market;
>
> *Restriction 2*: There is a block for commercial banks to engage in the business of financial investment (such as buying bonds).

We assume that these institutional restrictions can be practically implemented under the appropriate regulations within the commercial banking system. In the following analysis, we evaluate the effectiveness of monetary policy under such institutional restrictions within a macro dynamic model specified for a small and less developed economy. Section 2 lays out the model. Section 3 provides the analysis, which allows us to demonstrate our findings. Section 4 discusses further implications for a small and less developed economy .

2 The Model

Given the institutional restrictions suggested in the last section, we construct a simple macro-dynamic model that will provide a basis for evaluating the monetary policy of a small and less developed economy.

[4]The empirically observed deviations among different interest rates are often termed a risk premium.

2.1 The Interest Rates

Standard macroeconomic models often consider only one interest rate. As we have mentioned, this implies an institution which makes all financial assets perfectly substitutable. Yet under our two institutional restrictions, this substitution no longer holds, and therefore we have to consider in our model the various interest rates. These may include the bond rate, inter-bank rate (federal fund rate in the US), loan rate, the deposit rate at commercial banks, and the rates for loan and deposit at the discount window of the central bank. Also, we remark that the money in our hypothetical economy can be circulated beyond the domestic economy. This indicates that the monetary authority may have difficulty in controlling the money supply and therefore it simply leaves the money to be endogenuously determined by demand.

Because the economy is small while the capital market is "open" under the WTO framework, the bond rate $r_{b,t}$ should be connected to the world capital markets. To simplify our analysis we set this exogenous bond rate to a constant r_w. This indicates

$$r_{b,t} = r_w \tag{1}$$

The deposit rate at the commercial bank is assumed to be zero. The two discount rates can be regarded as policy variables, whose determination will be discussed in Section 2.3. Our major discussion is thus on how the commercial loan rate and inter-bank rate are determined.

In the first place, we can assume that there is a very close relationship between the inter-bank rate and loan rate. For example, we assume that

$$r_t = b_0 + b_r r_{i,t}, \qquad b_0, b_r > 0; \;\; r_t \geq r_{i,t} \tag{2}$$

where $r_{i,t}$ is the inter-bank rate. This assumption is supported by empirical evidence from those economies in which administrative bodies play no part in determining the commercial loan rate. To some extent, we can regard the process of loan supply as the commercial bank first getting the fund at the inter-bank market and then supplying the loan to the loan demander whose investment project is evaluated to be rewardable. This leaves us only to discuss how the inter-bank rate $r_{i,t}$ is determined. On the assumption that the commercial bank is permitted, at their discretion, to borrow or deposit their reserve at the discount window of the central bank, we may obtain the following proposition with regard to the determination of the inter-bank rate.

Proposition 1 *Let $d_{1,t}$ and $d_{2,t}$ denote the two discount rates, the deposit rate and the loan rate respectively, of the central bank with $d_{1,t} \leq d_{2,t}$. Under the two institutional restrictions as expressed in Section 1, the determination* *of the inter-bank rate will follow*

$$d_{1,t} \leq r_{i,t} \leq d_{2,t} \tag{3}$$

while the two discount rates can be set by the central bank at its own discretion.

The proof of this proposition is trivial. Assume that $d_{1,t}$ and $d_{2,t}$ are given while $r_{i,t} > d_{2,t}$. The demander at the inter-bank market will switch to the discount window to borrow the fund. This will reduce demand at the inter-bank market, causing the inter-bank rate to go down. On the other hand, if $r_{i,t} < d_{1,t}$, then the loan supplier will prefer to deposit money at the discount window rather than supply the money at the inter-bank market. This will reduce the loan supply, causing the inter-bank rate to go up.

It should be noted that this proposition will not hold without the institutional restrictions we have suggested. Specifically, the central bank cannot set the two discount rates at its "discretion" so it cannot use the two discount rates as policy variables for stabilization purpose even if the inequality (3) still holds. Here, for the purpose of stabilization, the two discount rates should be adjustable to any positive value without incurring any negative effect such as continual arbitrage by individuals or financial intermediates of first borrowing money from commercial banks or the central bank at a lower rate and then buying bonds for a higher return. For example, without Restriction 2, $d_{2,t}$ cannot be set below the bond rate r_w. Otherwise, commercial banks will borrow money from the central bank to invest in the bond market. Similarly, to avoid arbitrage by individuals or other financial intermediates of borrowing money from the commercial bank in order to buy bonds while Restriction 1 is absent, the interest rate on commercial loans r_t must be higher than the bond rate r_w. From (2) and (3), this also means that there is some lower boundary to set $d_{2,t}$ given the externally determined bond rate r_w.

Note that equation (3) in Proposition 1 also indicates that if $d_{1,t} = d_{2,t} = r_{d,t}$, then

$$r_{i,t} = r_{d,t} \tag{4}$$

Therefore, if the two discount rates merge, the inter-bank rate $r_{i,t}$ should also be equal to the merged discount rate $r_{d,t}$.

2.2 Price and Output

The real sector of the model resembles the standard Keynesian model of the AS-AD type. Following recent discussion on the Philips curve of the New

Keynesian model,[5] we may assume that the aggregate supply curve takes the form

$$p_t = \alpha_0 + \alpha_p p_{t-1} + \alpha_y y_{t-1} \tag{5}$$

where p_t is the inflation rate and y_t can be regarded as an output measure.[6]

With regard to output (or the aggregate demand curve), we may assume that it is determined by the real interest rate on commercial loans $r_t - p_t$ and the real exchange rate γ_t:

$$y_t = y(r_t - p_t, \gamma_t) \tag{6}$$

where $y_1^{'} < 0$, $y_2^{'} > 0$. Here, $y_1^{'}$ and $y_2^{'}$ are defined as follows. For function $y(x, z)$, we define $y_1' = \frac{\partial y(x,z)}{\partial x}$, $y_2' = \frac{\partial y(x,z)}{\partial z}$. Also note that we define the real exchange rate γ_t to be

$$\gamma_t = \frac{x_t}{P_t} P_t^f \tag{7}$$

where x_t is the nominal exchange rate (domestic currency over foreign currency); P_t is the domestic price index and P_t^f is the price index in foreign countries.

We remark that the determination of output could be more complicated. Yet our major concern in this paper is with monetary policy and exchange rate on the assumption that both of them may have an impact on the real side of the economy. For this purpose, our formulation here should be sufficient while keeping the analysis as simple as possible.

2.3 The Monetary Policy

When money is endogenous, the monetary authority will target interest rates for the purpose of stabilizing the domestic economy. Because it is the discount rates through Proposition 1 that determine the inter-bank rate, which further determines the loan rate via (2) and then output via (6), we suggest that the monetary authority should use the discount rate as a major instrument of its monetary policy. To simplify our analysis, we even assume one (rather than two) discount rate so that $d_{1,t} = d_{2,t} = r_{d,t}$. As expressed in (4) the inter-bank rate in this case is simply equal to the discount rate $r_{d,t}$. Suppose that the central bank sets the target inflation rate and target interest rate at p^* and r^* respectively. Then its interest rate rule (or Taylor rule) can be expressed as

$$r_{d,t} - r_{d,t-1} = \theta_p(p_{t-1} - p^*) - \theta_1(r_{t-1} - r^*) \tag{8}$$

[5] For the recent research on this topic, see Mankiw and Reis (2002, 2007), Gali and Gertler(1999), and Beyer et al. (2005), among others.

[6] As in the literature, y_t can be measured as (but not limited to) the percentage deviation of output from its trend at the steady state.

where $\theta_p, \theta_1 > 0$.

The equation (8) can only be regarded for stabilizing the domestic economy. The central bank may also have a responsibility to stabilize the exchange rate, and may trade on the foreign exchange market to serve this purpose. Suppose that the central bank targets the exchange rate at x^*. We find that

$$x_t = x^* \tag{9}$$

can be held via the central bank's trading on the foreign exchange market.

3 The Analysis

In this section, we shall provide analysis of the model constructed in the previous section.

3.1 The Intensive Form of the Model

We shall first transform the model into an intensive form. To keep our analysis simple, without affecting our major analytical result, we pose the following two assumptions:

$$P_t^f = (1 + \bar{p}_w) P_{t-1}^f \tag{10}$$

$$b_0 = 0, \;\; b_1 = 1 \tag{11}$$

Assumption (10) implies that we do not attempt to model the dynamics of P_t^f and thus simply assume that it grows at a constant rate $\bar{p}_w$. Assumption (11) is in regard to the parameters in (2). It indicates that the loan rate r_t is equal to the inter-bank rate and we therefore do not have to settle the parameters b_0 and b_1 in our dynamic analysis.

We now provide the following proposition regarding the intensive form of our model.

Proposition 2 *Given assumption (10) and (11), the intensive form of the model (2) - (9) can be written as*

$$r_t = \theta_0 + \theta_r r_{t-1} + \theta_p p_{t-1} \tag{12}$$

$$p_t = \alpha_0 + \alpha_p p_{t-1} + \alpha_y y(r_{t-1} - p_{t-1}, \gamma_{t-1}) \tag{13}$$

$$\gamma_t = \frac{(1 + \bar{p}_w)\gamma_{t-1}}{1 + \alpha_0 + \alpha_p p_{t-1} + \alpha_y y(r_{t-1} - p_{t-1}, \gamma_{t-1})} \tag{14}$$

where $\theta_0 = \theta_1 r^* - \theta_p p^*$ *and* $\theta_r = 1 - \theta_1$.

The proof of this proposition is given in the appendix. Apparently, equation (12)-(14) forms a standard discrete time dynamic system in three dimensions (p_t, γ_t, r_t). Given the solutions on p_t, γ_t and r_t, the output y_t can also be determined by the function $y(\cdot)$.

3.2 The Steady States

Next, we consider the steady state. From equation (14), it can be found that for the economy to possess a steady state, the steady state of domestic inflation $\bar{p}$ must be equal to p_w, the inflation in the world market. This further implies from (12) that the central bank has to set the target inflation rate p^* to the world inflation rate p_w. The following proposition regards the determination of the steady state.

Proposition 3 *For the dynamical system (12)-(14), there is a unique steady state equilibrium* $(\bar{r}, \bar{p}, \bar{\gamma})$ *with* $\bar{\gamma} \neq 0$ *given by*

$$\begin{aligned} \bar{p} &= \bar{p}_w \\ \bar{r} &= \frac{1}{1-\theta_r}(\theta_0 + \theta_p \bar{p}_w) \end{aligned}$$

and $\bar{\gamma}$ *solves* $\bar{y} = y(\bar{r} - \bar{p}, \bar{\gamma})$ *with*

$$\bar{y} = \frac{1}{\alpha_y}\left[-\alpha_0 + (1-\alpha_p)\bar{p}_w\right]$$

The proof of this proposition is again provided in the appendix.

3.3 Monetary Policy in a Closed Economy

To observe the transmission mechanism of monetary policy clearly, we first consider the simple case of a closed economy. In this case, only rule (8) is relevant, while $y(r_t - p_t, \gamma_t)$ can be replaced by $y(r_t - p_t)$. Suppose that the monetary policy (8) is not in action, that is, $r_t = r_0$. We first find that there might be a destabilizing mechanism which makes the economy inherently unstable:

$$p \uparrow, \quad (r-p) \downarrow, \quad y \uparrow, \quad p \uparrow \tag{15}$$

This instability can be found by observing equation (13) with $y(r_t - p_t, \gamma_t)$ to be replaced by $y(r_0 - p_t)$. In this case, the system is one dimensional: $p_t = p(p_{t-1})$. It is well known that if

$$p' = \alpha_p - \alpha_y y' > 1 \tag{16}$$

the system will be inherently unstable.

Adding monetary policy rule (8) will stabilize the economy by the usual transmission mechanism of monetary policy:

$$p \uparrow, \quad (r-p) \uparrow, \quad y \downarrow, \quad p \downarrow \tag{17}$$

As discussed in the literature,[7] this transmission mechanism may work well only if the adjustment speed θ_p is high enough so that the interest rate effect resulting from rule (8) and expressed in (17) can cover the price effect of destabilization as expressed in (15). On the other hand, the economy could be cyclically explosive if θ_p is too large. This indicates that a Hopf bifurcation may exist with regard to θ_p. The following proposition detects the existence of Hopf bifurcation for our model of a closed economy.[8]

Proposition 4 *Consider the dynamic system composed of (12) and (13) with $y(r_t - p_t, \gamma_t)$ to be replaced by $y(r_t - p_t)$. Let J be the Jacobian matrix of the system evaluated at the steady state and $\lambda_{1,2}$ as the two eigenvalues of J. Suppose that*

$$\theta_r + \alpha_p - \alpha_y y' < 2, \;\; (\alpha_p - \alpha_y y')\theta_r < 1 \tag{18}$$

where y' is the derivative of y evaluated at the steady state. Then there exists a θ_p denoted as θ_p^* such that in the neighborhood of θ_p^*

1. *$\lambda_{1,2}$ are complex conjugates;*
2. *the modulus of the complex conjugates denoted as $|\lambda_{1,2}|$ can be either below or above 1 depending on the castellation of the structure parameters. In particular,*
   (a) *$|\lambda_{1,2}| < 1$ when $\theta_p < \theta_p^*$;*
   (b) *$|\lambda_{1,2}| = 1$ when $\theta_p = \theta_p^*$;*
   (c) *$|\lambda_{1,2}| > 1$ when $\theta_p > \theta_p^*$;*
3. $\left.\frac{d|\lambda_{1,2}(\theta_p)|}{d\theta_p}\right|_{\theta_p=\theta_p^*} \neq 0.$

[7]See for instance, Chiarella, Flaschel, Gong and Semmler (2003).

[8]For the related references on the existence of Hopf bifurcation in a two dimensional system of discrete time dynamics, see Guckenheimer and Holmes (1986, p. 162).

According to this proposition, even if $\alpha_p - \alpha_y y' > 1$ so that the economy is inherently unstable without intervention from the government, adding monetary policy rule (8) can stabilize the economy if condition (18) is satisfied and θ_p is set at an appropriate level. Because the parameters θ_r and θ_p are always controllable, one thus finds that the central bank can always match the condition (18).

To illustrate the result, we provide the following simulation. The output function $y(\cdot)$ for this simulation is assumed to be linear:

$$y_t = C - B(r_t - p_t)$$

The parameters for this simulation are given in Table 1. Some of the parameters (those in interest rate (12) and price (13) equations) in this table are closed to the empirically estimated parameters for the current Chinese economy.[9]

Table 1: Parameters for Simulation

α_0	α_y	α_p	θ_0	θ_r	C	B
-0.002	0.1042	0.5781	0.0123	0.5262	0.1600	5.000

Given these parameters, we find that the instability condition (16) is satisfied. The initial conditions for these simulations are set as follows: $p_0 = 0.004$, $y_0 = 0.03$, $r_0 = 0.03$. With respect to the bifurcation parameter θ_p, we set it to 0.6, 0.05 and 0.9, respectively.

Here we illustrate the effectiveness of the monetary policy rule (8) as expressed in (17). If the interest rate effect resulting from the monetary policy rule (8) is sufficient, indicating an appropriate adjustment speed θ_p, the economy in this case can be stabilized (see Figure 1).

On the other hand, if the interest rate effect is not sufficient, indicating too low of an adjustment speed θ_p, the price effect of the destabilization mechanism as expressed in (15) will be dominant and the economy will be divergent (see Figure 2).

Finally, if the interest rate effect is too strong, indicating too high of a θ_p, the economy could be cyclically explosive (see Figure 3)

3.4 Monetary Policy in an Open Economy with a Fixed Exchange Regime

Now let us go back to the open economy and a system composed of (12) - (14) with the steady state given in Proposition 3. The following proposition regards its stability property.

[9]See Gao and Yang (2007).

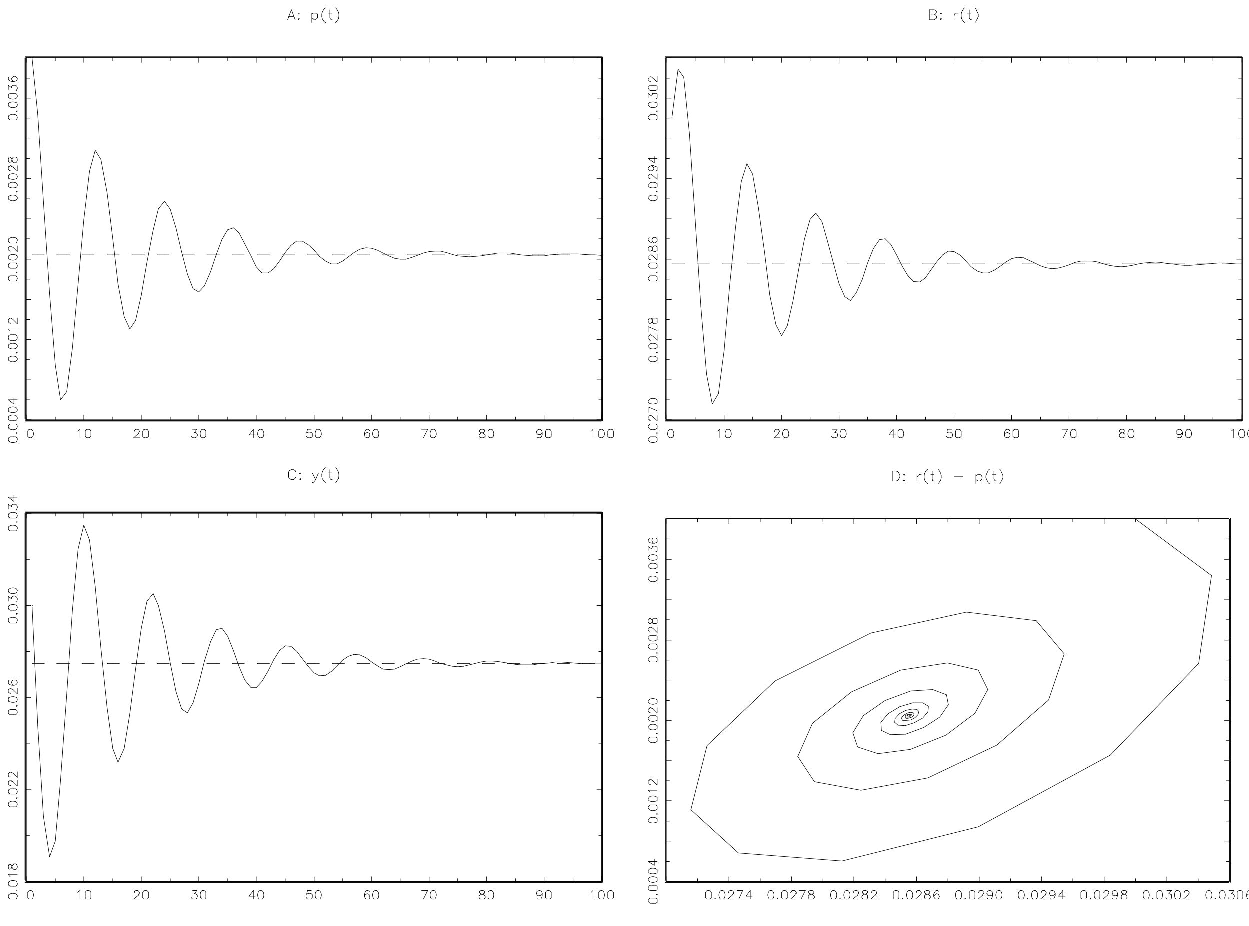

Figure 1: Sufficient Interest Rate Effect via Monetary Policy Rule (8): Closed Economy, $\theta_p = 0.6$

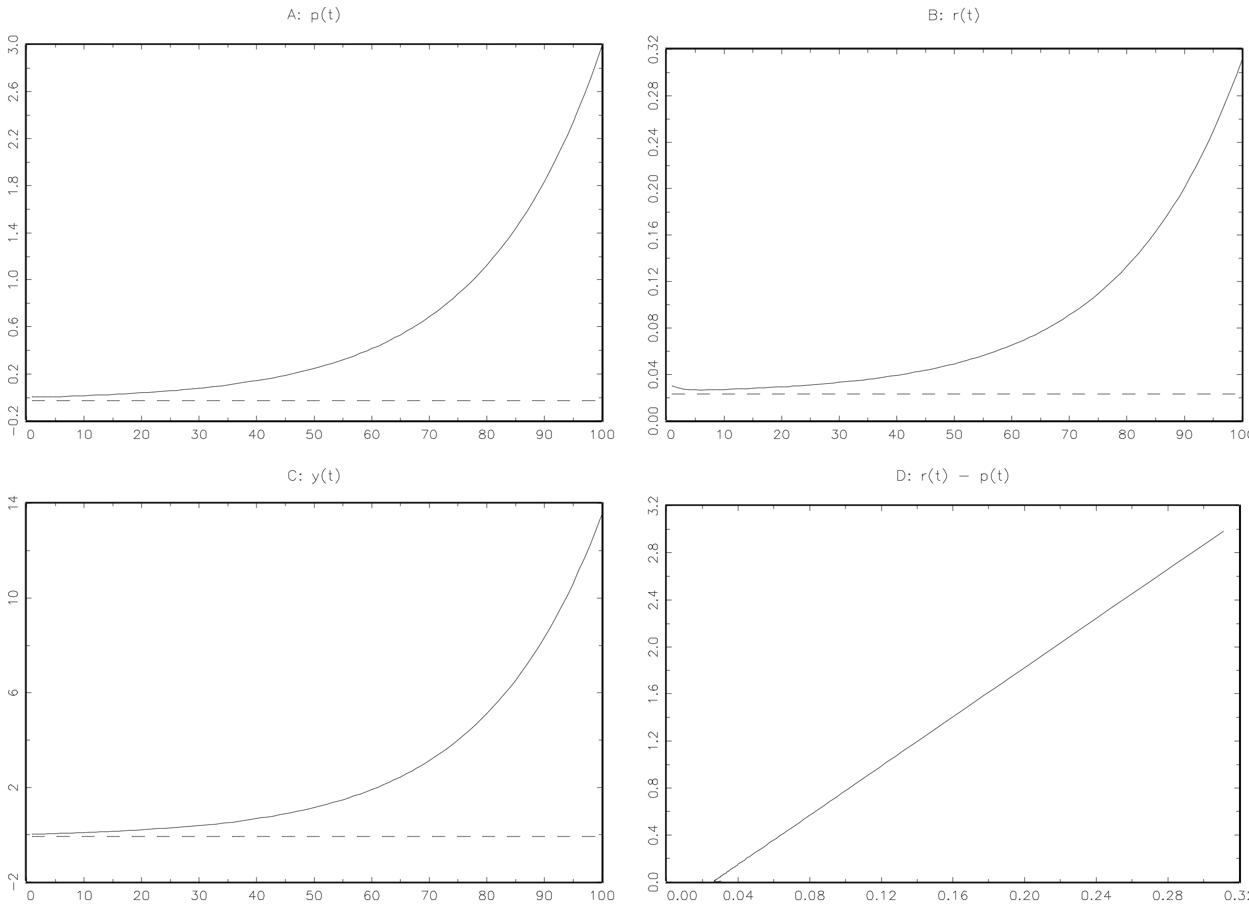

Figure 2: Insufficient Interest Rate Effect via Monetary Policy Rule (8): Closed Economy, $\theta = 0.05$

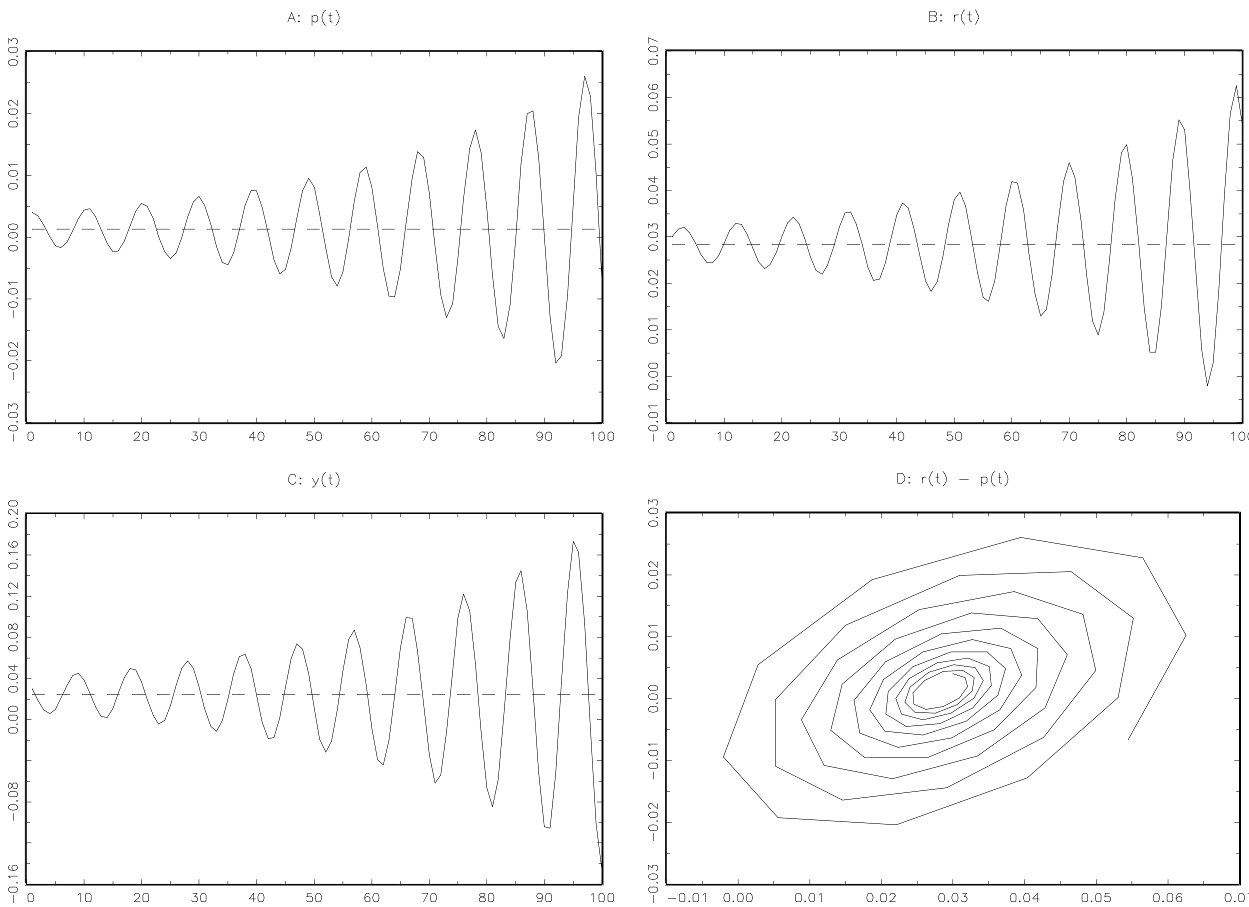

Figure 3: Over Sufficient Interest Rate Effect via Monetary Policy Rule (8): Closed Economy, $\theta_p = 0.9$

Proposition 5 *For the dynamical system (12)-(14), the steady state equilibrium* $(\bar{r}, \bar{p}, \bar{\gamma})$ *is locally asymptotically stable if, and only if,*

$$\theta_r < 1, \qquad \Pi_3 > 0, \tag{19}$$

$$\frac{\alpha_y y_2}{2} \frac{\bar{\gamma}}{1+\bar{p}} < 1 + \alpha_p - \left[1 + \frac{\theta_p}{1+\theta_r}\right] \alpha_y y_1, \tag{20}$$

where

$$y_1 = \left.\frac{y(x,z)}{\partial x}\right|_{x=\bar{r}-\bar{p}, z=\bar{\gamma}}, \qquad y_2 = \left.\frac{y(x,z)}{\partial z}\right|_{x=\bar{r}-\bar{p}, z=\bar{\gamma}}$$

and

$$\begin{aligned} \Pi_3 \;=\; & (1-\theta_r) - (1+\theta_r)(\alpha_p - \alpha_y y_1) + \theta_p \alpha_y y_1 + \alpha_y y_2 \theta_r \frac{\bar{\gamma}}{1+\bar{p}} \\ & -[\theta_p \alpha_y y_1 - \theta_r(\alpha_p - \alpha_y y_1)] \Bigg[(1+\theta_r) \\ & +(1-\theta_r)(\alpha_p - \alpha_y y_1) + \theta_p \alpha_y y_1 - \alpha_y y_2 \frac{\bar{\gamma}}{1+\bar{p}} \Bigg]. \end{aligned}$$

In addition, a Hopf bifurcation occurs when (20) holds and $\Pi_3 = 0$.

The proof of this proposition is given in the appendix.

Given that the $y_2 < 0$, condition (20) actually gives an upper boundary for the equilibrium real exchange rate.

$$\bar{\gamma} < \bar{\gamma}^* := 2\frac{(1+\bar{p})(1+\alpha_p)}{\alpha_y y_2} - 2(1+\bar{p})\frac{y_1}{y_2}\left[1 + \frac{\theta_p}{1+\theta_r}\right].$$

From equation (7), we also find that there is an upper boundary for the central bank to set its target nominal exchange rate x^* if it wants the steady state of economy to be stable.

The parameter Π_3 can be treated as a quadratic function in θ_p, that is,

$$\Pi_3(\theta_p) = \pi_0 + \pi_1 \theta_p + \pi_2 (\theta_p)^2$$

This indicates that $\Pi_3(\theta_p) = 0$ will occur two times and therefore there might be two Hopf birfurcations in θ_p. Note that the coefficient of the second order term π_2 is negative. Hence, if $\Pi_3(0) < 0$, then condition $\Pi_3(\theta_p) > 0$ is satisfied for $0 < \theta^*_{p,1} < \theta_p < \theta^*_{p,2}$. Further , Hopf bifurcation occurs at $\theta_p = \theta^*_{p,i}$ for $i = 1, 2$. This implies either a θ_p $(\theta_p < \theta^*_{p,1})$ that is too low or a θ_p $(\theta_p > \theta^*_{p,2})$ that is too high, may destabilize the economy. Only a moderate adjustment in θ_p can stabilize the economy.

Again we shall now provide the following simulation to prove the stability property as expressed in Proposition 5. The production function now takes the form

$$y_t = -0.16 - 4.5(r_t - p_t) + 0.05\gamma_t$$

We set p_w at 0.0025 while the other parameters are the same values as in Table 1. The initial conditions are set as follows: $p_0 = 0.002$, $y_0 = 0.03$, $r_0 = 0.026$, $\gamma_0 = 6$.

For the set of parameters given, the conditions $\theta_r < 1$ and (20) are always satisfied. The condition $\Pi_3 > 0$ is satisfied for $\theta_p \in (\theta^*_{p,1}, \theta^*_{p,2})$ with $\theta^*_{p,1} \approx 0.058$ and $\theta^*_{p,2} \approx 0.857$. In Figure 4-6, we choose $\theta_p = 0.0047$, 0.15 and 1.

Again, if the interest rate effect resulting from the monetary policy rule (8) is insufficient, indicating too small of an adjustment speed ($\theta_p = 0.0047$), the economy will be unstable (see Figure 4).

On the other hand, if the interest rate effect is sufficient, indicating an appropriate adjustment speed ($\theta_p = 0.15$), the economy will be stable (see Figure 5).

Finally, if the interest rate effect is excessive, indicating too high of an adjustment speed ($\theta_p = 1$), the economy will be cyclically explosive (see Figure 6).

We thus demonstrate that under our two institutional restrictions monetary policy (8) can be effective even if the nominal exchange rate x is fixed and the capital market is "open" under the WTO framework.

4 Discussion

This paper studies the monetary policy for a small and less developed economy when the capital market is "open" under the WTO framework and when stability in the exchange rate is pursued by the government. According to the theorem of open economy trilemma, it seems impossible to maintain the independence of monetary policy for stabilizing the domestic economy in this case. Yet the open economy trilemma is effective only under a certain type of institution. For this, we suggest two institutional restrictions. Given the institutional restrictions, we study the monetary policy in a model of an open small economy. We find that monetary policy will be effective in this case even if the capital market is "open" and the exchange rate is fixed. However, several factors still need to be clarified in this discussion.

First, this paper is not a challenge to the theorem of open economy trilemma. We only state that the capital market is "open" under the WTO framework. The suggested institutional arrangements should be interpreted

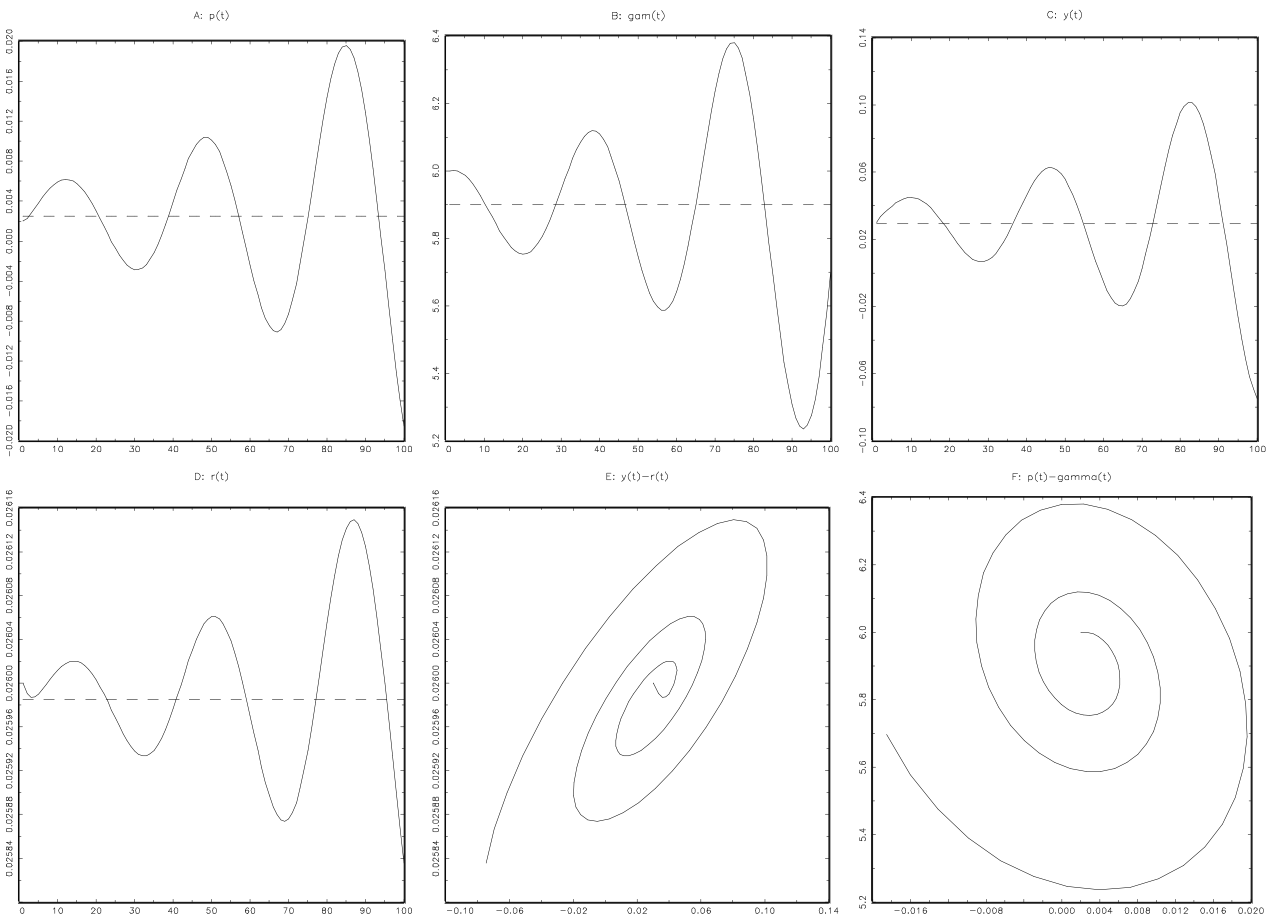

Figure 4: Insufficient Interest Rate Effect via Monetary Policy Rule (8): Closed Economy, $\theta_p = 0.0047$

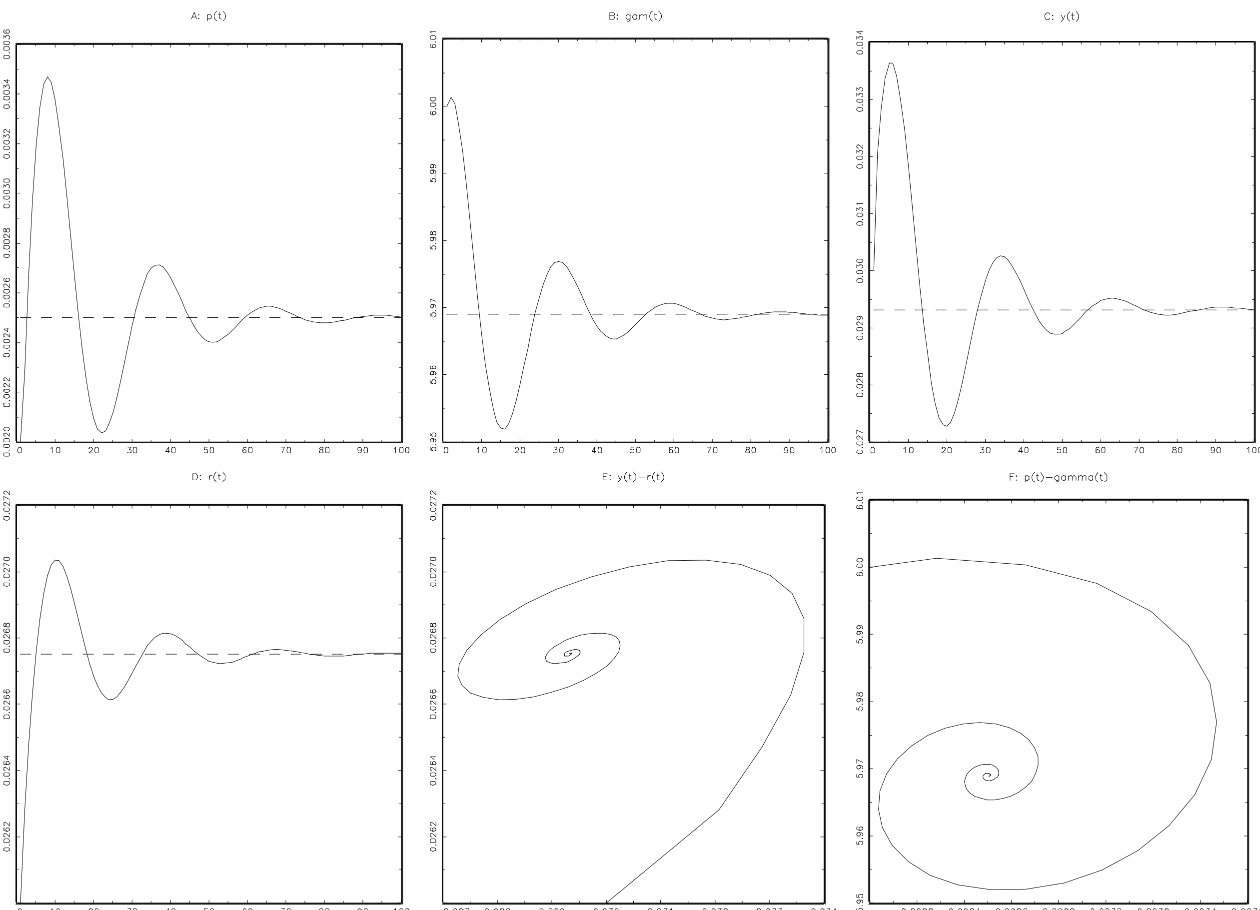

Figure 5: Sufficient Interest Rate Effect via Monetary Policy Rule (8): Open Economy, $\theta = 0.15$

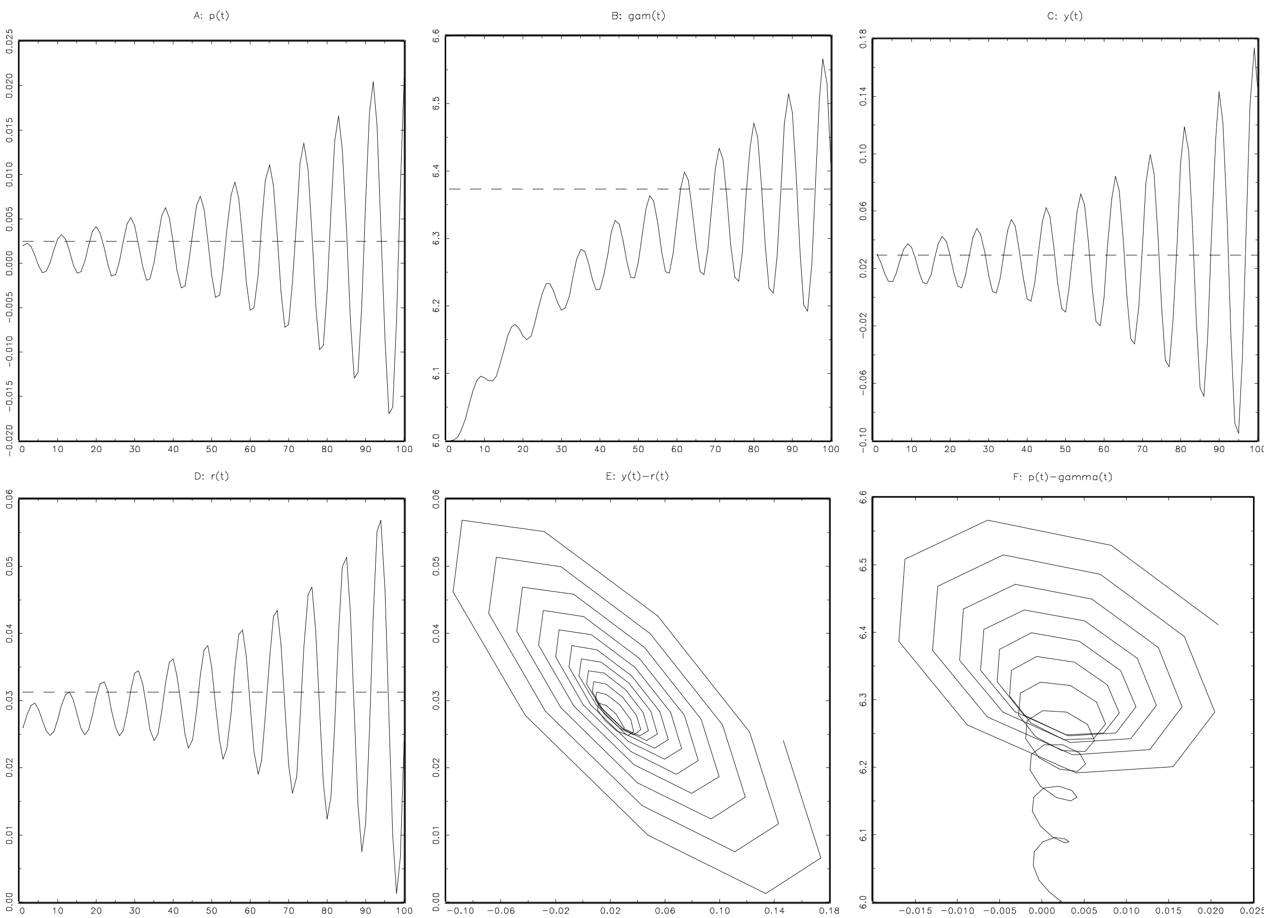

Figure 6: Excessive Interest Rate Effect via Monetary Policy Rule (8): Open Economy, $\theta_p = 1$

as friction that means the capital markets are not fully open. By restricting the financial decisions of the commercial banks, the full openness of the capital market is blocked.

Second, there is still a sustainability issue for a country to adopt the fixed exchange regime in that the target exchange rate could be unsustainable. Because the central bank has limited foreign exchange reserves, it cannot continually use its exchange reserves to stabilize its target exchange rate. However, this issue really depends on how the government sets the target exchange rate. Suppose that the government sets the target exchange rate x^* beyond the equilibrium level or deliberately under-values the domestic currency. Then, interventions from the central bank will in most cases appear to be the purchase of foreign currency with its domestic currency. This may effectively prevent a financial crisis, that is, the shortage of foreign exchange reserves while the target exchange rate is unsustainable. Indeed, the higher the target exchange rate is (or the more under-valued the domestic currency is), the greater the possibility for the exchange rate to be sustainable.

Finally, we discuss the welfare effect of such a system design. One can argue that the under-valuation of domestic currency may be at the expense of the welfare of domestic citizens while letting foreigners enjoy domestic products and services at a discount. While there might be a welfare loss for domestic citizens, a stable and under-valued domestic currency may accrue additional benefits from the under-valued economic resources of the domestic country. The under-valued economic resources may create an advantage by increasing the export of domestic products and in attracting foreign direct investment. All of these factors are important for a less developed economy where unutilized economic resources, such as labor in rural areas, are huge.[10] Export promotion and foreign direct investment may activate these economic resources. Consequently, the welfare of domestic citizens can be improved. Although the net result is not clear, we already know that such a design of the exchange rate regime will not necessarily cause a welfare loss for domestic citizens in a less developed economy.

5 Appendix

5.1 The Proof of Proposition 2

Equation (12) is simply from (8) and (4) when equation (11) holds and the two discount rates are merged. Equation (13) is trivial from (5) when y_{t-1} is

[10]This feature of a less developed economy is what Lewis, a 1979 Nobel laureate, described in his seminal paper on development economics (see Lewis 1954).

replaced by (6). To prove (14), we divide both sides of (7) by γ_{t-1} while in the left side using definition $\gamma_{t-1} = \frac{x_{t-1}}{P_{t-1}} P_{t-1}^f$:

$$\frac{\gamma_t}{\gamma_{t-1}} = \frac{x_t P_t^f / P_t}{x_{t-1} P_{t-1}^f / P_{t-1}}$$

Note that here $x_t = x_{t-1} = x^*$, $P_t = (1 + p_{t-1}) P_{t-1}$ and $P_t^f = (1 + \bar{p}_w) P_{t-1}^f$. We thus find (14).

5.2 The Proof of Proposition 3

Let $(\bar{r}, \bar{p}, \bar{\gamma})$ be the fixed point of the system (12)-(14). It then satisfies

$$\bar{r} = \theta_0 + \theta_r \bar{r} + \theta_p \bar{p}, \tag{21}$$

$$\bar{p} = \alpha_0 + \alpha_p \bar{p} + \alpha_y \bar{y}, \tag{22}$$

$$\bar{\gamma} = \frac{(1 + \bar{p}_w)\bar{\gamma}}{1 + \alpha_0 + \alpha_p \bar{p} + \alpha_y \bar{y}}, \tag{23}$$

where

$$\bar{y} = y(\bar{r} - \bar{p}, \bar{\gamma}). \tag{24}$$

Assume $\bar{\gamma} \neq 0$. Then, it follows from (23) and (22) that $1 + \bar{p} = 1 + \bar{p}_w$ and hence

$$\bar{p} = \bar{p}_w. \tag{25}$$

Substituting (25) into (22), we solve $\bar{y}$ as in the proposition. Substituting (24) and (25) into (21), we obtain $\bar{r}$ as in the proposition. Because of $\frac{\partial y(x,z)}{\partial z} > 0$, we can solve $\bar{\gamma}$ uniquely from equation (24). Hence, the system (12)-(14) has a unique steady state with $\bar{\gamma} \neq 0$.

5.3 The Proof of Proposition 4

For this closed economy, the dynamic system can be written as

$$r_t = \theta_0 + \theta_r r_{t-1} + \theta_p p_{t-1} \tag{26}$$

$$p_t = \alpha_0 + \alpha_p p_{t-1} + \alpha_y y(r_{t-1} - p_{t-1}) \tag{27}$$

with $y' < 0$.

The Jacobian matrix of the system (26) and (27) can be written as

$$J = \begin{pmatrix} \theta_r & \theta_p \\ \alpha_y y' & \alpha_p - \alpha_y y' \end{pmatrix}$$

Thus, the characteristic equation takes the form

$$\lambda^2 - a_1\lambda + a_2 = 0$$

where

$$a_1 = \theta_r + \alpha_p - \alpha_y y', \quad a_2 = (\alpha_p - \alpha_y y')\theta_r - \theta_p \alpha_y y'$$

Let us now derive the bifurcation θ_p^* on the assumption that the eigenvalues $\lambda_{1,2}$ are complex conjugates. This can be accomplished by assuming that the modulus $|\lambda_{1,2}|$ (which is a_2) equals 1, that is,

$$(\alpha_p - \alpha_y y')\theta_r - \theta_p \alpha_y y' = 1$$

Solving this equation for θ_p, we obtain

$$\theta_p^* = \frac{(\alpha_p - \alpha_y y')\theta_r - 1}{\alpha_y y'} \tag{28}$$

For $\theta_p^* > 0$, we request that

$$(\alpha_p - \alpha_y y')\theta_r < 1$$

which is one of the conditions in (18). Given this θ_p^*, we shall now prove that the eigenvalues $\lambda_{1,2}$ are complex conjugates. This requests that $a_1^2 - 4a_2 < 0$, that is,

$$(\theta_r + \alpha_p - \alpha_y y')^2 - 4\left[(\alpha_p - \alpha_y y')\theta_r - \theta_p \alpha_y y'\right] < 0 \tag{29}$$

Substituting (28) into (29), the request (29) becomes

$$(\theta_r + \alpha_p - \alpha_y y')^2 - 4 < 0$$

This inequality holds under the other condition in (18): $\theta_r + \alpha_p - \alpha_y y' < 2$. We therefore prove 1 in the proposition. Next,

$$\frac{d\,|\lambda_{1,2}(\theta_p)|}{d\theta_p} = -\alpha_y y' > 0$$

2 and 3 in the proposition are thus proved.

5.4 The Proof of Proposition 5

Following the notation of y_1 and y_2, the Jacobian matrix J of the system at the steady state can be written as

$$J = \begin{pmatrix} \theta_r & \theta_p & 0 \\ \alpha_y y_1 & \alpha_p - \alpha_y y_1 & \alpha_y y_2 \\ -\frac{\alpha_y \bar{\gamma} y_1}{1+\bar{p}} & -\frac{\bar{\gamma}}{1+\bar{p}}(\alpha_p - \alpha_y y_1) & 1 - \frac{\alpha_y \bar{\gamma} y_2}{1+\bar{p}} \end{pmatrix}$$

The characteristic equation takes the form

$$\Gamma(\lambda) \equiv \lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0, \tag{30}$$

where

$$\begin{aligned}
c_1 &= -(1 + \theta_r + \alpha_p - \alpha_y y_1) + \frac{\alpha_y \bar{\gamma} y_2}{1 + \bar{p}}, \\
c_2 &= \theta_r + (1 + \theta_r)(\alpha_p - \alpha_y y_1) - \theta_p \alpha_y y_1 - \frac{\alpha_y \theta_r \bar{\gamma} y_2}{1 + \bar{p}} \\
c_3 &= \theta_p \alpha_y y_1 - \theta_r(\alpha_p - \alpha_y y_1).
\end{aligned}$$

To finish the proof, we need the following lemma (see Elaydi 1996 and Sonis 2000).

Lemma 6 *The eigenvalues $\lambda_i (i = 1, 2, 3)$ of (30) satisfy $|\lambda_i| < 1$ for $i = 1, 2, 3$ if and only if $\Pi_j (j = 1, 2, 3) > 0$ and $c_3 < 3$, where*

$$\begin{aligned}
\Pi_1 &= 1 + c_1 + c_2 + c_3; \\
\Pi_2 &= 1 - c_1 + c_2 - c_3; \\
\Pi_3 &= 1 - c_2 + c_1 c_3 - c_3^2.
\end{aligned}$$

Further,

- *on $\Pi_1 = 0$ at least one of the eigenvalues is equal to 1;*
- *on $\Pi_2 = 0$ at least one of the eigenvalues is equal to -1;*
- *on $\Pi_3 = 0$ the three eigenvalues satisfy $\lambda_{1,2} \in \mathbb{C}$ and $\lambda_3 \in \mathbb{R}$ with $|\lambda_{1,2}| = 1$ and $\lambda_3 \in [-1, 1]$.*

In our case

$$\Pi_1 = \frac{\alpha_y \bar{\gamma} y_2}{1 + \bar{p}} [1 - \theta_r].$$

Hence, $\Pi_1 > 0$ iff $\theta_r < 1$.

$$\Pi_2 = 2[(1 + \alpha_p - \alpha_y y_1)(1 + \theta_r) - \theta_p \alpha_y y_1] - \frac{1}{1 + \bar{p}} [\alpha_y \bar{\gamma} y_2 (1 + \theta_r)]$$

Therefore, $\Pi_2 > 0$ iff condition (20) holds. Substituting c_i into Π_3, we get the expression in Proposition 5. Applying the above lemma completes the proof.

References

[1] Beyer, Andreas; Farmer, Roger; Henry, Jerome and Marcellino, Massimiliano (2005): Factor Analysis in a New Keynesian Model, Working Paper Series, No.510, European Central Bank.

[2] Chiarella, Carl; Peter Flaschel; Gang Gong and Willi Semmler (2003): Nonlinear Phillips Curve, Complex Dynamics and Monetary Policy in a Keynesian Macro Model, Chaos, Solutions and Fractals, Vol. 18, 613 – 634.

[3] Dornbusch Rudiger (1976): Expectations and Exchange Rate Dynamics. Journal of Political Economy, Vol. 84: 1161 - 1176.

[4] Elaydi, S. (1996): An Introduction to Difference Equations, Springer, New York.

[5] Fischer, Stanley (2001): Exchange Rate Regimes: Is the Bipolar View Correct?, Journal of Economic Perspectives, Vol. 15, 3-24.

[6] Fleming, Marcus (1962): Domestic financial policies under fixed and uder floating exchange rates. IMF Staff Papers, Vol. 3: 369 - 379.

[7] Gao, Jian and Yang, Nian (2007): The Aggregate Demand and Aggregate Supply Model for Chinese Economy: A Framework for Fiscal and Monetary Policy Analysis, Journal of Quantitative and Technical Economics, forthcoming (in Chinese).

[8] Gali, Jordi and Gertler, Mark (1999): Inflation Dynamics, a Structural Econometric Analysis, Journal of Monetary Economics, Vol. 44, 195-222.

[9] Guckenheiner, John and Philip Hommes (1986): Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, New York, Springer-Verlag.

[10] Lane, P. R. (2001): The New Open Economy Macroeconomics: A Survey, Journal of International Economics, Vol. 54, 235 - 266.

[11] Lewis, Sir Arthur (1954): Economic Development with Unlimited Supplies of Labour, The Manchester School, Vol. 22, 139 - 191.

[12] Mankiw, Gregory and Reis, Ricardo (2002): Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Philips Curve, Quarterly Journal of Economics, Nov., 1295-1328.

[13] Mankiw, Gregory and Reis, Ricardo (2007): Sticky Information in General Equilibrium, Journal of European Economic Association, forthcoming.

[14] Mundell, Robert A (1961): A theory of optimum currency areas. American Economics Review, 51: 509-517

[15] Mundell, Robert A (1963): Capital mobility and stabilization policy under fixed and flexible exchange rates. Canadian Journal of Economic and Political Science, 1963, 29: 475 - 485.

[16] Obstfeld, Maurice (2006): Pricing to Market, the Interest Rate Rule and the Exchange Rate, NBER Working Paper Series, No.12699.

[17] Obstfeld, Maurice and Rogoff, Kenneth (1995): Exchange Rate Dynamics Redux, Journal of Political Economy, Vol. 103, 624-660.

[18] Sonis, M. (2000): Critical bifurcation surfaces of 3d discrete dynamics, Discrete Dynamics in Nature and Society 4, 333-343.